Portfolio Choice and Self Control*

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Abstract

Experiments have shown that people are present biased when they make intertemporal decisions. However, for the most important financial decision of households - the housing and mortgage decision - the effects of being present biased have not been analyzed yet. In this paper I thus built a life-cycle model with housing and mortgages and allow households to suffer from temptation and self control. Agents with more severe problems of self control are predicted to overall save less and to save relatively more in illiquid assets. Moreover, agents with more severe problems of self control do not necessarily take on more debt and they tend to buy smaller houses. Contrary to what might be assumed they thus do not overinvest in housing. Finally, I show that financial regulation can improve welfare for present biased agents even without taking equilibrium effects into account.

1 Introduction

In psychological experiments of intertemporal choice people were found to exhibit time inconsistencies and preference reversals (inter alia, Thaler 1981; Green et al. 1994; Kirby and Herrnstein 1995; Casari 2009). This means that given the choice between a smaller earlier reward and a larger later reward, people choose the smaller earlier reward if it is paid out immediately but choose the larger later reward if both dates of

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payout are in the future. Such a present bias cannot be explained with the preferences and time discounting commonly employed in economic models. However, in recent years alternative preference specifications have been developed which can account for such a behavior. Most notably, these are Hyperbolic Discounting (Laibson, 1998) and Dynamic Self Control Preferences (Gul and Pesendorfer, 2001, 2004).

These preferences have been applied to different areas of household decisions. Selected examples include the choice of gym contracts (DellaVigna and Malmendier, 2006), drug addiction (Gul and Pesendorfer, 2007) and credit card debt (Laibson et al., 2007). In all these analyses, the behavior of present biased agents significantly differs from the behavior of standard agents. Moreover, including present bias in these models helps to reconcile the models’ predictions with observed data. However, an area where possible effects of present bias have so far not been analyzed is the housing and mortgage decision. At the same time, this decision is one of most important decisions a household has to make. Not only are houses very expensive relative to a household’s life-time income. Mortgages are also financial contracts with a very long repayment period which implies that the housing and mortgage decision has very long-term effects on the budget constraint of a household. For this reason it is important to understand if and in which way the degree of present bias affects the behavior and welfare of agents.

In this paper I aim to fill this gap. I build a model of portfolio choice, focusing on the housing and mortgage decision. Furthermore, I allow agents to have a problem with temptation and self control and analyze how the degree of this problem affects their portfolio decision. I particularly differentiate between the consumption and investment

1For an overview of time discounting and time preferences see Frederick et al. (2002).
2A detailed survey on decision theoretic models which accommodate present biased behavior can be found in Lipman and Pesendorfer (2011).
3Throughout the paper, I will refer to agents who are not present biased, i.e. whose behavior can be described by the preferences commonly employed in economic models, as standard agents.
4DellaVigna (2009) gives a broad overview over field experiments concerning behavioral aspects in household decisions.
motives for purchasing a house and show how each of these motives is affected by
problems with self control. Moreover, I analyze how the welfare of present biased
agents can be affected by financial regulation.

My model is related to literature in three main areas. First, it is related to other
portfolio choice models like, for example, models by Flavin and Yamashita (2002, 2011),
Cocco (2005), and Cocco et al. (2005). These models mainly focus on the decision be-
tween investment in stocks or in bonds and do not model the housing decision. If
housing is taken into account then it is assumed to be exogenous and its effects on the
remaining portfolio decisions are analyzed. Second, there are models of housing which
are close to my model (inter alia, Chambers et al. 2009, Chatterjee and Eyigungor
2011, Corbae and Quintin 2011, Iacoviello and Pavan 2011). These are general equi-
librium models which mostly focus on the determinants of foreclosure rates. In order
to be able to conduct a general equilibrium analysis they necessarily have to cut down
on the choice set of households. In particular, these models typically feature only a
small number of discrete house sizes and mortgage options. While having interesting
implications on the general equilibrium level, these models can thus not analyze the
portfolio decisions of households in a detailed way. Moreover, none of these models
allow for agents to exhibit present bias. Third, my model is related to other models
that allow agents to be present biased (inter alia, DellaVigna and Malmendier 2006;
Gul and Pesendorfer 2007, Laibson et al. 2007). But as already explained these models
do not analyze housing or mortgage decisions.

The model which is closest to mine is the model by Ghent (2011). Her model is a
life-cycle model with housing and mortgages where agents discount the future hyperbol-
ically. In this setting, she analyzes how equilibrium outcomes such as home ownership
rates and foreclosure rates are affected by the introduction of subprime mortgage prod-
ucts. However, in order to be able to focus on steady state equilibria, she uses a limited
choice set for the households, i.e. only a small number of discrete house sizes and only two possible mortgage contracts. Moreover, she analyzes the outcomes for one particular degree of present bias. In my model, I explicitly focus on the portfolio choice of households and thus have a very rich choice set for the agents. In particular, agents have a continuous choice for both house size and mortgage size. I can thus rule out that the observed optimal behavior might be driven by the discrete nature of an agent’s choice set. In addition to that, I analyze the portfolio decisions for varying degrees of present bias. This enables me to make clear predictions about how the degree of present bias affects the housing and mortgage decision of households.

I obtain three predictions about how the degree of present bias affects the portfolio decision. First, agents with a stronger problem of self control overall save less than agents with a smaller problem. Second, they invest a larger fraction of these savings in illiquid assets. Third, agents with more severe problems of self control do not necessarily take on more debt and they tend to buy smaller houses. Contrary to what might be assumed they thus do not overinvest in housing. Regarding the effects of financial regulation on the agents’ welfare I find that even without taking equilibrium effects intro account a minimum down payment requirement might increase the agents’ welfare if they are present biased.

