Estimating a Banking–Macro Model for Europe Using a Multi–Regime VAR*

Stefan Mittnik† and Willi Semmler‡

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Abstract

This paper introduces a Banking-Macro Model and estimates the linkages through a Multi-Regime VAR (MRVAR). We introduce a dynamic model which is akin to the Brunnermeier and Sannikov (BS) model (2010). The banking sector is exposed to instability due to adverse movements of asset prices and their impact on risk premia and credit spreads. In contrast to the standard model of the financial accelerator, exhibiting mean reversion, our model, similarly to BS (2010), exhibits local instability. Whereas the standard model leads, in terms of econometrics, to a one-regime VAR we argue for the use of a MRVAR. We estimate our model for EU countries with a MRVAR using a constructed financial stress index and industrial production for those countries. We undertake impulse-response studies with a MRVAR and explore regime dependency of shocks. We show that the shocks have asymmetric effects, depending on the growth regime of the economy, and on the size of the shocks. Small financial stress shocks may not matter, but large shocks are likely to have magnifying effects.

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†Dept. of Statistics and Center for Quantitative Risk Analysis, Ludwig Maximilians Universität München, Germany.

‡Dept. of Economics, New School for Social Research, 79 Fifth Avenue, New York, NY 10003.
1 Introduction

At the center of the large financial meltdown of the years 2007-2008 in the US was the banking system. As Reinhard and Rogoff (2009) and Gorton (2009, 2010) have demonstrated, the banking sector is often at the center of a financial crisis. Most crises have ended up as a meltdown of the banking sector, and the banking sector has usually exacerbated and amplified the crisis whatever origin it had. As Gorton (2010) shows, in the past, loan losses and bank runs where the conventional mechanisms by which the crises where triggered, but more recently, banking crises seem to be strongly related to adverse shocks in asset value losses and financial stress.

Important versions of such studies of the destabilizing effects of the banking sector were put forward after the "great recession" of the years 2007-2009 in the US.\textsuperscript{1} Not many of such studies can be found for Europe.\textsuperscript{2} It is of great interest now whether such adverse feedback movements could currently also occur to the European banking system. We want to study how this destabilizing mechanisms might work in a model of a banking-macro link and apply this to an EU Data set that is now made available from IMF (2011).\textsuperscript{3}

For the US there were recent studies that work with the financial accelerator to capture the financial-real linkage, but the destabilizing dynamics is not sufficiently captured in those models. Also, so far the financial accelerator theory has mainly been applied to firms and households. Bernanke, Gertler and Gilchrist (1999) have shown that the financial market can have amplifying effects. Yet, in the DSGE tradition there is only a locally magnifying effect, through collaterals. Collateral value rises at high level of economic activity, making credit available and cheap, and the reverse happens at low level of economic activity.

Technically, the models are solved through local linearizations about a unique and stable steady state, and the amplifying effects occur only with respect to deviations from the steady state. Also, mostly no debt dynamics is tracked.\textsuperscript{4} The departure from the steady state is eventually mean reverting. Although the economy is accelerating, it will revert back to the steady state. Empirically, this is often shown in a one-regime VAR, see Gilchrist et al. (2009, 2010), Christensen and Dib (2008), and Del Negro et al. (2010).

As the meltdown of the years 2007-8 has demonstrated, shocks to banks seem to be destabilizing rather then mean reverting.\textsuperscript{5} Important papers in this context are Brunnermeier (2009) and

\textsuperscript{1}See Brunnermeier and Sannikov (2010, 2011), He et al (2008), and Adrian et al (2010).
\textsuperscript{2}An interesting recent paper is Monnin and Jakipi (2010), who work with a distance of default model to study the instability of banking for selected EU countries.
\textsuperscript{3}See the IMF’s (2011) Financial Stability Index (FSI).
\textsuperscript{4}Empirically the debt to asset value ratio is predicted to fall in the boom and rise in recessions. This is for example empirically stated in Gilchrist et al. (2009). Yet, as Geanakoplos (2010) mentions, the empirical measure is distorted through the way the debt asset ratio is measured, namely as total assets over equity. Equity value rises in the boom and falls in a recession. A model of the interaction of asset prices and leveraging is presented by Semmler and Bernard (2011).
\textsuperscript{5}Many students of the great depression developed the perception that locally destabilizing effects, arising
Brunnermeier and Pederson (2009), that show that banks often have to liquidate their capital, when asset prices get depressed and margin requirements in the money market rise, which forces the financial intermediaries to take a hair cut and to delever further, with another subsequent fall of asset prices reinforcing the downward trajectory. This has started new research on financial instability putting asset prices and their volatility at the center.\(^6\)

Models attempting to capture such mechanisms often stress that the falling asset prices, generate by fire sales of assets by some intermediaries, have external effects on the financial industry. The possibility of a downward spiral then comes from interconnectedness, interlinkages and contagion. Such studies have started with Greenwald and Stiglitz (1996) and continued with Adrian et al. (2010), Gorton (2010), Geanakoplos (2010), Geanakoplos and Farmer (2009), and Brunnermeier and Sannikov (2010, 2011). Those papers argue that this dynamics will create an endogenous generated jump in risk which is usually triggered by large changes in asset price movements.

This process primarily works through the balance sheets of banks. Banks, in the first instance, may have loan losses.\(^7\) This may be arising from default of the firm or household sector, the foreign sector or resulting from sovereign debt. On the other hand, large shocks to asset prices and financial stress will affect banks – the asset and liability side of their balance sheets – reducing the availability of credit. As the financial stress rises, so will the risk premia, repo rates, the Ted spreads and credit spreads. These spillover effects to other intermediaries (as well as firms and households) creates what BS (2010) call endogenous risk. Following BS (2010), in a dynamic model of the banking sector, we show that such an unstable dynamics on the downside is likely to occur.

As to the empirics of such unstable dynamic one would expect that it is regime dependent: There will be high financial stress and a rise of credit spreads in a period of low economic activity, but low financial stress and narrow credit spreads in a period of high economic activity. To explore this instability empirically, we use a Financial Stress Index (FSI) recently provided by the IMF (2011).\(^8\) The FSI by the IMF is available for a large number of EU countries and the US. We will use this data set for the empirical part of our study.

In our model and empirical work on the banking-macro link, we refer less to asset prices and their

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\(^6\)We may include here what has been called by Gorton (2010) the shadow banking system, such as investment firms, brokers and money market dealers. Those have been growing rapidly in the last 15 to 20 years.

\(^7\)The exposure of banks to the borrowing of the public sector, private firms and households – as well other banks — is likely to matter for the insolvency or liquidity risk of banks, As Gorton (2010) shows, this is often accompanied by bank runs.

\(^8\)See IMF (2011) for EU countries. For other important financial stress measures, specifically for the US Economy, see the St. Louis Fed (2011), and Kansas City Fed (2011).
volatility, as BS (2010, 2011), in constructing the downward destabilizing but to the movement of risk premia and credit spreads to explore the downward instability. We can justify our focus on those measures of financial stress, since in theoretical as well as empirical studies it has been shown that factors such as large asset price falls, rising volatility, higher risk premia, and a run into liquidity are highly correlated with movements in discount rates. The different factors driving discount rates – and through that asset prices – are extensively discussed in Cochrane (2011).

