Trade-Offs in Means Tested Pension Design

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16th January 2012

Abstract

Inclusion of means testing into pension programs allows governments to better direct benefits to those most in need and to control funding costs by providing flexibility to control the condition for receiving pension benefits (extensive margin) and the benefit level (intensive margin). We investigate how the presence of the extensive margin influences the trade-off between the desirability of protecting low income of the elderly and the economic costs of distorting incentives to work and save of young individuals. We show that the means test effect via the extensive margin improves the insurance aspect but introduces opposing impacts on incentives, and that potentially leads to the positive welfare outcome depending upon how these effects play out. We also characterize combinations of two policy parameters: maximum pension benefit and taper rate (the rate at which the benefit declines), that balance out the negative incentive effect and the positive insurance effect.

JEL Classification: D9, E2, E6, H3, H5, J1.

Keywords: Means-Tested Pension, Social Security, Optimal Policy, Overlapping Generations, Dynamic General Equilibrium.

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*This research was supported by a grant from Australian Research Council and the National Health and Medical Research Council under the Ageing Well, Ageing Productively Strategic Initiatives program and by the Australian Research Council through its grant to the ARC Centre of Excellence in Population Ageing Research (CEPAR).

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1 Introduction

Unlike the U.S.A. and many developed countries in which pension systems are mainly universal and pay-as-you-go (PAYG), Australia has a unique means tested age pension system. It has the following distinct features: (i) coverage of the retirement benefits system is not universal in that only a fraction of the retiree population receives pension benefits; (ii) the pension benefits are dependent on economic status (assets and income) and are directed towards the poorer elderly; (iii) the pension benefits are independent of individuals’ contribution history; and (iv) the tax financing instrument is not restricted to payroll tax revenue collected from the current working population.

Similarly to a universal PAYG pension system, when uncertainty and market imperfections are present a means tested pension system provides individuals with a mechanism to smooth consumption over the life-cycle. Differently from a universal pension system, however, inclusion of means testing into pension programs allows governments to better direct benefits to those most in need and to control funding costs by providing flexibility to control the condition for receiving benefits (extensive margin) and the benefit level (intensive margin). These distinctive features of a means tested pension program result in a number of new aspects. First, limiting benefits towards old and poor agents strengthens the redistributive function of a pension system, with emphasis more on intra-generational redistribution, while keeping the distortionary effects of tax financing relatively small. Arguably, the presence of an extensive margin, which is completely absent in universal pension systems, strengthens the progressiveness of pension benefits, which, in turn, improves risk-sharing across households and generations. Second, on other hand, however, uncertainty in receiving pension benefits combines with high effective tax rates in retirement ages to distort individuals’ inter-temporal allocations of resources over the life cycle. That is, means tests distort individuals’ incentives to work and save.1 This, under certain conditions, might lead to unfavourable welfare outcomes, compared to a universal pension system.

The idea of using means tests to limit pension benefits towards old and poor agents is suggested by Friedman and Cohen (1972). Proponents of this approach argue that means testing instruments minimize the distortions of pension on incentives, strengthening the progressiveness of the pension benefits and improving welfare outcomes. However, Feldstein (1987) argues that a means-tested pension system may not be superior to a universal pension system as high

1Empirical evidence on the links between earnings tests on savings and labor decisions is well documented. Neumark and Powers (1998) and Neumark and Powers (2000) estimate the effects of means-tested Supplemental Security Income for old age individuals in the U.S.A. and find that these retirement benefits reduce savings and labor supply of those likely to participate in the program when approaching retirement age. There are also a number of studies exploring the effects of labor-earning tests on early retirement and the elderly’s working hours in the U.K. Disney and Smith (2002) find that an abolition of the earning test induces older male workers to work 4 more hours a week. This empirical result is also consistent with the result of previous study by Friedberg (2000). These studies confirm that earnings tests significantly affect savings and labor supply decisions in older ages, especially around the mandatory age from which individuals are eligible for retirement benefits. Previous analyses of the effects of means-tested, non-pension benefits (e.g., see Hubbard at el (1995), Powers (1998), Jonathan Gruber and Yelowitz (1999), Heer (2002) and Chou, Liu and Huang (2004)) also found that the asset test reduces saving incentives of low income households.
effective tax rates in retirement ages induce individuals to dissave further, reducing individual utility by more than under a universal program.

The basic problem of designing a means-tested pension system is to set the conditions for receiving benefits (extensive) and the level of benefits (intensive) in the way that efficiently trades off the desirability of protecting low income elderly (insurance) against the economic cost of distorting effects on labor supply and savings (incentive). In this paper, we study how the design of means testing instruments affects individuals' inter-temporal allocation decisions and utility. Our ultimate goal is to determine how the inclusion of means testing of pension benefits influences trade-offs between insurance and incentive effects and the conditions under which means tested pension systems lead to favourable welfare outcomes.

To that end, we first investigate the social insurance role of means tested pensions within the framework of a two period, partial equilibrium model based on Varian (1980). This model captures essential aspects of the problem that motivates a society's desire for social insurance, while providing a convenient framework with which to explain the two margins of interest when means-tested benefits are introduced - the intensive and extensive margins. Means testing of pensions involves the specification of the maximum pension benefit and the taper rate at which benefits are withdrawn for each unit of extra income beyond an income threshold. These policy parameters determine whether an individual receives benefits or not, depending on both the withdrawal or taper rate and the overall magnitude of the benefit. In our simple model, we are able to isolate two channels of effects of means tested pension program on individuals' incentives: the likelihood of being a pensioner (extensive margin) and the level of pension benefits (intensive margin).

Our analytical results point to dynamic interactions between these two margins, which result in opposing effects on savings incentive. First, the effects via the intensive margin when imposing taper rates on pensioners' interest income tend to discourage savings. Second, however, the presence of an extensive margin, which is only embedded in a means tested pension system, introduces a new channel of effects that potentially encourage individuals to save for their retirement. On one hand, the extensive margin tends to encourage agents to save more to prepare themselves for the possibility that they are not eligible for pensions; on other hand, it tends to induce agents to reduce saving to increase their chances of receiving a pension in retirement. Moreover, we find that the direction of the extensive margin effect depends on the strength of the intensive margin effect. If the intensive margin effect is relatively less generous, the extensive margin has a positive effect; otherwise, it has a negative effect. This indicates that the existence of extensive margin effects potentially mitigates the adverse intensive margin effects on savings. The total effect on individuals's savings incentives depends on how these interactions combine.

The final judgment regarding the value of a means tested pension program should be based on the welfare effects embodying the trade-off between welfare gains from strengthening the positive insurance effects and welfare losses due to the negative incentive effects. Means testing adds new dimensions to this trade-off between the insurance and incentive effects, but the final
welfare outcome depends upon how these new aspects interact with other features of the overall social insurance system and upon the nature of the economy. Qualitatively, it is not clear whether the introduction of means-testing instruments results in a favorable welfare outcome. More specifically, the final welfare effects are influenced by dynamic interactions between these two margins and fundamentals, including preferences, endowments, market structures and institutional features.

Accordingly, in the second part of the paper we focus on exploring quantitative aspects of these trade-offs in a more realistic framework, taking these fundamental factors into account. We follow the tradition of the dynamic general equilibrium literature on social security and formulate an incomplete market, overlapping generations economy with heterogeneous households, a perfect competitive representative firm and a government with a full commitment technology (e.g., Imrohoroglu et al., 1995). We incorporate the main features of Australia’s means tested age pension system and calibrate our benchmark model to match key features of the Australian economy. We conduct several policy experiments and our quantitative results are summarized as follows.

In our first experiment, we compare steady state results of an economy with means tested pension with an economy without a pension. The results reveal that a non-PAYG pension program with means testing instruments results in lower welfare outcomes than having no pension. This implies that means tested pension systems are not socially desirable in our dynamic, general equilibrium model economy, since the adverse effects on incentives dominate the positive social insurance effects of pensions even when they are means tested. Consequently, when the pension program is completely removed, efficiency gains from increases in savings and labor supply result in higher consumption and welfare. This finding is similar to that obtained in the PAYG social security literature.

Next, conditioning on the existence of a pension system, we compare steady state results when varying the generosity of the maximum pension and taper rates for the income means test. We find that, conditioning on the existence of a pension system, the introduction of means testing results in non-linear welfare effects of changes in the generosity of the pension system and taper rates. That is, when the maximum pension benefits are relatively small, the introduction of income tests (raising taper rates) always leads to a welfare gain as the positive welfare effects from strengthening risk-sharing and mitigating self-insurance disincentives are always dominant. However, once the pension benefits become more generous, the negative incentive effects become more pronounced as taper rates are increased. The underlying economic mechanism behind this outcome is that the economic distortions of taper rates as implicit taxes on life-cycle savings and labor supply are more severe when pensions are more generous. We find that there is an optimal of taper rate that balances these two forces, conditioning on the level of maximum pension benefits.

Our paper contributes to several strands of the macroeconomics and public finance literature. First, our work contributes to a theoretical analysis of the choice between a universal and a means-tested pension system. Our results indicate that the use of means testing instru-
ments mitigates the economic distortions of tax financing and pension benefits as suggested in Friedman and Cohen (1972). However, this conclusion is not generally true as argued in Feldstein (1987). Note that Feldstein uses a framework with heterogeneous preferences (myopes and cyclers) and a very simple means tested pension program. Differently, we conduct our analysis in a moral hazard model of Varian (1987) and in a general equilibrium life cycle model with elastic labor supply as in Imrohoroglu et al. (1995); and we also consider a more complex and realistic means testing rule. We find that a means-tested pension system may not be superior to a universal pension system, which is similar to the finding in Feldstein (1987). We also characterized the conditions under which means testing might lead to a favourable welfare outcome compared to a universal pension system. Indeed, our work extends the analysis in Feldstein (1987) and highlights that the key mechanism in Feldstein (1987) is still applicable in a more general framework.

Recent studies quantifying the effects of means tested pensions on savings, labor supply and welfare in a life cycle framework emphasize the effects of taper rates working through the intensive margin, i.e., imposing an implicit tax. Sefton and van de Ven (2008) use a calibrated multi-period overlapping generations model to analyze the effects of a means tested pension reform on life cycle savings and labor decisions in the U.K. They find that tightening the taper rate for the income test encourages poor individuals to save more and to delay retirement, while generating opposite effects on the savings and retirement decisions of the rich. Selton, van de Ven and Weale (2008) conduct a welfare analysis and find that means tested pensions are socially preferred to a universal pension in the U.K. as they deliver better welfare outcomes. Kumru and Piggott (2009) also find a welfare gain from introducing means tests in the U.K. social security system. Kudrna and Woodland (2011) analyze the general equilibrium effects of changing taper rates of the Australian pension system in a deterministic overlapping generations model. Maattanen and Poutvaara (2007) study welfare implication of introducing labor earnings tests to the PAYG social security system in the U.S.A. and find negative welfare effects because the adverse effects of the labor earnings tests on the elderly’s labor supply are significantly large. It is noteworthy that these papers abstract from an important channel of effects via the extensive margin. In contrast, our research extends these papers by highlighting the importance of the extensive margin effects. We show that the interactions between taper rates and the maximum pension benefit via the extensive margin results in opposing effects on individuals’ incentives. Subsequently, the welfare effects of changes in taper rates vary significantly over the levels of maximum pension benefits.

Our study is also related to the literature that undertakes dynamic, general equilibrium analyses of social security systems. That literature focuses upon universal PAYG social security systems and consistently finds negative welfare outcomes when accounting for general equilibrium effects. It implies that the adverse effects on incentives tend to dominate the insurance role so that the introduction of an unfunded PAYG social security system usually lowers welfare. The adverse effects of unfunded social security in dynamic, general equilibrium models have been well documented (e.g., see Auerbach and Kotlikoff (1987), Hubbard and Judd (1987),
Imrohoroglu, Imrohoroglu and Jones (1995), Conesa and Krueger (1999), Krueger (2006) and Fuster, Imrohoroglu and Imrohoroglu (2007)). This literature focuses on the U.S.A. social security system in which the coverage is universal, and it therefore excludes the effects coming from the extensive margin. Our study is complementary to that literature as we examine a pension system in which the extensive and intensive margins are both relevant. We show that interactions between these two margins are important and potentially lead to welfare gains.

Our paper is also linked to the literature on social insurance with means testing, which has focused mainly on disability insurance. Diamond and Mirrlees (1978) and Diamond and Mirrlees (1986) conclude that optimal benefits are structured so that the healthy are indifferent as to whether to mimic the disabled or continue working. In a more recent work on optimal disability insurance, Golosov and Tsyvinski (2006) argue that disability insurance benefits should be asset-tested to prevent individuals from claiming benefits when, optimally, they should not. Our paper follows a similar approach, but focuses on a pension program. Specifically, we analyze the role of means testing in enhancing the social insurance function of public pensions rather than disability insurance. Nevertheless, we reach a similar conclusion that means testing could be used to foster savings and working longer. However, we find that this statement is not universally correct for a pension program, since we identify some cases in which the introduction of means tests makes the adverse intensive margin effects more severe and results in a negative overall welfare effect.

The paper is structured as follows. In section 2 we present a simple model to highlight the role played by the intensive and extensive margins arising from a mean-tested old-age pension and to derive some analytical results. In section 3 we set up a dynamic, general equilibrium model that embodies endogenous retirement, earnings uncertainty and a means tested pension system. Section 4 describes details of our calibration of the model to the Australian economy and old-age pension scheme. Section 5 contains the discussion of a range of policy experiments and results relating to alternative means test parameters. We present conclusions in section 6. The Appendix provides mathematical details for the theoretical model, and the fiscal policy specification and solution algorithm for the dynamic general equilibrium model.

2 A simple model economy with a means tested pension

In this section, we specify a theoretical model and use it to highlight how the inclusion of means testing into the age pension benefit formula influences individuals’ incentives to save over the life cycle. This simple model, based on Varian (1980), captures essential aspects of the problem that motivates a society’s desire for social insurance and provides a convenient framework in which to explain that there are two margins of interest when means-tested benefits are introduced. In doing so, we are able to emphasize the essential role played by means testing on the intensive and extensive margins related to pension receipts by the elderly.
2.1 Environment

We consider a simple partial equilibrium economy comprised of agents living for two periods with endowment incomes of \( w_1 \) and \( w_2 \) in period 1 and 2, respectively. At the beginning of period 1 an agent receives income \( w_1 \) and makes a decision on consumption and saving to maximize expected utility, taking the income distribution \( f(w_2) \) in period 2 and the government pension policy as given.\(^2\) The individual agent’s optimization problem is

\[
\max_{c_1, c_2, s} \{ u(c_1) + pE u(c_2) : c_1 + s = w_1 - g(\tau, w_1) \text{ and } c_2 = w_2 + (1 + r)s_1 + P \},
\]

where \( p \) is survival probability, \( c_1 \) is consumption when young, \( s \) is saving, \( c_2 \) is consumption when old, \( P \) is an individual specific pension benefit, \( r \) is the market rate of return on savings and \( g(\tau, w_1) \) is the tax function with tax rate \( \tau \).\(^3\) The individual’s standard first order necessary condition for an optimal solution is given by

\[
-u'(w_1 - g(\tau, w_1) - s) + (1 + r)pE u'(w_2 + (1 + r)s_1 + P) = 0.
\]

The optimal savings decision rule, derived by solving this equation, is a function of the initial endowments (assumed the same for all agents for simplicity), the distribution of endowments when old and the old-age pension benefit and is indicated by \( s^* = s(w_1, f(w_2), P) \).