The remainder of this paper is structured as follows. In the next section I will briefly introduce the agents’ preferences and describe their implications for a simple consumption-saving decision. In section 3 I will then describe the portfolio choice model of housing and mortgages. Section 4 details the parametrization and solution method used to numerically solve the model. The effects of problems with self control are presented in section 5. Welfare implications of financial regulation are discussed in section 6. Section 7 concludes.
2 Dynamic Self Control Preferences

In this paper I allow agents to be present biased. In particular, I assume that households have Dynamic Self Control Preferences (Gul and Pesendorfer, 2001, 2004). Before applying these preferences to the model of housing and mortgages, in this section I first describe the preferences and its main driving forces for a simple consumption and saving decision. It is important to keep these analytical results in mind when analyzing the numerical solution of the more complex model of portfolio choice.

Dynamic Self Control (DSC) Preferences capture the idea that agents are subject to temptation and suffer from costs of self control if they want to resist this temptation. The detailed functional form of the per period utility is as follows:

\[
U(c_t) = \max_{c_t} \{u(c_t) + v(c_t)\} - \max_{\tilde{c}_t} v(\tilde{c}_t)
\]

where \(U(c_t)\) is the per period utility and \(u(c_t)\) and \(v(c_t)\) denote commitment utility and temptation utility, respectively. From the first term in equation \(1\) we see that the agent maximizes the sum of commitment utility and temptation utility. The second term reflects the effect of temptation. The agent’s utility is always reduced by the temptation of the most tempting alternative in the choice set. The preferences are hence not only defined over actual consumption but over the whole choice set. In particular, it is important to define the most tempting option in the choice set. Anytime that the agent does not choose this option she suffers from costs of self control.

In order to analyze the effect of problems of self control on consumption-saving decisions we have to specify the relation between commitment and temptation utility. For a generic consumption good more consumption should both give the agent more commitment utility and be more tempting. To keep the model as parsimonous as
possible, I thus choose the same functional form for both utilities:

\[ v(c_t) = \lambda \cdot u(c_t) \]  

(2)

This implies that commitment utility and temptation utility have the same ranking of consumption alternatives. The only difference is the scalar parameter \( \lambda \) which indicates the degree of the problem of self control. Applying this to an intertemporal problem of consumption and saving we obtain the following Bellman Equation:

\[
W(x_t) = \max_{s_t: x_t - s_t \geq 0} \left\{ U(x_t - s_t) + \beta W(s_t(1 + r)) \right\} 
= \max_{s_t: x_t - s_t \geq 0} \left\{ u(x_t - s_t) + \lambda \cdot [u(x_t - s_t) - u(x_t)] + \beta W(s_t(1 + r)) \right\} 
\]

(3)

where \( x_t \) denotes cash-on-hand in period \( t \), \( s_t \) are savings in period \( t \), \( \beta \) is the discount factor and \( r \) is the interest rate. Equation (3) differs from the standard Bellman equation only in the additional term \( \lambda \cdot [u(x_t - s_t) - u(x_t)] \) which corresponds to the costs of self control. These costs of self control are equal to the difference in utility between the actual consumption and the maximum possible consumption, which is equal to cash-on-hand \( x_t \). This difference is multiplied by \( \lambda \), which regulates the degree of the problem of self control: For \( \lambda = 0 \) the whole term drops out so that equation (3) simplifies to the Bellman Equation under standard preference. As \( \lambda \) increases, however, the the costs of self control also increase.

Optimal behavior in this simple model is characterized by the following intertemporal optimality condition:

\[
u'(c_t) = \beta (1 + r) \left[u'(c_{t+1}) - \frac{\lambda}{1 + \lambda} u'(x_{t+1})\right] \]

(4)

This equation differs from the standard Euler Equation due to the second term in
brackets. Since the marginal utility is always positive this term reduces the right-hand-side and hence savings for any $\lambda > 0$. We thus see that savings unambiguously decrease if the problem of self control, $\lambda$, increases. Moreover, rearranging equation (4) reveals the two driving forces of Dynamic Self Control Preferences:

$$ (1 + \lambda) \cdot u'(c_t) = \beta (1 + r) \left[ u'(c_{t+1}) + \lambda \left( u'(c_{t+1}) - u'(x_{t+1}) \right) \right] $$

First, we see that the marginal costs of giving up consumption today are higher when agents suffer from problems of self control. This is why they save less in order to avoid current costs of self control. Second, the marginal benefit of saving is increased by the marginal future costs of self control. Therefore, the optimal level of savings trades off current vs. future costs of self control.

From the brief introduction of DSC preferences in this simple consumption-saving context we thus make three observations. First, to understand the problem faced by the agent it is crucial to identify the most tempting option since all possible actions are evaluated against this temptation. Second, the driving force behind DSC preferences is the desire to limit both current and future costs of self control. The optimal decision will thus trade off these two costs. Finally, in this simple context savings unambiguously decrease if the problem of self control increases. It will be helpful to keep these findings in mind when interpreting the results of the more complex model of housing and mortgages.

### 3 Model of Housing and Mortgages

In this section I describe the portfolio choice model with housing and mortgages. It is a model of optimal household behavior where agents optimize their expected life-time...
utility. Their life-cycle is stylized in the sense that there are three phases in life. First, the analysis starts in the period where the agents buy their house and decide about the size of their mortgage. Second, after this purchasing period they enter working life where they get random income and repay their mortgage. Third, the last phase of life is retirement where the agents do not receive any income anymore. Moreover, the transition from working life to retirement and from retirement to death are random.

There are three assets in this model: a risk-free liquid asset, houses and mortgages. Liquid savings $s_t$ are equivalent to the savings vehicle in the simple consumption-savings model discussed in the previous section. They are here called liquid savings to distinguish them from houses which serve as illiquid investment. In fact, there are two separate motives why agents want to buy a house in this model. On the one hand, houses serve as illiquid retirement savings, where illiquidity is modelled in a strict sense: During working life agents cannot sell their house. However, at entry to retirement all agents sell their house. Home equity is hence equivalent to illiquid retirement savings. On the other hand, agents also gain utility from housing services.

