Housing Market Dynamics with Delays in the Construction Sector

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Abstract

Construction takes time—about six months for a typical single-family house, and over two years for many large commercial buildings. We show that allowing for such construction delays considerably improves the real-estate sector fit of an otherwise standard DSGE model.

JEL Classification: E32, E44, F34, F41
Keywords: House prices, construction, delays

Acknowledgement: We would like to thank Santanu Chatterjee, Jonathan Heathcote, Matteo Iacoviello, Bill Lastrapes, George Selgin, participants of Southern Economic Association meeting in San Antonio (especially Enrique Martinez-Garcia and Alex Nikolsko-Rzhevsky), Midwest Macroeconomics meeting in East Lansing and seminar participants at the University of Georgia, University of Wisconsin-Whitewater, Federal Reserve Bank of Richmond, Davidson College, Virginia Commonwealth University, and University of Richmond for valuable comments and suggestions. All remaining mistakes are our own.

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1 Introduction

Because real estate market developments appear to have played such an important part in the Great Recession, much subsequent research has been devoted to exploring the links between this market and the macroeconomy. Particular attention has been paid to the role of the government and the financial sector in contributing to the housing price increase of 1995-2007 and to evaluating policies that could prevent such events in the future. Most of that research, however, explores the demand side of the real estate market following the finding by Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), and Iacoviello (2005) that collateral constraints (often tied to the value of housing) on firms and/or households lead to amplification of business cycle dynamics. The supply of real estate has, on the other hand, generally been treated either as being fixed or as responding contemporaneously to demand. In the former case, the long-run response of real estate prices to demand innovations tends to be exaggerated, while in the latter the short-run response tends to be downplayed. Neither approach seems appropriate for properly accounting for housing price volatility observed in the data; consequently, most of the existing models of the real estate market are not well suited for policy analysis.

In fact the stock of new houses and commercial buildings does of course respond to growth in anticipated demand. But that response occurs with a lag that reflects both the time it takes to secure permission for new construction—on average about one month in the U.S.—and the time involved in construction itself—around 6 months for a single family home, and more than two years for many larger commercial projects.\(^1\) Moreover, most residential homes are built based on the expectation that a buyer will be found after the unit is built (so called "on spec" construction), thus causing construction companies to often take on the risk of real estate investment. During the 1963-2010 period, 74 percent of houses sold in any given month were either under construction or already completed.\(^2\) Because construction activity must therefore be forward-looking, and also because building costs tend to be “front-loaded,” with returns achieved only after the costs have been sunk, fluctuations in construction output can exacerbate the business cycle.

Our paper augments the strand of recent dynamic stochastic general equilibrium (DSGE) literature on house prices, which either assumes that supply of housing responds to demand contemporaneously (Iacoviello and Neri, 2010; Favilukis, Ludvigson, and Nieuwerburgh, 2011) or that it is fixed (Iacoviello, 2005). Davis and Heathcote (2005) construct a model allowing for a one-year lag in residential con-
struction, but do not study the bearing of this delay on their model’s dynamics, and do not succeed in matching the empirical record of housing market volatility. We extend the existing models by incorporating real estate construction delays with the specific aim of accounting for the historical volatility of the U.S. real estate market, paying particular attention to housing prices.\footnote{The notion that it takes “time to build” certain investment projects has already been pointed out by Kydland and Prescott (1982), Gomme, Kydland and Rupert (2001), Edge (2007) and Fisher (2007). These studies, however, treat residential construction as occurring instantaneously and do not study house prices.} We demonstrate that adding delays to real estate construction significantly improves business cycle properties of the model; this feature, consequently, should be incorporated when studying the interactions between housing market and macroeconomy.

More specifically, our model is a two-sector DSGE model with financial frictions and nominal rigidities in which the construction firms sell newly finished houses several periods after having committed resources necessary to build them. The addition of delays enables us to match the volatility of housing prices and real-estate production seen in the data. Financial frictions play an important role in our analysis; when credit constraints of households and firms are tied to the value of real estate, the response of house prices to shocks get amplified, and the effects of delays become more pronounced. The robustness checks suggest that lags in the real estate production matter more when housing and consumption is more complementary; land has a smaller share in the construction production; and the degree of price and wage rigidity is low.

2 The Basic Model

Our basic model is composed of a closed economy, which is populated by infinitely lived households, entrepreneurs and construction firms. Households consume real estate and all varieties of home goods; perfectly competitive entrepreneurs use labor and capital to produce a continuum of domestic goods. The stock of real estate is augmented by the construction sector.

2.1 Households

A representative household (here denoted by the superscript \( p \) to differentiate it from credit constrained household which we introduce in the full model) maximizes
expected lifetime utility

\[ U_t^p = E_0 \sum_{t=0}^{\infty} (\beta^p)^t \left\{ \left[ (1 - \gamma_t) \frac{C_t^p}{\xi - 1} + \gamma_t \frac{H_t^p}{\xi - 1} \right] \frac{1 - \rho_t}{1 - \Theta} - \frac{[L_t^p]^{1+\chi}}{1 + \chi} \right\}. \]  

(1)

Here \( C_t^p \) denotes the household’s consumption of the composite good, which is aggregated from home varieties using the Dixit-Stiglitz aggregator defined in section 3.3. \( H_t^p \) denotes the stock of housing. Households supply labor services \( L_t^p \) to entrepreneurs and the construction sector.\(^4\) We allow the parameter \( \gamma_t \), which measures the weight on housing in the consumption basket, to vary in order to capture shifts in consumer preferences.

Each household faces the following budget constraint:

\[ Q_t \Delta H_t^p + P_t (C_t^p + I_t^e + I_t^c) - B_t^p = W_t^p L_t^p - R_t^p B_{t-1}^p + R_{K,t}^e K_{t-1}^e + R_{K,t}^c K_{t-1}^c + D_t^c. \]  

(2)

The first term on the left-hand side captures the expenditure on additional housing. \( B_t^p \) represents household’s borrowing, given the nominal interest rate \( R_t^p \). Households own two types of capital, \( K_t^e \) and \( K_t^c \), which they rent out at the rates \( R_{K,t}^e \) and \( R_{K,t}^c \) to the entrepreneurs and the construction sector, respectively, and which evolve (subject to adjustment costs) according to

\[ K_i^t = (1 - \delta) K_{t-1}^i + I_i^t - P_t \frac{\psi}{2} \left[ \frac{I_i^t}{K_{t-1}^i} - \delta \right]^2 K_{t-1}^i, \quad i \in \{e, c\} \]  

(3)

The investment goods \( I_t^e \) and \( I_t^c \) have the same composition as consumption; in the rest of the paper we define \( I_t = I_t^e + I_t^c \). Finally, \( D_t^c \) represents the profits of the construction sector.

Households maximize utility (1) subject to the constraints (2) and (3) by choosing labor effort \( L_t^p \), consumption \( C_t^p \), and investment positions \( B_t^p \), \( \Delta H_t^p \), \( I_t^e \) and \( I_t^c \).