To aid the exposition, we assume that individuals have quadratic preferences given by \( u(c) = -c^2/2 + \chi c \), where \( \chi > 0 \), and that income in period 2 follows a uniform distribution \( f(w_2) = 1/w_2^{\text{max}} \). Thus, the expected wage income when old is \( E(w_2) = w_2^{\text{max}}/2 \equiv \bar{w}_2 \). For ease of exposition, we also assume (except in one case further below) that the rate of return on investment is \( r = 0 \) and that the survival probability is \( p = 1 \), guaranteeing that the economy is dynamically efficient so that the pension system fails to yield a higher rate of return. We next consider alternative designs of a public age pension program.

To set the scene, we begin with a universal PAYG pension program in which the government collects tax revenue from incomes of the young in period 1 (whence \( g(w_1, \tau) = \tau w_1 \)) and transfers to every old agent an equal amount of pension benefit, \( P = P^{\text{max}} \), i.e., a universal pension. Optimal saving for an agent is simply given by

\[
s^* = (1 - \tau) \frac{w_1 - \left(\bar{w}_2 + P^{\text{max}}\right)}{2}.
\]

Variable \( \bar{w}_2 = w_2^{\text{max}}/2 \) is the average (expected) endowment income when old and the term \( [\bar{w}_2 + P^{\text{max}}] \) is the expected income when old. This expression for optimal saving indicates that public pensions discourage individuals to save for retirement, since optimal saving responds negatively to the expected pension income in period 2, \( \partial s^*/\partial P^{\text{max}} < 0 \). Particularly, the more pension benefits individuals receive in period 2 the less they will save in period 1. It is well

\(^2\)In the following, we consider a typical agent and so do not distinguish between agents.

\(^3\)We abstract from the labor/leisure decision to keep the model sufficiently simple to highlight the channels by which the design of a means-test pension distorts the savings decision. The labor/leisure choice could readily be included, but at the cost of simplicity.
established in previous studies that pension benefits distort the saving decision of individuals who are rational enough to saving for their future. This is a classic crowding-out effect resulting from the introduction of a public pension program. Labor income tax as a financing instrument also lowers savings, since $\partial s^*/\partial \tau < 0$.

2.2 Alternative pension schemes and saving

We now examine the salient features of means testing instruments. We argue that these instruments result in two separate channels of effects: (i) the condition for receiving pension benefits, i.e., the number of agents participating in a public pension program (extensive margin); and (ii) the level of pension benefits (intensive margin). The latter is comprehensively analyzed in the PAYG social security literature, while the former is relatively new and only appears when means testing is introduced. Here, we investigate how these two margins can influence an individual’s savings decision.

2.2.1 A simple means tested pension

First, we consider the simplest means tested pension program, in which the government is allowed to discriminate between income groups to determine the receipt public pension benefits; that is, the government uses an income test to determine individuals’ pension benefits. To get some intuition, we start with the very simple means testing rule

$$P = \begin{cases} P_{\text{max}} & \text{if } w_2 < \overline{y}_2, \\ 0 & \text{if } w_2 \geq \overline{y}_2, \end{cases}$$

(3)

where $\overline{y}_2 \in (0, w_{\text{max}}^2)$ is the threshold level of income (here labour or endowment income only) separating pension recipients from non-recipients. This rule state that all agents with income endowments in period 2, $w_2$, below the income threshold, $\overline{y}_2$, are eligible for an equal amount of pension benefits. This pension rule separates the elderly population into two groups: one defined as relatively poor and one as relatively rich. With means testing and a uniform distribution for endowments when old, individuals face the probability of $(\overline{y}_2/w_{\text{max}}^2) < 1$ of being a pensioner. By discriminating amongst retirees by income, the government is better able to target poor retirees. Moreover, it also tends to encourage young individuals to save more for their old age compared to a universal pension scheme. Note that, since the government excludes individuals’ savings from testable income, it can directly control the number of individuals participating in the pension program.

With these means testing instruments, the government has two pension policy parameters that it can adjust: first, adjusting the income test threshold, $\overline{y}_2$, to determine the number of pensioners (extensive margin) and, second, adjusting $P_{\text{max}}$ to vary the generosity of pension benefits (intensive margin). In this simple model, the extensive margin disappears when the
government sets $y_2 = w_2^{\text{max}}$ or $y_2 = 0$. The household’s optimal saving rule is

$$s^* = \frac{\left[ w_1 - g(\tau, w_1) \right]}{2}$$

We now characterize the mechanism through which means testing complicates the way that public pension programs influence individuals’ saving incentives. We find that means testing adds another source of uncertainty to income in period 2 as the expected income in period 2 depends on the income threshold, $y_2$, set by the government, and this influences the individual’s saving decision. More specifically, as the government adjusts the income test threshold it affects the probability of being a pensioner and expected income, and thereby affects the individual’s incentive to save.

To identify these channels through which means testing instruments impact upon individuals’ saving incentives, we take the first derivatives of the saving function with respect to the maximum pension benefit, $\partial s^*/\partial P^{\text{max}}$, and the income test threshold, $\partial s^*/\partial y_2$. The former reflects the effect from the intensive margin, while the latter captures the effect from the extensive margin (hereafter called the intensive margin and extensive margin effects, respectively).

Not surprisingly, we find that the effect through the intensive margin is negative as $\partial s^*/\partial P^{\text{max}} < 0$. We conclude that a public pension program crowds out private savings via the intensive margin even with means testing. However, we find that the sign of the extensive margin effect is ambiguous, since

$$\frac{\partial s^*}{\partial y_2} = \frac{1}{2 \frac{w_2^{\text{max}}}{w_2}} \left( \frac{\bar{y}_2}{w_2} + P^{\text{max}} \right)$$

Indeed, it is dependent on the magnitude of the maximum pension benefit, $P^{\text{max}}$, relative to the average income in period 2, $\bar{w}_2$. This distance $(\bar{w}_2 - P^{\text{max}})$ also measures the generosity of the public pension program, i.e., relative strength of the intensive margin effect. As $P^{\text{max}}$ becomes relatively more generous, the strength of the intensive margin effect becomes relative larger. For example, when the maximum pension benefit is higher than the average income in period 2, $P^{\text{max}} > \bar{w}_2$, the pension system is very generous. The direction of the extensive margin effect depends on the strength of the intensive margin effect. If the intensive margin effect is relatively less generous ($P^{\text{max}} < \bar{w}_2$) the extensive margin effect is $\frac{\partial s^*}{\partial y_2} = \frac{1}{2 \frac{w_2^{\text{max}}}{w_2}} (\bar{w}_2 - P^{\text{max}})$, which is positive; otherwise, it is negative.

Feldstein (1987) emphasizes the negative effects on savings as individuals rationally reduce savings to qualify for pension benefits. Our result indicates that the existence of extensive margin effects potentially mitigates the adverse intensive margin effects on savings. However, the final effect on saving is not clear as it depends on how these two margins interplay. Our
result also points out that generosity of pension benefits relative to average income determine the direction of interactions between two margins.

2.2.2 Means testing with a taper rate

We now consider a more complex means testing rule under which the pension payment depends continuously upon the individual’s endowment income. Under this specification, the pension benefit declines by \( \omega \) for each additional unit of income received, where \( 0 \leq \omega \leq 1 \). Analytically, the pension benefit is determined by

\[
P = \begin{cases} 
  P_{\text{max}} - \omega w_2 & \text{if } w_2 < y_2, \\
  0 & \text{if } w_2 \geq y_2,
\end{cases}
\]

where the maximum income threshold is now determined by \( y_2 = P_{\text{max}}/\omega \). Note that we continue to assume that testable income excludes interest income and hence abstract from the effects of means test on effective rate of return on private capital.

The corresponding optimal savings function is given by

\[
s^* = \frac{[w_1 - g(\tau, w_1)] - \begin{bmatrix} w_2 + \frac{P_{\text{max}}}{\omega} & \frac{y_2}{w_2} + \frac{P_{\text{max}}}{\omega} \\
  \frac{y_2}{w_2} + \frac{P_{\text{max}}}{\omega} & 1 - \frac{y_2}{w_2} \end{bmatrix}}{2}.
\]

The expected income in period 2 now depends on three pension policy design parameters, two of which are independent - the maximum benefit, \( P_{\text{max}} \), the taper rate, \( \omega \), and the income test threshold, \( y_2 \). If an individual is a pensioner, \( \frac{y_2}{w_2} \) is the expected labor income endowment they will receive; therefore, total expected income in period 2 is \( \left[ (1 - \omega) \frac{y_2}{w_2} + P_{\text{max}} \right] \), in which the additional term \( -\omega \frac{y_2}{w_2} \) reflects the effects of the taper rate, i.e., an implicit tax on individuals’ income. A non-pensioner’s expected income is \( \left[ \frac{w_{\text{max}}}{w_2} + \frac{y_2}{w_2} \right] = \left[ \frac{w_2 + \frac{y_2}{w_2}}{w_2} \right] \).

Inclusion of the taper rate in the pension benefit formula provides the government with an additional tool to affect both the extensive and intensive margins of a pension program. The government can vary taper rates to determine the progressivity of the pension payment schedule. First, the government may use the taper rate to adjust the level of pension benefits, which directly tunes down the negative intensive margin effect on savings. Furthermore, it affects the extensive margin effect as the taper rate appears in the derivative

\[
\frac{\partial s^*}{\partial y_2} = \frac{1}{2} \frac{1}{w_2^{\text{max}}} \left( \frac{y_2}{w_2} - P_{\text{max}} + \omega \frac{y_2}{w_2} \right),
\]
Again, we find that the saving effect via the extensive margin is dependent on the relative strength of the intensive margin. The extensive margin effect is positive ($\frac{\partial s^*}{\partial y^2} > 0$) only if the pension benefit is sufficiently less generous, relative to average income in period 2, ($P^{\text{max}} - \omega \bar{y}_2 < \bar{w}_2$). Compared to the previous means testing policy, the presence of the taper rate weakens the strength of intensive margin effect. More specifically, the government can increase the taper rate to amplify the positive extensive margin effect, taking the level of the maximum benefit as given. Consequently, this increases the likelihood that the extensive margin effect is positive.

2.2.3 Means testing of total income

In the previous two cases, we assumed that the government’s means test was based solely upon labor endowment income in period 2. We now relax this assumption to consider the case where the government includes interest income from saving as a part of testable income. The pension payment then becomes

\[
P = \begin{cases} 
P^{\text{max}} - \omega (w_2 + rs) & \text{if } w_2 + rs < \bar{y}_2, \\ 0 & \text{if } w_2 + rs \geq \bar{y}_2, \end{cases}
\]

where $w_2 + rs$ is the testable income, which includes two components: labor endowment income in period 2 and interest from saving in period 1.

Optimal saving is now implicitly given by

\[
s = \frac{[1 + (1 - \omega) r] \left( \frac{\hat{w}_2}{2} + P^{\text{max}} - \omega \hat{w}_2 \right) \pi^p - (1 + r) \left( \frac{w_2^{\text{max}} + \hat{w}_2}{2} \right) (1 - \pi^p)}{1 + [1 + (1 - \omega) r]^2 \pi^p + (1 + r)^2 (1 - \pi^p)},
\]

where the probability of being a pensioner is $\pi^p = \frac{\hat{w}_2}{w_2^{\text{max}}}$, $\hat{w}_2 = \bar{y}_2 - rs$ and the tax function is assumed to be $g(w_1, \tau) = \tau w_1$. Note that the probability of being a pensioner, $\pi^p$, is now dependent on the individual’s saving, since the wage rate that separates pensioners from non-pensioners, $\hat{w}_2$, depends on the level of saving.\(^4\) Under this new means testing policy, the government can no longer directly control the number of pensioners in the economy, since the testable income used by the government to determine the number of agents eligible for the pension program is now dependent upon both the level of labor income endowment in old age and optimal savings of the agents when young (which is endogenous), i.e., upon $w_2 + rs$.

By including interest income from saving in the income test, the government is providing another (this time, direct) channel through which the means test impacts upon the saving decision. Under the two previously considered means tests for the old-age pension, the policy instruments affected the saving decision of the young indirectly through their impacts upon

\(^4\)See Appendix 7.1 for a complete equilibrium solution.
expected future income. While these indirect impacts remain operational, the new channel or impact upon the saving decision is direct. Higher saving directly reduces the probability of becoming a pensioner (extensive margin) and, if the individual is a pensioner, directly reduces the pension payment (intensive margin).

In responding to a means tested pension policy, individuals optimize their saving for retirement taking into account the effect of saving upon the expected pension payment through the effect on both the intensive and extensive margins. Individuals can manage their savings decision to increase the probability of being a pensioner by decreasing saving. In that sense, the effect of the means test on savings through the extensive margin tends to be negative. On other hand, decreasing the probability of being a pensioner lowers expected income in period 2, which may encourage individuals to save more. Feldstein (1987) emphasizes the negative effects on savings while abstracting from the potential positive effect. Thus, we argue that this aspect of the means test leads to two opposing effects on self-insurance incentives to save. The final effect on savings depends on which effect is dominant and how the intensive margin effect interacts with the extensive margin effect.

Moreover, an introduction of means testing instrument affects effective rate of return on private savings. That is, it is \((1 - \omega) r\) for pensioners rather than \(r\). For worker, the expected rate of return on private savings is \((1 - \omega) r \pi^p + r (1 - \pi^p)\) which is lower than market rate of return. This in return discourage individuals to save for their retirement.

### 2.3 The intensive and extensive margins and welfare

We now turn our analysis to the welfare implication of a means-tested pension program. In our simple setting considered here, every young agent is forced to pay the same amount of payroll tax but only a fraction of retirees get pension benefits. Since the tax financing instrument is non-progressive, the tax-transfer system as a whole provides social insurance from progressive transfers. Indeed, a means-tested pension system provides two types of social insurance: (i) insurance against income risk and (ii) insurance against longevity risk (annuity market). When when there is no lifetime uncertainty \((p = 1)\), the latter does not work but the former does. Hence, limiting pension benefits (extensive margin) to a sub-group of the retired population and providing progressive pension benefits (intensive margin) keeps the redistributive role of a tax-transfer program in play, strengthening the insurance role. Any changes in policy parameter values of a means-tested pension policy affect the progressiveness of this tax-transfer system and the insurance effect of means-tested pension system. On other hand, as discussed above, the introduction of a taper rate distorts saving and this might have negative welfare effects. When welfare gains from risk-sharing outweigh welfare losses resulting from distorted saving, there is a positive overall welfare effect and so there is a demand for means-tested pensions as social insurance.