Yet, as we also will show the triggering of the downward instability also depends on the constraints that are set for the banking sector, for example, on their asset accumulation and payouts, affecting risk taking, equity formation and leveraging of the banks. Higher payoffs, for instance, may encourage more risk taking and risk transfer, generating eventually higher aggregate risk and risk premia to be paid by all. So we will explore the behavior of our dynamic model for different constraints on the banks decision variables.

The remainder of the paper is organized as follows. Section 2 builds up a model that reflects the above features. Section 3 solves numerically some model variants using dynamic programming. Section 4 discusses the quality of empirical variables to capture the interaction of banking financial stress and real output. Section 5 presents the MRVAR estimation procedure and reports the impulse-response studies for VAR as well MRVAR for EU countries and the USA. Section 6 concludes the paper. The appendix describes our procedures and presents in detail the multi-country results.

2 The Banking Model and its Dynamics

Next, let us present the above developed ideas in a more formal model, which is closely related to the BS model (2010). We build this model on the balance sheets of the financial intermediaries.

2.1 Basic Model

We are introducing a basic model, akin to the model by BS (2010), by referring to the balance sheets of the banks. On the left hand side of table there are assets, valued at current asset prices. On the right hand side there is debt $d_t$ and net worth $n_t=p_tk_t-d_t$.

The equity might be divided up into inside equity $\alpha(p_tk_t-d_t)$ and outside equity $(1-\alpha)(p_tk_t-d_t)$. The latter may be state dependent.

Next we introduce the dynamics of the variables. The asset price, the capital stock and the debt may evolve as defined in equs (1)-(3).

$$dp_t = \mu_t p_t dt + \sigma_t p_t dZ_t$$ (1)
Table 1: The Balance Sheet of Banks

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t k_t$</td>
<td>$d_t$</td>
</tr>
<tr>
<td>$n_t = p_t k_t - d_t$</td>
<td></td>
</tr>
<tr>
<td>total assets</td>
<td>$\alpha (p_t k_t - d_t) + (1 - \alpha) (p_t k_t - d_t)$</td>
</tr>
</tbody>
</table>

$$dk_t = (\varphi(i_t/k_t) - \delta)k_t dt + \sigma_t k_t dZ_t$$  \hspace{1cm} (2)

$$dd_t = (r_t d_t - (a k_t - i_t))dt$$  \hspace{1cm} (3)

In BS (2010) the growth rate of asset prices in equ. (1) follows a geometric Brownian motion, but it is affected by time varying volatility, $\sigma_t$, for example as a result of rapid sales of assets. As mentioned, instead of using asset price movements and volatility we employ risk premia and credit spreads to capture those effects.\(^9\)

Assets of the financial intermediaries in equ. (2) will be increased by investment, $i_t/k_t$, the function $\varphi(i_t/k_t)$ includes some adjustment cost which is concave in the argument, and $\delta$ is a depreciation rate of assets.\(^10\) The actual gross capital of the bank increases at the rate $i_t/k_t$. The debt evolves at a rate that is determined by the excess spending of investment over capital income, which is defined here as $ak_t$. Investment in equ. (2) will increase the stock of assets for financial intermediaries, but the high rate of purchase of assets will also increase their debt, once the investment spending exceeds their income. The interest rate to be paid on debt, $r_t$, includes a risk premium reflecting asset price shocks and financial stress of banks. It will be made endogenous being state or time depending risk premium. Note that only Equations (1) and (2) are stochastic.

So far we have neglected payouts, bonus payments for executives, which can be viewed to serve the consumption stream of the executives.\(^11\) We can define the payouts as an optimal consumption stream. We can also have the investment being computed as optimal, with $g_t = i_t/k_t$. Then we have a dynamic decision problem such as:

$$V(k, d) = \max_c E \int_0^{\infty} e^{-\rho t} U(c_t) dt \quad \text{s.t. (2) and (3)}. \hspace{1cm} (4)$$

The latter model includes now payouts, $c_t$, which is used for a consumption stream.\(^12\) The future

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\(^9\)As mentioned above, in empirical studies it is shown that large asset price falls, rising volatility, higher risk premia as well as higher discount rates are highly correlated. The relationship of those are discussed in Cochrane (2011).

\(^10\)In their recent version, Brunnermeier and Sannikov (2011) write a model with capital assets, that could be banking capital or real capital. The model is equivalent of there is perfect substitution among them.

\(^11\)In Semmler and Bernard (2011) bonus payments of the six largest US investment banks are computed. Bonus payment, as a percent of revenues, went up from roughly 10 percent in 2000 to 35 percent in 2007.

\(^12\)In recent attempts of financial market reforms in Europe the cash payment of bonus payments is planned
payouts are discounted at a rate $\rho$. Note that we have here $g_t = i_t/k_t$. Note also that in equ. (3) if the excess of spending for new assets and payouts exceeds the income generated, then the debt of the financial intermediary will rise.\(^\text{13}\)

We want to remark that the above is a standard model of wealth management, now commonly used to study wealth management of financial institutions, see He and Krishnamurthy (2008). If we replace the constant income for a unit of wealth, $a$ in $a k_t$, by a weighted average of risky and risk free returns of a wealth fund $k_t$, then the remaining parts of the equations above are reasonably familiar from the wealth management literature, see also Semmler et al. (2009). Yet the explicit equation for the evolution of debt of the financial intermediary, as represented in equ. (3), is usually missing. This reflects the innovative part of the model by BS (2010) and other recent literature.\(^\text{14}\)

Now let us derive a dynamic equation for the debt-asset ratio.\(^\text{15}\) Let us take as the debt-asset ratio: $d_t/k_t$: We can rewrite this, for convenience, as $\omega = -(d_t/k_t)$.\(^\text{16}\) Taking log and time derivative of this, we can write the asset accumulation and debt dynamics with the previous objective function of the financial intermediaries as:\(^\text{17}\)

$$
V(\omega_t) = \max_{\tilde{c}_{tt}, g_t} \int_0^{\infty} e^{-\rho t} U(\tilde{c}_t) dt
$$

$$
d\omega_t = (\left(g_t - r_t + \sigma^2\right)\omega_t + a - \tau(g_t)) dt - \tilde{c}_t + \sigma_t \omega_t dZ_t
$$

Hereby $\tilde{c}_t$ is the new control variable.\(^\text{18}\) Term $\tilde{c}_t$ is the consumption wealth ratio, $\frac{c_t}{k_t}$. The expression $\tau(g_t)$ represents a convex adjustment cost which is affecting the size of borrowing to achieve a growth rate $g_t$. This is modeled by following the capital adjustment cost literature. Yet, of course only the growth of wealth $g_t$ appears in the equation for the evolution of assets $k_t$. The other expressions in equ. (6) are straight forward derivations from the negative of the growth rate of the debt-asset ratio as stated above.

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\(^\text{13}\)As mentioned before, for the problem of a social planner, which is equivalent to a monopoly problem of the financial intermediary, the prices are endogenous and do not play an additional role at first, see BS (2010)\(^\text{14}\)\(^\text{14}\)See for example Hall (2010) who also includes an equation for the evolution of debt. Note that recently, as also BS (2010) are arguing, financial intermediaries have been encouraged by more risk taking through transfer of risk to outside investors, for example through CDO’s and consequently the financial intermediaries will build up their debt – thus their default risk – more.\(^\text{15}\)Note that we use stocks of assets and debt, in contrast to Geanokoplos (2010) who uses flows as leverage measure, hereby then leveraging is highly positively correlated with booms.