### 2.2 Entrepreneurs

At time \( t \), each entrepreneur hires labor input \( N_t^e \) and capital stock \( K_{t-1}^e \) at the rates \( W_t^p \) and \( R_{K,t}^e \), respectively, to produce one of the varieties of the domestic good:

\[ Y_t = A_t^e \left[ K_{t-1}^e \right]^\mu [N_t^e]^{1-\mu} \]  

(4)

\(^4\)Following Woodford (2003), Chapter 3, we model monetary policy as directly targeting interest rates and therefore drop real balances from the consumer utility function.
Each entrepreneur maximizes

\[
E_0 \sum_{t=0}^{\infty} [\beta^{t}] E \left\{ \frac{[C_t]^1 - \Theta}{1 - \Theta} \right\},
\]

subject to (4) and the following flow of funds constraint:

\[
P_t^e Y_t = P_t^e C_t^e + W_t^p N_t^e + R_{K,t}^e K_{t-1}^e.
\]

\(C_t^e\) denotes the entrepreneur’s consumption, and \(P_t^e\) is the wholesale price of output.

### 2.3 Construction Sector

The perfectly competitive domestic construction sector adds to the stock of real estate by using capital, labor and (a fixed amount of) land to produce new buildings and structures \(S\):

\[
S_{t+k} = A_t^c \left[ K_{t-1}^c \right]^{\mu^c} \left[ N_t^c \right]^{1-\mu^c-\eta} T^\eta
\]

We assume that due to the lags in the construction sector, resources for new real estate production must be committed at time \(t\), but the buildings will not be completed until time \(t + k\). The construction sector in effect takes on the risk of unexpected future fluctuations in the price of real estate. The profits (which may deviate from zero because of this risk) therefore take the form

\[
D_t^c = Q_t S_t - W_t^p N_t^c - R_{K,t}^c K_{t-1}^c.
\]

We additionally assume full commitment on the part of the construction sector, in the sense that once a project is started, it must be carried out to completion regardless of the prevailing market conditions. According to the Bureau of Economic Analysis, 60 percent of the total cost of a two-quarter residential construction project is committed during the first quarter.\(^5\) Therefore, the assumption that all costs of a new project are incurred upfront does not compromise our results and allows us to study the price dynamics in a more succinct way.

It is worth noting that delays impart a different dynamic to real estate construction as compared to investment adjustment costs. While both lower the responsiveness of construction to economic shocks, time delays, coupled with full commitment of resources at the beginning of the project, expose the construction sector to the risk of not being able to recover the costs due to unforeseen shifts in demand. We

\(^5\)NIPA Handbook, Chapter 6.
demonstrate in section 4 that adjustment costs already imbedded in the structure of the construction sector (land in the production function and capital stock adjustment costs in equation (3)) cannot by themselves account for the observed volatility of housing prices.\footnote{\textit{As an exercise, we include adjustment costs directly in the construction sector production function. We find that, as a result, the volatility of house prices and construction output change only marginally.}}

We abstract from delays associated with real estate sales and assume that all new buildings that come on the market at time $t$ are sold in the same period. The aggregate stock of real estate available to households evolves according to the following transition equation:

$$H_{t+1}^p = (1 - \delta^h) H_t^p + S_t.$$ 

As a robustness check (and for easier comparison of our findings to the existing literature), we also consider a version of the model with a fixed stock of housing: $H_t^p = \bar{H}$. 

Construction firms maximize (6) subject to (5) by choosing $B^c_t$, $K^c_{t-1}$ and $N^c_t$.

### 2.4 Equilibrium

To close the model, we assume that the central bank credibly targets domestic inflation and the output gap by adopting a variation of the Taylor rule commonly used in the monetary literature:

$$\ln (R_t^p) = (1 - \rho_i) \ln (\bar{R}^p) + \rho_i \ln (R_{t-1}^p) + (1 - \rho_i) \left[ \rho_p \pi_t + \rho_g gdp_{t}^{gap} \right] + \varepsilon_{i,t} \quad (7)$$

$\bar{R}^p = \frac{1}{\beta^p}$ is the steady state level of the interest rate, and $\rho_i$ measures the degree of interest rate inertia. We define GDP as the sum of the two sectors’ outputs:

$$GDP_t = Y_t + Q_t S_t$$

The following market clearing conditions complete the description of the economy:

$$Y_t = C_t^p + C_t^e + I_t \quad (8)$$

$$L_t^p = N_t^c + N_t^e \quad (9)$$

Equilibrium in the economy is defined by (8)-(9) and the first order conditions of the agents (described in Appendix A), given the form of monetary policy rule described above. The model is solved numerically using Dynare (see Collard and Juillard, 2003).
2.5 Calibration

We follow the existing literature as closely as possible in setting most of the parameter values (summarized in Table 1). The choice of calibrated and estimated parameters comes from the full model, described in Section 3; we list their values below but defer the discussion of most of these parameters until the later section.

Each time period in the model corresponds to one quarter. We set the $\beta^p = 0.99$, resulting in a 4% annual real rate of return.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\beta^p$</td>
<td>Discount factor of patient households</td>
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<td>$\Theta$</td>
<td>Relative risk aversion</td>
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<td>$\chi$</td>
<td>Inverse of Frisch labor elasticity</td>
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<td>$\xi$</td>
<td>Elasticity of substitution between housing and consumption</td>
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<tr>
<td>$\mu$</td>
<td>Capital share in the entrepreneurial production</td>
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<td>$\psi$</td>
<td>Capital adjustment costs</td>
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<tr>
<td>$\mu^c$</td>
<td>Capital share in construction</td>
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<td>$\eta$</td>
<td>Land share in construction</td>
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<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
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<tr>
<td>$\delta^h$</td>
<td>Real estate depreciation rate</td>
<td>0.013</td>
</tr>
</tbody>
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Table 1: Benchmark parameter values

Households’ preferences are described by $\chi = 3$ and $\Theta = 1$, both of which are fairly standard in the literature. The value of $\xi$, on the other hand, is more contentious. Davis and Heathcote (2005), Iacoviello (2005) and Favilukis, Ludvigson and Van Nieuwerburgh (2010) set $\xi = 1$. Gete (2009) uses $\xi = 0.9$, while several other studies estimate the parameter to be well below unity: $\xi = 0.59$ as reported in Song (2010), 0.45 in Stokey (2009) and 0.3 used in Kahn (2008). Moreover, we note that the parameters $\xi$ and $\gamma$ (the latter captures the share of income spent on housing) jointly determine the steady state ratio of housing to GDP, which is equal to 1.33 for the U.S. (average over the 1974-2007 period, see Table 3). In our model, the combinations of $\{\xi, \gamma\}$ required to match this target are $\{0.9, 0.1\}$, $\{0.5, 0.3\}$ and $\{0.3, 0.7\}$. Based on the Consumer Expenditure Survey, the share of income spent on housing has fluctuated between 26 and 29 percent during the 1984-2009 period. We simulate the benchmark version of the model for $\{\xi, \gamma\} = \{0.9, 0.1\}$ and defer robustness checks of our results to these values until Section 4.

We set $\mu = 0.35$ and $\mu^c = 0.20$. We calibrate the adjustment parameter $\psi$ to
match the volatility of capital investment in the data and set it equal to 4. The amount of land available to the construction sector (which serves as an investment adjustment cost) is set to unity: $T = 1$.

We follow Taylor (1993) in setting the parameters of the monetary response function: $\rho_i = 0$, $\rho_\pi = 1.5$ and $\rho_y = 0.5$, and compute the shocks $\varepsilon_{t,t}$ as the difference between the actual interest rate and that implied by (7). It is worth noting that, in the absence of frictions, monetary policy shocks do not affect real variables in the basic model. Finally, the estimates of the three other shocks processes are listed in Appendix C.