The trade off between the risk sharing effects and the saving-disincentive effects, which underlies how much social insurance should be provided through a tax-transfer system, has been analyzed in the literature. Notably, a seminal work by Varian (1980) shows that social insurance
can be provided via a progressive tax system and that an optimal progressive income tax system efficiently trades off the benefit due to redistributive taxation as risk-sharing mechanism against the costs due to distortion on saving incentives. In the Varian tax-transfer system, the tax side has a redistributive function but the transfer side does not. Low income agents contribute less in absolute amount of tax payments but receive the same amount of transfers. This redistributive tax-transfer system provides social insurance via progressive contribution. Overall, the tax-transfer system has redistributive function as the net tax-transfer is relatively higher for "unlucky" agents. In our model, the mechanism at work is similar to that in Varian (1980). Our result reveals that means-tested pension systems can achieve a similar outcome, but the mechanism for risk-sharing within and between households and generations is provided through progressive pension benefit rather progressive tax.

2.4 A numerical example

In this section, we quantitatively explore how means testing instruments influence the trade-off between insurance and incentive effects. This provides a concrete example and supplements the theoretical analysis above.

**Parameterization and solution method.** We assume individual agents have standard CRRA preferences in the form of $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 2$. Regarding endowments, we assume that in period 1 agents are homogeneous in terms of productivity, $w_1 = 2.7$, but heterogeneous in period 2 with $w_2$ following a log normal distribution with mean $\mu_{w_2} = 1$ and standard deviation $\sigma_{w_2} = 2$. The interest rate normalized to zero ($r = 0$) for simplicity and we assume credit market imperfections such that individuals are not allowed to borrowed. The solution method is simple. For any combination of maximum pension benefit, $P_{\text{max}}$, and taper rate, $\omega$, we numerically search for an equilibrium allocation that solves the household problems and clears the government budget. Note that the number of pensioners is endogenously determined in equilibrium.

**Experiments.** We conduct various experiments to quantify the welfare effects of a means-tested pension program. More specifically, we numerically explore how different designs of a means-tested pension program, by choosing different policy parameter values of maximum pension payment $P_{\text{max}}$ and taper rates $\omega$, affects social welfare. We index the maximum pension payment, $P_{\text{max}}$, to average income, $y$, by specifying $P_{\text{max}} = \Psi y$, where $\Psi$ is a replacement rate. We consider a range of replacement rates $\Psi$ between 0 and 0.8, $\Psi = [0, 0.1, \ldots, 0.8]$ and a range of taper rates between 0 and 1, $\omega = [0, 0.1, \ldots, 1]$. We also assume that the government uses payroll tax as a financing instrument. By doing so, we completely assume away the insurance role of progressive tax and are able to identify the insurance role of progressive benefits. In our setting, individuals are ex-ante identical and we use the expected utility of a young agent as a measure of social welfare.

**Results.** We report the results in table 1. We present different policy choices of maximum pension by columns while presenting different policy choices of taper rates by rows. The first column is the case when maximum pension is nil ($\Psi = 0$), which means there is no social
security. The first row is the case when taper rate is nil (ω = 0), which means the social security system is universal with uniform benefit levels. Figures 1-3 provide graphical illustrations of the contents of the table.

<table>
<thead>
<tr>
<th>Taper (ω)</th>
<th>Replacement Rate (Ψ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0 -0.8639 -0.8639 -0.8639 -0.8680 -0.9431 -1.1634 -1.8092 -2.7757 -7.0430</td>
</tr>
<tr>
<td>1</td>
<td>0.1 -0.8639 -0.8555 -0.8550 -0.8554 -0.9073 -1.0786 -1.5458 -2.1370 -3.7981</td>
</tr>
<tr>
<td>2</td>
<td>0.2 -0.8639 -0.8515 -0.8479 -0.8471 -0.8791 -1.0121 -1.3622 -1.7590 -2.6502</td>
</tr>
<tr>
<td>3</td>
<td>0.3 -0.8639 -0.8511 -0.8435 -0.8410 -0.8579 -0.9600 -1.2288 -1.5117 -2.0670</td>
</tr>
<tr>
<td>4</td>
<td>0.4 -0.8639 -0.8523 -0.8414 -0.8369 -0.8429 -0.9195 -1.1290 -1.3393 -1.7173</td>
</tr>
<tr>
<td>5</td>
<td>0.5 -0.8639 -0.8539 -0.8410 -0.8345 -0.8337 -0.8885 -1.0531 -1.2141 -1.4868</td>
</tr>
<tr>
<td>6</td>
<td>0.6 -0.8639 -0.8556 -0.8416 -0.8334 -0.8291 -0.8654 -0.9500 -1.1208 -1.3255</td>
</tr>
<tr>
<td>7</td>
<td>0.7 -0.8639 -0.8570 -0.8428 -0.8334 -0.8279 -0.8485 -0.9503 -1.0501 -1.2082</td>
</tr>
<tr>
<td>8</td>
<td>0.8 -0.8639 -0.8582 -0.8443 -0.8341 -0.8277 -0.8368 -0.9156 -0.9956 -1.1206</td>
</tr>
<tr>
<td>9</td>
<td>0.9 -0.8639 -0.8592 -0.8459 -0.8352 -0.8282 -0.8295 -0.8888 -0.9530 -1.0533</td>
</tr>
<tr>
<td>10</td>
<td>1 -0.8639 -0.8600 -0.8475 -0.8366 -0.8291 -0.8258 -0.8685 -0.9199 -1.0008</td>
</tr>
</tbody>
</table>

Table 1: Welfare effects for alternative pension parameters.

First, consider social welfare in an economy with no pension as in the first cell of row 1 of table 1 and introduce a universal social security program by raising $P_{max}$. As seen in row 1 of table 1, the introduction of a universal pension program always lead to lower welfare, i.e., social welfare is a decreasing function of maximum pension benefits. The intuition is as follows. In our setting, the implicit rate of return in a universal pension program is less than the rate of return from private savings. Moreover, the universal pension program does not have any distributional effect as agents contribute identical amount of tax income to the program and receive identical benefits regardless of their income status (i.e., no insurance effect). Consequently, since the universal pension program distorts the savings incentive while providing no risk-sharing mechanism, it results in lower welfare.

Second, we now deviate from a universal pension program and allow the government to use income tests to determine social security benefits: individuals with higher incomes receive smaller benefits. This discrimination makes social security schemes progressive and, hence, triggers trade-offs between the insurance effect and the incentive effect. Varying taper rates affects coverage of social insurance programs (extensive margin). That is, the presence of extensive margin, which is completely absent in universal pension systems, strengthens the progressiveness of the pension benefits, which improves risk-sharing across households and generations. Comparing results for $\omega = 0.1$, 0.2 and 0.3 in rows 2 to 5 (means-tested pension) with those for $\omega = 0$ in row 1 (universal pension) in table 1, we find that a means-tested social security program results in higher social welfare compared to a universal pension. This indicates that means-tested social security programs are strictly preferred to universal pension programs for a wide range of policy parameter values in our model.

Third, as argued above, the social welfare outcome is determined by trade off between
insurance and incentive effects working through extensive and intensive margins in our framework. To explore it quantitatively, we first keep maximum pension benefits unchanged and vary taper rates. By examining each column in table 1 or the panels in figure 1 for various maximum pension benefit levels, we find a trade-offs between insurance and incentive effects as taper rates increase. When maximum pension payments are small (for example, when \( \Psi \leq 0.4 \)), increases in the taper rate generates higher welfare initially and then lower welfare for higher taper rates. This implies that there is a optimal taper rate that yields highest social welfare, taking the maximum pension as given. However, this result is not true when the maximum benefit payment is sufficiently large (\( \Psi > 0.4 \)), in which case increases in the taper rate always results in higher welfare. This means that when the incentive effect is already dominant, the government could increase taper rates to neutralize some of these distortions.

Figure 1: Social welfare effects when fixing maximum pension benefit \( P^{\text{max}} \) and varying taper rate \( \omega \). Note that \( \Psi \) is a replacement rate used to calculate the maximum pension benefit and horizontal line is welfare for no pension.

Fourth, we examine the effects of increasing maximum pension benefits while keeping the taper rate unchanged. The welfare effects are indicated by examining each row of table 1 or the panels of figure 2 for various taper rates. As previously noted, social welfare declines as \( P^{\text{max}} \) increases for the case when \( \omega = 0 \) (row 1). However, when \( \omega > 0 \) (all other rows of the table and panels 2 to 6 of the figure) we see hump-shaped patterns of the social welfare function, initially rising as a function of the maximum pension benefit and then declining. This illustrates numerically the interplay of the insurance and incentive effects upon social welfare.
When the insurance effect is dominant, increasing the maximum pension raises social welfare; when the incentive effect is dominant, higher maximum pension benefits result in lower welfare. More specifically, when the maximum pension is relatively small the insurance effects are still dominant and therefore increases in maximum pension still lead to higher welfare. On other hand, when the maximum pension is relatively large, the disincentive effects are a dominating force and therefore further increases in the maximum pension benefit decrease welfare.

Figure 2: Social welfare effects when fixing taper rate $\omega$ and varying maximum pension benefit $P^{\text{max}}$. Note that the maximum pension benefit is calculated by $P^{\text{max}} = \Psi P$ and horizontal line is welfare for no pension.

Overall, we find in our numerical example that a means tested pension system results in a superior social welfare outcome, compared to either no pension or a universal pension system. It is always the case that means testing (at any taper rate) is better than a universal pension (each row element is larger than the first row in every column). For all maximum pension benefits with a replacement rate less than $\Psi \leq 0.5$ there are taper rates yield better welfare outcomes than obtained with no pension. However, we also find cases when maximum pension benefit is very generous that social welfare is lower than obtained with no pension or with a universal pension. Thus, an optimal design of a means tested pension program is obtained when the system efficiently trade off between insurance and incentive effects as seen in figure 3. From table 1, we determine that the optimal pension parameters for this economy are $\Psi = 0.5$ and $\omega = 1$. 

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Figure 3: Social welfare effects of means-tested pension. Note that maximum pension is given by $P_{\text{max}} = \Psi \bar{y}$.

2.5 Discussion

We have demonstrated via our simple theoretical model and illustrated via our numerical example that means tested pensions create two channels of effects on incentives for individuals: the probability of being a pensioner (extensive margin) and the level of pension benefits (intensive margin). We have demonstrated that dynamic interactions between these two margins result in opposing effects on savings and utility, and that the total effect depends in how these interactions interact in the economy. Importantly, these interactions will depend upon fundamentals of the economy, such as preferences, endowments, market structure and institutional settings. To make a judgment on the effects of a means tested pension program, one should seriously account for these fundamentals. In the next section, we develop a dynamic, general equilibrium model of an economy in which we take into account these factors.

3 A dynamic general equilibrium model

We consider an overlapping generations dynamic general equilibrium model, which consists of heterogeneous households, a perfect competitive representative firm, and a government with full commitment technology.

Demographics. The economy is populated by agents (households) whose ages are denoted
by \( j \in [1,\ldots,J] \). Each period a continuum of agents of age 1 are born. The population grows at an exogenous annual rate, \( n \). All agents face an age-dependent survival probability, \( sp_j \), and live at most \( J \) periods. When the demographic pattern is stationary, as assumed here, the population share of the cohort age \( j \) is constant at any point in time and can be recursively defined as \( \mu_j = \mu_{j-1} sp_j / (1 + n) \). The share of agents who do not survive to age \( j \) is \( \tilde{\mu}_j = \mu_{j-1} (1 - sp_j) / (1 + n) \).

**Preferences.** All agents have identical lifetime preferences over consumption \( c_j \geq 0 \) and leisure \( l_j \), where household leisure time per period for household \( j \) is constrained by \( 0 \leq l_j \leq 1 \). Preferences are time-separable with a constant subjective discount factor \( \beta \) and are given by the expected utility function

\[
E \left[ \sum_{j=1}^{J} \beta^j sp_j u(c_j, l_j) \right].
\]  

(9)

Instantaneous utility obtained from consumption and leisure is defined as

\[
u(c_j, l_j) = \left( \left( (1 + dp_j)\xi \right)^\gamma \left( l_j \right)^{1-\gamma} \right)^{1-\sigma} / (1 - \sigma),
\]  

(10)

where \( \gamma \) is the weight on utility from consumption relative to that from leisure, \( \sigma \) is the coefficient of relative risk aversion, \( dp_j \) is the number of dependent children at age \( j \) and \( \xi \) is the demographic adjustment parameter for consumption.

**Endowment.** Agents are endowed with 1 unit of labor time in each period of life that has labor efficiency (or working ability or labor productivity) denoted by \( e_j \). The value of an agent’s period effective labor services is \( h_j = (1 - l_j) e_j \). When the agent chooses to allocate all time to leisure \( (l_j = 1) \), the agent exits the labor market and has retired. There is no mandatory retirement age so agents may stay in the labor force as long as they choose. The retirement age is endogenously determined. However, retirement is not required to be irreversible since households may re-enter the labor market. The efficiency unit \( e_j \) is age dependent and follows a Markov switching process with \( \pi_j (e_{j+1}|e_j) \) denoting the conditional probability that a person of working ability \( e_j \) at age \( j \) will have working ability \( e_{j+1} \) when at age \( j + 1 \). According to this specification, agents have working abilities that vary by age and change stochastically over the life cycle; they therefore face idiosyncratic earnings risk, which is assumed to be non-insurable.

**Technology.** The production sector consists of a large number of perfectly competitive firms, which is formally equivalent to one aggregate representative firm that maximizes profits. The production technology of this firm is given by a constant returns to scale production function \( Y = F(K, L) = AK^\alpha L^{1-\alpha} \), where \( K \) is the input of capital, \( L \) is the input of effective labor services (human capital) and \( A \) is the total factor productivity, assumed to be growing at a constant rate, \( g \). Capital depreciates at rate \( \delta \). The firm chooses capital and labour inputs to maximize its profit according to \( \max_{K,L} \left\{ AK^\alpha L^{1-\alpha} - qK - wL \right\} \), given rental rate, \( q \), and market wage rate, \( w \).

**Means tested pension.** In the benchmark economy, the government operates a means
tested pension system similar to the current Australian system. The old-age pension (social insurance) system is not universal but targets households who have low private retirement incomes through the use of income and assets means tests. The amount of pension benefit \( P(a_j, y_j) \) receive at age \( j \) varies across individuals and depends on the asset and income tests as

\[
P(a_j, y_j) = \min \{ P^a(a_j), P^y(y_j) \},
\]

where \( P^a(a_j) \) is the asset test pension and \( P^y(y_j) \) is the income test pension. Accordingly, the pension benefit is the smaller of the two pension rates; the strictest test binds. The pension benefit arising from the asset test is given by

\[
P^a(a_j) = \begin{cases} 
P^\text{max} & \text{if } a_j \leq \overline{a}_1, \\
P^\text{max} - \omega_a (a_j - \overline{a}_1) & \text{if } \overline{a}_1 < a_j < \overline{a}_2, \\
0 & \text{if } a_j \geq \overline{a}_2,
\end{cases}
\]

(12)

where \( \overline{a}_1 \) and \( \overline{a}_2 = \overline{a}_1 + P^\text{max}/\omega_a \) are the asset thresholds and \( \omega_a \) is the asset taper rate indicating the amount by which the pension is decreased for each additional unit of asset above the low asset threshold. Similarly, the pension benefit based on the income test is given by

\[
P^y(y_j) = \begin{cases} 
P^\text{max} & \text{if } y_j \leq \overline{y}_1, \\
P^\text{max} - \omega_y (y_j - \overline{y}_1) & \text{if } \overline{y}_1 < y_j < \overline{y}_2, \\
0 & \text{if } y_j \geq \overline{y}_2,
\end{cases}
\]

(13)

where \( \overline{y}_1 \) and \( \overline{y}_2 = \overline{y}_1 + P^\text{max}/\omega_y \) are the income thresholds, \( \omega_y \) is the income taper rate indicating the amount by which the pension is reduced for each additional unit of income above the low income threshold, \( \overline{y}_1 \).