\(^\text{16}\)For a similar approach, see BS (2010) and Hall (2010).\(^\text{17}\)A derivation of a dynamic equation in the stochastic case, using Itoh’s lemma, is given in BS (2010). The term $\sigma^2$ comes in through Itō’s lemma.
2.2 Some Specifications

Next we undertake some specifications of the basic model that may help to highlight the inherent amplification mechanisms. A first type of specification pertains to the impact of financial stress on the banks decisions, in particular to role risk premia or credit spreads, banks are facing. BS (2010) provide basic proofs of the probability of instability with endogenous asset prices.\(^\text{19}\)

Though, at first sight, one might look at asset price volatility but what actual might magnify the downward spiral, is the financial stress and credit spreads that the capital market requires.\(^\text{20}\) A first simple way to capture such varying credit spreads is to introduce state dependent credit spreads, determined by leverage ratios. We make credit spreads a function of leveraging, but this will be a bounded function, with a floor and ceiling. The floor will be a risk free interest rate, and the ceiling will be given by an upper bound.\(^\text{21}\) We take take for the credit spread

\[
 r_t = r_\varpi(\arctan(\omega_t)).
\]  

(7)

The \(\arctan\) function above avoids the extreme instabilities arising in the asset price leveraging dynamics as might be implied in Geanokoplos and BS (2010), and it makes the credit spreads depending on leveraging \(\omega_t\). Here the spread is made endogenous but it has bounds. Banks face credit spreads, but the impact on the debt dynamics is initially low, and then rising with leverage. We thus have that the decisions on consumption and asset growth are depending on state dependent credit spreads that the financial intermediaries are exposed to.\(^\text{22}\) This will be built into our DP solution algorithm presented in Appendix 1.

Yet, as above mentioned, there are other factors affecting financial stress and risk premia, due to externalities and contagion effects from asset price movements.\(^\text{23}\) So, the alternative is to

\(^\text{19}\)Here, the role of heterogeneous expectations and trading strategies for market price movements could be explored as well, as in Chiarella et al. (2009: chs 6-9). This could be related then to risk premia and credit spreads. There could also be more general effects such as liquidity problems, fire sales of assets and market dysfunctions, all giving rise to higher risk premia, see Geneva Report (2009). The rising volatility has also been included by numerous the financial stress indices developed by the KCFSI and the IMF FSI, to be discussed in sect. 4, where it is as own to affect credit spreads. Volatility is also relevant in a distance to default model where it is shown that the distance to default shrinks with rising volatility, and thus the risk premia rises, see Semmler (2011, ch. 19), and Bonnin et al (2010).

\(^\text{20}\)Furthermore, it is very likely that positive and negative asset price shocks may have asymmetric effects. This is also discussed in Basel III. BS (2010) have studied also the effect of a rising volatility \(\sigma_t\) on the spread.

\(^\text{21}\)See Beja and Goldman (1980), and for a recent use see Chiarella et al. (2002), and Chiarella (2009, ch. 6 and 8, where extensions to heterogeneous agents and the stochastic case are provided.

\(^\text{22}\)Since the publication of the financial accelerator principle by Bernanke et al (1999) the economists have been greatly concerned with the fact that borrowing cost moves counter-cyclically, and the ease of lending standards pro-cyclically. Accordingly, we have proposed a state dependent risk premium \(r(\omega_t)\). The risk premium, and thus the credit spread, is hereby made state dependent thus \(r(\omega_t)\) rises with the leveraging. If the involved parameters are appropriately chosen, the risk premium goes to zero and a constant (risk free) interest rate will re-emerge. The constant interest rate, as assumed in some version of BS (2010), is a limit case of the above scenario. For a model with a state dependent risk premium, see Gruene et al. (2004).

\(^\text{23}\)Adrian et al. (2010) have defined such a risk premium as a macro economic risk premium. They summarize
take a time varying risk premium that captures those factors. In order to capture those effects we extract low frequency components from our financial stress index and use this as proxy for time varying risk premia.\textsuperscript{24} This is obtained by using spectral analysis to estimate low frequency movements in the IMF (2011) FSI.\textsuperscript{25} In our DP of appendix 1 we would then have one additional state variable such as
\begin{equation}
dx = 1dt
\end{equation}
This represents a time index to capture time varying risk premia. The low frequency components in the credit spread is indexed on the variable \(x\), representing time in the DP algorithm. It can be computed and included in the numerical procedure. We would thus have in equ.(6):
\begin{equation}
r_t = r_x(x).
\end{equation}
Formally, our stochastic dynamic decision problem will then have two decision variables and three state variables, the leverage ratio \(\omega_t\), the time index \(x_t\) and the stochastic term \(dZ_t\) in the above variant, with \(r_t\) time varying. Details of the estimation are discussed in appendix 2.

Another type of specification pertains to constraints on decision variables, for example on payoffs, resulting in consumption stream, \(\tilde{c}_t\). We could assume that payoffs are be constrained by some financial market regulation; for example, if the net worth, as a ratio of net worth to total assets, falls below a certain safe threshold, then the payouts could be reduced. Equivalently we could postulate that if the debt to asset ratio moves below some threshold, lets say \(\omega = -(d_t/k_t) \leq \varpi\), then the payouts are decreased or set to zero. It could hold that the payouts are used to give the managers an incentive to reduce leverage, so when the leverage is lower, a higher payouts payments could be allowed.\textsuperscript{26} This might be considered as a penalty on risk taking and high leveraging – the former resulting from leveraged asset purchases.\textsuperscript{27} The dynamics of the debt-wealth ratio, once those constraints on decision variables are introduced, are likely to change.\textsuperscript{28}

\textsuperscript{24} As mentioned, important components of the financial stress index are credit spreads, for example the BAA/AAA spread or the BAA/T-Bill spread. Many studies have worked with those measures, see for example, Gilchrist et al. (2009), see also the important role of credit spread in the IMF (2012) FSI.

\textsuperscript{25} This has been done in Semmler and Hsiao (2009) to estimate time varying asset returns and can be employed here, see appendix 2.

\textsuperscript{26} This is for example planned by Basil III, where it refers to “linkages of the total variable compensation pool to the need...to maintain a sound capital base”.

\textsuperscript{27} A further issue might be that the financial intermediaries have in fact transferred risk to outside investors through securitization, i.e. through pooling and tranching of mortgage debt or other kind of liabilities, through MBSs or CDOs. Successfully undertaking the transfer of risk encourages them to take on more risk, but passes the verification cost on to someone else. The verification cost usually defines the amount that financial intermediaries have to pay, but if it is passed on, they can generally borrow at a lower risk premium, see BS (2010: sect. 4) for details of such considerations. Higher bonus payouts may encourage more risk taking and risk transfer, generating eventually higher aggregate risk and greater risk premia.

\textsuperscript{28} One could consider a further modification that takes into account the availability of funds for the financial
3 Solution Method and Numerical Results

As BS (2010), the dynamics for a model such as represented by equs. (5)-(6) should not be studied by common linearization techniques. The first or even second order Taylor approximations to solve for the local dynamics of a model such as (2)-(3) or (5)-(6) will not properly capture the global instabilities of the model in particular in some regions of the state space. We have used the dynamic programming method by Gruene and Semmler (2004) to study the dynamics of the stochastic version of the model (5)-(6). Here, the debt to asset ratio is the state variable, and the control variables are the growth rate of assets and payouts for consumption, for which we will introduce constraints, and we will use state dependent and time dependent credit spreads as specified in sect. 2.2.