### 2.6 Impact of Construction Sector Delays

We illustrate the basic mechanism through which construction delays affect the dynamics of housing variables using the example of the housing demand shock; to this end, Figure 1 plots the response of the real estate stock and real housing prices to a one-standard-deviation increase in $\gamma$ for three different model specifications: construction sector with no delays, and 2- and 4-period delay.

In the version of the model with $k = 0$, construction sector responds immediately to the increase in demand; housing stock going up on impact and continues to grow for the next 40 periods, finally leveling off (before reverting to its steady state level) at 0.5 percent. Increasing $k$ to two or four quarters not only delays the response of the construction sector (and consequently of the housing stock), but also mutes its magnitude: construction firms endogenize the fact that demand for new housing will decline by the time they are able to bring new buildings to the market. Aggregate housing stock is lower (by about a fifth) vis-à-vis the no-delay case, causing housing prices to be higher in each of the 40 periods.

Even though qualitatively the mechanism is quite intuitive, its quantitative effect on housing sector variables seems small. The basic model presented in the previous section lacks the well-documented financial accelerator mechanism that amplifies the responsiveness of most key variables to economic disturbances. This mechanism (first outlined in Bernanke, Gertler and Gilchrist, 1996, and later applied by Iacoviello, 2005, to housing prices) ties borrowing limits of credit-constrained agents - be it consumers or businesses - to the price of real estate. Thus, a positive development in the economy increases the demand for housing both through the positive wealth effect and, as a result of higher real estate prices, through the collateral effect by increasing borrowing limits of households and/or firms.

In the next section, we show that construction sector delays, embedded in a richer model of the economy, lead to very significant changes in the business cycle properties
Figure 1: Impulse response functions of housing stock and prices following a housing demand shock: the impact of construction delays. Vertical axis shows percent deviations from the steady state.

of housing market variables and enable us to bring their theoretical moments much closer to the data.

3 The Full Model

While the basic model, outlined in the previous section, is useful for illustrating the impact of construction delays on the dynamics of housing prices and the stock of real estate, it cannot be expected to match the data to a reasonable degree. Therefore, we extend the basic model to incorporate several features that have been shown to be important in replicating the behavior of the macroeconomy. Following Bernanke, Gertler and Gilchrist (1999) and Iacoviello (2005), we add collateralized borrowing on the three agents in the model, and introduce nominal (price and wage) rigidities that enable the monetary policy to influence the real economy. From the modeling point of view, our paper is closely related to Iacoviello and Neri (2010), who develop a New Keynesian DSGE model that links real estate and borrowing constraints in
a general equilibrium framework. We extend their setup by carefully modeling the features of the construction sector discussed in the previous section.

More specifically, the model of the previous section is augmented with the financial accelerator mechanism. We assume that a portion of households, which we call impatient, are credit-constrained; in addition, entrepreneurs and construction firms are also able to participate in the loanable funds market using real estate as collateral. We model financial frictions by assuming that banks protect themselves from the risk of borrowers’ default by requiring that all loans be collateralized, as in Kiyotaki and Moore (1997).

We introduce price and wage rigidity by assuming that entrepreneurial goods and household labor are resold by retailers and labor unions, respectively, at a mark-up; retail prices and wages are sticky à la Calvo (1983).

In the full model, construction sector output is meant to capture all new buildings (residential or commercial) that are produced in the economy in any given period. The term housing will be used to describe the stock of buildings held by households; we will use the term commercial structures to refer to real estate holdings of businesses. Finally, capital investment will denote additions to the stock of software and equipment owned by businesses. Thus, our division of the economy into the two sectors (construction and entrepreneurial output) is somewhat different from that used in the papers, mentioned in the introduction, that study time delays: in the latter, the distinction is made between investment in (residential) housing and that in the combined measure of commercial structures and software and equipment. Therefore, some caution should be used when comparing business cycles of our model to those of the existing studies.

Since the framework we use is well documented in the literature, we refer the reader to Appendix A for the equilibrium equations of the model.

### 3.1 Impatient Households

Impatient households maximize utility

$$U_t^m = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left[ (1 - \gamma_t) \left[ C_t^m \right]^{\frac{\theta-1}{\epsilon}} + \gamma_t \left[ H_t^m \right]^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1-\epsilon}{\epsilon}}}{1 - \Theta} - \frac{[L_t^m]^{1+\chi}}{1 + \chi} \right\}$$

subject to

$$Q_t \Delta H_t^m + P_t C_t^m + R_{t-1} m B_{t-1} = \tilde{W}_t^m L_t^m + B_t^m,$$  \hspace{1cm} (10)
and an additional credit constraint that limits the amount of borrowing, which cannot exceed a fraction $l_t^m$ of the expected discounted value of their housing:

$$B_t^m \leq l_t^m E_t \left[ \frac{Q_{t+1} H_t^m}{R_t^m} \right].$$

(11)

Mortgage loans are refinanced each period. We allow $l_t^m$ to be time-dependent so that we can study the effects of financial liberalization that took place in the U.S. during the last twenty years.

We assume that $\beta^p$ is sufficiently high to induce patient households to save in the steady state. Moreover, setting $\beta^m < \beta^p$ implies that impatient households discount future more heavily than the patient households and guarantees that the former are constrained in and around the steady state. We make sure that the impatience level of households will outweigh the interest rate for loans. Specifically, the fact that $1/\beta^m > R^m$ causes the Lagrange multiplier on the credit constraint (11) to be greater than zero. Therefore, the borrowing constraint will hold with equality

Impatient households maximize utility subject to the constraints (10) and (11) by choosing labor effort $L_t^m$, consumption $C_t^m$, and investment positions $B_t^m$ and $\Delta H_t^m$.

### 3.2 Entrepreneurs and Construction Firms

Both production functions are augmented to reflect the addition of impatient households. We now assume that $N_t^e = [N_t^{p,e}]^\alpha [N_t^{m,e}]^{(1-\alpha)}$, where $N_t^{p,e}$ and $N_t^{m,e}$ are the patient and impatient household labor efforts in the entrepreneurial sector. Similarly, $N_t^c = [N_t^{p,c}]^\alpha [N_t^{m,c}]^{(1-\alpha)}$. Parameter $\alpha$ measures the relative size of each group. We let hours of the two households enter the production function in a Cobb-Douglas fashion, which implies complementarity across the labor skills of the two groups. This form for the production function is also motivated by the fact that credit-constrained households typically have low incomes. Hence, their share in labor income, $1 - \alpha$, is lower than that of the patient households.\footnote{The version of the model with perfectly substitutable labor inputs, $N_t^i = N_t^{p,i} + N_t^{m,i}$ for $i \in \{e, c\}$, produces a much worse fit of housing construction vis-à-vis the data, and significantly overestimates the volatility of capital investment.}

In addition to labor and capital, entrepreneurs use the stock of structures in the production function, which now takes the form

$$Y_t = A_t^e \left[ K_{t-1}^e \right]^\mu \left[ N_t^e \right]^{1-\mu-v} \left[ H_{t-1}^e \right]^{v}.$$  

We assume that entrepreneurs and construction firms can borrow at the rates $R_t^e$ and $R_t^c$, respectively, but the amount of borrowing available to the two sectors sector
is limited to a fraction of the present discounted value of their real estate:

\[ B^c_t \leq l^c E_t \left[ \frac{Q_{t+1}^c}{R^c_t} \right], \]

\[ B^c_t \leq l^c E_t \left[ \frac{Q_{t+1}^c S_{t+1-k}}{R^c_t} \right]. \]

In the presence of credit constraints, entrepreneurs can choose to postpone consumption and quickly accumulate enough capital so that their credit constraint becomes nonbinding. In order to make sure that entrepreneurial self-financing does not arise, we assume that entrepreneurs discount the future more heavily than patient households: \( \beta^e < \beta^p \). As for impatient households, this guarantees that the credit constraint is binding in and around the steady state.