**Market structure.** Markets are incomplete and households cannot insure against the idiosyncratic labor income and mortality risks by trading state contingent assets. They can, however, hold one-period riskless assets to imperfectly self-insure against idiosyncratic risks. We assume that agents are not allowed to borrow against future income, i.e., \( a_j \geq 0 \) for all \( j \).
The economy is assumed to be small in the sense that all agents in the economy take the world prices for traded goods and the world interest rate on bonds, \( r \), as given and independent of the amount of trade in these goods and bonds. The free flow of financial capital ensures that the domestic interest rate is equal to the world interest rate, which is assumed to be constant. An implication is that the rental price of capital is then given by \( q = r + \delta \).

**Household problem.** Households are heterogeneous with respect to their state variables including age, working ability and asset holdings. Let \( x_j = (e_j, a_j) \) denote the household’s state variables at age \( j \). At the beginning of age \( j \) the household realizes its individual state \( x_j = (e_j, a_j) \) and chooses its optimal consumption, \( c_j \), leisure time, \( l_j \), or working hours, \( (1 - l_j) \), and the end-of-period asset holdings, \( a_{j+1} \), taking the transition law for working ability, \( \pi_j (e_{j+1}|e_j) \), conditional survival probabilities, \( sp_j \), the wage and interest rates, and
government tax and pension policies as given.

Households have three sources of incomes: labor earnings, savings and transfers. First, if households decide to work they supply \((1 - l_j) e_j\) units of effective labor service to the labor market, attract a wage rate \(w_t\) and so earn a gross wage income or labor earnings of \((1 - l_j) e_j w_t\). Second, households have the cash balance from savings income available to spend in the amount \((1 + r) a_j\). Third, eligible households may receive old-age pension transfers from the government in amount \(P_j\). Specifically, agents who are \(J_1 = 65\) years of age or older are entitled to receive the old-age pension. There is a maximum amount of pension income, \(P^{\text{max}}\), but the actual amount of pension benefits varies across individuals and depends on the asset and income tests as \(P_j = P(a_j, y_j)\), where assessable income for the pension income test is simply labour and interest earnings, \(y_j = e_j (1 - l_j) w + ra_j\). Finally, households receive accidental bequests, \(b_j\), as a lump-sum transfer from the government.

Formally, the life-cycle expected utility maximization problem of agent \(i\) can be expressed recursively as

\[
V^j (x_j) = \max_{c_j, l_j, a_{j+1}} \left\{ u(c_j, l_j) + \beta s p_j E \left[ V^{j+1} (x_{j+1}) | e_j \right] \right\}
\]  

subject to the following constraints for every \(j \in J\)

\[
a_{j+1} = \frac{1}{(1 + g)} [a_j + e_j (1 - l_j) w + ra_j + b_j + P(a_j, y_j) - T(y_j) - (1 + \tau^c) c_j],
\]

\[
a_1 = 0, ~ a_J = 0, ~ a_j \geq 0,
\]

\[
0 < l_j \leq 1,
\]

where \(E \left[ V^{j+1} (x_{j+1}) | e_j \right]\) is the expected value function, \(T(y_j)\) is income tax payment and \(\tau^c\) is the consumption tax rate. Note that individual quantity variables, except for working hours, are normalized by the steady state per capita growth rate, \(g\).

**Fiscal policy.** The government levies taxes on consumption and income to finance general government consumption and the old-age pension program. The consumption tax rate is set at \(\tau_c\). Income tax is progressive and compactly written as

\[
T(y_j) = T_k + \tau_k (y_j - \bar{y}_k), ~ y_j \in [\bar{y}_k, \bar{y}_{k+1}],
\]

where the parameters of this tax function are the marginal tax rates, \(\tau_k\), the tax payment thresholds, \(T_k\), and the tax bracket income thresholds, \(\bar{y}_k\). It is assumed that \(\tau_1 = 0, ~ T_1 = T_2 = 0\) and \(T_k = T_{k-1} + \tau_k (\bar{y}_k - \bar{y}_{k-1})\). This specification corresponds to a standard segmented-linear income tax schedule with an initial tax free threshold and marginal tax rates that rise with taxable incomes. The income tax is set so that the consolidated government budget constraint is satisfied every period, whence

\[
\sum_j T(y_j) \mu(x_j) + \sum_j c_j \mu(x_j) = \sum_j P^y(x_j) \mu(x_j) + \hat{G},
\]  

(17)
where, $\mu(x_j)$ is the measure of agents in state $x_j$.

**Equilibrium.** Given government policy settings for tax rates and the old-age pension system, the population growth rate, world interest rate, a steady state competitive equilibrium is such that

(a) a collection of individual household decisions $\{c_j(x_j), l_j(x_j), a_{j+1}(x_j)\}_{j=1}^J$ solve the household problem (14);\textsuperscript{5}

(b) the firm chooses labour and capital inputs to solve the profit maximization problem;

(c) the total lump-sum bequest transfer is equal to the total amount of assets left by all the deceased agents, $B = \sum_{j \in J} \mu_j \int \Phi(a_j(x_j)) d\Lambda_j(x_j)$;

(d) the current account is balanced and foreign assets, $FA$, freely adjust so that $1 + r = R^w$;

(e) the markets for capital and labor clear

$$K = \sum_{j \in J} \mu_j \int \Phi(a_j(x_j)) d\Lambda_j(x_j) + B + FA,$$

$$H = \sum_{j \in J} \mu_j \int \Phi((1 - l_j)e_j(x_j)) d\Lambda_j(x_j),$$

and factor prices are determined competitively, i.e., $w = F_L(K, L)$, $q = F_K(K, L)$ and $r = q - \delta$; and

(f) the government budget constraint defined in Eq. (17) is satisfied.

### 4 Calibration

This section describes the calibration and parameterization of the model. We calibrate our benchmark model to match the Australian economy and report the values of key parameters of the benchmark model in Table 2.

**Demographics.** One model period corresponds to 5 years. Households become economically active at age 20 ($j = 1$) and live up to the maximum age of 90 years (equal to the maximum model period $J = 14$). The survival probabilities are calculated from life tables for Australia. The annual growth rate of the new born agents (households) is assumed to be 1.2%, which is the long-run average population growth in Australia.

**Working abilities.** We use estimates of labor productivities and other key life cycle profiles obtained using data drawn from the Household, Income and Labour Dynamics in Australia (HILDA) longitudinal survey (see Wooden and Watson (2002) for more details) for our model calibration. HILDA is a broad social and economics longitudinal survey, with particular

\textsuperscript{5}In the following, endogenous variables for the household of age $j$ are shown with dependence on the vector of state variables, $x_j = (e_j, a_j)$, at that age.
Parameters | Model | Observation/Comment/Source
--- | --- | ---
**Preferences**
Annual discount factor | $\beta = .99$ | to match $I/Y$
Inverse of inter-temporal elasticity of substitution | $\sigma = 4$ |  
Share parameter for leisure | $\gamma = 0.18$ | to match labor supply profile

**Technology**
Annual growth rate | $g = 0.025$ | 2.65%
Total Factor Productivity | $A = 1$ |  
Share parameter of capital | $\alpha = 0.4$ |  
Annual depreciation rate | $\delta = 0.055$ | 5.5%

**Demography**
Maximum lifetime | $J = 14$ | equivalent to 70 years
Maximum working periods | $J_w = 9$ | equivalent to 45 years
Annual population growth | $n = 0.012$ | 1.2%

**Government**
Income taxes | $\tau_j, T_j, \bar{f}_j$ | tax schedules in 2007
Medicare levy | $\tau^\text{Med} = 0.015$ | 1.5%
Consumption tax | $\tau^c$ | endogenous
Pensions | $P^\text{max}, \text{taper}$ | pension rules in 2007
Government consumption | $\Delta_G = 0.14$ | to match government size

Table 2: Preference and policy parameters

attention paid to family and household formation, income and work. We use data from the first 7 waves of HILDA surveys in this paper.

Working ability corresponds to the hourly average wage rate, defined as gross labor income divided by total hours worked. We estimate age-dependent hourly wage rates from HILDA data. The Markov transition matrix that characterizes the dynamics of working abilities over life cycle is estimated by a counting method. To make the transition matrix more persistent we use the average of these estimates. We also make an assumption that labor productivities from 65 decline at a constant rate, reaching zero at age 80 years.\(^6\)

**Preferences.** The utility function has the constant relative risk aversion (CRRA) form. We follow previous studies (e.g., Auerbach and Kotlikoff, 1987) and set the relative risk aversion coefficient to $\sigma = 4$, which implies an inter-temporal elasticity of substitution of 0.25. We follow Nishiyama and Smetters (2007) and set $\xi = 0.6$. The number dependent children $dp_j$ is calculated from HILDA data, using the average numbers of children of ages 0 – 19 in each age group, $j$. We calibrate $\gamma$ to match work hours on average. The subjective discount factor $\beta$ is calibrated to match Australia’s net investment to GDP ratio, which has averaged around 0.27 since 1990 according to Australian Bureau of Statistics (ABS) data.

**Technology.** We set the capital share of output $\alpha = 0.4$. The depreciation rate for capital is determined by the steady state condition and is $\delta = 0.055$. The average annual GDP

\(^6\)More details on the data and estimation methods provided in the Technical Appendix available at https://sites.google.com/site/chungqtran.
per capita growth rate in Australian is 3.3 percent so we set \( g = 0.033 \). The total factor of productivity \( A \) is a scaling parameter.

**Fiscal policy.** We use the tax and pension policy parameter values in 2007 to calibrate fiscal policy in the model. The maximum pension is set at \( P_{\text{max}} = $13,314.60 \). The income test threshold income is set at \( y_{\text{p1}} = $3,328 \) and the income taper rate is \( \omega_y = 0.4 \). For the asset test, the design is relatively more complicated. There are separate asset tests for renters and homeowners in Australia. In our model, there is no difference between residential and non-residential assets so we are not able to directly use the statutory asset test thresholds. Instead, we choose \( \omega_a \) to match the fraction of pensioners at age 65 years. Assets over this threshold reduces pension by $1.50 per fortnight for every $1000 above the limit, implying a taper rate for asset tests is \( \omega_a = 0.0015 \).

The government collects tax from consumption and income to cover spending on pension and other government spending programs. The consumption tax rate is set at 10 percent, which is the statutory goods and services (GST) rate in Australia. The details of pension and income tax schedule are reported in the Appendix.

**Small open economy.** The budget constraint for the small open economy may be expressed in steady state form as \( 0 = rFA + TB \), where \( FA \) and \( TB \) are the net holding of foreign assets and trade balance respectively. The right hand side is the current account balance consisting of net interest receipts plus the balance of trade (value of exports minus the value of imports) and the left hand side is net capital flows, which are zero. In a steady state, the stock of foreign asset holding is constant and so \( 0 = rFA + TB \), meaning that there is a current account balance with interest on foreign assets (if \( FA < 0 \)) matched by a positive trade balance. We normalize the world price to 1 and assume that the world (and domestic) interest rate is \( r = 5\% \). The Australian trade balance in the last 15 years is about \(-1.3\) percent of GDP. Using this fact in the context of a steady state, the net foreign asset is calculated as \( FA = TB/r = 0.013 \times Y/r > 0 \), which implies that Australia is a net investor in the world capital market. However, data on Australia’s international position reveals the opposite - Australia is a net borrower from the world capital market. Since our benchmark economy is in steady state, it cannot accommodate both facts. In the model, we assume that Australia is a net borrower with 19% of total national assets being foreign-owned.

### 5 Policy simulations and analysis

In this section, we first present the calibration result of the benchmark model and discuss how our model solution matches the data describing the Australian economy. Next, we specify, present and discuss various policy experiments constructed to explore the implications of alternative designs of a means tested pension for macroeconomic variables and household welfare.
5.1 Benchmark model

Our benchmark model economy is able to match some key features of the Australian economy. We summarize our calibration results in Figure 4.

**Asset profiles.** In our life-cycle model with income uncertainty and incomplete markets, individuals accumulate assets in early stages of a life cycle. As seen in panel 1 of Figure 4, our model is able to generate a hump-shaped pattern of asset holdings over the life-cycle that broadly matches in the data drawn from the HILDA panel data set.\(^7\) However, individuals draw down savings faster in the model than observed in the data because they do not have other motives to save, such as for bequests or to accommodate other life cycle shocks. De Nardi, French and Jones (2010), for example, show that bequest motives and health expenditure shocks are the main determinants of savings behavior of elderly American households. Also, we do not have compulsory retirement savings via superannuation or housing in our model. Incorporating these factors would potentially improve the match between model and data generated asset profiles.

**Labor market behavior.** Our model can match the observed life cycle pattern of labor market behavior and does a good job of capturing life cycle trends in labor force participation rates. However, it generates more young individuals participating in the labor force in early stages of the life cycle. This is primarily due to the assumption of no bequest motive. Since agents are born with no assets our model, there is very little wealth effect on labor supply decisions at young ages. Consequently, the new born agents optimally choose to work to maintain consumption. However, as agents accumulate more assets in middle and older ages, our model captures the labor force participation rates quite well. Agents between ages 20 and 40 years, on average, supply around 30 hours of work per week. Starting from the late 40s, agents decrease work hours and when they reach 70 years of age there is virtually no labor supplied. The model also captures the observed life cycle pattern of labor earnings.

\(^7\)Although HILDA is a longitudinal survey, not all questions are asked in every wave. Since waves 2 and 6 collect information on household assets, we construct the age profiles of asset holdings based on data from these two waves.
Figure 4: The benchmark model and the data.
5.2 Policy experiments

We now examine how the salient features of a means tested pension influence individuals’ incentives to work and save, macroeconomic aggregates and welfare. Our primary focus is upon the choice of parameters of the Australian old-age pension system and, more specifically, upon whether they can be optimally chosen by the government to maximize the steady state expected lifetime utility that accrues to an individual.

The design of a means tested pension program as described above involves the setting of three policy parameters: the maximum pension benefit that an old-age pensioner may receive, the threshold below which the maximum pension is, in fact, received, and the taper rate that reduces the pension above the threshold level. While the Australian system, as modeled here, has two tests - the income and asset tests - each of which has three such parameters, our policy experiments will simplify the analysis by concentrating on the design of the income test alone, keeping the assets test unchanged. In short, our concern is with the choice of values of the maximum pension, the income test threshold and the income test taper rate.