3.1 Solution Method and Model Variants

When we use a dynamic programming method to explore the local and global dynamics we use a coarse grid for a larger region of the state space, and then employing grid refinement for smaller region. The DP in the appendix 1 can provide us with information on the truly global dynamics in a larger region of the state space without losing much accuracy (see Becker et al., 2007). When we study the basic model variants we explore the stability properties of each variant.

When state dependent credit spreads is defined as a function of leveraging, \( \omega_t \) we define the risk premia and credit spreads as

\[
 r_t = \kappa \arctan(\omega_t). \tag{10}
\]

as mentioned, the \( \arctan \) function above, with \( \kappa > 0 \), avoids the extreme instabilities arising in the asset price leveraging dynamics as might be implied in Geanokoplos (2010) and BS (2010). We restrict our considerations here to the simplest case, as in equ (10), where the bank pays a risk premium formulated in a simple way\(^{29}\) which is built into our DP algorithm.

The alternative is to introduce a time varying credit spread which is detailed in appendix 2. We take here the case of Germany and proxy the time varying risk credit spread by the IMF intermediaries. There might be a fraction of households that accumulate risky assets, which will provide funds for the financial intermediaries. A fraction of funds could also come from capital inflows, see Caballero and Krishnamurthy (2009). In this context, the inflow of funds from the Central Bank could be considered, which for example took place in the US in the years 2008 and 2010 when the Fed employed an unconventional monetary policy, called quantitative easing, buying bad — and rapidly declining — assets from the financial intermediaries. The ECB provision of a three year low interest rate liquidity for the EU banks in December 2011 is a similar case. This has a mitigating effect on the unstable forces generated by the banking system. An estimation of this effect will be presented in section 5. On the other hand, the precautionary motives of households (and firms), the “run into high quality assets”, would lead to a reduction of financial funds for the financial intermediaries.

\(^{29}\)As mentioned above, we use the same \( \arctan \) function-type for the risk premium. This has a lower limit, the risk free rate, and an upper limit. The upper limit of a premium charged is justified, since, as Stiglitz has always argued, with higher default premia, the lender might have loan losses at greater credit spreads.
(2012) FSI.\textsuperscript{30} which by and large reflects the varying risk premia. We estimate low frequency components of the credit spreads by:

\[ r_t = \alpha_1 - \alpha_2 (t - t_0) + \sum_{i=1}^{n} \left( a_i \sin \left( \frac{2\pi}{\tau_i} (t - t_0) \right) + b_i \cos \left( \frac{2\pi}{\tau_i} (t - t_0) \right) \right). \tag{11} \]

Note that the first two terms in the above equation represent a constant and time trend of credit cost, the next terms are the low frequency components. Appendix 2 reports how many periodic components are needed to properly proxy the actual time series of the credit spread and the coefficients of equ. (11).

As to the payouts\textsuperscript{31}, we introduce alternatively broader and narrower constraints, which will, however, always be non-negative.\textsuperscript{32} When we define the payouts for, \( \tilde{c}_t \), we let the choice to be taken from and interval: \( c_{\min} < \tilde{c}_t < c_{\max} \). So, the payout is always positive but is constrained.\textsuperscript{33}

For the case of less constrained payouts we assume \( 0.01 < \tilde{c}_t < 0.3 \) and for constrained payoffs, we assume that \( 0.01 < \tilde{c}_t < 0.05 \). As concerning the asset growth we constrain the growth of assets to \( -0.1 < g_t < 0.1 \).

### 3.2 State-dependent Credit Spreads

We here report the results of the state-dependent credit spread and larger interval for payoffs. As to the parametrization of our model we take: \( a = 0.25 \), \( \alpha = 0.3 \), \( \sigma = 0.008 \), and \( \gamma = 0.03 \) and \( \rho = 0.03 \).

Figure 1 shows on the horizontal axis the state variable \( \omega \) and on the vertical axis the stochastic shocks to the state variable \( \omega \). Since we have stochastic shocks, with pre-defined standard deviation \( \sigma = 0.008 \), the path of \( \omega \) varies in the state space, and thus there is no unidirectional vector field, i.e. the path of \( \omega_t \) is not a straight line. In our numerical procedure the shocks are drawn from a distribution having a pre-defined standard deviations \( \sigma = 0.008 \). As visible from the numerical solution path in Figure 1a, there is an unstable steady state roughly about a low

\textsuperscript{30} Note that the major components of the financial stress indices of the IMF (2011) FSI as well as the KCFSI, and the STLFSI, are variables that are capturing risk premia and thus credit spreads.

\textsuperscript{31} As to the constraints on the the growth rate of assets (or certain types of assets) acquired by the banks, those are probably hard to constrain, unless there are borrowing constraints introduced as Geanopoplos (2010) seems to suggest. So we have used here rather broad constraints for the growth rate of assets.

\textsuperscript{32} Regarding the payoffs, we want to remark that BS (2010) conjecture that when the bonus payouts are chosen less constrained “the system is relatively stable near its “steady state” ... but becomes unstable below the steady state... “(BS, 2010:17). The reason for the result from unconstrained endogenous payout is: “With endogenous payout, the steady state naturally falls in the relative unconstrained region where amplification is low, and amplification below the steady state is high” (BS, 2010:18). BS make this statement with respect to the ratio of net worth to assets. Since we take the negative of the debt to asset ratio, the statements can be immediately translated into the properties of our model using the debt to asset ratio.

\textsuperscript{33} Note we also could allow for dividend payments, in fact as our model is constructed the bonus payments can encompass dividend payments.
level of debt to asset ratio $\omega^* = -0.85$. In other words small initial leveraging will eventually end up at low level of debt to asset ratio, because the credit spread is low, but larger leveraging – a shock that moves the leverage above $\omega^* = -0.85$ makes both the leveraging as well as the credit spread rising, a vicious cycle: higher leverage creates higher credit spreads and higher credit spreads results in higher leveraging. On the other hand, there is a small domain of attraction: if the financial intermediary starts with low leveraging and low credit spread, both may be reduced further.

It is the debt to asset ratio, and its accompanying credit spread, as well as the payouts, that are amplifying. Thus, under those conditions a leveraging ratio beyond a threshold, is likely to be dynamically unstable, as BS (2010) predict.

### 3.3 Time-depending Credit Spreads

Next we consider time dependent credit spreads but also tighter constraints on payouts. BS (2010:32) state that allowing the debt to asset ratio rise too much, driven by the incentives of

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34 Above the large credit spreads and larger pay-offs make the debt to asset ratio rise, moving the $\omega_2$ toward another steady state, which is roughly at $\omega^{**} = -3.8$

35 Note that we do not pursue the issue here at what leverage ratio bankruptcy would occur. This depends on the distance to default, which is defined by the KMV model by the distance of the asset value of the bank to the debt, divided by the standard deviation (volatility) of the asset value. We are not pursuing this question here, since we do not explicitly computing the asset value of the financial intermediary. This is issue is pursued in Gruene and Semmler (2005).