Agents have Dynamic Self Control Preferences which are now defined over both consumption and housing services. As described in the previous section, commitment utility and temptation utility have the same functional form except for the degree of the problem of self-control $\lambda$:

$$v(c_t, h_0 P_t) = \lambda \cdot u(c_t, h_0 P_t) \quad (6)$$

where $h_0$ is the size of the house the agents buy in the purchasing period and $P_t$ is the house price in period $t$. Hence the utility that agents derive from housing services

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5As a model of optimal household behavior this paper does not aim at explaining equilibrium outcomes in the housing market. Instead, it focuses on explaining the driving forces behind the households portfolio choice. In this sense it is close to Campbell and Cocco (2011) who do a similar exercise in explaining the determinants of mortgage default.
increases with the value of their house. The reason behind this formulation is that in this model it is assumed that house prices increase because they yield more utility. Moreover, utility is assumed to be of the Constant Relative Risk Aversion (CRRA) form:

$$u(c_t, h_0P_t) = \frac{(c_t(h_0P_t)^{\theta})^{1-\sigma}}{1 - \sigma}$$

(7)

where $\theta$ indicates the weight of housing services in the utility function and $\sigma$ is the coefficient of relative risk aversion.

The third kind of assets are mortgages, which are modelled as fixed rate mortgages with mortgage payment $m$. Choosing a mortgage payment is equivalent to choosing the size of the mortgage since the mortgage term is assumed to be constant. To keep the model as simple as possible, it is assumed that mortgages cannot be refinanced. It is possible, however, to default on the mortgage. If agents default they immediately lose their house and their remaining mortgage balance is set to zero. In addition, they will never be able to purchase another house.

The model has to be solved by backward induction starting in retirement and moving backwards through the agents’ life-cycle. In the following subsections I will thus describe in detail the agents’ optimization problem in each phase of their life.

### 3.1 Retirement

During retirement the agents never own a house since it is assumed that they sell their house at entry to retirement. Moreover, they do not receive any income anymore. Hence they face a consumption-saving problem which is similar to the one discussed in the

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6Utility from housing services is comparable to dividends from stocks. The reasoning behind equation (6) is thus similar to the notion that stock prices increase because expected future dividends increase.
previous section:

\[
W_{ret}(x_t) = \max_{s_t} \left\{ u(x_t - s_t, 0) + \lambda \cdot [u(x_t - s_t, 0) - u(x_t, 0)] + \beta(1 - p_D)W_{ret}(s_t(1 + r_S)) \right\}
\]

s.t. \quad x_t - s_t \geq 0 \tag{9}

\quad s_t \geq 0 \tag{10}

Given the only state variable, their current cash-on-hand \( x_t \), the agents have to decide about their liquid savings. The difference to the problem discussed before is that now there is a positive probability, \( p_D \), that the agent will die after the current period. Also, the interest rate on liquid savings is now denoted by \( r_S \) because there will be different interest rates on savings and mortgages.

For this subproblem of optimal behavior in retirement it is in fact possible to find a closed form solution if logarithmic utility is assumed. The solution is derived in Appendix A and is given by

\[
s(x_t) = (1 - \psi)x_t \tag{11}
\]

\[
W_{ret}(x_t) = A_0 + A_1 \log(x_t) \tag{12}
\]

where \( \psi = \frac{(1 - \beta(1 - p_D))(1 + \lambda)}{\beta(1 - p_D) + (1 - \beta(1 - p_D))(1 + \lambda)} \tag{13} \)

\[
A_0 = \frac{1 + \lambda}{1 - \beta(1 - p_D)} \log(\psi) + \frac{\beta(1 - p_D)}{[1 - \beta(1 - p_D)]^2} \log \left( (1 - \psi)(1 + r_S) \right) \tag{14}
\]

\[
A_1 = \frac{1}{1 - \beta(1 - p_D)} \tag{15}
\]

We see that the optimal savings function is linear in cash-on-hand and that the savings share \( (1 - \psi) \) decreases with the problem of self control \( \lambda \). For the more complex subproblems earlier in the agents’ life, however, the optimal policy function will no longer
be linear and it won’t be possible to find closed form solutions. For the main analyses in this paper I will thus employ numerical solution methods instead of analytical ones.

### 3.2 Working Life

The problem during working life is somewhat more complicated. In particular, we have to differentiate between the case where the agent does not have a house and the case where she has a house. In the former case, the problem still looks similar to the problems analyzed so far. The difference is that now we have to account for the fact that transition from working life to retirement is random and that the agent will receive random income if she is still working next period.

\[
W_{\text{work}}^{\text{nohouse}}(x_t) = \max_{s_t} \left\{ \begin{array}{l}
    u(x_t - s_t, 0) + \lambda \cdot [u(x_t - s_t, 0) - u(x_t, 0)] \\
    + \beta \cdot E \left[ (1 - p_R) \cdot W_{\text{work}}^{\text{nohouse}}(s_t(1 + r_S) + y_{t+1}) \\
    + p_R \cdot W_{\text{ret}}(s_t(1 + r_S)) \right] \end{array} \right\} 
\]

\[\text{s.t.} \quad x_t - s_t \geq 0 \quad s_t \geq 0\]  

(16)  

If the agent has a house, however, she has a second choice variable: She can decide if she wants to default or not. It is optimal for the agent to default if the value of defaulting is higher than the value of not defaulting:

\[
W_{\text{work}}^{\text{house}}(x_t, h_0 P_t, M_t, m_t) = \max \left[ W_{\text{work}}^{\text{def}}(x_t, h_0 P_t, m_t), W_{\text{work}}^{\text{nodef}}(x_t, h_0 P_t, M_t, m_t) \right] 
\]