### 3.3 Nominal Rigidities

To introduce price rigidity, we model a retail sector following Bernanke, Gertler and Gilchrist (1996). Each of the monopolistically competitive retailers (indexed by \( r \) on the interval \([0,1]\)) buys one of the varieties produced by the entrepreneurs and resells it to the consumers and investors at an optimally determined price \( P_t(r) \). These varieties are bundled into a composite good using the Dixit-Stiglitz aggregator

\[ Y_t = \left[ \int_0^1 Y_t(r) \frac{\sigma-1}{\sigma} dr \right]^{\frac{1}{\sigma-1}}, \]

where \( \sigma > 1 \). This composite good can then be used for consumption or investment.

As in Calvo (1983), retailers reset their prices each period with a constant probability \((1 - \omega_p)\); otherwise, the old prices remain in effect. If a retailer \( r \) gets to announce a new price in period \( t \), she chooses \( \tilde{P}_t(r) \) to maximize her expected discounted future profits

\[ E_t \sum_{j=t}^{\infty} [\beta^p]^{j-t} \frac{\partial^p_j}{\partial^p_t} \omega_p^{j-t} \left\{ \left[ \tilde{P}_t(r) - P_j^c \right] Y_j^d(r) \right\}, \]

where \( \partial^p_t \) is the patient households’ marginal utility of consumption. Given the price-setting behavior of individual firms, the aggregate price index of the consumption bundle can be written as

\[ P_t^{1-\sigma} = (1 - \omega_p) \tilde{P}_t^{1-\sigma} + \omega_p P_{t-1}^{1-\sigma}. \]
Wage rigidity is modeled analogously. Households sell their labor services to unions, who then bundle and resell them to entrepreneurs and construction sector at the optimally determined rate $\bar{W}_t^j(u)$ for $j \in \{p, m\}$. Labor varieties are bundled into a composite good according to $L_t^j = \left[ \int_0^1 L_t^j(u) \frac{\phi - 1}{\phi} du \right]^{\frac{1}{\phi-1}}$. Labor unions reset their wages each period with a constant probability $(1 - \omega_w)$; otherwise, the old wages remain in effect. Similar to the price of the bundle of goods, the expression for aggregate wage can be written as

$$[W_t^j]^{1-\phi} = (1 - \omega_w) [\bar{W}_t^j]^{1-\phi} + \omega_w [W_{t-1}^j]^{1-\phi}.$$ 

The profits of retailers and unions get rebated as lump-sum payments to patient households.

### 3.4 Market Clearing

The banks link domestic savers to borrowers. The banks maximize their period $t + 1$ profits

$$B_t^m R_t^m + B_t^e R_t^e + B_t R_t^c - \kappa (B_t^m + B_t^e + B_t^c) - (-B_t^p) R_t^p$$

subject to

$$B_t^m + B_t^e + B_t^c \leq (-B_t^p),$$

by choosing the deposit/loan portfolio $(B_t^p, B_t^m, B_t^e, \text{and } B_t^c)$. We assume that banks face a fixed cost of intermediation $\kappa$ when lending to borrowers. Perfect competition implies that equilibrium bank profits are zero; solving the maximization problem, we derive the following expression for the interest rate spread:

$$R_t^m = R_t^p + \kappa$$  \hspace{1cm} (12)

We assume for simplicity that $R_t^m = R_t^e = R_t^c$.

Finally, we require the labor and housing markets to clear:

$$L_t^p = N_t^p + N_t^p$$
$$L_t^m = N_t^m + N_t^m$$
$$H_t = H_t^p + H_t^m + H_t^e,$$

where $H_{t+1} = (1 - \delta^h) H_t + S_t$.  

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8Modeling wage rigidity is necessary to generate a decline in residential investment following a domestic interest rate shock (observed in the data). This result is in line with findings of Barsky, House and Kimball (2007) and Iacoviello and Neri (2010).
3.5 Calibration

The goal of this paper is to test how well the model (variants of which are frequently used for policy analysis) can replicate the U.S. data on housing and construction. To this end, we now carefully describe our choice of the structural parameters and the estimation techniques used to obtain stochastic processes that drive the model dynamics.

Parameters describing household preferences are the same as in Table 1. We next turn to the discount factors of impatient households, entrepreneurs and the construction sector. While the values of these parameters have limited effects on the dynamics of the model, it is important to set them such that the credit constraints are binding in and around the steady state. We set $\beta^m = 0.97$, which lies within the range of empirical estimates (see Iacoviello, 2005, and the papers referenced therein). For simplicity, we assume $\beta^m = \beta^e = \beta^c$.

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<tbody>
<tr>
<td>$\beta^m$, $\beta^e$, $\beta^c$</td>
<td>Discount factors of constrained agents</td>
<td>0.97</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Income share of patient households</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of substitution between goods varieties</td>
<td>3.9</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of substitution between labor varieties</td>
<td>4.3</td>
</tr>
<tr>
<td>$v$</td>
<td>Housing share in the entrepreneurial production</td>
<td>0.10</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Capital share in the entrepreneurial production</td>
<td>0.25</td>
</tr>
<tr>
<td>$\omega_p$, $\omega_w$</td>
<td>Degree of nominal rigidity</td>
<td>0.67</td>
</tr>
<tr>
<td>$l^m$</td>
<td>LTV ratio for impatient households</td>
<td>0.77</td>
</tr>
<tr>
<td>$l^e$, $l^c$</td>
<td>LTV ratio for entrepreneurs and the construction sector</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Table 2: Parameter values of the extended model.

The parameters on the production side are chosen to bring the steady state of the model as close as possible to several target ratios we compute from the data, while still keeping these parameters within the range used by business cycle literature. The share of housing in production, $\nu$, controls the ratio of commercial structures to GDP. The value of this parameter is constrained by the observed share of labor income in output, which on average has been just above 70% since the 1970, as reported in Gomme and Rupert (2004); we set $\nu = 0.1$, resulting in a 35% share of capital.

---

9In order for the borrowing constraints to be binding at all times, we must require that the Lagrange multipliers $\Lambda^m$, $\Lambda^e$ and $\Lambda^c$, described in Appendix A, be positive. The necessary condition, which follows from combining Euler equation of impatient households with equation (12), is $\beta^m < \left[\frac{1}{\sigma} + \kappa\right]^{-1}$ in the steady state. The same inequality applies to $\beta^e$ and $\beta^c$.
Variable Interpretation Data Model
---
$C/GDP$ Consumption share of GDP 80% 79%
$S/GDP$ Construction share of GDP 10% 11%
$I/GDP$ Software & equipment investment 10% 10%
$qH_t^c/(4xGDP))$ Commercial structures, % of GDP 0.86 0.72
$q(H_t^p+H_t^c)/(4xGDP))$ Housing, % of GDP 1.37 1.33

Table 3: Steady state targets used in model calibration

income. With $\nu = 0.1$, we lower the capital share $\mu$ to 0.25. Following Kahn (2008), we let $\eta = 0.5$; this parameter represents the fixed inputs in construction and serves to bring the volatility of construction output in line with the data. As is common in the business cycle literature, we assume $\delta = 0.02$; we set the housing depreciation rate at 1.3 percent per quarter, as in Greenwood, Hercowitz and Krussell (1997). Based on Iacoviello and Neri (2010), we set $\alpha$, the wage income share of the patient households, to 0.79.