36 Since the the shape of the value function for this case is similar to the next case, it will be discussed in sect. 3.3.
the intermediaries to take on too much risk for the sake of short term profits, allowing for high payouts, and neglecting externalities may lead to damages and downturns. In their view the triggering of the downturn in the financial, product and labor markets, and the higher asset price volatility, results from not taking into account the full extent of the externalities.

They thus state that limiting payouts should be welfare improving. More explicitly they say: “We would like to argue that a regulator can improve social welfare by a policy that limits bonus payments within the financial sector. Specifically, suppose that experts are not allowed to pay themselves as long financial intermediaries are not sufficiently capitalized” (BS, 2010: 32). This type of regulatory effort would keep sufficient capital within the banking system and make it more stable.  

This conjecture can also be shown to hold using our DP solution algorithm. In order to explore this variant with time varying credit spreads and tighter payouts, we, as before, allow for negative and positive growth rates of the assets purchased by the financial intermediaries to be in the range $-0.1 < g_t < 0.1$, but we constrain the consumption to capital ratio by $0.01 < \tilde{c}_t < 0.05$. Again, the latter is always positive but it is constrained not to be too large. Under the condition that the growth rate of assets and the consumption rate can be chosen optimally, yet payouts will be constrained to be low.

Figure 2 shows the dynamics where the domain of attraction is increased, the steady state is now roughly at $\omega^* = -3.9$, which is also repeller: with lower leverage and low payouts, the debt to

\[ \frac{dZ}{d\omega} \]

\[ 0.030 \]

\[ 0.012 \]

\[ 0.006 \]

\[ -0.006 \]

\[ -0.018 \]

\[ -0.006 \]

\[ 0.012 \]

\[ 0.030 \]

\[ \omega \]

\[ t \]

\[ 33.333 \]

\[ 0.000 \]

\[ -10.000 \]

\[ -8.000 \]

\[ -6.000 \]

\[ -4.000 \]

\[ -2.000 \]

\[ 0.000 \]

\[ \omega^* = -3.9 \]

A similar view is present in the Geneva Report (2009, sect. 6.2) and Basel III.
Figure 3: Value function for time-varying credit spread, small payouts, and large domain of attraction

asset ratio will go to zero. The domain of attraction of the zero debt to asset ratio is considerable enlarged. The dangers of large externalities, financial stress and meltdowns are reduced. So, now a high debt to asset ratio can be stabilized if risk premia and payouts are small.

As to our Figure 2, the shock $dZ_t$ moves the trajectories, along the vertical axis, whereas the debt dynamics with credit spread moves the trajectories along the axis $\omega$. Note that we have here now the shock drawn from the range $-0.1 < dZ_t < 0.1$, and we have used $-10 < \omega_t < 0$. For our third dimension, our time index $x_t$, the range is defined as $0 < t < 100$. But because we have a movement in a three dimensional space, the time axis, the third variable, the one that goes to the back, is fixed. We are showing here the projection of the trajectories to a 2 dimension space.

Figure 3 shows the corresponding value function, revealing the result that total welfare (for the financial intermediaries) is rising with lower debt to asset ratio.

Overall, the risk premium, and thus the credit spread, is, in our two variants, made state-and time-dependent. The state dependent credit spread is likely to trigger a vicious cycle. If the spread goes to zero interest rates tend to reflect the short term interest rate set by the central

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38 There appears that an initial debt to asset ratio below that new threshold $\omega^* = -3.9$, with low payouts, and time varying risk premia, will produce always stability.

39 The rise of the value function to the right of the steady state is reasonable, since the welfare from lower credit spread should be higher. It was computed through our numerical solution procedure. The value function for the case of Section 3.2. was similar to the case above. It also increased with the debt to asset ratio falling, i.e., the value function to the right of $\omega = -0.85$ in Figure 1 was rising. Note that the shape of the value functions in the cases of Section 3.3. as well as 3.2. are roughly the same as shown in BS (2010) in their Figure 7, though we have negative values on the vertical axis, since we are taking log $\tilde{c}_t$, not $\tilde{c}_t$, in the preferences.
bank. For the state-dependent credit spread and relatively larger payouts, we can observe that for a leverage beyond a threshold, the leverage will finally be built up quickly through a vicious cycle, and for small payouts and time-dependent credit spread there is a much larger domain of the zero attractor of the debt to asset ratio. Thus, larger shocks are likely not to matter as much as in the first case.

The downward instability and depends not only on the financial stress (and thus the size of the risk premia), but also on the constraints of payouts: With high payouts, the high leverage equilibrium can become an attractor, whereas with tighter constraints and low payouts the zero level leveraging can become the attractor for a large domain of attraction.

4 Financial Stress and Output Measures

Given our model variants with smaller and larger domains of attraction, a study of the study of the range of instability- or stability- is an important empirical issue to be explored next. Yet, what measures can one utilize to empirically evaluate the predictions of the model and to conduct empirical estimates. One issue is to measure the financial stress of banks, and the other linkages of the banking-macro feedback.

Note that our model variants above, may suggest to take leverage ratios as measure triggering banking instability: So high leverage implying high financial stress and low leverage the reverse. However, there is an issue whether the ratio of net worth to capital assets, or the reverse measure, the leveraging $\omega$, can be used as good measure of financial stress. This measure is greatly affected by the market valuation of assets as well as liabilities. In particular, asset valuation is heavily impacted by the confidence and estimate of income streams the asset generates, as well as presumed discount rates, and the liabilities such as bonds or short and long term loans are strongly affected by their corresponding risk premia. Moreover, credit constraints, for example,
as measured by the Fed index of changes in credit standards to determine the ease and tightness of obtaining credit as well as default premia and credit spreads and short term liquidity, are also important financial stress factors for financial intermediaries. All this will affect credit demand and supply of financial intermediaries. We thus need more extensive measures than only leverage to evaluate financial stress.

In contrast, as mentioned above, we propose to measure financial stress empirically by taking the IMF’s (2011) financial stress index, the FSI. This is available for a large number of EU countries. The IMF’s (2011) FSI refers to three major sources and measures of instability, namely: 1) a bank related index – a banking beta as 12-month rolling beta of bank stock index and a Ted or interbank spread, 2) a security related index – a corporate bond yield spread, an inverted term spread, and a monthly stock returns (measured as declines), six-month rolling monthly squared stock returns and finally, 3) an exchange rate index – a six-month rolling monthly squared change in real exchange rates. All three sets of variables are detrended and scaled with their standard deviations in order to normalize the measures.

As measure for the performance of the macroeconomy we take a monthly production index for the different countries, or what is more proper in the context of our model, the growth rate of the monthly production index of the various countries we are considering.

As concerning the IMF FSI, combining the three groups of variables with appropriate weight in a stress index and contrasting it with the monthly production index, one can observe clearly a counter-cyclical behavior. This is illustrated in Figure 4, where the variables are shown for a three–month moving average.