(19)

Here, in addition to cash-on-hand \(x_t\), we have three more state variables: the current value of the agent’s house \(h_0 P_t\), outstanding mortgage balance \(M_t\) and the mortgage payment \(m_t\) that the agent has to make in order not to default. This payment is equal
to the fixed mortgage payment that the agent chose in the purchasing period as long the mortgage has not been fully repaid yet:

\[
m_t = \begin{cases} 
m & \text{if } M_t > 0 \\ 0 & \text{if } M_t = 0 \end{cases}
\]  

(20)

If the agent decides to default her value function looks as follows:

\[
W_{work}^{def}(x_t, h_0 P_t, m_t) = \max_{s_t} \left\{ u(x_t - s_t, 0) + \lambda \cdot [u(x_t - s_t, 0) - T(x_t, h_0 P_t, m_t)] \\
+ \beta \cdot E \left[ (1 - p_R) \cdot W_{work}^{nohouse}(s_t(1 + r_S) + y_{t+1}) \\
+ p_R \cdot W_{ret}(s_t(1 + r_S)) \right] \right\}
\]

(21)

s.t.  
\[
x_t - s_t \geq 0 
\]

(22)

\[
s_t \geq 0 
\]

(23)

\[
T(x_t, h_0 P_t, m_t) = \max \left[ u(x_t, 0), u(x_t - m_t, h_0 P_t) \right] 
\]

(24)

As described above, if the agent defaults she loses her house immediately. This implies two things: First, already in this period she does not receive any utility from the house. Second, she will not have a house in the next period either. She will hence either be working without a house or she will be retired.

The only way the value of defaulting is affected by the house and the mortgage is through the temptation \( T(x_t, h_0 P_t, m_t) \) the agent faces. As explained in section 2, the temptation is always the utility of the most tempting alternative. Here, this can be either of the following two options. On the one hand, it could be most tempting not to make the mortgage payment, hence to lose the house and the utility from housing services, and to consume the whole cash-on-hand. On the other hand, it could also be most tempting not to default, i.e. to make the mortgage payment and therefore
to get the utility from housing services, and to consume the remaining cash-on-hand \( x_t - m_t \). Which of these two options will be most tempting will depend on the size of the required mortgage payment relative to the value of the house. Intuitively, if the value of the house and the resulting utility from housing services is too low compared to the mortgage payment required to get this utility, then it will be most tempting to default.

If the agent does not default in this period, her value function has the following form:

\[
W_{work}^{modef}(x_t, h_0P_t, M_t, m_t)
= \max_{s_t} \quad u(x_t - m_t - s_t, h_0P_t) + \lambda \cdot [u(x_t - m_t - s_t, h_0P_t) - T(x_t, h_0P_t, m_t)]
+ \beta \cdot E\left[(1 - p_R) \cdot W_{work}^{house}(s_t(1 + r_S) + y_{t+1}, h_0P_{t+1}, M_{t+1}, m_{t+1}) + p_R \cdot W_{ret}(s_t(1 + r_S) + h_0P_{t+1} - M_{t+1})\right]
\]

s.t. \( x_t - m_t - s_t \geq 0 \) \hspace{1cm} (26)
\( s_t \geq 0 \) \hspace{1cm} (27)
\( T(x_t, h_0P_t, m_t) = \max \left[u(x_t, 0), u(x_t - m_t, h_0P_t)\right] \) \hspace{1cm} (28)
\( M_{t+1} = (M_t - m_t) \cdot (1 + r_M) \) \hspace{1cm} (29)

In this case she has to make the mortgage payment and gains utility from housing services. What is also different is the expected value in the next period. If the agent does not retire, she will still be working and will also still own the house. The value of the house will change, however, and so will the outstanding mortgage balance. In particular, the remaining mortgage balance will grow with the mortgage rate \( r_M \). But there is also a change if the agent retires. In that case she sells her house and prepays any outstanding mortgage. This implies that the cash-on-hand with which she enters
retirement consists of both her liquid savings and the amount of home equity that she has accumulated in her house up to retirement. Having a house, or to be more precise, having home equity increases the retirement savings.

### 3.3 Purchasing period

The purchasing period is the period in which the agent makes her portfolio decision. In particular, she chooses the house size $h_0$, the size of the fixed mortgage payment $m$, and how much she wants to save in liquid assets $s_0$. Her optimization problem has the following form:

$$
W_0(x_0, P_0) = \max_{h_0, m, s_0} u(x_0 - D(h_0 P_0, m) - s_0, h_0 P_0)
$$

$$
+ \lambda \cdot [u(x_0 - D(h_0 P_0, m) - s_0, h_0 P_0) - T_0(x_0, P_0)]
$$

$$
+ \beta \cdot E\left[W_{\text{house}}^{\text{work}}(s(1 + r_S) + y_1, h_0 P_1, m_1, M_1)\right]
$$

s.t. 

$$
x_0 - D(h_0 P_0, m) - s_0 \geq 0
$$

$$
s_0 \geq 0
$$

$$
D(h_0 P_0, m) = h_0 P_0 - m \cdot \frac{1 - (1 + r_M)^{-TERM}}{r_M}
$$

$$
m \leq (1 - \gamma)h_0 P_0 \cdot \frac{r_M}{1 - (1 + r_M)^{-TERM}}
$$

$$
M_1 = (h_0 P_0 - D(h_0 P_0, m)) \cdot (1 + r_M)
$$

$$
T_0(x_0, P_0) = \max_{h_0, m, s_0} u(x_0 - D(h_0 P_0, m) - s_0, h_0 P_0)
$$

s.t. 

$$
(31) - (34)
$$

where $D(h_0 P_0, m)$ is the amount of down payment that the agent makes when buying the house. From equation (33) we see that this amount of down payment depends on the overall purchase price, $h_0 P_0$, and the size of the constant mortgage payment, $m$. 