Next, we calibrate elasticities of labor supply and varieties substitution. We set $\sigma = 3.9$ and $\phi = 4.3$ (the latter two resulting in a 35 percent and 30 percent markups of price over marginal cost and wage over labor disutility, respectively). We set the price and wage stickiness parameters $\omega_p = \omega_w$ to 0.67, which implies that prices and wages are reset on average every three quarters, well within the range reported by Christiano, Eichenbaum and Evans (2005).

The financial sector in the model is described by the loan-to-value (LTV) ratios of the different sectors and the bank overhead costs. To estimate the LTV ratio for entrepreneurs and the construction sector, we use the value of credit market instruments as a fraction of total financial assets in the nonfinancial business sector (Table L.101 of the Flow of Funds Accounts of the United States). This ratio declined from around 100 percent to just over 60 percent between 1974 and 2010; we set $l^e = l^c = 0.68$. We conservatively assume that $\kappa = 0.009$, somewhat higher than the 233 basis point (annualized) average spread between effective Federal Funds Rate and Bank Prime Lending Rate during 1974Q1-2007Q4, to bring the volatility of business investment relative to output closer to the data.

Overall, the model does quite well in matching the steady state targets, with the exception of commercial structures (to match this value, we would have to increase the share of buildings in construction, which would consequently lower the share of labor income in output to below the observed values).

We describe the shock estimation process in Appendix C; here we briefly summa-
rize the results of the estimation. We set $\sigma (\varepsilon_i) = 0.0028$. For the housing demand shock process, we find $\rho_\gamma = 0.98$ and $\sigma (\varepsilon_\gamma) = 0.02$. Impatient households’ LTV is set to $l^m = 0.77$ during the 1974Q1-1982Q4 period; we increase it to 0.89 for 1983-1989, and finally let $l^m = 0.92$ after 1990. Finally, technology in the two sectors evolves according to

$$\begin{pmatrix}
\ln \hat{A}_t^e \\
\ln \hat{A}_t^c
\end{pmatrix} = \begin{pmatrix}
0.66 & 0.00 \\
0.00 & 0.66
\end{pmatrix} \begin{pmatrix}
\ln \hat{A}_{t-1}^e \\
\ln \hat{A}_{t-1}^c
\end{pmatrix} + \begin{pmatrix}
\varepsilon_{a,t}^e \\
\varepsilon_{c,t}^c
\end{pmatrix}$$

with $\sigma (\varepsilon_a^e) = 0.007$ and $\sigma (\varepsilon_a^c) = 0.020$.

4 Delays in the Full Model

4.1 Business Cycle Properties

We use the policy functions resulting from the numerical solution to the linearized model to simulate the dynamic behavior of housing market variables by feeding in the series of the five shocks described above. We begin our analysis of the results by first studying the impact of construction delays on business cycle properties of housing prices and construction output. We then test the robustness of our findings to several key parameter values.

Table 4 compares moments of the data with those of the model for three different construction lags ($k = 0, 2, 4$) and the no construction set up ($k = \infty$). Although in the rest of the paper we focus on house prices and construction output dynamics, here we report a fairly comprehensive set of business cycle moments.

The version of the model with a fixed stock of real estate overstates the volatility of both consumption and housing prices, and is, of course, unequipped to explain the moments of the construction output. By introducing a contemporaneous response of construction to economic developments, we are able to bring the volatility of consumption closer to the data, but at the cost of underestimating the variance of capital investment and housing prices. Moreover, the simulated series of construction output is too volatile.

We find that longer delays in construction can address these mismatches: the house price volatility (relative to GDP) increases by a third from 0.70 to 0.94, the value observed in the data. This increase is a result of the decline in the responsiveness of the construction production to economic shocks, which, at 2.82 percent,

---

10 We also study the performance of the model for the case of the small open economy. Adding the estimated shock process for the world interest rate to the model produces quantitatively negligible impact on the moments or dynamics of the variables of interest.
is just slightly above the value in the data. The reason behind this result is quite simple: following, for example, an unexpected adverse shock, houses already under construction continue to be built, thus lowering the volatility of construction sector output. At the same time, this sluggishness in supply, coupled with much faster response of demand, increases housing price volatility.

The model is not as successful in matching the variables correlations. In the data, GDP and construction output comove positively: the contemporaneous correlation is equal to 0.88. Since longer delays lower the responsiveness of the construction sector to economic fluctuations, they lead to smaller correlation between GDP and $S$. Delays help to lower the correlation between house prices and GDP from 0.92 to 0.86, but even so it remains significantly higher than its counterpart in the data. Finally, the model with 2- and 4-period delay in construction underpredicts the correlation between $q$ and $S$.

The closest parallel to this exercise in existing literature can be found in Davis and Heathcote (2005), who incorporate a one-year delay in construction into an RBC model and study its dynamics using shocks computed from the data. While the focus of their study is somewhat different from ours, the authors are unable to replicate the observed high volatility of housing prices in the U.S. Our model extends their work in two dimensions. First, we incorporate credit constraints that are linked to the value of agents’ real estate holdings à la Iacoviello (2005). As our results indicate, both construction delays and the financial accelerator are important in matching the dynamics of housing market variables. Second, we add more shocks to capture the changes in monetary policy and housing demand that are present in the data.
Our framework is closely related to that of Iacoviello and Neri (2010), who are able to match the business cycles of housing market variables in the U.S. without assuming construction sector delays. The authors’ approach is different from ours: they estimate a large number of shocks (some of which are unobservable) using Bayesian techniques, whereas we calculate the shocks directly from the data. While our methodology limits the number of shocks we can estimate, it allows us to study the importance of construction delays by changing the lag structure in the construction sector while keeping all other parameter and shock values fixed.

4.2 Parameter Sensitivity

We have demonstrated that the model with two period delays in construction, simulated using several series of shocks computed from the data, can generate the observed high volatility of housing prices and construction output (in contrast, the specification with no delays in construction understates housing price volatility while generating too much variability in the construction sector). Here we test the sensitivity of our results to several important parameters of the model. To this end, Figure 2 plots impulse responses of real housing prices and housing stock to a positive productivity shock in the entrepreneurial sector.\footnote{For space considerations, we choose to display the impulse responses only for a positive shock to $A^e$, which essentially works as a demand shock for housing. In our full model we have two other demand shocks—preferences $\gamma$ and monetary policy shock—that qualitatively generate similar responses.} Higher real wages that result from the increase in $A^e$ push up the demand for housing. The increase in demand raises both house prices and the returns to housing investment, thus causing the housing stock to rise.