As the comparison of the smoothed growth rate of the production index and the stress index in Figure 4 show there is less financial stress in good times, but more in bad times. Financial intermediaries are clearly doing better in economic booms than in recessions. Given the apparent

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45 As noted the Federal Reserve Bank of Kansas City and the Fed St. Louis have also developed a general financial stress index, called KCFSI and STLFSI respectively. The KCFSI and the STLFSI, take into account the various factors generating financial stress. The KC index is a monthly index, the STL index a weekly index, to capture more short run movements, see also Hatzius et al (2010). Those factors can be taken as substitutes for the net worth or leverage ratios as measuring financial stress of financial intermediaries. See also the Bank of Canada index for Canada, i.e. Illing and Lui (2006). Both the KCFSI and STLFSI include a number of variables and financial stress is related to an: 1) increase the uncertainty of the fundamental value of the assets, often resulting in higher volatility of the asset prices, 2) increase uncertainty about the behavior of the other investors, 3) increase the asymmetry in information, 4) increase the flight to quality, 5) decrease the willingness to hold risky assets, and 6) decrease the willingness to hold illiquid assets. The principle component analysis is then used to obtain the FSI. Linear OLS coefficients are normalized through their standard deviations and their relative weights computed to explain an FSI index. A similar procedure is used by Adrian and Shin (2010) to compute a macro economic risk premium. We want to note that most of the variables used are highly correlated with credits spread variables. They also have the highest weight in the index, for details see Hakkio and Keeton (2009, tables 2-3.).

46 This is published for advanced as well for developing countries, see IMF (2008) and IMF FSI (2011)

47 This coincides also with the empirical study by Gorton (2010) that there is more insolvency of financial institutions in bad times.
Figure 4: Financial stress and output for Germany: Financial stress index (IMFFSI, lower graph) plotted against growth rates of industrial production (3 month moving average, upper graph).

linkages between the FSI and economic activity, we would also expect a strong linkage between net worth, or leveraging, of financial intermediaries and economic activity, since the financial stress is affecting the balance sheets of financial intermediaries.\(^{48}\)

A “one-regime VAR” has been used frequently to study the financial accelerator.\(^{49}\) Yet those “one-regime VAR” studies presume only local behavior of the variables, symmetry effects of shocks and mean reversion after the shocks. What we will pursue here is an MRVAR. Our MRVAR analysis\(^{50}\) takes the IMF FSI as empirical measure of financial stress, and the growth rate of the monthly production index—the latter is also used as a threshold variable to define growth regimes for a selected EU countries.

5 Empirical MRVAR Analysis

To empirically examine whether or not local instabilities are present and, if so, to assess their consequences, we require a modeling framework that can accommodate varying dynamic patterns across alternative states of the economy. The multi–regime vector autoregression (MRVAR) approach we adopt permits us to detect the presence of multiple regimes and to investigate regime–dependence in the responses to shocks to the system. Shocks might occur during a

\(^{48}\)We want to note that the financial stress index can also be linked to some broader index of economic activity, see Hakkio and Keeton (2009) see Hakkio and Keeton (2009).

\(^{49}\)Estimating the financial accelerator for the macroeconomy with a “one regime VAR”, see Christensen and Dib (2008). For the application of the financial accelerator to study financial intermediaries in a “one regime VAR”, see Hakkio and Keeton (2009) and Adrian et al. (2010).

\(^{50}\)For an application of MRVAR modeling see also Mittnik and Semmler (2009) and Ernst, Mittnik and Semmler (2010).
regime with great instability, as, for example, in the case of a high steady-state leverage ratio (or high stress) discussed in Section 3 (see Figure 1) the effects will be larger as compared to a regime with a large domain of attraction for the zero leverage ratio (and low stress), see Figure 2. As a consequence, responses to positive and negative shocks may have different effects, as may variations in the size of the shocks.

We estimate MRVAR models for six countries: the EU countries Germany, France, Italy, Spain, and the UK as well as the U.S. We will discuss the results for Germany in some detail those for the other countries are graphically summarized in Appendix 3 and discussed briefly below.

5.1 MRVAR Approach

To assess the dependence of the responses to shocks to the stress index, we employ an MRVAR approach. A major limitation of conventional linear VAR models is that shock responses are independent of the state of the economy at the time a shock occurs. Also, VAR response profiles are invariant with respect to the sign and size of a shock. That is, responses to positive and negative shocks are mirror images; and responses to shocks of different sizes are simply scaled versions of the response to a shock of size one. To capture state dependencies and asymmetries of shock responses, a nonlinear model needs to be specified. The “mildest” form of generalizing a linear, constant-parameter VAR is to adopt a piecewise linear VAR, such as Markov-switching autoregressions (Hamilton, 1989) or threshold autoregressions (Tong, 1978, 1983). A characteristic of Markov–switching autoregressions is that the states are unobservable and, hence, do not necessarily have an obvious interpretation. Also, a given observation cannot directly be associated with any particular regime. Only conditional probabilistic assignments are possible via statistical inference based on past information.

For our purposes, namely state–dependent response analysis, states are associated with specific stages of the business cycle as measured, for example, in terms of output growth. MRVAR models in the form of threshold autoregression models of Tong (1978, 1983) or, in a vector setting, of multivariate threshold autoregressions (Tsay, 1998) are obvious candidates. In contrast to Markov–switching autoregressions or standard multivariate threshold autoregressions, our approach assumes that we can, based on some observable variable, define upfront a meaningful set of regimes, which are not a result of some estimation procedure, but rather motivated by the objective of the empirical analysis. This is preferable in our setting, where we are interested in evaluating the potential effectiveness of policy measures for specific states of the economy.

The MRVAR specification adopted here is given by

$$y_t = c_i + \sum_{j=1}^{p_i} A_{ij} y_{t-j} + \varepsilon_{it}, \quad \varepsilon_{it} \sim (0, \Sigma_i), \quad \text{if } \tau_{i-1} < r_{t-d} \leq \tau_i, \quad \text{for } i = 1, \ldots, M, \tag{12}$$
where \( r_{t-d} \) is the value of the threshold variable observed at time \( t - d \); and regimes are defined by the (prespecified) threshold levels \(-\infty = \tau_0 < \tau_1 < \cdots < \tau_M = \infty\). In the following analysis we estimate a two-regime VAR, with the output-growth rate as the threshold variable, and the average growth rate defining the threshold for the sample.

In addition to the more straightforward regime interpretation, MRVAR models are also more appealing than Markov-switching autoregressions as far as estimation is concerned. Rather than EM-estimation, MRVARs with predefined threshold levels resemble conventional VARs and can be estimated regime by regime, using common least-squares or Bayesian techniques.

Response analysis for linear VAR models is straightforward. Point estimates and asymptotic distributions of shock response can be derived analytically from the estimated VAR parameters (see Mittnik and Zadrozny, 1993). In nonlinear settings, this is, in general, not possible, and one has to resort to Monte Carlo simulations. Following Koop et al. (1996), the so-called generalized impulse responses, which depend on the overall state, \( z_t \), the type of shock, \( v_t \), and the response horizon, \( h \), are defined by \( GIR_h(z_t, v_t) = E(y_{t+h} | z_t, u_t + v_t) - E(y_{t+h} | z_t, u_t) \), where the overall state, \( z_t \), reflects the relevant information set. For a Markov-switching VAR process, \( z_t \) comprises information about the past realizations of \( y_t \) and the states; for an MRVAR process with known threshold levels, only information about past realizations \( y_{t-1}, \ldots, y_{t-p_{\text{max}}} \), with \( p_{\text{max}} = \max(p_1, \ldots, p_M) \), is required.

To understand the differences in the dynamic characteristics between the different regimes, regime-specific response analysis as in Ehrmann et al. (2003) is helpful. MRVAR models assume that the process remains within a specific regime during the next \( h \) periods. This is reasonable when regimes tend to persist or when we are interested in short-term analysis and, particularly, helps to understand regime-specific dynamics.

