Two-sided Learning in New Keynesian Models: Dynamics, Convergence and the Value of Information

Christian Matthes * Francesca Rondina†

April 29, 2012

Abstract

This paper investigates the role of two-sided learning in a New Keynesian framework populated by private agents and a central bank that have asymmetric imperfect knowledge about the true data generating process. We assume that all agents employ the data they observe (which can be different for different sets of agents) to form beliefs about the aspects of the true model of the economy that they don’t know, use these beliefs to decide on actions, and revise them through a statistical learning algorithm as new information becomes available. We study the short-run dynamics of the model and the optimal policy recommendations along this pattern. We find that two-sided learning can generate large departures of beliefs and policy decisions from their rational expectations equilibrium values, and can alter the behavior of the variables in the model in a significant way.

1 Introduction

This paper studies the role of asymmetric information and learning in a New Keynesian framework in which both private agents and the monetary authority have imperfect knowledge about the true model of the economy. In particular, we focus on the short run dynamics that can arise when given their respective beliefs and information sets, policymakers optimally choose the policy rule to be implemented and private agents form their expectations rationally. In this environment, a rich and complex variety of interactions between beliefs and actions can potentially arise, with important consequences on the patterns of the variables of the model.

*Universitat Pompeu Fabra and Barcelona GSE, email: christian.matthes@upf.edu
†Institute for Economic Analysis, CSIC and Barcelona GSE, email: francesca.rondina@iae.csic.es
In a large number of situations and contexts, it is reasonable to assume that two or more interacting parts have asymmetric information about the environment in which they operate. In this work, we consider a model of the economy in which private agents do not observe a policy shock and the monetary policy rule implemented by the central bank, while the monetary authority does not observe a technology shock and the beliefs private agents have when forming their expectations. We assume that agents employ all available information to estimate the aspects of the true data generating process that they do not know, and use a statistical learning algorithm to revise their beliefs as new data becomes available. In each period, these updated beliefs will be the base for policymakers’ optimal policy decisions and for private agents’ expectations about the future values of the variables of interest.

An extensive literature in economics focuses on the analysis of monetary policy in environments characterized by imperfect knowledge and learning (some recent contributions are Barnett and Ellison, 2011; Bullard and Mitra, 2002; Cho et al., 2002; Cogley et al., 2011; Evans and Honkapohja, 2006; Honkapohja and Mitra, 2005; Marcet and Nicolini, 2003; Milani, 2008; Orphanides and Williams, 2005; Bullard and Eusepi, 2005; for additional references, Evans and Honkapohja, 2009, provide an extensive review of this literature). A large part of this research studies the conditions under which the economy converges to a determinate and learnable rational expectations equilibrium (REE), and the role of monetary policy in attaining this result. Bullard and Mitra (2002), for instance, investigate this issue for a variety of alternative policies under the assumption that the central bank adopts Taylor-type instrument rules. On the other hand, Evans and Honkapohja (2003, 2006) focus on expectations-based targeting rules obtained from the optimization of the central bank’s objective function.\footnote{Svensson (2002) strongly argues that these rules are superior to Taylor-type policy rules because they reflect optimal behavior on the part of the central bank.} Our paper departs from most of this literature with respect to the fundamental assumption of policymakers’ knowledge of the true model of the economy and of its REE. More specifically, in our framework both private agents and the monetary authority have incomplete knowledge of the true data generating process, and they adopt the same approach to deal with their lack of full information: they form some beliefs using the data that they have available in each period, and they decide optimally based on these beliefs. The substantially different assumptions about the central bank’s knowledge and behavior in our environment also implies that the focus of our analysis is not on the ability of the monetary authority to enforce a particular equilibrium of the model, but rather on the short run dynamics that the interactions of beliefs and actions between private agents and policymakers can generate. To our knowledge, the study of optimal policymaking and two-sided learning in an environment characterized by asymmetric information has not received much
attention yet, particularly in the context of the business-cycle dynamics of a New Keynesian model. Given that this framework is often used for policy analysis, we believe that our paper fills an important gap of the literature in economic policy.

With respect to learning, we assume that both private agents and the monetary authority use statistical models to estimate and predict the behavior of the variables for which they do not know the true data generating process. In addition, we allow them to update their estimates as new data becomes available using a recursive learning algorithm. These assumptions follow a large branch of the learning literature originating from Marcet and Sargent (1989), Cho et al. (2002), and Evans and Honkapohja (1998). However, as in Cogley et al. (2011), we allow the agents’ perceived laws of motion to incorporate the cross-equation restrictions originating from their respective decisions on actions. In our specific case, these restrictions will reflect rational expectations for private agents, and optimal policy decisions for the monetary authority.

Because of the complex relationships between parameter updates and optimal decisions, the actual law of motion of the variables in the framework under analysis cannot be characterized analytically. For this reason, we study the impact of asymmetric information and learning in a number of simulations that are performed using standard parameter values for this model. Our results show that two-sided learning can significantly alter the short-run dynamics of the model. More specifically, we find that it can generate large departures of the variables of interest from their rational expectations equilibrium values, and changes in their behavior in terms of autocorrelations and correlations with the other variables. Our exercises also suggest that, in this environment, information communication by the central bank or infrequent re-optimization of the implemented policy rule do not seem effective in reducing the impact of asymmetric information and learning on the equilibrium dynamics.

The remainder of the paper is organized as follows. Section 2 presents the model of the economy under analysis and derives its equilibrium. Section 3 describes the empirical exercises that we perform, and presents their results. Section 4 concludes.