Thus, our design of a means tested pension program involves setting three policy parameters: the maximum pension benefit, $P_{\text{max}}$, the income threshold, $y_1$, and income taper rate, $\omega_y$. To further simplify the analysis, we restrict attention to the study of the effects of social security reforms along just two dimensions: the maximum pension benefit, $P_{\text{max}}$, and the taper rate, $\omega_y$. For convenience, we recall the income test pension payment function

$$P^y(y) = \begin{cases} P_{\text{max}} & \text{if } y \leq y_1, \\ P_{\text{max}} - \omega_y (y - y_1) & \text{if } y_1 < y < y_2, \\ 0 & \text{if } y \geq y_2, \end{cases}$$

(18)

where $y$ is assessable income.

In order to understand how a choice of these two policy instruments influence individuals’ inter-temporal allocations of consumption and hours of work, the insurance-incentive trade off and welfare consequences, we implement a number of hypothetical policy reforms. We start from the benchmark economy with the maximum pension benefit $P_{\text{max, benchmark}}$ set equal to 25% of average labor income and the taper rate set at $\omega = 0.4$. We then consider alternative model economies in which we change the values of these two policy parameters. We would like to emphasize that this part of the paper explores quantitatively a range of policy parameter values in which a means-tested pension program is preferred to a universal pension program. We therefore focus on a steady state analysis, compared to Imrohoroglu et al (1995) and Fuster, Imrohoroglu and Imrohoroglu (2003). Of course, the final welfare consequences of social security reform depend upon transitional costs and other factors.

The effects of maximum pension benefits. In a general equilibrium model, changes in the levels of maximum pension benefits affect not only the generosity of pension benefits (intensive margin) but also the number of pensioners in the economy (extensive margin). However, the effects via the former tend to be strong. To understand the effects of the maximum
benefits we simulate a number of alternative model economies in which we vary the levels of the maximum pension benefits, while keeping the taper rate unchanged at its benchmark level.

Technically, we index the maximum pension benefit in an alternative economy to that in the benchmark economy as

\[ P_{\text{max}}(\varphi) = \varphi P_{\text{max, benchmark}}, \tag{19} \]

where \( P_{\text{max}}(\varphi) \) denotes the maximum pension benefits in the economy after the reform and \( \varphi \geq 0 \) is a parameter. Note that there are several special cases: when \( \varphi = 0 \) the government closes the pension program, and when \( \varphi = 1 \) it is the benchmark economy. In our experiments, setting \( \varphi < 1 \) implies a lower maximum pension benefit than in the benchmark economy, while \( \varphi > 1 \) implies a higher maximum pension benefit. Any financial discrepancy between the government’s consolidated tax revenues and expenditures are financed by a higher or lower income rate

We report the main aggregate and welfare effects of these experiments in Table 3. The first column specifies the maximum pension benefits relative to the maximum pension in the benchmark economy. Note that we normalized capital, labor, output (but not expected utility) in the benchmark model (\( \varphi = 1 \)) to 100 and so the entries in the table show these variables relative to 100 for the benchmark model. We format the benchmark values in italics in Table 3.

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>Capital</th>
<th>Labor</th>
<th>Output</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>351.3</td>
<td>112.6</td>
<td>177.5</td>
<td>-0.3666</td>
</tr>
<tr>
<td>0.2</td>
<td>314.1</td>
<td>111.6</td>
<td>168.8</td>
<td>-0.3837</td>
</tr>
<tr>
<td>0.4</td>
<td>250.1</td>
<td>109.3</td>
<td>152.2</td>
<td>-0.4086</td>
</tr>
<tr>
<td>0.6</td>
<td>187.9</td>
<td>106.2</td>
<td>133.4</td>
<td>-0.4346</td>
</tr>
<tr>
<td>0.8</td>
<td>136.2</td>
<td>102.8</td>
<td>115.1</td>
<td>-0.4617</td>
</tr>
<tr>
<td>1.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>-0.4867</td>
</tr>
<tr>
<td>1.25</td>
<td>69.3</td>
<td>96.5</td>
<td>84.5</td>
<td>-0.5193</td>
</tr>
<tr>
<td>1.5</td>
<td>47.6</td>
<td>93.4</td>
<td>71.3</td>
<td>-0.5602</td>
</tr>
</tbody>
</table>

Table 3: Aggregate effects when adjusting maximum pension benefits. Note that we keep the taper rate unchanged at the benchmark level (0.4). The expected utility is calculated for a new born agent at age 20. The values of capital, labor and output in the benchmark model are normalized to 100 and so the entries in the table show these variables relative to 100 for the benchmark model. We format the benchmark values in italics.

In all the experiments reported in Table 3, we consistently find that capital stock, labor supply and output monotonically increase as the government decreases the generosity of pension benefits. This indicates that public pension programs result in adverse effects on individuals’ incentives to save and work, thus crowding out savings, labor supply and output. Conversely, cutting the generosity of a public pension program improves efficiency and hence income. We also run the extreme experiment in which the government closes down the public pension program (\( \varphi = 0 \)), shown by the bolded row in Table 3. We find that when the public pension
program is completely removed ($\varphi = 0$), efficiency gains from completely removing economic distortions of public pensions on savings and labor supply lead to the highest attainable income. These large crowding out effects on savings found in our experiments are primarily due to our small open economy model assumption. Since the domestic interest rate is equal to the world interest rate, which is assumed constant, general equilibrium interest rate adjustments are removed.

We now turn our attention to the welfare effects. As established in the previous literature, a social security system is often justified as a mechanism for sharing longevity and income risks (social insurance) across households and generations, which potentially improves welfare when markets imperfections are present. On other hand, however, social security systems are often criticized as being detrimental to capital accumulation, labor supply and growth because they distort savings and labor supply decisions (through adverse incentives), resulting in efficiency and welfare losses. The welfare outcomes of a social security system depends how the system trades off the insurance effect against the incentive effect.

In our quantitative experimental results reported in column 5 of Table 3, we find that decreasing the generosity of pension benefits (reducing $\varphi$) always leads to increases in the expected utilities of individuals so that expected utility is maximized when the public pension ceases ($\varphi = 0$). This indicates that the adverse effects on incentives always dominate the insurance effect even when means testing is present. It seems that means testing strengthens risk-sharing and incentives via extensive margin effects, but fails to overturn the negative intensive margin effects.

We conclude that a means tested pension is not socially desirable in our dynamic general equilibrium economy as expected utility is highest in an economy with no public pension. This is perhaps not surprising as we learnt from previous studies that general equilibrium adjustments magnify the crowding out effects of social security systems without means testing and that negative welfare outcomes are likely. Indeed, the PAYG social security literature using a dynamic general equilibrium model consistently finds negative welfare effects because the adverse effects on incentives dominate the insurance effect (Auerbach and Kotlikoff (1987) and Imrohoroglu et al (1995)), leading to the recommendation that governments privatize their PAYG social security systems. In that sense, our finding for an old-age pension scheme with means testing is consistent with the previous results in the literature of general equilibrium analysis of social security without means testing.

The effects of taper rates. We now consider the implications of alterations in the taper rate for the income test, keeping the maximum pension level unchanged. We start our analysis with the benchmark economy and vary the taper rate, $\omega_y$, over the interval between 0 and 1. Any financial discrepancy between the government’s consolidated tax revenues and expenditures are financed by a higher or lower income tax rate. Specifically, our experiments include two special cases. When the taper rate is nil, $\omega_y = 0$, the government provides a universal pension. On other hand, when the taper rate is unity, $\omega_y = 1$, the government imposes a 100 percent tax rate on pensioners’ incomes above the income threshold - any extra
income obtained is taxed so there is no incentive to earn extra income from working more or to have extra interest income.

As already argued, the introduction of a taper rate to the pension design results in two opposing effects. First, since the resulting means test targets lower income agents (extensive margin), it mitigates self-insurance disincentives, lowers the deadweight loss of tax financing, and strengthens intra- and inter-generational risk-sharing. Second, it creates economic distortions as it imposes a higher implicit income tax (by the amount of the taper rate) on savings and labor incomes of pensioners. When the former effect is dominant, the welfare effects are positive; otherwise, the welfare effects will be negative. In this experiment, we examine how these two effects interplay. Note that in these experiments we only focus on the effects triggered by taper rates as we keep the maximum pension level unchanged.

We report the results of these experiments in Table 4. Column 1 specifies the various values of the income taper rate. Columns 2 to 5 present the values of aggregate variables including capital, labor, output and expected utility. Again, we normalized the values of aggregate variables in the benchmark economy \((\omega_y = 0.4)\) to 100, which are shown in italics in row 5, and report those in alternative economies relative to the benchmark.

<table>
<thead>
<tr>
<th>Taper rates ((\omega_y))</th>
<th>Capital</th>
<th>Labor</th>
<th>Output</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>92.3</td>
<td>99.6</td>
<td>96.6</td>
<td>-0.4867</td>
</tr>
<tr>
<td>0.1</td>
<td>95.9</td>
<td>99.7</td>
<td>98.1</td>
<td>-0.4826</td>
</tr>
<tr>
<td>0.2</td>
<td>98.2</td>
<td>99.7</td>
<td>99.1</td>
<td>-0.4802</td>
</tr>
<tr>
<td><strong>0.3</strong></td>
<td>99.9</td>
<td>100.04</td>
<td>99.99</td>
<td><strong>-0.4785</strong></td>
</tr>
<tr>
<td>0.4</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>-0.4787</td>
</tr>
<tr>
<td>0.5</td>
<td>99.3</td>
<td>99.92</td>
<td>99.7</td>
<td>-0.4797</td>
</tr>
<tr>
<td>0.6</td>
<td>97.5</td>
<td>99.8</td>
<td>98.9</td>
<td>-0.4820</td>
</tr>
<tr>
<td>0.7</td>
<td>96.8</td>
<td>99.8</td>
<td>98.6</td>
<td>-0.4829</td>
</tr>
<tr>
<td>0.8</td>
<td>96.1</td>
<td>99.81</td>
<td>98.2</td>
<td>-0.4840</td>
</tr>
<tr>
<td>0.9</td>
<td>95.2</td>
<td>9.82</td>
<td>97.9</td>
<td>-0.4851</td>
</tr>
<tr>
<td>1.0</td>
<td>94.6</td>
<td>99.8</td>
<td>97.7</td>
<td>-0.4860</td>
</tr>
</tbody>
</table>

Table 4: Aggregate effects when adjusting the taper rate. Note that we keep the maximum pension benefits unchanged at the benchmark level. The expected utility is calculated for a new born agent at age 20. The values of capital, labor and output in the benchmark model are normalized to 100 and so the entries in the table show these variables relative to 100 for the benchmark model. We format the benchmark values in italics.

First, we analyze whether the current means tested pension system in Australia would deliver a more favourable outcome than a universal pension system like the one in the U.S. We compare row 2 \((\omega_y = 0)\) and row 6 \((\omega_y = 4)\) and find that removing the income test results in a lower capital stock and labor supply, causing output to drop by 3.5 percent. We also find that expected utility is lower in the economy with a universal pension system than in the benchmark economy, meaning that newly born agents would prefer to live in the benchmark economy. We conclude that the means tested pension system in the benchmark economy is socially preferred to the universal pension system.
Second, we consider a wider range of alternative means test parameters and find that changing taper rates results in non-linear effects on individuals’ savings and labor supply behavior and macroeconomic aggregates. When the government raises the taper rate from 0.4 to 1, there is a decrease in the capital stock and labor supply. This suggests that the economy is in a region in which the adverse effects of the taper rate as an implicit tax dominate the effects of the taper rate via the extensive margin. Raising the taper rate therefore discourages individuals from saving more or working longer, as they face a higher effective marginal income tax rate on labour and interest earnings at higher ages. The increase in the taper rate therefore induces individuals to save more or work extra hours. This result is consistent with empirical evidence documented in previous empirical studies, such as Neumark and Powers (1998), Neumark and Powers (2000), Disney and Smith (2002) and Friedberg (2000). This result is also consistent with Selton, van de Ven and Weale (2008), who analyze a calibrated multi-period overlapping generations model of the U.K. However, when the government raises the taper rate from 0.0 to 0.4, there is an increase in capital stock and labor supply. This implies that the positive effect of taper rates via the extensive margin dominates the negative effects resulting from higher implicit tax rates. The hump-shaped pattern of capital stock and labor supply over a range of parameter values of taper rates could be explained by the dynamic interaction between the extensive and intensive margins described in our analytical results in section 2 further above.

Third, we find that the welfare effects have a hump-shaped pattern, a result consistent with that found for our numerical example of the simple mode in section 2. Starting from the benchmark taper rate, \( \omega_y = 0.4 \), the expected utility for a household decreases as taper rates are increased, implying that the adverse incentive effects of the more stringent income test dominant the insurance effects in this policy parameter range. On other hand, however, we find the opposite outcome as the taper rate is reduced from 0.4 to 0. Looking at the whole range for the taper rate, we observe that the introduction of, and increase in, a small taper rate at first improves expected utility for the household, reaches a maximum, and then decreases welfare at higher taper rates.

This non-linear pattern of welfare effects of changes in the income taper rate clearly indicates a trade off between the insurance and incentive aspects of means testing. When the economy is in a region where the insurance effects are dominant, increases in the taper rate induce more self-insurance by working longer hours and increasing saving, which, in turn, lead to efficiency gains and a positive welfare outcome. However, when the taper rate becomes bigger, distortions arising from having higher effective marginal tax rates become more severe, which, in turn, reduce savings and labor supply. Aggregate capital, labor supply and income decrease and welfare subsequently decreases.

The point at which expected utility reaches a maximum is around \( \omega_y = 0.3 \). This indicates that the introduction of means testing (via a taper rate) is socially desirable in our model, conditioning the pre-existence of a pension system with the benchmark level of the maximum pension benefit. Sefton and Ven (2009) conduct a welfare analysis in a partial equilibrium model of the U.K. pension system and also find that means tested pensions are socially desirable.
analysis of the Australian pension system in a general equilibrium framework also reaches a similar conclusion. This suggests that the conclusions of Sefton and Ven obtained with a partial equilibrium model might well be confirmed when accounting for dynamic general equilibrium adjustments.

**Interactions between maximum pension benefits and taper rate.** We now turn our attention to interactions between maximum pension benefits and taper rate, and derive implications for the insurance-incentive trade off and welfare. We numerically characterize two steady states economies with two different levels of the maximum pension benefits: low $\varphi = .5$ and high $\varphi = 1.5$. In each alternative economy, the government keep the maximum pension benefits unchanged and the government varies the taper rate between 0 and 1.

We report the effects of alternative taper rates and maximum pension benefits in the design of means testing of the old-age pension on the aggregate capital stock and labor supply in Table 5.