### 5.2 Estimation

For our bivariate analysis of the six countries we use monthly data on industrial production (IP) and the IMF’s Financial Stress Index (FSI) covering more or less the period from mid 1981 to mid 2011.\(^{51}\) Below, we focus on the results for Germany. The results for the other countries are detailed in Appendix 3.

We estimate a standard VAR and an MRVAR model for the IP-growth rate and absolute changes in FSI, and define \( y_t = (100 \Delta \log IP_t, \Delta FSI_t,)' \). We use the AIC for model selection. For

\(^{51}\)We use seasonally-adjusted industrial-production data from the OECD (2011); the FSI data were provided from the IMF (2011).
The MRVAR model (12), the AIC is given by

\[ AIC (M, p_1, \ldots, p_M) = \sum_{j=1}^{M} \left[ T_j \ln |\hat{\Sigma}_j| + 2n \left( np_j + \frac{n + 3}{2} \right) \right], \tag{13} \]

where \( M \) is the number of regimes; \( p_j \) is the autoregressive order of Regime \( j \); \( T_j \) reflects the number of observations associated with Regime \( j \); \( \hat{\Sigma}_j \) is the estimated residual covariance matrix for Regime \( j \); and \( n \) denotes the number of variables in vector \( y_t \). Formulation (13) differs from that in Chan et al. (2004) in that we account for possible heterogeneity in the constant terms, \( c_j \), and residual covariance, \( \Sigma_j \), across regimes.\(^{52}\)

For the case of Germany, the AIC suggests a fourth–order linear VAR, \( p = 4 \). Specifying a two–regime MRVAR with the threshold, \( \tau \), set to the sample mean of the monthly IP–growth rate, given by 0.1469%, we assign observations associated with below–mean (above–mean) growth rates to the High–Regime (Low–Regime). Then, the AIC suggests an autoregressive order of three for both regimes. Although the MRVAR has quite a few more free parameters than the fitted VAR (35 vs. 21 parameters), the AIC favors the two–regime MRVAR with AIC \( (M = 2, p_{lo} = 3, p_{hi} = 3) = 393.2 \) (and regime–specific sample sizes \( T_{lo} = 176 \) and \( T_{hi} = 186 \)) over a standard VAR with AIC \( (M = 1, p = 4) = 691.1 \).

The model specifications for Germany as well as the other five countries are summarized in Table 2. For all countries, the AIC strongly favors the MRVAR model over the conventional, one–regime VAR. The mean of the six VAR–AICs is 458.4 and substantially exceeds that of the MRVAR–AIC average of 188.9.

### 5.3 Response Analysis for Germany

To assess the effects of linear versus nonlinear model specification, we first look at the estimates of the cumulative unit–shock responses for the VAR model and the regime–specific responses for the MRVAR model. To derive structural responses, we assume that a shock to IP simultaneously affects the FSI, whereas IP reacts with a one–period delay to an FSI shock. The cumulative responses due to a unit shock implied by the estimated VAR model are shown in Figure 5.\(^{53}\)

The results for the conventional VAR model (Figure 5) suggests that a positive one–standard–deviation stress shock has an increasingly negative cumulative IP–growth effect which settles at \(-0.37\%\) after about seven months. The cumulative response of IP to a unit shock in IP itself

\(^{52}\)When employing (13) to discriminate between an MRVAR and a standard VAR specification (i.e., a one–regime MRVAR), we need to include the \( n \) parameters in the intercept vector, \( c \), and the \( n(n+1)/2 \) parameters in the residual covariance matrix for an equivalent parameter count.

\(^{53}\)In the following initial discussion discussion of the general results for the response analysis we, first, focus solely on the responses’ point–estimates. We will consider interval estimates when we discuss the results specific to the question under investigation.
Table 2: Specifications of VAR and MRVAR models

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>France</th>
<th>Italy</th>
<th>Spain</th>
<th>UK</th>
<th>U.S.</th>
</tr>
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<td><strong>VAR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>6</td>
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<tr>
<td>AIC</td>
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<td>667.4</td>
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<tr>
<td>$T$</td>
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<td>357</td>
<td>361</td>
<td>360</td>
<td>361</td>
<td>362</td>
</tr>
<tr>
<td><strong>MRVAR</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Threshold</td>
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<td>0.0526</td>
<td>0.0346</td>
<td>0.0642</td>
<td>0.0572</td>
<td>0.1600</td>
</tr>
<tr>
<td>$p_{lo}$</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$p_{hi}$</td>
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<td>2</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>AIC</td>
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<td>170.4</td>
<td>364.5</td>
<td>439.9</td>
<td>31.27</td>
<td>-266.0</td>
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<tr>
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<td>174</td>
<td>183</td>
<td>177</td>
<td>167</td>
<td>166</td>
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<tr>
<td>$T_{hi}$</td>
<td>186</td>
<td>183</td>
<td>178</td>
<td>183</td>
<td>194</td>
<td>196</td>
</tr>
</tbody>
</table>

settles at 1.16%; and the stress index responds positively to a positive IP shock (0.19%), but more so to a positive FSI shock itself (0.65%).

Next, we explore the MRVAR within–regime response behavior. Clearly, the assumption to stay within a particular regime for an extended period is not very realistic as shocks and regime dynamics may induce regime migration. Also, by looking at the within–regime dynamics, we solely focus on the regime–specific autoregressive parameters and ignore the level effects resulting from differences in the regime intercepts. Any differences in the regimes’ intercepts will induce additional variation in the overall dynamics when the process switches between regimes. However, a regime–specific response analysis is useful as it helps to better understand the short–term dynamics associated with the estimated regimes.

The within–regime MRVAR responses are presented in MRVAR in Figure 6. Here, the cumulative responses to unit shocks implied by each of the two MRVAR regimes are somewhat different. Within the high IP–growth regime (upper half in Figure 6) a positive one–standard–deviation stress shock has practically no impact on IP (-0.02%). Compared to the VAR analysis, IP responds less positively to a unit shock in IP itself, settling at 0.71% rather than unity. The stress index responds somewhat higher to a positive IP shock (0.33%); and the cumulative MRVAR response to a positive FSI shock with 0.74% versus 0.65% is marginally higher than the VAR–implied. Within the low IP–growth regime (lower half in Figure 6) a stress shock has a weak negative impact on IP (-0.17%). With 1.82%, the IP response to positive shock in IP is much stronger than in the high–growth regime or the VAR–implied case. The stress index hardly reacts an IP shock (0.07%); and, with 0.70%, the response to an FSI corresponds to that obtained for the high–growth regime.

Given our objective, namely the evaluation of the impact of banking–sector stress on economic
growth, measured in terms of IP growth, subsequent analysis will focus on the response of IP to shocks to the financial stress index. In order to investigate the economy’s overall growth effects due to FSI shocks we, first, simulate generalized cumulative response functions to unit–impulse shocks. We do this for specific states at which the shock is assumed to occur. The two specific states we select are the sample averages observed for each of the two regimes, as they can be viewed as representatives for low– and high–growth states of the economy. The low–growth regime mean is $\bar{y}_{\text{low}} = (-1.0542, -0.0216)'$, and that for high–growth $\bar{y}_{\text{high}} = (1.2834, 0.0051)'$. From these we simulate both a positive and a negative unit–shock to the stress–index. The mean cumulative responses to IP together with one–standard deviation confidence bands are shown in Figure 7.