2 The true model of the economy

The true model of the economy is a standard New Keynesian framework as developed in Gali (2008). We assume perfect indexation of prices that cannot be reset to past inflation, as in Christiano et al. (2001), which makes sure that the pricing equations would be unaffected by the presence of positive trend inflation, so that the steady state level of output would still be independent of the steady state level of inflation.\(^2\)

\(^2\)See Ascari (2004) for a discussion.
Given these assumptions, private agents’ side of the economy can be described by the following equations:

\[ y_t = E^P_t (y_{t+1}) - \frac{1}{\sigma} \left( i_t - E^P_t (\pi_{t+1}) - r^n_t \right) \]  

(1)

\[ \pi_t = \frac{1}{(1 + \beta)} \pi_{t-1} + \frac{\beta}{(1 + \beta)} E^P_t (\pi_{t+1}) - \frac{\kappa}{(1 + \beta)} (y_t - \bar{y}) \]  

(2)

\[ r^n_t = \pi + u_t \]  

(3)

\[ u_t = \rho_u u_{t-1} + \varepsilon^u_t \]  

(4)

where \( y_t \) is output, \( \pi_t \) is the inflation rate, \( i_t \) is the nominal interest rate, and \( r^n_t \) is the natural rate of interest. This last variable is assumed to be the sum of the steady state real interest rate \( \pi \) and a technology shock \( u_t \), or shock to the real side of the economy, which evolves according to the AR(1) process described by (4). All variables are in logs. \( \bar{y} \) is the steady state level of output (all other steady variables drop out in the equations above). The parameters \( \sigma \), \( \beta \) and \( \kappa \) have standard interpretation, and are obtained from the underlying problem of consumers and firms; see Gali (2008) for further details. Equations (1) and (2) have the standard interpretation of a IS equation and Phillips curve equation. Differently from the standard New Keynesian framework, the superscript \( P \) in the expectation operator \( E^P_t (\cdot) \) in (1) and (2) denotes the fact that at each time \( t \) private agents will form these expectations based only the information that they have available, which will generally be different from the information available to policymakers.

In addition to the private sector, the economy is populated by a central bank or public authority, which is assumed to have some control over the nominal interest rate and to use it as its policy instrument. More specifically, the central bank is assumed to be able to set the value of \( i_t \) up to a monetary policy shock \( v_t \). Let \( x_t \) be the value of the policy instrument chosen by the central bank for time \( t \), then the interest rate will be:

\[ i_t = x_t + v_t \]  

(5)

where \( v_t \) is assumed to follow the AR(1) process:

\[ v_t = \rho_v v_{t-1} + \varepsilon^v_t \]  

(6)

Private agents and policymakers do not have full knowledge of the economy. In particular, the central bank does not observe the technology shock and does not know how private agents form expectations about the future values of the variables of interest. On the other hand, private agents ignore the policy rule that the central bank uses to set \( x_t \), and do not observe
the monetary policy shock. A more thorough description of the information set available to each side, and of its changes over time through learning, is given next.

3 Information and decisions on actions

The imperfect and asymmetric information that private agents and the monetary authority employ in the decision making process are the central features that differentiate this work from the previous literature. However, these features require that we make some additional assumptions about the way in which each side will use its specific knowledge to estimate the aspect of the economy that are not known, to take decisions in each period, and finally to update its information based on the new data that can be observed.

The state and noise vectors considering all the information available in the economy are:

\[ z_t = \begin{bmatrix} y_t & \pi_t & i_t & u_t & v_t & 1 \end{bmatrix}' \]  
\[ \varepsilon_t = \begin{bmatrix} \varepsilon^u_t & \varepsilon^v_t \end{bmatrix}' \]  
\[ (7) \]

However, as previously mentioned, the vectors \( z_t \) and \( \varepsilon_t \) are not perfectly observed. In particular, private agents do not observe the policy shock \( v_t \), while the central bank does not observe the technology shock \( u_t \). Thus, the vectors of variables that each side will use in their decision process can be written as:

\[ z^P_t = \begin{bmatrix} y_t & \pi_t & i_t & u_t & 1 \end{bmatrix}' \]  
\[ z^{CB}_t = \begin{bmatrix} y_t & \pi_t & i_t & v_t & 1 \end{bmatrix}' \]  
\[ (8) \]

or:

\[ z^P_t = M^P z_t \]
\[ z^{CB}_t = M^{CB} z_t \]

where \( M^P \) and \( M^{CB} \) are just selection matrices that pick the relevant variables from the overall state \( z_t \).

Private agents will use the vector \( z^P_t \) to estimate policymakers’ interest rate rule and to predict future values of the nominal interest rate. In the same way, the monetary authority will employ \( z^{CB}_t \) to approximate and predict the behavior of output and the inflation rate. We assume that agents will make use of reduced-form VARs for this purpose, thus estimating a simple linear relationship between the variables for which they have limited knowledge and
Given this framework, the decision process is composed of two steps. First, private agents and policymakers will need to estimate the parameters of the model using the available data. Second, they will use their perceived model of the economy, together with the estimates of its parameters, to make their respective decisions. These steps will be updated in each period according to the new information that they can observe over time.

3.1 Estimation and learning

Private agents do not know the interest rate rule that the central bank uses to set the value of the interest rate. However, they know that the nominal interest rate affects output and the inflation rate through equations (1) and (2). For this reason, in order to be able to form their expectations on the future values of these variables, they will need to make some conjecture about the relationship between $i_t$ and the variables that they can observe.

We assume that private agents behave like econometricians and estimate the simple linear relationship:

\[ i_t = \psi_0 z_{t-1}^{P} + \omega_t^{P} \]

\[ i_t = \psi_{i0} + \psi_{i1} y_t + \psi_{iu} u_t + \omega_t^{P} \]  

where the error term $\omega_t^{P}$ just captures all the determinants of the nominal interest rate that are orthogonal to the information included in the state vector $z_{t-1}^{P}$.