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Recall that the term of the mortgage, \textit{TERM}, is the same for all mortgages so that in fact the agent not only chooses the fixed mortgage payment but at the same time also the size of the mortgage. Moreover, from equation (34) we see that there is an upper bound for the mortgage payment. In particular, the mortgage payment has to be chosen such that the absolute size of the mortgage is at most equal to the share \((1 - \gamma)\) of the purchase price. In other words, there is a down payment requirement of at least \(100\gamma\%\).

It is also important to be clear on the temptation that the agent faces in the purchasing period. Equation (36) shows that the most tempting option is the combination of choice variables which gives the highest utility in the purchasing period while still being feasible with respect to equations (31)-(34). If there is no down payment requirement, \(\gamma = 0\), then it is always most tempting to buy the biggest house available, to fully finance this purchase with a mortgage, and to consume all cash-on-hand. If \(\gamma > 0\), however, then the agent always has to make a down payment if she wants to buy a house. In this case it is hence not clear which house size combined with the minimum down payment will lead to the highest within period utility.

4 Parametrization and Solution Algorithm

Table I summarizes the parameter values used to analyze the model. The coefficient of relative risk aversion is assumed to be equal to be one, \(\sigma = 1\), which implies that utility is logarithmic: \(u(c, h) = \log(c) + \theta \log(h)\). The weight \(\theta\) of housing is set to \(1/3\). Moreover, for the benchmark specification it is assumed that there is no interest rate spread such that the interest rate on liquid savings is equal to the mortgage rate \(r_s = r_M = 0.02\). Also, the discount rate \(\rho\) is set equal to the interest rate so that in the benchmark model, the standard agent is not impatient. This makes it possible to see
all the effects which are inherent in the problems of self control. As it turns out parts of these effects are similar to impatience even if there is no impatience assumed itself. Furthermore, the house price is assumed to grow deterministically each period,

\[ P_{t+1} = P_t \cdot (1 + g_H) \quad \forall t, \tag{37} \]

and the growth rate of house prices is assumed to be identical to the interest rate on liquid savings. This choice makes it possible to analyze the effects of illiquidity separately from trade-offs with respect to the return on investment. Finally, income is assumed to be iid lognormal in each working life period with expected income equal to one. This choice normalizes the analysis in the sense that all quantities can be regarded as relative to income.

The model has to be solved by backward induction, starting from retirement. There the functional equation detailed in equation (8) contains as only function the value of being retired, \( W_{\text{ret}}(\cdot) \). For retirement, the value function can thus be determined by value function iteration independently from the rest of the model. As the next step, the solution to \( W_{\text{ret}}(\cdot) \) is needed as input to the value function iteration of not having a house during working life, \( W_{\text{nohouse work}}(\cdot) \), as described in equation (16). From equation (21) we see that both these value functions are then needed for the value function iteration of having a house and defaulting. The next step is thus to solve for the value of defaulting, \( W_{\text{def work}}(\cdot) \). Only after that it is possible to solve for the value of having a house, \( W_{\text{house work}}(\cdot) \), as shown in equation (19). In fact, the value of having a house, \( W_{\text{house work}}(\cdot) \), and the value of having a house and not defaulting, \( W_{\text{nodef work}}(\cdot) \), have to be solved for simultaneously. The reason for that is that the agent retains the option to default in the future if she does not default now. This can be seen in equation (25) where the expected value next period depends on \( W_{\text{house work}}(\cdot) \). The last step to solve the
model is then to optimize the value in the purchasing period, \( W_0(\cdot) \), over the three choice variables house size \( h_0 \), mortgage payment \( m \), and liquid savings \( s_0 \) as seen in equation (30).

For the value function iterations I used grid search over both state and control variables. In particular, the grids for cash-on-hand and liquid savings were constructed by grids which are logarithmic for low values and linear for higher values. This choice was necessary to accommodate DSC preferences which are very sensitive to the choice of the lower end of the grid. I employed a full grid search in order to ensure that I obtain global optima. This is a critical issue in this model since both the default decision and the temptation that agents face exhibit non-convexities. As described before, the most tempting option during working life can either include default or no default and temptation in the purchasing period can include buying a house or not. Hence temptation is not a smooth function and the numerical solution algorithm has account for that.

5 Effects of Self Control on Portfolio Choice

In this section I describe the numerical results of the model in detail. In order to explain all the channels through which problems of self control affect the portfolio decisions of households I will do this in two steps. First, I will assume agents do not get any utility from housing services. I will further distinguish between the case where mortgages are not possible and the case where the agent can take out a mortgage. This part of the analysis will thus give detailed insight into the investment motive of housing and how it interacts with a possible present bias. In the second step, I will then analyze the full model, i.e. the model with both investment and utility motive for purchasing a house. This makes it possible to understand how being present biased affects the utility of a
house.

5.1 Model without utility from house

If the agent does not receive any utility from having a house, the house can be thought of as a form of financial illiquid investment. Moreover, if we further assume that it is not possible to finance the purchase by a mortgage, then buying a house is equivalent to putting some part of wealth in a lockbox which can only be accessed after retirement. This is the first case I will analyze. In the second case I will add mortgages to the model.

5.1.1 Setting without mortgages

In the setting without utility from the house and without mortgages, buying a house is equivalent to putting money in a lockbox which will only be accessible after retirement. In such a setting, Figure 1(a) shows the optimal level of overall savings in the purchasing period depending on current cash-on-hand. The figure plots these policy functions for four different degrees of the problem of self control. In particular, the bold solid line depicts the behavior of the standard agent who does not have any problem of self control. The other three lines show the optimal behavior of agents with increasing problems of self control $\lambda$.