We first discuss how our results change depending on the value of $\xi$, the elasticity of substitution between housing and consumption. For $\xi > 1$, the two goods are substitutes. In response to a positive productivity shock in the consumption sector, the price of the consumption good declines; households find it easier to shift their demand towards the consumption good when $\xi > 1$, thus lowering their demand for housing and so the volatility of $q$, the price of housing relative to consumption. Consequently the addition of delays in the construction sector generates a smaller increase in relative house price volatility when $\xi$ is greater than one (vis-à-vis the benchmark calibration reported in Table 4). Nonetheless, we find that delays still matter for the dynamics of housing market variables. For example, setting $\xi = 1.5$, we achieve a 25 percent increase in house prices volatility (relative to GDP) when we increase construction sector delays from zero to two periods.

Next, we turn to the impact of $\eta$, the land share in construction. In our model,
land is fixed and so limits the extent to which the housing stock can be adjusted in any given period. As the second row of Figure 2 illustrates, when the share of land is higher, relative house prices are more volatile, and housing stock responds less to economic shocks compared to the setup with \( \eta = 0.1 \). Since construction sector response is already limited by the high share of the fixed inputs, adding delays leads only to a marginal increase in \( q \) and a small drop in \( H \). Conversely, when we lower the land share, construction delays become more important in generating volatility in house price. For example, when we set \( \eta = 0.1 \), the relative house price volatility increases by 46 percent going from \( k = 0 \) to \( k = 2 \) (as compared to 34 percent increase when \( \eta \) is equal to 0.5, reported in Table 4).

Finally, we turn to the role of price and wage rigidities in driving the differences in house price volatility for different delay specifications. Lower degree of nominal rigidities amplifies the response of housing prices to economic disturbances; it also makes the impact of construction delays more pronounced. This result is largely due to the decline in output volatility when wages and prices are flexible.
5 Conclusion

A documented but underutilized property of the construction sector is the existence of time delays between the commencement and completion of a project, which can vary from 6 months for a single-family house to over two years for a large commercial building. We introduce this feature into an otherwise standard monetary DSGE model of housing market in order to better understand the joint dynamics of construction output and housing prices in the U.S.

We find that the model with two period delays in construction, simulated using several series of shocks computed from the data, can generate the observed high volatility of housing prices and construction output (in contrast, the specification with no delays in construction understates housing price volatility while generating too much variability in the construction sector). Our main results turn out to be quite robust to changing key parameter values used in calibrating the model. Thus, it appears that delays in the construction sector should be an essential feature in any model that investigates the relationship between housing market variables and aggregate economic activity.

Our paper opens several avenues for future research. The framework can be extended to explain the observed differences between residential and commercial construction. It is likely that capital, labor and land intensities differ between the two subsectors, and the duration of construction delays is longer in the commercial construction. Insofar as the share of labor in the production of real estate has important implications for its dynamics (since it is the most volatile of the inputs), it may be possible to explain the differences between residential and commercial investment by carefully calibrating their production functions. Moreover, we hope to extend the results of the reported exercises to the international setting and contribute to the debate about the differences in the behavior of housing market variables across the OECD countries.
References


A Solution to Model

Patient household FOCs.

\[ \frac{1}{R_t^p} = \beta^p E_t \left[ \left( \frac{C_t^p}{C_t^p} \right)^{\frac{1-\theta}{\xi}} \left( \frac{C_{t+1}^p}{C_t^p} \right)^{-\frac{1}{\xi}} \frac{1}{\Pi_{t+1}} \right] \]

\[ [L_t^p]^\gamma = (1 - \gamma) \frac{C_t^p}{[C_t^p]}^{\frac{1-\theta}{\xi}} \frac{1}{\Pi_{t+1}} \frac{W_t^p}{P_t} \]

\[ (1 - \gamma) \left( C_t^p \right)^{-\frac{1}{\xi}} \frac{Q_t}{P_t} \left( 1 + \phi^p \left( \frac{H_t^p}{H_{t-1}^p} - 1 \right) \right) = \gamma \left( H_t^p \right)^{-\frac{1}{\xi}} + \]

\[ (1 - \gamma) \beta^p E_t \left[ \left( \frac{C_{t+1}^p}{C_t^p} \right)^{\frac{1-\theta}{\xi}} \left[ \frac{C_{t+1}^p}{C_t^p} \right] \right]^{-\frac{1}{\xi}} \left\{ 1 + \phi^p \left( \frac{H_{t+1}^p}{H_t^p} - 1 \right) \frac{H_{t+1}^p}{H_t^p} - \frac{\phi^p}{2} \left( \frac{H_{t+1}^p}{H_t^p} - 1 \right)^2 \right\} \]

\[ \left\{ 1 + \psi \left( \frac{K_t^p}{K_{t-1}^p} - 1 \right) \right\} = \beta^p E_t \left[ \frac{C_{t+1}^p}{C_t^p} \right]^{-\frac{1}{\xi}} \left[ \frac{C_{t+1}^p}{C_t^p} \right]^{-\frac{1}{\xi}} \left\{ 1 - \delta + \frac{R_{K,t-1}^p}{P_t} + \psi \left( \frac{K_{t+1}^p}{K_t^p} \right)^2 - 1 \right\} \]

We have defined

\[ C_t^j \equiv (1 - \gamma) \left( C_t^j \right)^{\frac{1-\theta}{\xi}} + \gamma \left( H_t^j \right)^{\frac{1-\theta}{\xi}}, \quad j \in \{p, m\} \]

A.1 Impatient Households FOCs

\[ [C_t^h]^{-\theta} = \beta^h R_t^h E_t \left[ \frac{[C_{t+1}^h]^{-\theta}}{\Pi_{t+1}} \right] + \lambda_t^h R_t^h \]

\[ [L_t^h]^\gamma = [C_t^h]^{-\theta} \frac{W_t^h}{P_t} \]

\[ [C_t^h]^{-\theta} \frac{Q_t}{P_t} \left\{ 1 + \phi^h \frac{\Delta H_t^h}{H_t^h} \right\} - \lambda_t^h m_t^h E_t Q_{t+1} = \]

\[ = \gamma \left( H_t^h \right)^{-\theta} + \beta^h E_t [C_{t+1}^h]^{-\theta} \frac{Q_{t+1}}{P_t} \left\{ 1 + \phi^h \frac{\Delta H_{t+1}^h}{H_{t+1}^h} \right\} \]

A.2 Entrepreneurs FOCs

The first-order conditions for the entrepreneur’s problem are the consumption Euler equation, the demand equations for labor of patient and impatient households, the
capital demand equation, and the real estate demand:

\[
[C^e_t]^{-\Theta} = \beta^e R^e_t E_t \left[ \frac{[C^e_{t+1}]^{-\Theta}}{\Pi_{t+1}} \right] + \lambda^e_t R^e_t
\]

\[
W^p_t N^p_{e,t} = \alpha (1 - \mu - \nu) P^e_t Y_t
\]

\[
W^h_t N^h_{e,t} = (1 - \alpha) (1 - \mu - \nu) P^e_t Y_t
\]

\[
R_{K,t} K^e_{t-1} = \mu P^e_t Y_t
\]

\[
[C^e_t]^{-\Theta} Q_t \left\{ 1 + \phi^e \frac{\Delta H^e_t}{H^e_{t-1}} \right\} - \lambda^e_t m^e_t E_t Q_{t+1} =
\]

\[
= \beta^e E_t \left[ C^e_{t+1} \right]^{-\Theta} \left( \nu \frac{P^e_{t+1} Y_{t+1}}{P^e_t H^e_t} + \frac{Q_{t+1}}{P^e_{t+1}} \left\{ 1 + \frac{\phi^e}{2} \left[ \left( \frac{H^e_{t+1}}{H^e_t} \right)^2 - 1 \right] \right\} \right)
\]