The central bank has imperfect knowledge on the private side of the economy. However, the policy decision process requires the monetary authority to have some beliefs on the way in which the nominal interest rate affects the variables of interest. As for private agents, we assume that policymakers behave like econometricians and estimate a simple reduced-form VAR relationship between $y_t$ and $\pi_t$ and the state vector $z_{t-1}^{CB}$ which includes the variables that they can observe:

\[ y_t = c_{yt} z_{t-1}^{CB} + \omega_{yt}^{CB} \]  

\[ \pi_t = c_{\pi t} z_{t-1}^{CB} + \omega_{\pi t}^{CB} \]  

As new data becomes available, private agents will update their estimates of the vector of coefficients $\psi$, and the central bank will update its estimates of the $c_{yt}$ and $c_{\pi t}$. We will start with a simple scenario and assume that this process of learning is developed through a standard recursive least squares algorithm (see, for instance, Evans and Honkapohja, 2001). We will investigate the case of constant gains, as well as the case of decreasing gains in which the beliefs on the values of $\psi$, $c_{yt}$ and $c_{\pi t}$ converge to OLS estimates. Further details about the learning approach that we adopt in this work are provided in the Appendix; the study
of additional learning approaches in one of the directions of our future research.

We augment our learning algorithm with a projection facility, which ensures that parameter estimates remain within a predetermined region of values which we regard as suitable.\(^3\) In the specific, we allow agents to make use of three types of projection facilities. The first type refers to the coefficients on the inflation rate and output in the perceived and actual the policy rules, which are restricted to assume only positive values. The second type of facilities allow private agents to disregard estimates of the policy rule coefficients for which the solution of the expectational difference equation that they need to solve in their decision process does not exist or is not unique.\(^4\) Finally, the third projection facility allows policymakers to rule out estimates of (12) and (13) that would cause the perceived law of motion of the variables of interest to be nonstabilizable. A more formal description of the impact of projection facilities on the learning algorithm adopted in this paper is provided in the Appendix.

We are aware that projection facilities might rule out some potentially interesting dynamics of the variables of interest. However, in the environment under analysis, which is characterized both by asymmetric imperfect information and two-sided learning, it might be very difficult for agents to correct explosive patterns of beliefs and policy decisions in case they should arise. This happens because agents’ decisions are based on beliefs which are in turn affected by the other side’s decisions. It follows that it is very well possible that unreasonable beliefs over the estimated coefficients keep reinforcing each other instead of being redirected towards more sensible values. In any case, the empirical section of the paper will provide further discussion about the role of projection facilities in our simulations.

Notice that because we allow for the presence of trend inflation in (2), the true long-run level of inflation in this model is not known, and it is not constant. However, private agents can estimate trend inflation in each period as function of the estimated policy rule and the steady state level of the real interest rate. It follows that this framework has all the features to allow us to investigate the impact of the uncertainty in the long-run level of inflation on

\(^3\)For a more thorough discussion of the use of projection facilities in a number of learning algorithms, see Carceles-Poveda and Giannitsarou (2007).

\(^4\)While it seems reasonable to assume that agents would rule out parameter estimates for which a stable pattern of the variables in \(z^P_t\) does not exist, the case in which the solution is indeterminate is a little bit more complex. The analysis of learning in environments in which multiple equilibria can potentially arise requires not only to take a stand on the way in which one of the alternative solutions should be selected, but also to model the way in which agents should account for the indeterminacy of the equilibrium when updating their beliefs. The learning patterns emerging in this environment could potentially be very complicated. For this reasons, we decided to start with a simpler scenario, and we allowed private agents to behave conservatively and disregard parameter estimates that would lead to indeterminate equilibria. Nonetheless, we do believe that the study of two-sided learning in the case of indeterminate equilibria is very interesting, and we aim to extend our research in this direction in the future.
current decisions, which is a direction we are interested in pursuing.

Before moving to the description of the decision process, we would like to address one issue related to the structure of the information set and learning approach that we assume in this paper. One possible objection to the framework that we adopt would be that the central bank should know the learning problem of the agents just by introspection, given that the central bank decision makers are private agents after all. However, the model could be rewritten under the assumption that each private agent does not know a priori that all the other agents use the same forecasting scheme. Rather, each household (or firm) $i$ could be endowed with a conditional expectations operator indexed by $i$, $E_i^t$. We can then assume that all those expectation operators are indeed equal, and since we work on a linearized model, by integrating over $i$ we would still get the standard aggregate equilibrium conditions.

### 3.2 Policy decisions and expectations formation

The actual law of motion of the variables in the model depends on the decisions of private agents and policymakers. More specifically, private agents will use their knowledge of the private side of the economy and their beliefs about the interest rate rule to form expectations, which in turn will affect the behavior of $y_t$ and $\pi_t$ through (1) and (2). On the other hand, the central bank will use its beliefs on the way in which the policy instruments impact the processes for $y_t$ and $\pi_t$ to set the value of the policy instrument $x_t$.

With respect to the private sector, we assume that decisions follow the same timing as in Cogley, Matthes and Sbordone (2011). Private agents will estimate the parameters of the policy rule (11) using information up to and including time $t-1$. Then, they will observe current period shocks and the value of the policy instrument, and use them, together with the previously available information, when making decisions on actions. This approach means that agents enter time $t$ with predetermined parameter estimates, but then use the current period shock to form expectations.

The central bank, on the other hand, has the power to decide the value of $x_t$ in (5). We assume that the policy rule for $x_t$ will be chosen by minimizing the expected discounted quadratic loss function:

$$E_{t-1}^{CB} \sum_{j=0}^{\infty} \beta^j [(\pi_{t+j})^2 + \lambda_y(y_{t+j})^2 + \lambda_i(i_{t+j} - i_{t+j-1})^2]$$

given (12) and (13), and the estimated values of $c_{yt}$ and $c_{\pi t}$. The parameters $\lambda_y$ and $\lambda_i$ represent the weight attached to the output variable relative to inflation, and the relative cost of changing the nominal interest rate. The subscript in $E_{t-1}^{CB}$ means that expectations
are taken with respect to the information set available to the central bank.