As can be seen from the graph, the optimal level of overall savings decreases as the problem of self control increases. This result is similar to the result in the simple consumption-savings problem above where savings unambiguously decrease as the problem of self control increases. We now see that this result holds for overall savings even if agents have the opportunity to save in illiquid assets.

However, the same is not true when liquid and illiquid investments are analyzed separately. Figure 1(b) shows the optimal illiquid investment for different degrees of
self control \( \lambda \). There are two things to be seen. First, for the standard agent it is not optimal to invest in illiquid assets at all. They do not have any advantage from illiquidity since they do not suffer from costs of self control. At the same time, income is random so that illiquidity is in fact a disadvantage: Liquid savings can be used as precautionary savings while illiquid investment cannot. In the model without utility from the house it is hence optimal for a standard agent to fully invest his savings in liquid assets.

Second, the level of optimal illiquid investment is not monotonic in the degree of self control \( \lambda \). The reason for that lies in two distinct motives for saving: retirement savings and consumption smoothing. Retirement savings can both be made by liquid and by illiquid investments. If agents suffer from costs of self control, they prefer illiquid savings to liquid ones because they want to avoid the costs of being tempted to spend the savings each period. Regarding the level of optimal retirement savings, it is important to remember that DSC agents are effectively more impatient since their marginal costs of giving up current consumption are higher. But as costs of self control increase, i.e. as effective impatience increases, optimal retirement savings and hence illiquid investment decrease.

Consumption smoothing, on the other hand, refers to the desire to smooth consumption within working life. This means that agents who are very rich in the purchasing period relative to their expected income during working life not only want to move resources to retirement but also want to save some of their wealth in order to smooth their consumption before retirement. These savings can only be made by liquid savings since illiquid investment cannot be accessed before retirement. However, the optimal level of consumption smoothing also depends on self control. Indeed, the higher the costs of self control, the more costly is consumption smoothing. This is because in each working life period, the agent will be tempted to spend all her available resources in-
stead of saving to smooth consumption. Hence, the stronger the problem of self control, the lower is the optimal level of consumption smoothing.

Taking both motives into account we see that the optimal level of illiquid investment is concave in cash-on-hand. Moreover, the degree of concavity is less pronounced the higher the problems of self control since the desired liquid savings for consumption smoothing decrease. Combining this with the decreasing desire for retirement savings we observe that the optimal level of illiquid investment is not monotonic in the degree of self control.

What is monotonic, however, is the share of illiquid investment in overall savings. As we can see in Figure 1(c), the share of illiquid investment increases as problems of self control increase. This implies that liquid savings decrease particularly strongly compared to illiquid savings. The reason for that is that liquid savings lead to both current and future costs of self control while illiquid savings only lead to current costs. In a world where illiquid investments are available, consumption smoothing is thus more costly than saving for retirement if agents suffer from costs of self control. The amount of savings for consumption smoothing is hence even more reduced than the amount of retirement savings.

From this simple setting without utility from the house and without mortgages we can thus draw two conclusions. First, as problems of self control become more severe, agents overall save less. Second, they predominantly save in illiquid assets such that the illiquid share in overall investment increases. The absolute value of illiquid investment, however, is not monotonic in problems of self control. These results will also hold when mortgages and the utility motive are added to the model. I will thus not explicitly present the results about overall savings and illiquid investment share in the following sections. Instead I will focus on the implications for the housing and mortgage choice.
5.1.2 Setting with mortgages

If now mortgages are added to the model, the house size will no longer be identical to the illiquid investment the agent makes in the purchasing period. Instead, the agent can decide how much of the purchase price to pay up front as down payment (which is then the illiquid investment in this period) and how much to finance through a mortgage. This then determines the mortgage payment the agent has to make during the repayment phase of the mortgage.

Figure 2(a) shows the policy function for the house size, given different degrees of self control. As before, the standard agent does not buy a house at all since she does not benefit from its illiquid nature. Compared to the house size in the setting without mortgages, the optimal house size for DSC agents is now bigger for lower levels of cash-on-hand. In this region, agents do not pay the whole purchase price up front as a down payment but finance the purchase by a mortgage. Since houses still do not yield utility, the only purpose of a house is still that it serves as retirement savings. As explained above, a stronger problem of self control leads to higher effective impatience and hence to lower optimal retirement savings. This can again be seen here in the fact that for lower levels of cash-on-hand the house size decreases with the problem of self control $\lambda$.

Figure 2(b) depicts the optimal mortgage payment of the agents. Recall that since the term of all mortgages is fixed, this is equivalent to analyzing the overall mortgage size. We see that the optimal level of borrowing decreases with the problem of self control $\lambda$. The reason for this result is that without utility from the house, default is always more tempting than making the mortgage payment. This implies that throughout the repayment phase of the mortgage, the agent will be tempted not to make the mortgage payment and thus has to exercise self control in order not to default. Mortgage payments are therefore more costly the stronger the problem of self control. The optimal mortgage size hence decreases with the problem of self control.
5.2 Model with utility from house

In this section I now analyze the full model where agents have two motives for buying a house: the investment motive already analyzed above and the utility motive. Figure 3 shows the policy functions for the house size and the size of the mortgage payment in this setting.

From Figure 3(a) we see that now also the standard agents purchase a house because they can gain utility throughout working life in addition to saving for retirement. Moreover, for the DSC agents we see that even in presence of the utility motive the earlier results regarding the optimal house size hold: For lower levels of cash-on-hand the house size decreases with problems of self control. Over the whole range of wealth, however, the relationship is still not monotonic. The reason for that is that the optimal house size according to the utility motive decreases with problems of self control just as the optimal retirement savings do. Agents with problems of self control are effectively more impatient and hence their optimal per period utility profile is decreasing over time. Houses, however, lead to an increasing stream of utility due to increasing house prices. The discounted utility from the house is therefore lower for agents with a stronger problem of self control which leads to a smaller optimal house size. Hence we see, contrary to what might be assumed, that problems of temptation and self control do not lead to overinvestment in housing.