### A.3 Construction Sector FOCs

\[
\lambda^c_t = \frac{1}{R^c_t} - \beta^c
\]

\[
W^p_t = \alpha (1 - \eta) (\beta^c)^{k-1} \frac{I^c_t}{N^p_{e,t}} \left( \Lambda^c_{t+k-1} m^c_{t+k-1} + \beta^c \right) E_t Q_{t+k}
\]

\[
W^h_t = \frac{(1 - \alpha) (1 - \eta) (\beta^c)^{k-1}}{N^h_{e,t}} \frac{I^c_t}{N^p_{e,t}} \left( \Lambda^c_{t+k-1} m^c_{t+k-1} + \beta^c \right) E_t Q_{t+k}
\]

Here \( \beta^c \) is the rate at which the construction sector discounts the future; in order to make the credit constraint binding at all times, we require that \( \beta^c \) be sufficiently low relative to the lending interest rate in the Home country.

### B Data Sources and Description

Below we describe the construction of time series used to calculate the moments of the data. All constructed variables (except for interest rates and price indices) are logged and HP-filtered using \( \lambda = 1600 \). Data source: Bureau of Economic Analysis (BEA), unless indicated otherwise.

Since the model is solved in real terms (normalized on \( P_t \)), and we abstract from population growth, all nominal variables in the data have to be deflated by population and a measure of the aggregate price level (we use CPI, which is the closest data equivalent of \( P_t \)). Moreover, we have to remove all government variables from the
data so as to match the shares of consumption and investment generated by the model.

\( P_t \): Personal Consumption Expenditures: Chain-type Price Index (series ID: PCECTPI), 2005=100.

\( \Psi_t \): Gross Domestic Product: Implicit Price Deflator (series ID: GDPDEF), 2005=100.

\( Q_t \): Constant Quality (Laspeyres) Price Index of New One-Family Houses Sold Including Value of Lot (source: U.S. Bureau of Census), 2005=100.


\( GDP_{pop}^t \): Real Gross Domestic Product (series ID: GDPC96) divided by Real Potential Gross Domestic Product (series ID: GDPPOT, source: Congressional Budget Office)

\( C_t \): Personal Consumption Expenditures (series ID: PCEC) deflated by PCECTPI and CNP16OV.

\( I_t \): Equipment and Software (Table 1.1.5, line 11) deflated by PCECTPI and CNP16OV.

\( S_t \): Structures (Table 1.1.5, line 10) plus Private Residential Fixed Investment (series ID: PRFI), deflated by PCECTPI, CNP16OV and \( Q_t \).

\( Y_t \): GDP\(_t\) less real construction output, \( \frac{Q_t}{P_t} \cdot S_t \)

\( H_t^{pop} + H_t^{rem} \): Households Owner-Occupied Real Estate Including Vacant Land and Mobile Homes at Market Value (series ID: FL155035015, source: U.S. Flow of Funds Accounts, Table B.100, line 4), deflated by PCECTPI, CNP16OV and \( Q_t \).

\( H_t^{r} \): Nonfarm Nonfinancial Corporate and Non-Corporate Business Real Estate at Market Value (series ID: FL105035005 and FL115035033, source: U.S. Flow of Funds Accounts, Table B.102, line 3 and Table B.103, line 5), deflated by PCECTPI, CNP16OV and \( Q_t \).

\( K_t^{r} \): Nonfarm Nonfinancial Corporate and Non-Corporate Business Equipment and Software, Current Cost Basis (series ID: FL105015205 and FL115013265, source: U.S. Flow of Funds Accounts, Table B.102, line 4 and Table B.103, line 8), deflated by PCECTPI and CNP16OV.

\( K_t^{c} \): 0.02\( K_t^{r} \) (see discussion in Iacoviello and Neri (2010))

\( N_t^{e} \): All Employees: Total Private Industries (series ID: USPRIV, source: BLS) less All Employees: Construction (series ID: USCONS, source: BLS), quarterly averages.

\( N_t^{c} \): All Employees: Construction (series ID: USCONS, source: BLS), quarterly
averages.

πₜ: quarter-on-quarter change in \( P_t \).

\( R^p_t \): Effective Federal Funds Rate (series ID: FEDFUNDS), divided by 400 to turn it into quarterly yield in units.

C Estimation of the Exogenous Processes

We use U.S. quarterly data from 1974 (post Bretton-Woods) to 2007, so as to exclude the fallout from most recent financial crisis. We strive to achieve the maximum correspondence between the model variables and their data equivalents, so that the estimated shocks, when fed into the model, have the highest chance of explaining the observed U.S. macroeconomic dynamics; for example, we exclude government purchases and taxes from the computation of real GDP. The detailed description of the empirical counterparts to our model variables can be found in Appendix B.

C.1 Monetary policy

Following the seminal paper by John Taylor (1993), we set the parameters in equation (7) to \( \{ \rho_t, \rho_\pi, \rho_y \} = \{ 0, 1.5, 0.5 \} \) and compute monetary policy shocks \( \tilde{\varepsilon}_{i,t} \) as the difference between the actual interest rate and the one implied by these three parameters, the observed output gap and inflation rate:

\[
\tilde{\varepsilon}_{i,t} = R^p_t - \frac{1}{4} \left[ r + \pi^a_t + 0.5 gdp^{gap}_t + 0.5 \left( \pi^a_t - \pi \right) \right]
\]

In this equation, \( R^p_t \) is the Federal Funds Rate (at quarterly rate), \( \pi^a_t \) is annualized growth of CPI, and \( gdp^{gap}_t \) measures the percent deviation of GDP from its potential value (reported by the Congressional Budget Office). Implicit in the calculation is Taylor’s assumption that long-run annualized rates of inflation \( \pi \) and real interest \( r \) are both equal to 2%, which maps quite naturally into our model’s specification of \( \pi = 0 \) and \( r = \frac{1}{3}p = 4\% \). We set \( \sigma(\varepsilon_i) = 0.0028 \).

We check the sensitivity of the model by changing the Taylor Rule parameters to their more commonly used values (in particular, by introducing inertia into the monetary response) and find that this leaves the main results of our paper unchanged.

C.2 Housing demand shocks

The more preferential tax treatment of homeowners in the 1990’s and early 2000’s, as well as more readily available sources of financing a home purchase may have led
the U.S. consumers to place higher value on houses relative to other goods in their consumption baskets; see, for example, Campbell and Hercowitz (2005) and Favilukis, Ludvigson, and Van Nieuwerburgh (2010) for an overview of housing finance developments. Within the model, such a shift in preferences can be represented by a shock to the parameter $\gamma$, the share of income spent on housing.