In their decisions, private agents and policymakers are assumed to behave as anticipated utility decision makers (Kreps, 1998), which means that they will treat parameter estimates as true values, thus disregarding parameter uncertainty and the effects of learning. This assumption is common in the literature on learning in macroeconomics (see, for instance, Evans and Honkapohja, 2001).

4 Model solution

Policymakers and private agents base their decisions on their respective perceived law of motion (PLM) for the variables of interest. However, their decisions will affect the true model of the economy, i.e. the actual law of motion (ALM) of these variables. This section provides more details about the agents’ PLMs, their decision process, and the resulting ALM.

4.1 PLM for the central bank

The central bank’s PLM for output and inflation is defined by equations (12) and (13), and can be rewritten in state space form using the vector $z_{t}^{CB}$ as:

$$A_{t}^{CB}z_{t}^{CB} = B_{t}^{CB}x_{t} + C_{t}^{CB}z_{t-1}^{CB} + D_{t}^{CB}e_{t}^{CB}$$

The time subscripts in the matrices of parameters emphasize the fact that the estimates of $c_{yt}$ and $c_{xt}$ in (12) and (13) are updated over time even if, as previously mentioned, the assumption of anticipated utility implies that policymakers will not take these updates into account in their decision process. The problem of the central bank is then to find the sequence $\{x_{t}\}$ that minimizes (14) subject to (15) under the assumption of constant parameter values.

It is well known that, under standard conditions, the solution to this problem is linear in the state $z_{t-1}^{CB}$, i.e.:

$$x_{t} = -F_{t}z_{t-1}^{CB}$$

$$= f_{0t} + f_{yt}y_{t-1} + f_{yt}y_{t-1} + f_{yt}y_{t} + f_{yt}y_{t}$$

so that the expression for the nominal interest rate will be:

$$i_{t} = f_{0t} + f_{yt}y_{t-1} + f_{yt}y_{t-1} + f_{yt}y_{t} + f_{yt}y_{t} + v_{t}$$
If the matrices of parameters in (15) were constant over time, standard results in the optimal control literature would deliver a time-invariant optimal policy vector \( F \). However, because in our setup the optimization problem is repeated in every period given updated values of \( c_{yt} \) and \( c_{xt} \) in (12) and (13), the optimal policy vector will actually depend on the current period estimates of these parameters.

Given estimates of the parameters for time \( t \) and the chosen policy rule, the PLM for the central bank can be rewritten as:

\[
A^{CB} z_t^{CB} = (C_t^{CB} - B_t^{CB} F_t) z_{t-1}^{CB} + D_t^{CB} z_t^{CB} + 1 + C_t^{CB} z_t^{CB} + D_t^{CB} z_t^{CB}
\]

or

\[
z_t^{CB} = \Phi_{1,t} z_{t-1}^{CB} + \Phi_{2,t} z_t^{CB}
\]

where \( \Phi_{1,t} = (A_t^{CB})^{-1} (C_t^{CB} - B_t^{CB} F_t) \) and \( \Phi_{2,t} = (A_t^{CB})^{-1} D_t^{CB} \).

The central bank will implement the policy rule for the interest rate defined by (16), so that the chosen value of the vector of coefficients \( F_t \) will in fact have an impact on the ALM of the variables of interest.

### 4.2 PLM for private agents

The PLM for private agents can be obtained from equations (1) – (4) and the perceived interest rate rule expressed by (11). In matrix form, this PLM can be written as:

\[
A^P z_t^P = B^P E_t^P (z_{t+1}^P) + C_t^P z_{t-1}^P + D_t^P z_t^P
\]

where the time subscript in the matrix \( C_t^P \) emphasizes the fact that the estimated coefficients of the perceived policy rule are updated over time. The matrices of coefficients \( A^P, B^P, C_t^P \) and \( D^P \) are specified in the Appendix.

Private agents will use (18) as the basis to solve the expectation term \( E_t^P (z_{t+1}^P) \). As previously mentioned, we use the same approach as in Cogley, Matthes and Sbordone (2011). In more detail, in every period private agents will estimate the coefficients of the perceived policy rule, and then solve the vector-valued expectational difference equation that features the equilibrium conditions including the estimated policy rule. This approach has the important consequence that agents do take into account cross-equations restrictions when forming forecasts. In addition, because of this timing structure, the PLM is just the reduced form VAR associated with (18), with reduced form coefficients that are time-varying because they depend on the estimate of the policy rule coefficients. For this reason, we can guess a solution
of the form:

\[ z_t^P = \Gamma_{1,t} z_{t-1}^P + \Gamma_{2,t} \varepsilon_t^P \]

which we can plug in for the expectation to get:

\[ (A^P - B^P \Gamma_{1,t}) z_t^P = C_t^P z_{t-1}^P + D^P \varepsilon_t^P \]

This gives the following equation for the reduced form matrices:

\[ \Gamma_{1,t} = (A^P - B^P \Gamma_{1,t})^{-1} C_t^P \]
\[ \Gamma_{2,t} = (A^P - B^P \Gamma_{1,t})^{-1} D^P \]

We use Sims’ (2001) Gensys program to find the values of \( \Gamma_{1,t} \) and \( \Gamma_{2,t} \). When \( z_t^P \) is determinate, this program delivers the unique nonexplosive solution for these matrices of parameters. When \( z_t^P \) is indeterminate, the program delivers one of the many possible nonexplosive solutions. As previously mentioned, we will endow private agents with a projection facility allowing them to rule out coefficient estimates for which a stable solution does not exist. In addition, we allow private agents to employ an additional projection facility according to which they will disregard estimates of the parameters in \( A^P, B^P, C_t^P \) and \( D \) that would lead to an indeterminate outcome for \( z_t^P \). This facility has the consequence of preventing the Gensys program from randomly selecting the solution that private agents should implement.