The results regarding the optimal size of the mortgage payment, however, are affected by adding the utility motive to the model as can be seen in Figure 3(b). For the standard agent, the size of the mortgage payment is only defined for lower levels of cash-on-hand. There, the agent prefers not to make any down payment at all. In-

\footnote{This result does not depend on utility being defined over the \textit{value} of the house instead of the house \textit{size}. If utility was defined over the house size, the utility stream from housing services would be constant instead of increasing. This would still lead to a smaller discounted utility as agents become effectively more impatient.}
vestment in housing is risky in the sense that agents can default if hit by very bad income shocks. Agents thus have an incentive to postpone paying the purchase price in order not to risk losing their investment. Since there is no interest rate spread and the standard agent does not suffer from costs of self control, she can costlessly postpone her payments. As she gets richer, however, the likelihood of a default becomes smaller and smaller so that the standard agent becomes indifferent to the timing of the payments. The size of the mortgage payment is hence not determined anymore.

For the DSC agent, the optimal mortgage size also changes when houses yield utility. Without utility, it was always more tempting to default than to make the mortgage payment and hence mortgage payments always lead to costs of self control. Now, however, mortgage payments only lead to costs of self control if the mortgage payment is too high relative to the utility the agent receives from keeping the house. The DSC agent thus has to ensure that her mortgage payment is not too big relative to the house size she chooses. This constraint is binding for lower levels of cash-on-hand where the optimal house size is particularly small. Since house sizes decrease with problems of self control, this thus leads to decreasing mortgage sizes for low levels of cash-on-hand.

For higher levels of cash-on-hand, however, this restriction is not binding anymore. There the agent has to trade off current costs of self control with future costs of self control. The former arise if she makes a down payment while the latter occur due to the increased level of required precautionary savings to avoid default with higher mortgage payments. For this region the relationship between mortgage size and self control is thus not monotonic anymore. While we hence cannot determine the direction of the self control effect on the optimal mortgage size for all levels of wealth we still see that leverage does not necessarily increase for more severe problems of self control.
6 Self Control and Financial Regulation

In this section I analyze how the welfare of present biased agents can be affected by financial regulation. In particular, I analyze if agents always benefit from having easy access to credit. In standard economic models with agents who have standard preferences, the agents’ welfare cannot be decreased by adding the possibility to leverage the investment position. This is not true, however, for agents who suffer from temptation and costs of self control. For them it is not clear if they benefit from easy access to credit because it enables them to buy a house they could otherwise not purchase or if they merely suffer from having to resist buying a house with a high mortgage that they in fact cannot afford.

To analyze this question I solve the model both without a down payment restriction and with a down payment requirement of at least 20%. Figure 4 plots the difference in expected life-time utility that arises due to this down payment restriction. Positive values indicate welfare improvement due to the restriction while negative values imply that welfare is reduced.

As expected, the standard agent cannot benefit from a minimum down payment restriction. Such a restriction removes some of the options from her choice set so that her welfare is decreased if she chooses this option in the unrestricted model and unaffected otherwise.

This is not true for an agent who suffers from costs of self control. In fact, we see that there are three groups of agents which can be distinguished based on their cash-on-hand in the purchasing period. The poorest agents are worse off with the down payment requirement. The reason for that is that they cannot afford the down payment for a house or only for a much smaller house than they would otherwise choose. The very rich agents, on the other hand, benefit from the down payment requirement. In the unrestricted model they would not choose a down payment smaller than 20% anyway.
so their choices are not affected by the down payment restriction. What is affected, however, is the temptation that they face. In fact, the temptation is significantly reduced by the introduction of a minimum down payment restriction. From a practical point of view, this implies that these agents are better off because without the down payment restriction they see all these very big houses which theoretically they could buy with a big mortgage but which they know are too expensive for them. With the down payment restriction, these houses are not in the budget set anymore so that the agents do not need to exercise self control to resist their temptation.

The third group of agents has a level of cash-on-hand which is neither very low nor very high. When there is no down payment restriction then these agents give in to the temptation and buy a house with a smaller down payment. Hence the introduction of a down payment requirement forces these agents to change their behavior. Nevertheless, these agents do not give in to the temptation completely and hence have to exercise self control. They thus benefit from facing a smaller temptation. This benefit is big enough that they are better off with a down payment restriction even though they have to alter their behavior.

From the analysis in this section we thus see that some present biased agents are better off with a minimum down payment restriction even without considering equilibrium effects. This is strikingly different to the standard agent who can ceteris paribus never be better off if her choice set is restricted.8

8This result is similar to the the result of Amador et al. (2006) who show for a different setup that a minimum savings rule can be optimal for consumers who have a desire for both commitment and flexibility.
7 Conclusion

In this paper I build a model of portfolio choice with focus on the housing and mortgage decision and allow agents to be present biased. I am thus able to analyze how temptation and problems of self control affect the optimal investment portfolio and the housing and mortgage decision in particular. In order to avoid limitations due to a non-convex choice set I model both the housing and the mortgage choice as a continuous control variable. Moreover, I explicitly distinguish between the investment and the utility motive for buying a house and explore for both motives how they interact with the problem of self control. In addition to that I analyze how financial regulation can affect the welfare of present biased agents.

Regarding the effects of temptation and self control on the portfolio decision of households I obtain three predictions. First, the agents overall save less as the problem of self control increases. Second, they predominantly save in illiquid assets such that the illiquid share in overall savings increases. The absolute amount of illiquid savings, however, is not monotonic in self control. Third, as the problem of self control becomes stronger agents tend to buy smaller houses and they do not necessarily choose higher mortgages. Contrary to might be assumed a more severe problem of self control thus does not lead to overinvestment in housing.