Since the stock of housing does not depreciate quickly and yields utility over multiple periods, consumers have to take into account the entire expected future path of housing prices when making their real estate consumption decisions. While we have data on actual housing prices, we lack information on housing price expectations in every quarter. Instead, if we abstract from the expected future services of housing stock as a simplification, we arrive at the following relationship between consumption, housing and relative prices:

$$H_i^t = \gamma_i \left[ \frac{Q_t}{P_t} \right]^{-\xi} \left[ (1 - \gamma_t) \left[ C_i^t \right]^{-\xi \frac{1}{\xi}} + \gamma_t \left[ H_i^t \right]^{-\xi \frac{1}{\xi}} \right]^{\xi \frac{1}{\xi}} \text{ for } i \in \{p, m\}$$ (13)

Here $P_t$ measures the overall price of the consumption basket, inclusive of housing. Given our assumption about the value of $\xi$, we can use the data on residential housing, consumption and housing prices (relative to the aggregate price index like the GDP deflator) to infer the value of $\gamma_t$ in any quarter between 1974Q1 and 2007Q4. More specifically, we use a non-linear equation solver to compute the series $\{\hat{\gamma}_t\}$ for the three values of $\xi$ reported in Section 3.5, given the observations on $\{H_i^t\}$, $\{C_i^t\}$, $\{Q_t\}$, and $\{P_t\}$. Both $Q_t$ and $P_t$ are reported as indices, which of course implies a presence of a scaling factor on the right side of equation (13). To address this issue, we re-estimate the demand shocks by calibrating the unobserved scaling factor such that on average, $\gamma_t = \gamma$ used in parameterizing the steady state of the model. This approach does not affect the estimate of the persistence of the shock, but does increase its standard deviation from 0.012 to 0.020.

By ignoring the expected future services of consumer housing stock, we are likely to overestimate the value of $\gamma_t$ in equation (13) relative to our model calibration. However, if we assume that this parameter is overestimated by the same amount in each period, we can detrend the time series for $\hat{\gamma}_t$ and interpret its cyclical component as period-by-period housing demand shock $\{\hat{\varepsilon}_{\gamma,t}\}$, which we feed into the model:

$$\log \hat{\gamma}_t = (1 - \rho_\gamma) \gamma + \rho_\gamma \log \hat{\gamma}_{t-1} + \hat{\varepsilon}_{\gamma,t}$$

The results of estimating the housing demand shock process are virtually identical for $\xi = \{0.3, 0.5, 0.9\}$. Therefore, we report only the estimates for $\xi = 0.9$: $\rho_\gamma = 0.98$ and $\sigma(\varepsilon_\gamma) = 0.02$. Our estimates of the volatility of the demand shocks are somewhat
more conservative than some of the existing studies (for example, Iacoviello and Neri, 2010, estimate the persistence parameter to be 0.96, and find that $\sigma (\hat{e}_{\gamma,t}) = 0.04$); thus, we are not at risk of placing too much weight on the housing demand shock in driving the dynamics of the model.

Alternatively, one can use the consumers’ Euler equation describing the consumption/housing margin to estimate the demand shocks; this approach incorporates the expectations about future house prices into the purchasing decisions of households. We estimate the log-linearized Euler equation:

$$\hat{q}_t = \hat{c}_t + (1-\beta^p)\hat{h}_t + \beta^p(\hat{q}_{t+1} - \hat{c}_{t+1}) + e_t,$$

where $e_t = (1-\beta^p)\hat{\gamma}_t$, by instrumenting for forward-looking variables using their own lags ($t-1, ... t-4$). We derive the housing demand shocks as residuals by estimating $\hat{\gamma}_t = \rho_{\gamma}\hat{\gamma}_{t-1} + \tilde{\epsilon}_{\gamma,t}$. The business cycle properties of the model remain unchanged compared with the numerical approach to computing $\hat{\epsilon}_{\gamma,t}$; however, the fit between the data and the simulated series deteriorates. The results of the exercise are available upon request.

### C.3 Productivity

We calculate the levels of technology in the two sectors using the standard Solow residual approach:

$$\hat{A}_t^e = \frac{Y_t}{[K_t^e][N_t^e]^{1-\mu-\nu}[H_t^e]^{\nu}}$$

$$\hat{A}_t^c = \frac{S_t}{[K_t^c]^\mu[N_t^c]^{-1-\mu-\eta}}$$

The data in this exercise are logged and filtered, since we want to abstract from trend growth in productivity (thus, we do not have to worry about estimating the stock of land, which is assumed to be constant). Our next step is to estimate the underlying technological innovations, which we assume follow an autoregressive process of the form:

$$\begin{pmatrix} \ln \hat{A}_t^e \\ \ln \hat{A}_t^c \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \ln \hat{A}_{t-1}^e \\ \ln \hat{A}_{t-1}^c \end{pmatrix} + \begin{pmatrix} \varepsilon_{e,t}^e \\ \varepsilon_{c,t}^c \end{pmatrix},$$

where $\varepsilon_{e,t}^e$ and $\varepsilon_{c,t}^c$ are uncorrelated i.i.d. shocks.

We begin by setting $A_{12} = A_{21} = 0$. While this is obviously a simplification, this specification is useful for calibrating the version of the model with no construction sector. We refer to this specification as "technology A." We find that productivity shocks in the construction sector tend to be more volatile, but the persistence of the...
two processes is quite comparable:

\[
\begin{bmatrix}
\ln \hat{A}_t^e \\
\ln \hat{A}_t^c
\end{bmatrix} =
\begin{bmatrix}
0.66 \\ 0.71
\end{bmatrix}
\begin{bmatrix}
\ln \hat{A}_{t-1}^e \\
\ln \hat{A}_{t-1}^c
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{a,t}^e \\
\varepsilon_{a,t}^c
\end{bmatrix}
\]

\( R^2 = 0.44, \sigma (\varepsilon_{a}^e) = 0.007, R^2 = 0.54, \sigma (\varepsilon_{a}^c) = 0.020 \)

If we do not place any constraints on the 2 \times 2 matrix \( A \), but rather allow the two technological processes to have spillovers across sectors, we obtain a slightly different set of estimates, which we call "technology B". The data indicate that there exists a lagged spillover from the entrepreneurial productivity to the construction sector; however, the degree of persistence or volatility is virtually unchanged compared with the previous estimation:

\[
\begin{bmatrix}
\ln \hat{A}_t^e \\
\ln \hat{A}_t^c
\end{bmatrix} =
\begin{bmatrix}
0.66 & 0.00 \\ 0.71 & 0.66
\end{bmatrix}
\begin{bmatrix}
\ln \hat{A}_{t-1}^e \\
\ln \hat{A}_{t-1}^c
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{a,t}^e \\
\varepsilon_{a,t}^c
\end{bmatrix}
\]

\( R^2 = 0.44, \sigma (\varepsilon_{a}^e) = 0.007, R^2 = 0.58, \sigma (\varepsilon_{a}^c) = 0.020. \)

### C.4 Household LTV ratio

Beginning in mid-1980’s, the U.S. economy has experienced several waves of financial deregulation, some of which created easier and cheaper access to mortgage financing for households. While the thorough overview of the deregulation process is beyond the scope of this paper, we want to capture its effects on the household balance sheets. According to the Federal Housing Finance Agency data, the average loan-to-price value in the U.S. increased from 73 to almost 80 percent between 1974 and 2007. Unfortunately, the data do not differentiate between patient and credit-constrained consumers. We combine the empirical findings of Campbell and Hercowitz (2005), Iacoviello and Neri (2010) and Jappelli and Pagano (1994) and set \( l^m = 0.77 \) during the 1974Q1-1982Q4 period, increase it to 0.89 for 1983-1989, and finally let \( l^m = 0.92 \) after 1990.