### 4.3 ALM

The ALM for the variables in the model involves equations (1) – (4), which describe the true behavior of the private sector, together with the true interest rate rule expressed by (16). In matrix form, this ALM can be written as:

\[ A z_t = B E_t^P (z_{t+1}^P) + C_t z_{t-1} + D \varepsilon_t \]

The matrices \( A, B, C \) and \( D \) are defined in the Appendix. Notice that the matrix \( C_t \) is time-variant because it includes the true policy coefficients, which the central bank will update in each period.

From the PLM for the private sector, we know that:

\[ E_t^P (z_{t+1}^P) = \Gamma_{1,t} z_{t-1}^P \]
\[ = \Gamma_{1,t} M^P z_t \]
which implies:

$$A z_t = B \Gamma_{1,t} M^P z_t + C_t z_{t-1} + D \varepsilon_t$$ (19)

It follows that the ALM of the model can be written as:

$$z_t = \Psi_{1,t} z_{t-1} + \Psi_{2,t} \varepsilon_t$$

where:

$$\Psi_{1,t} = (A - B \Gamma_{1,t} M^P)^{-1} C_t$$

$$\Psi_{2,t} = (A - B \Gamma_{1,t} M^P)^{-1} D$$

These last two expressions, which define the matrices of coefficients in the ALM of the economy, do not include the matrix $\Gamma_{2,t}$. This implies that the inclusion of a perceived policy shock in the PLM of agents is actually unnecessary.

5 Empirical Exercises

The model that we have described in the previous section involves complex interactions of beliefs and actions between private agents and policymakers. In particular, while the learning procedure that agents use to update their beliefs has a recursive structure, policymakers’ optimization approach and private agents’ expectation formation process are highly nonlinear. For this reason, the equilibrium pattern implied by the learning and decision sequence assumed in this paper cannot be characterized analytically. The main goal of this section is then to offer some empirical insights about the role of asymmetric information and two-sided learning in the context of the New Keynesian framework described in the previous section.

We focus on the short run dynamics of the endogenous variables of the model, and we investigate the patterns, magnitude and length of the departures of these variables from their values in a rational expectation equilibrium (REE). We define the REE as the one emerging from an environment in which policymakers set a fixed policy rule for the instrument $x_t$ in (5) and maintain this policy for the entire simulation period, while private agents still learn and compute expectations based on the procedure described in the previous section. The fixed policy rule that we use in this case is a standard Taylor-type rule in the form:

$$x_t = 0.5 y_{t-1} + 1.5 \pi_{t-1} + 0.5 i_{t-1}.$$  

In addition to the benchmark scenario incorporating learning and optimal decisions as described in the previous section, we also perform two additional exercises. In the first one, we allow the central bank to communicate its perceived steady state value of the nominal
interest rate to private agents. More specifically, we assume that at the end of period $t$, the central bank announces $\tilde{i}_t$, their time $t$ estimate of the long run value of the nominal interest rates, which can be calculated using the policy rule computed in period $t$. Because of anticipated utility, private agents will treat this announced level of the steady state nominal interest rate as fixed, and use it in their regressions. In more detail, they will estimate the policy rule with the time $t$ left hand side variable being $i_t - \tilde{i}_t$ and the right hand side variables being deviations from the implied steady state. When making their decisions next period, the private agents still use $\tilde{i}_t$, even though the central bank will update its estimate to $\tilde{i}_{t+1}$ that period. We believe that this setup is interesting because it allows us to study the impact that a reduction in the asymmetry of the information available to agents has on the process of learning.

The second exercise that we perform is in the direction of investigating the extent to which policymakers’ optimization procedure affects the pattern and speed of private agents’ learning on the true policy parameters. In this exercise, we keep the information sets and the learning approach as in the benchmark case, with no communications between the central bank and private agents, but we assume that policymakers are able to re-optimize and change their policy rule only every $k^{th}$ periods.

In all the simulations, we set the parameters of the true model of the economy and those in policymakers’ loss function as: $\sigma = 1; \kappa = -0.01; \beta^P = \beta^{CB} = 0.99; \tau = 1/\beta^P - 1; \lambda_y = 1/16$ and $\lambda_i = 0.5$. For the real shock $u_t$ and the policy shocks $v_t$, we assume normal distributions with parameters: $\sigma_u^2 =; \sigma_v^2 =; \sigma_{u,v}^2 = 0.008; \rho_u = 0.5; \rho_v = 0.5$ In order to initialize the learning and decisions procedure, we need to set an initial value for agents’ beliefs. We do this by using population regressions and population moments from the rational expectation solution of the model obtained using the same fixed policy rule adopted in the RE scenario. Since the stability of the patterns of the variables of interest seems to be affected by the choice of these initial beliefs, we believe that this approach is to be preferred to the adoption of arbitrary values.\footnote{Carceles-Poveda and Giannitsarou (2007) provide a discussion of different methods that can be employed to initialize agents’ beliefs in frameworks characterized by adaptive learning.}

We report the results for the case of a recursive learning algorithm with decreasing gains, as described in the appendix.\footnote{We also experimented with constant gains, $g = 0.015$, and we found a much larger volatility of the variables of interest and more frequent deviations from the RE equilibrium in this case. As previously mentioned, the analysis of the learning patterns under alternative algorithms is one of the extensions that we intend to pursue.} The value of $t_0$ was set equal to 12. In all the exercises, we set the period length to $T = 400$, and we performed $N = 1000$ simulations. We study the impact of asymmetric information and two-sided learning in the New Keynesian model.
under analysis by looking at the distributions of the patterns of the variables of interest and of the policy parameters obtained from the different simulations. We show the median, and 15th and 85th percentile bands of these distributions, and we report their relevant statistics.