On possible limitation of this model is that it does not allow agents to use their house as collateral for borrowing other than through a mortgage. In particular, home equity lines of credit are not part of this model. However, the results of this paper should not be altered if it was possible to take out short term loans against home equity. As Laibson (1997) argues, the possibility to borrow against an illiquid investment reduces its’ effectiveness as commitment device. For the model analyzed here this would imply that even the illiquid investment is at least to some extent accessible in each period and hence leads to temptation and costs of self control. This would reduce the benefits
that a present biased agent gets from investing in a house which hence would further decrease the optimal house size. Adding home equity lines of credit would thus lead to even a stronger negative relation between optimal house size and problems of self control.

Regarding the possible scope for financial regulation I find that a minimum down payment requirement might improve the welfare of present biased agents. In particular, I show that we can differentiate three types of agents depending on their level of cash-on-hand in the purchasing period. The poorest agents have lower welfare if there is a minimum down payment requirement since in this case they cannot afford a house at all or only a very small house. The richest agents gain welfare since their behavior is not affected at all by the restriction but the temptation they face is reduced. Finally, the agents with moderate levels of wealth gain from the down payment restriction even though they have to adjust their behavior. This implies that they give in to the temptation of a low down payment if it is available and would be better off if they didn’t have to face this temptation.

The analysis in this paper hence shows that financial regulation might be able to improve the households’ welfare if agents are present biased. However, it is important to remember that the model here only looks at the optimal household behavior without considering the response of other market participants like banks, rental markets or the housing sector. Without further analysis it is therefore not clear if this result generalizes to a setting where equilibrium effects are taken into account. Moreover, as seen in this analysis, there are both agents who gain and agents who are harmed by restrictions set by financial regulation. In order to judge if a minimum down payment restriction can be beneficial to the society as a whole redistributive considerations and aggregate effects would have to be taken into account. The analysis in this paper does not aim to answer this question. What I want to stress here is rather that even without considering
equilibrium effects present biased agents potentially benefit from financial regulation. This is strikingly different to agents without present bias who can ceteris paribus never be better off if their choice set is restricted by financial regulation.
A Closed Form Solution for Retirement

The closed form solution for the subproblem of retirement can be derived by standard guess and verify methods. From the maximization problem during retirement described in equations (8) and (9) we can derive the intertemporal optimality condition as

\[(1 + \lambda) \cdot u'(c_t) = \beta(1 - p_D)(1 + r_s) \left[ u'(c_{t+1}) + \lambda \left( u'(c_{t+1}) - u'(x_{t+1}) \right) \right] \]

Under the assumption of logarithmic utility this simplifies to

\[(1 + \lambda) \frac{1}{c_t} = \beta(1 - p_D)(1 + r_s) \left[ \frac{1}{c_{t+1}} + \lambda \left( \frac{1}{c_{t+1}} - \frac{1}{x_{t+1}} \right) \right] \]

Using the guess of a linear consumption function \( c(x_t) = \psi \cdot x_t \) and inserting it in the above equation we obtain

\[(1 + \lambda) \frac{1}{\psi x_t} = \beta(1 - p_D)(1 + r_s) \left[ \frac{1}{\psi(1 - \psi)x_t(1 + r_s)} + \lambda \left( \frac{1}{\psi(1 - \psi)x_t(1 + r_s)} - \frac{1}{(1 - \psi)x_t(1 + r_s)} \right) \right] \]

This expression can be solved for the consumption share \( \psi \)

\[\psi = \frac{(1 - \beta(1 - p_D))(1 + \lambda)}{\beta(1 - p_D) + (1 - \beta(1 - p_D))(1 + \lambda)} \]

which is indeed constant.

For the value function we guess that \( W_{ret}(x_t) = A_0 + A_1 \cdot \log(x_t) \). Inserting this together with the linear consumption into equation (8) and again assuming logarithmic
utility we obtain

\[ A_0 + A_1 \cdot \log(x_t) = (1 + \lambda) \log(\psi x_t) - \lambda \log(x_t) \]

\[ + \beta (1 - p_D) \left( A_0 + A_1 \cdot \log((1 - \psi)x_t(1 + r_S)) \right) \]

Simplifying and collecting terms then gives us the results that

\[ A_0 = \frac{1 + \lambda}{1 - \beta(1 - p_D)} \log(\psi) + \frac{\beta(1 - p_D)}{[1 - \beta(1 - p_D)]^2} \log((1 - \psi)(1 + r_S)) \]

\[ A_1 = \frac{1}{1 - \beta(1 - p_D)} \]

This confirms our guess and completes the proof.
References


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Table 1: Parameter Values in Benchmark Model

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>risk aversion</td>
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<tr>
<td>weight of housing in utility</td>
<td>$\theta$</td>
<td>$1/3$</td>
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<td>interest rate on savings</td>
<td>$r_S$</td>
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<tr>
<td>mortgage rate</td>
<td>$r_M$</td>
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</tr>
<tr>
<td>discount rate</td>
<td>$\rho$</td>
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</tr>
<tr>
<td>growth rate of house price</td>
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<tr>
<td>house price in purchasing period</td>
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</tr>
<tr>
<td>expected income</td>
<td>$E(y_t)$</td>
<td>1</td>
</tr>
<tr>
<td>minimum down payment requirement</td>
<td>$\gamma$</td>
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</tr>
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</table>
Figure 1: Optimal decisions for different degrees of problems of self control; house does not give any utility, mortgages are not possible
Figure 2: Optimal decisions for different degrees of problems of self control; house does not give any utility, mortgages possible

Figure 3: Optimal decisions for different degrees of problems of self control; house gives utility, mortgages possible
Figure 4: Difference in expected life-time value between the model with minimum down payment requirement of 20% and the model without down payment restriction.