<table>
<thead>
<tr>
<th>Taper rate</th>
<th>Capital</th>
<th>Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>Benchmark</td>
</tr>
<tr>
<td>0.0</td>
<td>1.3490</td>
<td>.7842</td>
</tr>
<tr>
<td>0.1</td>
<td>1.4605</td>
<td>.8148</td>
</tr>
<tr>
<td>0.2</td>
<td>1.5315</td>
<td>.8346</td>
</tr>
<tr>
<td>0.3</td>
<td>1.5760</td>
<td>.8491</td>
</tr>
<tr>
<td>0.4</td>
<td>1.5974</td>
<td>.8499</td>
</tr>
<tr>
<td>0.5</td>
<td>1.6125</td>
<td>.8443</td>
</tr>
<tr>
<td>0.6</td>
<td>1.6161</td>
<td>.8284</td>
</tr>
<tr>
<td>0.7</td>
<td>1.6257</td>
<td>.8230</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6309</td>
<td>.8165</td>
</tr>
<tr>
<td>0.9</td>
<td>1.6371</td>
<td>.8093</td>
</tr>
<tr>
<td>1.0</td>
<td>1.6405</td>
<td>.8039</td>
</tr>
</tbody>
</table>

Table 5: Aggregate capital stocks and labor when adjusting taper rates in three different economies: low, benchmark and high maximum pension benefits.

We find that the effects of changes in the taper rates on the aggregate capital stock and labor supply vary significantly across the economies. In the economy where the level of maximum pension benefits is relatively low, the taper rate that maximizes the capital stock and labor supply is 1, which is much higher than in the benchmark economy. On other hand, in the economy where the level of maximum pension benefits is relatively high, the taper rate that maximize the levels of aggregate capital and labor is around 0.3. This indicates that the effects of means testing on incentives to work and to save are dependent of the levels of maximum pension benefits. When the levels of maximum pension benefits are relatively low, tightening the taper rate leads to an increase in the capital stock and labor supply. The intuition for this result can be explained by the prediction in our simple model. That is, when the pension benefits $P_{\text{max}}$ are relatively less generous the positive extensive margin effect is positive and always dominates the negative intensive margin effects. On other hand, in the economy where the levels of maximum pension benefits are relatively generous (benchmark or high) there is
a trade off between two opposing forces. The positive extensive margin effect tends to be a
dominant force when the rates are small, but loses ground to the negative intensive margin
effects as the taper rate becomes sufficiently high (0.4 or above in the benchmark economy).
This result confirms that the existence of the extensive margin embedded in a means tested
pension system potentially mitigates the adverse intensive margin effects on savings.

We now analyze the welfare outcome in which the interactions between the insurance and
incentive effects are taken into account. Our results for the effect of these different policy
settings upon expected utility are summarized in Table 6.

<table>
<thead>
<tr>
<th>Taper rate</th>
<th>Low (50%)</th>
<th>Benchmark (100%)</th>
<th>High (150%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.4086</td>
<td>-0.4867</td>
<td>-0.5602</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.4012</td>
<td>-0.4826</td>
<td>-0.5580</td>
</tr>
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<td>0.2</td>
<td>-0.3969</td>
<td>-0.4802</td>
<td>-0.5577</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.3947</td>
<td>-0.4785</td>
<td>-0.5581</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.3940</td>
<td>-0.4787</td>
<td>-0.5593</td>
</tr>
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<td>0.5</td>
<td>-0.3935</td>
<td>-0.4797</td>
<td>-0.5607</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.3927</td>
<td>-0.4820</td>
<td>-0.5631</td>
</tr>
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<td>-0.3917</td>
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<td>-0.5669</td>
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<td>1.0</td>
<td>-0.3900</td>
<td>-0.4860</td>
<td>-0.5727</td>
</tr>
</tbody>
</table>

Table 6: The effects on expected utilities when adjusting taper rates in three different eco-
nomies: low, benchmark and high maximum pension benefits.

We find that the welfare effects of varying the taper rate are different across the three
economies and, hence, dependent upon the levels of the maximum pension benefit. In the first
economy where the maximum pension benefits are relatively less generous (Low), increases in
the taper rate lead to monotone increases in capital stock, labor supply, national income and,
therefore, expected utility. This implies that the effects of higher taper rates in mitigating self-
insurance disincentives and strengthening risk-sharing are always dominant so that the welfare
effects are always positive. The optimal taper rate in this economy is $\omega_y = 1$. There is no
clear trade off between insurance and incentive effects as the taper rate increases. However, as
pointed in the previous analysis, the positive extensive margin effects tend to be a dominant
force.

In the third economy where the maximum pension benefits are assumed to be 150% more
generous than in the benchmark economy (High), we again find a hump-shaped pattern of wel-
fare effects. This is indicative of the two opposing effects of means testing at work: mitigating self-insurance disincentives and strengthening risk-sharing versus distortions of higher effective
marginal income tax rates of the higher taper rate. When the former is dominant the welfare
effects are positive; otherwise, they are negative. The insurance and incentive effects are evenly
balanced around $\varphi = 0.2$, which is the optimal taper rate in this economy. Note that the taper
rate that delivers the best welfare outcome is not necessarily the one that results in highest
levels of capital stock, labor supply and output. The difference is partly due to the fact that means testing strengthens the social insurance role of the pension system.

To enable a more detailed examination of the welfare and macroeconomic implications of alternative pension design parameters, we simulate a number of alternative economies for a wider range of maximum pension benefits. We summarize the results of these policy experiments on aggregate variables in Tables 20, 21, 22 and 23. Table 23 shows that the level of expected utility is greatest when the taper rate is unity for old-age pension replacement rates up to 0.6, indicating that it is optimal for pensioners to only receive the pension for incomes less than the income threshold. The optimal taper rate is 0.5 when the replacement rate is 0.8, drops to 0.3 for replacement rates of unity and 1.25, and further to 0.2 when the replacement rate is 1.5. Thus, the optimal taper rate falls as the pension becomes more generous. Overall, we find from these tables that the interaction between the maximum pension benefit and the taper rate magnifies the disincentive effects of the taper rate as an implicit tax on life-cycle savings and labor supply.

In summary, our results point out the importance of accounting for the interaction between these two pension policy instruments and of analyzing the economic mechanisms that explain these nonlinear effects. Our results point to a conclusion that the welfare effect of introducing and increasing an income test taper rate is nonlinear and dependent of the level of the maximum pension benefit. The interaction between these two policy variables is important as it has different implications for individuals’ inter-temporal allocation of resources, macroeconomic aggregates and welfare.

6 Conclusion

Inclusion of means testing into the pension benefit formula allows governments to have additional policy instruments to affect the number of public pensioners (extensive margin) and the benefit level (intensive margin). The former is aimed at strengthening risk sharing across individuals and generations and to mitigate the adverse effects of self-insurance incentives. In this paper, we analyzed the welfare implications of these salient features of old-age pension design for the trade off between insurance and incentive effects. We find that the extensive margin strengthens the insurance effect but introduces two opposing effects on incentives, and that the magnitude of the positive extensive margin effect depends on relative strength of the intensive margin. The final welfare outcome depends how two opposing effects on incentives play out in the economy.

We investigate these trade-offs in a dynamic general equilibrium model with heterogeneous agents that is calibrated to the Australian economy. We find that the introduction of a taper rate leads to positive welfare outcomes and that the pattern of welfare effects varies, depending on the level of maximum pension benefits. More specifically, when the maximum pension benefit is relatively less generous, increases in taper rates always leads to a welfare gain as the insurance effect together with the positive incentive effect are always dominant. However, when
the maximum pension benefits are relatively more generous, there is an optimal taper rate at which the insurance and positive incentive effects efficiently trade off with the negative incentive effects and at which expected utility is maximized. Importantly, our results reveal that the interactions between the levels of maximum pension benefits and taper rates are critical in forming the direction of the welfare effects.

Our results carry important policy implications. Countries that are interested in introducing means testing to their currently universal pension systems should take into account the potential interactions between the choice of taper rates and the choice of the levels of maximum pension benefit. Our results highlight the point that the effects of a higher taper rate on savings, labor supply and household welfare are nonlinearly dependent on the level of the maximum pension benefit.

References


### 7 Appendix

#### 7.1 Simple model: Solving the model

We provide a solution for a model in which savings is incorporated in the income test formula and the government finances its pension program via a tax on the labor income of the young.

**Household.** The individual agent’s optimization problem is

\[
\max_{c_1, c_2, s} \{ u(c_1) + pE u(c_2) \text{ st. } c_1 + s = (1 - \tau) w_1 \text{ and } c_2 = w_2 + (1 + r) s + P \},
\]

where \( P \) is the pension benefit defined as

\[
P = \begin{cases} 
  P^{\text{max}} - \omega [w_2 + rs] & \text{if } w_2 + rs < y_2 \\
  0 & \text{if } w_2 + rs \geq y_2.
\end{cases}
\]

Let \( y_2 = w_2 + rs \) be testable income and follows an uniform distribution. Assuming that \( u(c) = -\frac{c^2}{2} + \chi c \) is the functional form for individual preferences, the individual’s first order necessary condition for optimality is

\[-c_1 + \chi = pE \left[ u'(c_2) \frac{\partial c_2}{\partial s} \right],\]

where

\[
E \left[ (c_2)^{-\sigma} \frac{\partial c_2}{\partial s} \right] = \int_{y_2^{\text{min}}}^{y_2^{\text{max}}} (-c_2 + \chi) \left( \frac{\partial c_2}{\partial s} \right) f(y_2) dy_2,
\]

\[
= \chi - \int_{y_2^{\text{min}}}^{y_2^{\text{max}}} c_2 \left( \frac{\partial c_2}{\partial s} \right) f(y_2) dy_2;
\]

\[
f(y_2) = \frac{1}{w_2^{\text{max}}}: \text{uniform } \sim [y_2^{\text{min}} = rs, y_2^{\text{max}} = rs + w_2^{\text{max}}].
\]

The individual’s consumption in period 2 is

\[
c_2 = \begin{cases} 
  (1 - \omega) w_2 + [1 + (1 - \omega) r] s + P^{\text{max}} & \text{if } P > 0 \\
  w_2 + (1 + r) s & \text{if } P = 0,
\end{cases}
\]
and the first derivative with respect to saving is

\[
\frac{\partial c_2}{\partial s} = \begin{cases} 
1 + (1 - \omega) r & \text{if } P > 0 \\
(1 + r) & \text{if } P = 0.
\end{cases}
\]

Using this expression for consumption when old, expected marginal utility may be expressed as

\[
E \left[ (c_2)^{-\sigma} \frac{\partial c_2}{\partial s} \right] = \chi - [1 + (1 - \omega) r] p \int_{\frac{y_2}{y_2}}^{\frac{y_2}{y_2}} [(1 - \omega) y_2 + s + P_{\text{max}}] f(y_2) dy_2
\]

\[- (1 + r) p \int_{\frac{y_2}{y_2}}^{\frac{y_2}{y_2}} [y_2 + s] f(y_2) dy_2.
\]

The individual’s first order necessary condition becomes

\[(1 - \tau) w_1 - s = \begin{cases} 
[1 + (1 - \omega) r] p \left\{ \int_{\frac{y_2}{y_2}}^{\frac{y_2}{y_2}} [(1 - \omega) y_2 + s + P_{\text{max}}] \frac{1}{w_2^2} dy_2 \right\} + (1 + r) p \left\{ \int_{\frac{y_2}{y_2}}^{\frac{y_2}{y_2}} [y_2 + s] \frac{1}{w_2^2} dy_2 \right\}.
\end{cases}
\]

Let \( \tilde{w}_2 = \frac{y_2}{w_2} - rs \) denote the level of income endowment in period 2 that separates pensioners from non-pensioners, taking saving, \( s \), as given. Noting that \( dy_2 = dw_2 \), we obtain the expression

\[(1 - \tau) w_1 - s = \begin{cases} 
\frac{[1 + (1 - \omega) r] p}{w_2^2} \left\{ \int_{\frac{y_2}{w_2}}^{\frac{y_2}{w_2}} [(1 - \omega) y_2 + s + P_{\text{max}}] dw_2 \right\} + (1 + r) p \left\{ \int_{\frac{y_2}{w_2}}^{\frac{y_2}{w_2}} [y_2 + s + P_{\text{max}}] dw_2 \right\}.
\end{cases}
\]

This equation may be solved for the optimal level of saving function, yielding the implicit
expression

\[
s = \frac{(1 - \tau) w_1 - \frac{[1 + (1 - \omega) r] p}{w_2^\max} \left( (1 - \omega) \left( \frac{\hat{w}_2}{2} \right)^2 + P^\max \hat{w}_2 \right) - (1 + r) p \left( \frac{w_2^\max + \hat{w}_2}{w_2^\max} \right)^2 - \left( \frac{\hat{w}_2}{w_2^\max} \right)^2}{1 + [1 + (1 - \omega) r] p \frac{\hat{w}_2}{w_2^\max} + (1 + r)^2 p \frac{\hat{w}_2}{w_2^\max}},
\]

\[
= \frac{(1 - \tau) w_1 - [1 + (1 - \omega) r] p \left( \frac{\hat{w}_2}{2} + P^\max - \omega \frac{\hat{w}_2}{2} \right) \left( \frac{\hat{w}_2}{w_2^\max} \right) - (1 + r) p \left( \frac{w_2^\max + \hat{w}_2}{w_2^\max} \right) \left( 1 - \frac{\hat{w}_2}{w_2^\max} \right)}{1 + [1 + (1 - \omega) r] p \frac{\hat{w}_2}{w_2^\max} + (1 + r)^2 p \left( 1 - \frac{\hat{w}_2}{w_2^\max} \right)}
\]

where \( \hat{w}_2 = \bar{y}_2 - rs \).

**Government.** The government budget clearing condition is

\[
\frac{w_1}{p} \tau = \int_{y_2^\min}^{\bar{y}_2} Pf(y_2) dy_2,
\]

\[
= \int_{y_2^\min}^{\bar{y}_2} (P^\max - \omega y_2) f(y_2) dy_2,
\]

\[
= \int_{0}^{\hat{w}_2} (P^\max - \omega (w_2 + rs)) f(w_2) dw_2,
\]

\[
= \frac{1}{w_2^\max} \left( P^\max w_2 - \omega \left( \frac{w_2}{2} + rs \right) w_2 \right) |_{w_2^\min}^{\hat{w}_2},
\]

\[
\frac{w_1}{p} \tau = \frac{\hat{w}_2}{w_2^\max} \left( \frac{P^\max - \omega \left( \frac{\hat{w}_2}{2} + rs \right) w_2}{w_2^\max} \right)
\]

where \( s \) is optimal saving and \( \hat{w}_2 = \bar{y}_2 - rs \).