Figures 1 – 3 and tables 1 – 2 provide clear evidence about the fact that the impact of two-sided learning in this environment is significant. Compared to their distribution in the RE case, all the variables exhibit patterns that are much more volatile when asymmetric information and two-sided learning are included. This happens for all the variables, and in all the exercises that we performed. In addition, the magnitude of this increase in volatility is very large. Table 1 shows that the median standard deviation of the output variables is more than twice its value in the RE case, and the median standard deviation of the annualized inflation rate is almost 5 times its value in the RE case. This tables also highlights the fact that neither reducing asymmetric information through central bank communications, or helping private agents’ learning by restricting the central bank re-optimization process only to some periods, can decrease the volatility originated by two-sided learning. On the contrary, these two cases produce median standard deviations that are even higher than in the benchmark case.

In figure 5, we report the distribution of the autocorrelations of the variables of interest in the 4 alternative scenarios under analysis. This figure shows that, relative to the RE case, two-sided learning can potentially increases the persistence of the variables in the model at all orders, in some cases in a substantial way. This result holds in all the exercises that we performed, but is much less pronounced in the case in which we allow the central bank to communicate its perceived steady state value of the nominal interest rate. Tables 3 – 6 reports the correlations between the variables for the different cases. While the correlations between the output gap and inflation and between the output gap and the nominal interest rate don’t seem to be much affected by the introduction of two-sided learning, those between the inflation rate and the nominal interest rate are. This correlation is negative in the RE case, it is still negative but smaller in the benchmark and infrequent re-optimization cases, and becomes positive in the case of central bank communication. Thus, two-sided learning has the potential to affect not only the persistence of the variables in the models, but also their contemporaneous relationships.

In terms of policy parameters, we report the distribution of the actual and estimated coefficient attached to the lagged inflation rate (figure 4). The convergence of these two values seem to be more pronounced in the benchmark case relative to the other two scenarios that we consider. In all cases the optimal policy coefficient selected by the monetary authority exhibits a very volatile pattern, and for the benchmark case this is especially true in the first part of the simulation period. We find that this happens because the projection facility that
requires the optimal policy parameters to assume reasonable values (in this case, a value greater than zero), is invoked in a larger number of the simulations in this initial part of the learning and decision process. It is common for the value of the estimated parameters to be subject to larger revisions at the beginning of the simulation period because the relative weight of each additional observation is greater when the length of the available data is limited. Infrequent re-optimizations or communication of the estimated long run value of the nominal interest rates might help reduce this phenomenon, and this might explain why the optimization process suggests more reasonable policy rules in these two cases. It is also interesting to notice that when policymakers communicate their perceived steady state value of the nominal interest rate, the private agents’ learning process might be slower but the dispersion of their estimates of the policy response to lagged inflation is reduced relative to the other cases. Regarding the other parameters in the policy rule, we find that they exhibit patterns that are similar to those reported in figure 4.

In all, our exercises suggest that the role of asymmetric information and two-sided learning in the context of a New Keynesian model of the economy is significant. We find that the pattern of the variables of the model can change considerably in terms of volatility and autocorrelations, and that the correlations between variables are affected as well. The monetary authority can attain patterns of the variables of interest that are closer to the RE case by communicating its perceived steady state value of the nominal interest rate. In our simulations, this information seems to help private agents’ learning of the true value of the parameters in the optimal policy rule. However, even in this case, we find that the dispersion of the potential patterns of the variables in the model would still be much higher than in the RE scenario.

6 Conclusions

This work represents a first attempt at investigating the role of asymmetric information and two-sided learning in a New Keynesian model of the economy. The assumption that both monetary authorities and private agents have imperfect knowledge of the true data generating process, and try to learn over time from the new information that becomes available, seems to reflect fairly well what we observe in the real world scenario. For this reason, we believe that the study of the way in which this learning process can potentially alter the dynamics of a New Keynesian framework, which is often used as the basis for policy analysis, is of primary importance. The results of our simulations support this idea by showing that two-sided learning can cause large departures of beliefs and decisions from their RE values.

The analysis in this paper can be extended in a number of directions. First, the impact
of alternative assumptions about agents’ learning approach, for instance the use of different learning algorithms, could be investigated. Second, we think that it would be interesting to further analyze the effects that communication between the agents might have on their process of learning about the true data generating process. In this work, we have studied the case in which policymakers inform the private agents about their perceived long run value of the nominal interest rates. Communications of different types of information, or communications from private agents to policymakers, could also be investigated. Third, the framework that we employ in this paper allows us to investigate the impact of the uncertainty about the long run level of inflation on current beliefs and decisions, and this is also a direction that we are interested in pursuing.