**Equilibrium.** The equilibrium conditions for this simple economy reduce to

\[
s^* = \frac{(1 - \tau^*) w_1 - [1 + (1 - \omega) r] p \left( \frac{\hat{w}_2^*}{2} + P^\max - \omega \frac{\hat{w}_2^*}{2} \right) \left( \frac{\hat{w}_2^*}{w_2^\max} \right) - (1 + r) p \left( \frac{w_2^\max + \hat{w}_2^*}{w_2^\max} \right) \left( 1 - \frac{\hat{w}_2^*}{w_2^\max} \right)}{1 + [1 + (1 - \omega) r] p \frac{\hat{w}_2^*}{w_2^\max} + (1 + r)^2 p \left( 1 - \frac{\hat{w}_2^*}{w_2^\max} \right)}
\]

\[
\tau^* = \frac{p}{w_1} \left[ \frac{\hat{w}_2^*}{w_2^\max} \left( P^\max - \omega \left( \frac{\hat{w}_2^*}{2} + rs^* \right) \right) \right],
\]

\[
\hat{w}_2^* = \frac{\hat{y}_2 - rs^*}{2}
\]

These equilibrium conditions simultaneously determine the solutions for \((s^*, \tau^*, \hat{w}_2^*)\). The first is the optimal saving function. The second equation determines the tax rate, \( \tau^* \), that ensures a government budget balance. The final equation determines the period 2 (extensive margin) wage rate, \( \hat{w}_2^* \), that separates pensioners from non-pensioners. Note that \( P^\max \), \( \omega \) and \( \bar{y}_2 \) are exogenously set by the government.
7.2 Simple model: A numerical example

Suppose the function form for individual preferences is \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \). The equilibrium is characterized by the household optimal savings and the government budget clearing condition and mean testing rule as

\[
f(s^*) = p(1 + (1 - \omega)r) \int \left( (c_2^i)^{-\sigma} f(w_2^i) \, dw_2^i \right) - \left( (1 - \tau^{ss}) w_1 - s \right)^{-\sigma} = 0, \quad (a)
\]

\[
\tau^{ss} = \frac{p \left( P_{\text{max}} - \omega \int_0^{w_1} w_2^i f(w_2^i) \, dw_2^i \right)}{w_1}, \quad (b)
\]

\[
\bar{w}_2(\tau^{ss}, P_{\text{max}}, \omega) = \frac{P_{\text{max}} - \omega s^*}{\omega}. \quad (c)
\]

We use the following algorithm to solve the model numerically:

1. Set the exogenous interest rate, \( r \), and endowment income when young, \( w_1 \), and draw 10,000 observations of \( w_2^i \) from a lognormal distribution, \( f(w_2^i) \).
2. Specify a policy choice \( (P_{\text{max}}, \omega) \).
3. Guess the tax rate, \( \tau^{ss} \).
4. Find \( s^* \) that solves the household problem and consumption allocation given by (a).
5. Find the threshold \( \bar{w}_2(\tau^{ss}, P_{\text{max}}, \omega) \) using (c).
6. Find new tax rate \( \tau^{ss} \) using (b).
7. Using the updated \( \tau^{ss} \), repeat steps 4-6 until the government budget constraint (b) clears.
8. Calculate the social welfare corresponding to the policy choice \( (P_{\text{max}}, \omega) \) specified at step 2.
9. Specify a new policy choice \( (P_{\text{max}}, \omega) \) and repeat steps 3-8 to get the complete set of solutions.

7.3 General equilibrium model: fiscal policy

Means tested pension. The Australian government runs a means tested old-age pension program. The maximum pension is set at \( P_{\text{max}} = $13,314.60 \) in 2007, which is technically is calculated by the formula \( P_{\text{max}} = 0.25 \times MTAWE \), where \( MTAWE \) is the Male Total Average Weekly Earnings. We assume that is \( MTAWE \) is the average labor income \( \bar{y} \) and the replacement rate \( \Psi = 0.25 \). In our benchmark model, the maximum pension is defined by \( P_{\text{max}} = 0.25 \bar{y} \). In 2007-8 the income test threshold is set at $3328 and incomes over these
amounts reduce pension by $0.4 for every $1. We therefore choose $\tilde{y}_1 = $3328 and $\omega_y = 0.4$. The pension benefit using the income test is given by

$$P^y(y_j) = \begin{cases} 
13,314.6 & \text{if } j \geq 60 \text{ and } y_j \leq \tilde{y}_1 = 3328, \\
\max[0, (13314.6 - 0.4(y_j - \tilde{y}_1))] & \text{if } j \geq 60 \text{ and } y_j > \tilde{y}_1 = 3328.
\end{cases}$$

There are two separate asset tests for renters and homeowners in Australia. For renters, the asset test threshold was $171,750 in 2007. For homeowners, residential assets are excluded from the assets test and so the lower bound threshold for the asset test for homeowners is set higher at $296,250. Assets above the asset test threshold reduce the old-age pension by $1.5 per fortnight for every $1000 above the limit, which implies a taper rate for the asset test of $\omega_a = 1.5/1000 = 0.0015$. In our model, there is no difference between residential and non-residential assets, so we are not able to use the statutory asset test threshold directly. Instead, we choose $\bar{a}_1$ to match the observed fraction of pensioners at age 65 years. The pension benefit using the asset test is given by

$$P^a(a_j) = \begin{cases} 
13314.6 & \text{if } j \geq 65 \text{ and } a_j \leq \bar{a}_1, \\
\max[0, (13134.6 - 0.0015(a_j - \bar{a}_1))] & \text{if } j \geq 65 \text{ and } a_j > \bar{a}_1.
\end{cases}$$

The government collects tax from consumption and income to cover spending on pensions and other government spending programs. The consumption tax rate is set at the statutory 10 percent.

**Income tax function.** The Australian income tax schedule is progressive. We use the tax schedules for 2007-8 in the benchmark model so that the tax function is given by

$$T(y) = \begin{cases} 
0 & \text{if } y < 6,000, \\
0.15(y - 6,000) & \text{if } 6,000 < y \leq 25,000, \\
3,600 + 0.3(y - 30,000) & \text{if } 25,000 < y \leq 75,000, \\
17,100 + 0.4(y - 75,000) & \text{if } 75,000 < y \leq 150,000, \\
47,100 + 0.45(y - 150,000) & \text{if } y > 150,000,
\end{cases}$$

where $y$ is taxable income.

**Senior Tax offset.** The maximum amount of senior tax offset is $2230 and for every income dollar above the income limit of $24,876 the tax offset reduces by 12.5 cents so we set $SATO_{\text{max}} = $2,230, $\bar{y}_{\text{SATO}} = $24,876 and $\omega_{\text{SATO}} = 0.125$. The senior tax offset function is given by

$$SATO(y) = \begin{cases} 
2230 & \text{if } j \geq J_1 \text{ and } y \leq \bar{y}_{\text{SATO}} = 24,876, \\
\max[0, 2230 - 0.125(y - \bar{y}_{\text{SATO}})] & \text{if } j \geq J_1 \text{ and } y > \bar{y}_{\text{SATO}} = 24,876.
\end{cases}$$

for households of pensionable age $j \geq J_1$ and zero otherwise.
7.4 General equilibrium model: Algorithm to solve the model

We follow the algorithm in Auerbach and Kotlikoff (1987) to solve the model. The general procedure to solve for general equilibrium is summarized as follows:

1. Discretize the state space of assets as \([a_0, ..., a_{max}]\).

2. Guess an initial wage rate, \(w\), and endogenous government policy variables while taking the world interest rate as given.

3. Work backwards from period \(J\) to period 1 to obtain decision rules for consumption, savings, labor supply, and the value and marginal value functions of the household.

4. Iterate forwards to obtain the measure of households across states, using the household decision rules and the laws of motion for working ability shocks and mortality shocks and taking the distribution of agents of age 1 as given.

5. Aggregate labor supply and clear the labor market to get a new wage rate; balance the government budget to determine endogenous government variables.

6. Check the relative change in aggregate variables after each iteration and stop the algorithm when the change is sufficiently small (10^{-4} percent). Otherwise, repeat steps from 3 to 6.

7.5 General equilibrium model: Estimation of the lifecycle profiles of labor efficiency

We use data drawn from the Household, Income and Labour Dynamics in Australia (HILDA) longitudinal survey (see Wooden and Watson (2002) and Watson and Wooden (2010) for more details) to estimate labor productivities and other key life cycle profiles for our model calibration. HILDA is an Australian household panel survey with focus on families, income, employment and well-being, and contains detailed information on individuals' current labour market activity including labour force status, earnings and hours worked, and employment and unemployment histories. We use data from the first 7 waves of HILDA surveys in this paper.

**Estimation of labor efficiency.** We follow the approach in Nishiyama and Smetters (2007) to approximate the dynamics of labor productivities of Australians over the life cycle. We define working ability/labor productivity as the hourly average wage rate, defined as gross labor income divided by total hours worked. We estimate age-dependent hourly wage rates using data drawn from the HILDA longitudinal household survey. We group individuals from age of 20 to 90 into 14 cohorts of five-year age categories, (For example, all individuals of age between 20 and 24 are in age category 1). We then classify individuals in each of these 14 age groups into 5 quintiles of the common hourly wage rate. We assume that individuals in each age-quintile group have a common working ability, so there are 5 discrete levels of working
abilities in each age cohort. To measure these discrete levels of working ability we use the average hourly wage rate, conditioning on quintile and age, defined by

$$e^i_j = \frac{\sum_{i=1}^{N^j} \text{Hourly wage rate}^i_j}{N^j},$$

where $i$ and $j$ denote the wage rate quintile and age cohort, respectively, $e^i_j$ is the level of working ability/labor productivity, and $N_j$ is the total number of individuals of age $j$. We make an assumption that labor efficiencies from 65 decline at a constant rate.

Table 7 reports the resulting labor efficiencies by age groups for 5 labor productivity quintiles.

**Transition matrix.** In our model, mobility of individuals across different quintiles of labor efficiency from one age to the next is captured by Markov transition matrices. The calculation of these transition matrices recognizes that we have five-year age cohorts in our model and is therefore based on two data points with time gap of 5 years. For example, to calculate the transition probabilities of labor efficiencies of individuals between the age of 20 and 24 and the age of 25 and 29 we need a panel data with 5 year lag. We use the data from waves 1 and 6 and the data from waves 2 and 7 of HILDA to estimate age-specific labor efficiencies and the transition matrices, and use the average these Markov matrix estimates. We also assume that the transition matrix from the age of 55 − 59 to the age of 60 − 64 to characterize the dynamics of labor efficiencies from age 65 onwards.

To estimate the transition matrices, we record individuals’s efficiency quintiles over age cohorts and count how many of these individuals stay in the same quintile and how many move to different quintiles. The transition probability from labor productivity level (quintile) $v$ at age $j$ to labor productivity level $k$ at age $j + 1$ is defined by

$$\pi_{j,j+1}(e^i_{j+1}|e^i_j = v) = \frac{n^{i=k}_{j+1}}{N^v_j} \text{ for } i = 1, \ldots, 5,$$

where $N^v_j = \sum_{k=1}^{5} n^{i=k}_{j+1}$ is the total number of individuals with working ability $i = v$ at age $j$, $n^{i=k}_{j+1}$ is the number if individuals in the pool $N^v_j$ with the working ability $i = k$ and age $j + 1$.

Tables 8 to 15 report the transition matrices for the various adjacent age cohorts.

**Labor Efficiencies and transition matrices.**
<table>
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<tr>
<th></th>
<th>$e^1$</th>
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<th>$e^3$</th>
<th>$e^4$</th>
<th>$e^5$</th>
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<tr>
<td>20-24</td>
<td>1.000</td>
<td>1.397</td>
<td>1.623</td>
<td>1.894</td>
<td>2.595</td>
</tr>
<tr>
<td>25-29</td>
<td>1.164</td>
<td>1.632</td>
<td>2.043</td>
<td>2.510</td>
<td>3.363</td>
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<td>30-34</td>
<td>1.189</td>
<td>1.693</td>
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<td>35-39</td>
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<td>1.678</td>
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<td>40-44</td>
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<td>70-74</td>
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<td>75-79</td>
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<td>85-90</td>
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<td>0.016</td>
<td>0.019</td>
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Table 7: Age specific labor efficiency profiles

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<tr>
<th></th>
<th>$e^1(j + 1)$</th>
<th>$e^2(j + 1)$</th>
<th>$e^3(j + 1)$</th>
<th>$e^4(j + 1)$</th>
<th>$e^5(j + 1)$</th>
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<tr>
<td>$e^1(j)$</td>
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<td>0.234</td>
<td>0.177</td>
<td>0.171</td>
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<tr>
<td>$e^2(j)$</td>
<td>0.199</td>
<td>0.288</td>
<td>0.192</td>
<td>0.176</td>
<td>0.144</td>
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<tr>
<td>$e^3(j)$</td>
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<td>0.249</td>
<td>0.236</td>
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<td>0.140</td>
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<tr>
<td>$e^4(j)$</td>
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<td>0.175</td>
<td>0.229</td>
<td>0.236</td>
<td>0.217</td>
</tr>
<tr>
<td>$e^5(j)$</td>
<td>0.108</td>
<td>0.121</td>
<td>0.166</td>
<td>0.223</td>
<td>0.382</td>
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Table 8: Markov Transition Matrix between 20-24 and 25-29

<table>
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<tr>
<th></th>
<th>$e^1(j + 1)$</th>
<th>$e^2(j + 1)$</th>
<th>$e^3(j + 1)$</th>
<th>$e^4(j + 1)$</th>
<th>$e^5(j + 1)$</th>
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<tr>
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<td>0.332</td>
<td>0.124</td>
<td>0.094</td>
<td>0.040</td>
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<td>$e^2(j)$</td>
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<td>0.284</td>
<td>0.244</td>
<td>0.159</td>
<td>0.070</td>
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<tr>
<td>$e^3(j)$</td>
<td>0.192</td>
<td>0.227</td>
<td>0.266</td>
<td>0.187</td>
<td>0.128</td>
</tr>
<tr>
<td>$e^4(j)$</td>
<td>0.065</td>
<td>0.120</td>
<td>0.255</td>
<td>0.320</td>
<td>0.240</td>
</tr>
<tr>
<td>$e^5(j)$</td>
<td>0.050</td>
<td>0.075</td>
<td>0.114</td>
<td>0.239</td>
<td>0.522</td>
</tr>
</tbody>
</table>

Table 9: Markov Transition Matrix between 25-29 and 30-34

<table>
<thead>
<tr>
<th></th>
<th>$e^1(j + 1)$</th>
<th>$e^2(j + 1)$</th>
<th>$e^3(j + 1)$</th>
<th>$e^4(j + 1)$</th>
<th>$e^5(j + 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^1(j)$</td>
<td>0.439</td>
<td>0.283</td>
<td>0.172</td>
<td>0.086</td>
<td>0.020</td>
</tr>
<tr>
<td>$e^2(j)$</td>
<td>0.305</td>
<td>0.317</td>
<td>0.177</td>
<td>0.132</td>
<td>0.070</td>
</tr>
<tr>
<td>$e^3(j)$</td>
<td>0.139</td>
<td>0.250</td>
<td>0.287</td>
<td>0.230</td>
<td>0.094</td>
</tr>
<tr>
<td>$e^4(j)$</td>
<td>0.066</td>
<td>0.103</td>
<td>0.251</td>
<td>0.346</td>
<td>0.235</td>
</tr>
<tr>
<td>$e^5(j)$</td>
<td>0.041</td>
<td>0.062</td>
<td>0.111</td>
<td>0.206</td>
<td>0.580</td>
</tr>
</tbody>
</table>