Finally, we believe that there are two major areas in which this work could be improved and extended. The first one is the characterization of the ordinary difference equation (ODE) emerging from agents’ learning and decision process, along the lines of the analysis of Marcet and Sargent (1989). The study of this ODE would allow us to provide more insights on the equilibrium pattern of the learning algorithm, and on the escape dynamics that could arise. However, the interactions between beliefs and optimal decisions in our framework are complex, and the dynamics of the model are highly nonlinear. For this reason, the ODE characterizing agents’ learning process cannot be obtained analytically, and its analysis needs to be performed numerically, which is considerably more involving. The second route that we would like to explore is in the direction of estimating our framework using real world data, which would allow us to employ this model to provide an interpretation of past event and to offer more punctual policy recommendations.
Learning algorithm

Let the equations to be estimated by agents be written in general terms as:

\[ q_t = p_{t-1}^\prime \phi_t + \eta_t \]

where \( q_t \) is the dependent variable or a vector of dependent variables, \( p_{t-1} \) a vector or matrix of regressors, \( \xi_t \) the residual(s) and \( \phi_t \) the vector of parameters of interest. In the case of private agents, this equation corresponds to (11), while for policymakers it encompasses (12) and (13). Using this notation, the learning algorithm can be written as:

\[ R_t = R_{t-1} + g_t (p_{t-1}p_{t-1}^\prime - R_{t-1}) \]

\[ \phi_t = \phi_{t-1} + g_t R_{t-1}^{-1} p_{t-1} (q_t - p_{t-1}^\prime \phi_{t-1}) \]

where \( g_t \) represents the gain. In the empirical part of the paper, we mainly focus on Recursive Least Squares (RLS) learning, in which \( g_t = \frac{1}{t_{0}+t} \). However, we also perform some comparative exercises using Constant Gain (CG) learning, in which \( g_t \) is assumed to be a constant positive and small number, i.e. \( g_t = g, 0 < g < 1 \). For a more thorough description of these learning algorithms and their properties, see Evans and Honkapohja (2001); for a discussion of their performance in a few standard macroeconomic models, see Carceles-Poveda and Giannitsarou (2007).

As mentioned in the main text, the basic learning algorithm will be augmented with a number of projection facilities. We assume that whenever the value of the estimated parameters by private agents and policymakers "hits" the projection facility, i.e. moves outside the predetermined parameter region \( Q \), agents will set current period’s estimates to the values of the previous period. Thus, the algorithm can be rewritten as:

\[ \hat{R}_t = \hat{R}_{t-1} + g_t \left( p_{t-1}p_{t-1}^\prime - \hat{R}_{t-1} \right) \]

\[ \hat{\phi}_t = \hat{\phi}_{t-1} + g_t \hat{R}_{t-1}^{-1} p_{t-1} (q_t - p_{t-1}^\prime \hat{\phi}_{t-1}) \]

\[ \left( \hat{\phi}_t, \hat{R}_t \right) = \begin{cases} (\phi_t, R_t) & \text{if } (\phi_t, R_t) \in Q \\ (\hat{\phi}_{t-1}, \hat{R}_{t-1}) & \text{if } (\phi_t, R_t) \notin Q \end{cases} \]

The specific types of projection facilities that we consider in this paper were discussed in the main text.

\( t_0 \) is set to 12 quarters in our simulations.
Matrices in the PLMs and ALM

The matrices of the PLM for the central bank can easily be obtained using the state space representation (15) and the policy rule (16) emerging as a result of the optimization problem. We have that:

$$A^{CB}z_t^{CB} = (C_t^{CB} - B^{CB}F_t)z_t^{CB} + D^{CB}e_t^{CB}$$

or more explicitly:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
y_t \\
\pi_t \\
i_t \\
v_t \\
1
\end{pmatrix}
= 
\begin{pmatrix}
c_{1yt} & c_{2yt} & c_{3yt} & c_{4yt} & c_{5yt} \\
c_{1\pi t} & c_{2\pi t} & c_{3\pi t} & c_{4\pi t} & c_{5\pi t} \\
-f_{\pi t} & -f_{yt} & -f_{it} & -f_{vt} & -f_{ot} \\
0 & 0 & 0 & \rho_v & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
y_{t-1} \\
\pi_{t-1} \\
i_{t-1} \\
v_{t-1} \\
1
\end{pmatrix}
+ 
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\omega_{yt}^{CB} \\
\omega_{\pi t}^{CB} \\
\varepsilon_t^v
\end{pmatrix}
$$

The PLM for private agents is given by the true equations (1) – (4) together with the perceived interest rate rule expressed by (11).

$$y_t = E_t^P (y_{t+1}) - \frac{1}{\sigma} (i_t - E_t^P (\pi_{t+1}) - \pi - u_t)$$

$$\pi_t = \frac{1}{(1+\beta)} \pi_{t-1} + \frac{\beta}{(1+\beta)} E_t^P (\pi_{t+1}) - \frac{\kappa}{(1+\beta)} (y_t - \bar{y})$$

$$i_t = \psi_{0t} + \psi_{\pi t} \pi_{t-1} + \psi_{yt} y_{t-1} + \psi_{it} i_{t-1} + \psi_{ut} u_{t-1} + \omega_t^P$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

These equations can be rewritten in matrix form as:
The ALM for the variables in the model can be obtained from the true equations (1) – (4) together with the true interest rate rule expressed by (16), and can be written in matrix form as:

\[
\begin{pmatrix}
1 & 0 & \frac{1}{\sigma} & -\frac{1}{\sigma} & -\frac{1}{\sigma}\gamma
\end{pmatrix}
\begin{pmatrix}
y_t
\pi_t
i_t
u_t
1
\end{pmatrix}
\begin{pmatrix}
1 & \frac{1}{\sigma}
0 & \frac{\beta}{(1+\beta)}
0 & 0
0 & 0
0 & 0
\end{pmatrix}
E_t^p
\begin{pmatrix}
y_{t+1}
\pi_{t+1}
i_{t+1}
u_{t+1}
1
\end{pmatrix}
+ 
\begin{pmatrix}
0 & 0 & 0 & 0 & 0
0 & \frac{1}{(1+\beta)} & 0 & \frac{\kappa}{(1+\beta)}y
0 & 0 & \rho_u & 0 & 0
0 & 0 & 0 & \rho_v & 0
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
y_{t-1}
\pi_{t-1}
i_{t-1}
u_{t-1}
1
\end{pmatrix}
+ 
\begin{pmatrix}
0 & 0
0 & 0
0 & 0
1 & 0
0 & 0
\end{pmatrix}
\begin{pmatrix}
\varepsilon_t^P
\omega_t^P
\end{pmatrix}
\]

or:

\[A^Pz_t^P = B^P E_t^p (z_{t+1}^P) + C^Pz_{t-1}^P + D^P \varepsilon_t^P\]

The ALM for the variables in the model can be obtained from the true equations (1) – (4) together with the true interest rate rule expressed by (16), and can be written in matrix form as:

\[
\begin{pmatrix}
1 & 0 & \frac{1}{\sigma} & -\frac{1}{\sigma} & -\frac{1}{\sigma}\gamma
\end{pmatrix}
\begin{pmatrix}
y_t
\pi_t
i_t
u_t
1
\end{pmatrix}
\begin{pmatrix}
1 & \frac{1}{\sigma}
0 & \frac{\beta}{(1+\beta)}
0 & 0
0 & 0
0 & 0
\end{pmatrix}
E_t^p
\begin{pmatrix}
y_{t+1}
\pi_{t+1}
i_{t+1}
u_{t+1}
1
\end{pmatrix}
+ 
\begin{pmatrix}
0 & 0 & 0 & 0 & 0
0 & \frac{1}{(1+\beta)} & 0 & \frac{\kappa}{(1+\beta)}y
0 & 0 & \rho_u & 0 & 0
0 & 0 & 0 & \rho_v & 0
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
y_{t-1}
\pi_{t-1}
i_{t-1}
u_{t-1}
1
\end{pmatrix}
+ 
\begin{pmatrix}
0 & 0
0 & 0
0 & 0
1 & 0
0 & 0
\end{pmatrix}
\begin{pmatrix}
\varepsilon_t^P
\omega_t^P
\end{pmatrix}
\]

or:

\[Az_t = BE_t^p (z_{t+1}^P) + C_t z_{t-1} + D_t \varepsilon_t\]
References


Figures and Tables

Figure 1 - Log output

Note: Median value and 15th and 85th percentile bands of the simulated pattern for log output. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication; 4) infrequent central bank optimization.

Figure 2 - Annualized inflation

Note: Median value and 15th and 85th percentile bands of the simulated pattern for the annualized inflation rate. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication; 4) infrequent central bank optimization.
Figure 3 - Annualized nominal interest rate

Note: Median value and 15th and 85th percentile bands of the simulated pattern for the annualized interest rate. The panels report the following scenarios: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication; 4) infrequent central bank optimization.
Figure 4 - Selected policy coefficients - response to the inflation rate

Note: Median value and 15th and 85th percentile bands of the simulated actual and estimated policy response to the inflation rate. Each row reports a different scenario: 1) REE; 2) benchmark asymmetric information and learning case; 3) central bank communication; 4) infrequent central bank optimization.
Figure 5 - Autocorrelations

Note: Median value and 15th and 85th percentile bands of the autocorrelations of the variables of interest in the performed simulations. Each row reports a different scenario: 1) RE; 2) benchmark asymmetric information and learning case; 3) central bank communication; 4) infrequent central bank optimization.
Table 1 - Median standard deviations relative to the RE case

<table>
<thead>
<tr>
<th>variable</th>
<th>benchmark</th>
<th>CB comm.</th>
<th>infrequent CB opt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>2.3146</td>
<td>3.2525</td>
<td>2.5704</td>
</tr>
<tr>
<td>annualized inflation</td>
<td>4.8544</td>
<td>6.4617</td>
<td>6.8849</td>
</tr>
<tr>
<td>annualized interest rate</td>
<td>1.7662</td>
<td>1.9320</td>
<td>1.8160</td>
</tr>
</tbody>
</table>

Table 2 - Standard deviations, RE case

<table>
<thead>
<tr>
<th>variable</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>0.0134</td>
</tr>
<tr>
<td>annualized inflation</td>
<td>0.8000</td>
</tr>
<tr>
<td>annualized interest rate</td>
<td>3.8500</td>
</tr>
</tbody>
</table>

Table 3 - Correlations in the RE case

<table>
<thead>
<tr>
<th></th>
<th>$y_t$</th>
<th>$\pi_t$</th>
<th>$i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>1.0000</td>
<td>0.6172</td>
<td>-0.7361</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>0.6172</td>
<td>1.0000</td>
<td>-0.3548</td>
</tr>
<tr>
<td>$i_t$</td>
<td>-0.7361</td>
<td>-0.3548</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 4 - Correlations in the benchmark case

<table>
<thead>
<tr>
<th></th>
<th>$y_t$</th>
<th>$\pi_t$</th>
<th>$i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>1.0000</td>
<td>0.6054</td>
<td>-0.7428</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>0.6054</td>
<td>1.0000</td>
<td>-0.2288</td>
</tr>
<tr>
<td>$i_t$</td>
<td>-0.7428</td>
<td>-0.2288</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 5 - Correlations in the case of central bank communication

<table>
<thead>
<tr>
<th></th>
<th>$y_t$</th>
<th>$\pi_t$</th>
<th>$i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>1.0000</td>
<td>0.5099</td>
<td>-0.7252</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>0.5099</td>
<td>1.0000</td>
<td>0.0122</td>
</tr>
<tr>
<td>$i_t$</td>
<td>-0.7252</td>
<td>0.0122</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Table 6 - Correlations in the case of optimization every $k$ periods

<table>
<thead>
<tr>
<th></th>
<th>$y_t$</th>
<th>$\pi_t$</th>
<th>$i_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>1.0000</td>
<td>0.5938</td>
<td>-0.7051</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>0.5938</td>
<td>1.0000</td>
<td>-0.0389</td>
</tr>
<tr>
<td>$i_t$</td>
<td>-0.7051</td>
<td>-0.0389</td>
<td>1.0000</td>
</tr>
</tbody>
</table>