Table 10: Markov Transition Matrix between 30-34 and 35-39
<table>
<thead>
<tr>
<th>(e^1(j + 1))</th>
<th>(e^2(j + 1))</th>
<th>(e^3(j + 1))</th>
<th>(e^4(j + 1))</th>
<th>(e^5(j + 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^1(j))</td>
<td>0.485</td>
<td>0.271</td>
<td>0.143</td>
<td>0.060</td>
</tr>
<tr>
<td>(e^2(j))</td>
<td>0.260</td>
<td>0.355</td>
<td>0.238</td>
<td>0.102</td>
</tr>
<tr>
<td>(e^3(j))</td>
<td>0.132</td>
<td>0.192</td>
<td>0.275</td>
<td>0.291</td>
</tr>
<tr>
<td>(e^4(j))</td>
<td>0.083</td>
<td>0.113</td>
<td>0.226</td>
<td>0.336</td>
</tr>
<tr>
<td>(e^5(j))</td>
<td>0.042</td>
<td>0.068</td>
<td>0.117</td>
<td>0.211</td>
</tr>
</tbody>
</table>

Table 11: Markov Transition Matrix between 35-39 and 40-44

<table>
<thead>
<tr>
<th>(e^1(j + 1))</th>
<th>(e^2(j + 1))</th>
<th>(e^3(j + 1))</th>
<th>(e^4(j + 1))</th>
<th>(e^5(j + 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^1(j))</td>
<td>0.500</td>
<td>0.281</td>
<td>0.134</td>
<td>0.065</td>
</tr>
<tr>
<td>(e^2(j))</td>
<td>0.318</td>
<td>0.336</td>
<td>0.226</td>
<td>0.089</td>
</tr>
<tr>
<td>(e^3(j))</td>
<td>0.072</td>
<td>0.229</td>
<td>0.363</td>
<td>0.236</td>
</tr>
<tr>
<td>(e^4(j))</td>
<td>0.086</td>
<td>0.089</td>
<td>0.202</td>
<td>0.394</td>
</tr>
<tr>
<td>(e^5(j))</td>
<td>0.024</td>
<td>0.065</td>
<td>0.075</td>
<td>0.216</td>
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</table>

Table 12: Markov Transition Matrix between 40-44 and 45-49

<table>
<thead>
<tr>
<th>(e^1(j + 1))</th>
<th>(e^2(j + 1))</th>
<th>(e^3(j + 1))</th>
<th>(e^4(j + 1))</th>
<th>(e^5(j + 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^1(j))</td>
<td>0.496</td>
<td>0.261</td>
<td>0.147</td>
<td>0.050</td>
</tr>
<tr>
<td>(e^2(j))</td>
<td>0.287</td>
<td>0.384</td>
<td>0.190</td>
<td>0.084</td>
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<tr>
<td>(e^3(j))</td>
<td>0.145</td>
<td>0.230</td>
<td>0.387</td>
<td>0.179</td>
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<tr>
<td>(e^4(j))</td>
<td>0.034</td>
<td>0.059</td>
<td>0.215</td>
<td>0.439</td>
</tr>
<tr>
<td>(e^5(j))</td>
<td>0.038</td>
<td>0.068</td>
<td>0.059</td>
<td>0.250</td>
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Table 13: Markov Transition Matrix between 45-49 and 50-54

<table>
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<tr>
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<th>(e^2(j + 1))</th>
<th>(e^3(j + 1))</th>
<th>(e^4(j + 1))</th>
<th>(e^5(j + 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^1(j))</td>
<td>0.503</td>
<td>0.270</td>
<td>0.129</td>
<td>0.061</td>
</tr>
<tr>
<td>(e^2(j))</td>
<td>0.270</td>
<td>0.374</td>
<td>0.264</td>
<td>0.049</td>
</tr>
<tr>
<td>(e^3(j))</td>
<td>0.079</td>
<td>0.220</td>
<td>0.335</td>
<td>0.287</td>
</tr>
<tr>
<td>(e^4(j))</td>
<td>0.068</td>
<td>0.086</td>
<td>0.210</td>
<td>0.377</td>
</tr>
<tr>
<td>(e^5(j))</td>
<td>0.080</td>
<td>0.049</td>
<td>0.061</td>
<td>0.227</td>
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Table 14: Markov Transition Matrix between 50-54 and 55-59

<table>
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<th>(e^1(j + 1))</th>
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<th>(e^3(j + 1))</th>
<th>(e^4(j + 1))</th>
<th>(e^5(j + 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^1(j))</td>
<td>0.472</td>
<td>0.250</td>
<td>0.125</td>
<td>0.083</td>
</tr>
<tr>
<td>(e^2(j))</td>
<td>0.222</td>
<td>0.375</td>
<td>0.194</td>
<td>0.181</td>
</tr>
<tr>
<td>(e^3(j))</td>
<td>0.155</td>
<td>0.268</td>
<td>0.296</td>
<td>0.183</td>
</tr>
<tr>
<td>(e^4(j))</td>
<td>0.014</td>
<td>0.083</td>
<td>0.236</td>
<td>0.375</td>
</tr>
<tr>
<td>(e^5(j))</td>
<td>0.062</td>
<td>0.093</td>
<td>0.155</td>
<td>0.183</td>
</tr>
</tbody>
</table>

Table 15: Markov Transition Matrix between 55-60 and 60-64

<table>
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<tr>
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<th>(e^2(j + 1))</th>
<th>(e^3(j + 1))</th>
<th>(e^4(j + 1))</th>
<th>(e^5(j + 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^1(j))</td>
<td>0.474</td>
<td>0.158</td>
<td>0.211</td>
<td>0.095</td>
</tr>
<tr>
<td>(e^2(j))</td>
<td>0.222</td>
<td>0.333</td>
<td>0.333</td>
<td>0.056</td>
</tr>
<tr>
<td>(e^3(j))</td>
<td>0.111</td>
<td>0.167</td>
<td>0.333</td>
<td>0.278</td>
</tr>
<tr>
<td>(e^4(j))</td>
<td>0.063</td>
<td>0.095</td>
<td>0.158</td>
<td>0.316</td>
</tr>
<tr>
<td>(e^5(j))</td>
<td>0.071</td>
<td>0.106</td>
<td>0.176</td>
<td>0.294</td>
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</table>

Table 16: Markov Transition Matrix between 65-69 and 70-74
Table 17: Markov Transition Matrix between 75-79 and older

<table>
<thead>
<tr>
<th></th>
<th>$e^1(j+1)$</th>
<th>$e^2(j+1)$</th>
<th>$e^3(j+1)$</th>
<th>$e^4(j+1)$</th>
<th>$e^5(j+1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^1(j)$</td>
<td>0.474</td>
<td>0.158</td>
<td>0.211</td>
<td>0.053</td>
<td>0.105</td>
</tr>
<tr>
<td>$e^2(j)$</td>
<td>0.167</td>
<td>0.389</td>
<td>0.222</td>
<td>0.148</td>
<td>0.074</td>
</tr>
<tr>
<td>$e^3(j)$</td>
<td>0.093</td>
<td>0.185</td>
<td>0.333</td>
<td>0.278</td>
<td>0.111</td>
</tr>
<tr>
<td>$e^4(j)$</td>
<td>0.053</td>
<td>0.105</td>
<td>0.158</td>
<td>0.456</td>
<td>0.228</td>
</tr>
<tr>
<td>$e^5(j)$</td>
<td>0.059</td>
<td>0.118</td>
<td>0.176</td>
<td>0.216</td>
<td>0.431</td>
</tr>
</tbody>
</table>
7.6 General equilibrium model: Additional tables and graphs

<table>
<thead>
<tr>
<th>Maximum Pension - $\varphi$</th>
<th>Y</th>
<th>K</th>
<th>H</th>
<th>W</th>
<th>$\tau_C%$</th>
<th>Gini</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>3.859</td>
<td>2.012</td>
<td>0.955</td>
<td>2.389</td>
<td>0.017</td>
<td>0.548</td>
<td>-0.396</td>
</tr>
<tr>
<td>0.2</td>
<td>3.653</td>
<td>1.777</td>
<td>0.946</td>
<td>2.281</td>
<td>0.025</td>
<td>0.560</td>
<td>-0.406</td>
</tr>
<tr>
<td>0.4</td>
<td>3.336</td>
<td>1.450</td>
<td>0.932</td>
<td>2.116</td>
<td>0.040</td>
<td>0.578</td>
<td>-0.425</td>
</tr>
<tr>
<td>0.6</td>
<td>3.046</td>
<td>1.188</td>
<td>0.914</td>
<td>1.969</td>
<td>0.059</td>
<td>0.589</td>
<td>-0.443</td>
</tr>
<tr>
<td>0.8</td>
<td>2.774</td>
<td>0.973</td>
<td>0.894</td>
<td>1.835</td>
<td>0.081</td>
<td>0.599</td>
<td>-0.462</td>
</tr>
<tr>
<td>1.0</td>
<td>2.526</td>
<td>0.795</td>
<td>0.875</td>
<td>1.706</td>
<td>0.104</td>
<td>0.605</td>
<td>-0.482</td>
</tr>
<tr>
<td>1.2</td>
<td>2.341</td>
<td>0.669</td>
<td>0.864</td>
<td>1.601</td>
<td>0.125</td>
<td>0.614</td>
<td>-0.499</td>
</tr>
<tr>
<td>1.4</td>
<td>2.178</td>
<td>0.574</td>
<td>0.849</td>
<td>1.516</td>
<td>0.147</td>
<td>0.623</td>
<td>-0.515</td>
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</table>

Table 18: Aggregate variables: varying maximum pension benefits while keeping taper rate unchanged

<table>
<thead>
<tr>
<th>Taper Rates</th>
<th>Y</th>
<th>K</th>
<th>H</th>
<th>W</th>
<th>$\tau_C%$</th>
<th>Gini</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>2.554</td>
<td>0.811</td>
<td>0.879</td>
<td>1.717</td>
<td>0.118</td>
<td>0.593</td>
<td>-0.479</td>
</tr>
<tr>
<td>0.100</td>
<td>2.558</td>
<td>0.816</td>
<td>0.878</td>
<td>1.722</td>
<td>0.113</td>
<td>0.595</td>
<td>-0.478</td>
</tr>
<tr>
<td>0.200</td>
<td>2.560</td>
<td>0.820</td>
<td>0.876</td>
<td>1.727</td>
<td>0.110</td>
<td>0.597</td>
<td>-0.478</td>
</tr>
<tr>
<td>0.300</td>
<td>2.577</td>
<td>0.831</td>
<td>0.878</td>
<td>1.734</td>
<td>0.107</td>
<td>0.599</td>
<td>-0.476</td>
</tr>
<tr>
<td>0.400</td>
<td>2.564</td>
<td>0.823</td>
<td>0.876</td>
<td>1.729</td>
<td>0.106</td>
<td>0.600</td>
<td>-0.478</td>
</tr>
<tr>
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<td>2.547</td>
<td>0.811</td>
<td>0.875</td>
<td>1.719</td>
<td>0.106</td>
<td>0.602</td>
<td>-0.480</td>
</tr>
<tr>
<td>0.600</td>
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<td>0.804</td>
<td>0.875</td>
<td>1.714</td>
<td>0.105</td>
<td>0.603</td>
<td>-0.481</td>
</tr>
<tr>
<td>0.700</td>
<td>2.526</td>
<td>0.795</td>
<td>0.875</td>
<td>1.706</td>
<td>0.104</td>
<td>0.605</td>
<td>-0.482</td>
</tr>
<tr>
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<td>2.519</td>
<td>0.790</td>
<td>0.875</td>
<td>1.702</td>
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<td>0.607</td>
<td>-0.483</td>
</tr>
<tr>
<td>0.900</td>
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<td>0.874</td>
<td>1.697</td>
<td>0.103</td>
<td>0.609</td>
<td>-0.484</td>
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<tr>
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<td>2.494</td>
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<td>1.686</td>
<td>0.103</td>
<td>0.611</td>
<td>-0.486</td>
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</tbody>
</table>

Table 19: Aggregate variables: varying taper rates while keeping maximum pension benefits unchanged
Table 20: Aggregate capital: all experiments

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>.2</th>
<th>.4</th>
<th>.6</th>
<th>.8</th>
<th>ψ=1</th>
<th>1.2</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>196.981</td>
<td>161.570</td>
<td>135.668</td>
<td>114.664</td>
<td>98.628</td>
<td>86.369</td>
<td>75.173</td>
</tr>
<tr>
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<td>205.170</td>
<td>169.826</td>
<td>141.171</td>
<td>116.291</td>
<td>99.193</td>
<td>86.338</td>
<td>74.732</td>
</tr>
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<td>208.212</td>
<td>173.434</td>
<td>144.793</td>
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<td>99.706</td>
<td>86.428</td>
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</table>

Table 21: Aggregate labor: all experiments

<table>
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<th>.4</th>
<th>.6</th>
<th>.8</th>
<th>ψ=1</th>
<th>1.2</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>106.495</td>
<td>104.704</td>
<td>103.026</td>
<td>101.592</td>
<td>100.299</td>
<td>99.353</td>
<td>98.608</td>
<td>98.052</td>
</tr>
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<td>.1</td>
<td>106.495</td>
<td>105.291</td>
<td>103.562</td>
<td>102.007</td>
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<td>100.000</td>
<td>99.070</td>
<td>98.326</td>
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Table 22: Aggregate income: all experiments

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      & 0  & .2 & .4 & .6 & .8 & $\psi=1$ & 1.2 & 1.4 \\
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0    & 83.088 & 86.759 & 90.516 & 93.866 & 97.203 & 100.193 & 102.888 & 105.849 \\
.1   & -0.000 & 85.986 & 89.466 & 93.001 & 96.882 & 100.074 & 102.900 & 105.996 \\
.2   & -0.000 & 85.720 & 89.094 & 92.484 & 96.218 & 99.983  & 102.896 & 106.144 \\
.3   & -0.000 & 85.495 & 89.014 & 92.395 & 96.216 & 99.712  & 103.010 & 106.268 \\
$\omega = .4$ & -0.000 & 85.430 & 89.084 & 92.377 & 96.217 & 100.000 & 103.268 & 106.598 \\
.5   & -0.000 & 85.316 & 89.070 & 92.506 & 96.354 & 100.392 & 103.509 & 106.858 \\
.6   & -0.000 & 85.188 & 89.089 & 92.589 & 96.586 & 100.635 & 104.151 & 107.403 \\
.7   & -0.000 & 85.076 & 88.956 & 92.787 & 96.730 & 100.926 & 104.524 & 107.854 \\
.8   & -0.000 & 85.110 & 88.918 & 92.836 & 96.916 & 101.097 & 104.907 & 108.111 \\
.9   & -0.000 & 84.998 & 88.913 & 92.924 & 97.159 & 101.287 & 105.260 & 108.693 \\
1    & -0.000 & 84.976 & 88.909 & 92.894 & 97.423 & 101.724 & 105.584 & 109.120 \\
\hline
\end{tabular}
\caption{Aggregate welfare: all experiments}
\end{table}