The estimated unit–shock responses strongly suggest that the impact of an FSI shock on output varies with the state of the economy. A positive unit–shock in the average high–growth state (top left plot in Figure 7) causes IP to drop by about -1.9% after two years. However, only for the first–month response does the confidence band border the zero line, indicating some significance. The same shock applied in the average low–growth state (bottom left plot), results in an IP contraction, which is twice as large (-0.41%). This result possesses more significance, as the confidence band excludes the zero line for 12 months. Thus, in a recession period an increase in financial stress curbs German IP more severely than during a boom.

If the same FSI–unit–shock is negative, we obtain pretty much the reverse results. In absolute terms, a negative FSI shock during low growth (bottom right plot) has a stronger impact than
Figure 6: MRVAR responses for Germany in a high-growth (top half) and low-growth regime (bottom half)
during high growth. Thus, at least for one–standard–deviation shocks, the German IP responses are mildly sign–asymmetric. Though, only in the low–growth regime do the confidence bands suggest significance beyond the first month.

Next, we investigate to what extent the size of the shock to financial stress matters. In addition to simply assuming a unit shock to the stress index, we simulate the cumulative IP responses to FSI shocks with different sizes. Specifically, we impose positive and negative shocks from one through six standard deviations.

The consequences of positive shocks after 24 months differ quite dramatically with the magnitude of the shocks. Figure 8 compares the response profiles. Large negative FSI shocks in the low–growth regime (top right graph in Figure 8) boost IP growth by about 3.5 times as much as in an average high–growth period (top left graph). On the other hand, in a low–growth state, large positive shocks reduce IP only about 2.5 times as much as in a high–growth period. Not only do large positive shocks have quite a different relative impact compared to small positive shocks. Their relative effect also varies strongly with the regime during which the shock occurs. Finally, the left panel in Figure 8 indicates that IP responses are more or less size–proportional and symmetric with respect to the sign of FSI shocks.

The response properties our findings suggest appear particularly relevant for policy action. Monetary policy shocks—in particular what has recently been called unconventional monetary pol-
icy\textsuperscript{54}—are more likely to be effective when the shocks are sufficiently large and applied during low–growth regimes.

Our empirical MRVAR results strongly suggest that the timing of policy actions affecting financial stress is very influential on the success of such measures. The findings are compatible with recent studies which argue that unconventional monetary policy is needed in a depressed economy that is accompanied by a sharp rise in credit spreads, which, more so than asset–price volatility, constitute the dominant component of the stress index.\textsuperscript{55} The results suggest that not only a decrease in the interest rate but also a reduction in financial stress and in credit spreads are required to induce significant expansionary effects.

5.4 Response Analysis for Selected EU Countries and the U.S.

With a look on the smoothed FSI and IP plots as well as our MRVAR response analysis, we now briefly elaborate on the empirical findings for the countries under investigation.\textsuperscript{56}

Altogether, the response analyses for these countries suggest that there are certain common patterns but also some country–specific differences. We summarize the results as follows.

Common Features

1. Regime Effects: Shocks the financial–stress index have asymmetric effects and are regime dependent, with positive and negative shocks of the same size resulting in different effects—an exception is the UK. A reduction of financial stress (negative shock) in the low–growth regime has a stronger effect on output than in the high–growth regime.

2. Shock–size effects: Small shocks often show small effects, and large shocks produce over-proportionally effects. This points to some corridor stability: given small shocks the process is more likely to stay in the present regime; in the presence of large shock it is more likely to switch the regime and, thus less likely to revert. Thus, in the low–growth regime, large shocks (i.e., a large stress reduction) is required to sizeably affect output. This holds for most countries under investigation, with the UK being again the exception.

Country Heterogeneity

We observe quite some heterogeneity between countries as to the way how negative or positive financial stress shocks affect output in the low– or high–growth regimes and to what extent the

\textsuperscript{54}For example, quantitative easing as pursued by the Fed since 2008.
\textsuperscript{55}See, for example, Curdia and Woodford (2009).
\textsuperscript{56}Results from VAR and MRVAR estimation are summarized in Table 2, reporting for each country the lag order, AIC values, threshold values, and the number of observations associated with high– and low–growth regimes.
Figure 8: Cumulative MRVAR responses to negative (top) and positive (bottom) stress-index shocks in average high– (left) and low–growth states (right)
size of financial shocks matters.

1. For Germany, looking at the FSI and IP index, we find that the output reaction to the 2007-8 financial meltdown shows very strong effects on FSI and output (as strong as for the U.S.). The responses are regime–specific, as responses to FSI shocks are much smaller during high growth. Moreover, larger shocks in the high–growth regime do not have over–proportional effects relative to small shocks. There are, however, over-proportional effects of positive and negative shocks in low growth; and the reduction of stress in the low–growth regime is considerably more effective given large shocks. This could be attributable to the fact that Germany has a strong manufacturing sector, which is more resilient to variations in financial stress. In a low–growth regime, German output is more sensitive to financial stress. However, large negative stress shocks have a more sizeable positive impact on output than positive shocks reduce output. Overall, the MRVAR impulse-response behavior is similar to the U.S.

2. The FSI of France shows less variation during the financial–market meltdown in 2007-8; and output falls less than in Germany. This is reflected in the response diagrams. There are some sizeable effects from negative shocks during high growth, but there are only little effects from positive shocks during low growth. There seem to be some form of shock absorbers in the French economy, prevent output from falling overly dramatic when financial stress rises. On the other hand, in Germany an FSI reduction induces a substantially higher output growth than in France. In the high–growth regime, France and Germany show rather similar and fairly symmetric responses. Overall, it seems that, in a low–growth state, France is in some sense more buffered against financial shocks.\(^{57}\)

3. Italian FSI responded little during the meltdown but, similar to Germany, output dropped sharply by about 2%. During low growth IP reacts, relative to Germany, less pronounced and fairly symmetrically. In the high–growth regime large positive shocks have over–proportionally negative consequences on growth, so that shocks to the FSI can break Italy’s expansion paths.

4. Before the financial–market meltdown Spain showed an exceptionally long period of very low financial stress. In the crisis, output fell as much as in Germany and Italy, but FSI rose very little. During high growth, Spain’s IP remains virtually unaffected by financial stress. There are effects during low growth, but these are pretty much sign–symmetric. Overall, Spain’s real economy seems to be more decoupled from financial stress than is the case in most other economies.\(^{58}\)

\(^{57}\)The reason for this might be France’s larger state sector, which has significant involvement in areas, such as (nuclear) power generation (nuclear), utilities, the automobile industry, and transport.

\(^{58}\)This may be related to the real estate boom in Spain which was somewhat self–sustained for some longer time period.
5. Surprisingly different results are obtained for the UK. During the 2007-08 crisis it shows a big rise in financial stress as the U.S. and a fall in output as Germany and Italy, roughly 2%. However, the MRVAR analysis for the UK shows for either growth regime little output sensitivities with respect to (negative or positive) financial shocks. This could be due to the facts that UK’s financial market is large relative to GDP, that it experiences contagion effects from the U.S., and that the real economy (manufacturing) is relatively decoupled from the financial sector. Since its actual manufacturing base, measured by industrial production over GDP has become relatively small, but its export share to the EU is 57%. The UK, therefore, has suffered from spillover effects from the Euro Area crisis.

6. The U.S. has had the strongest rise in FSI and a substantial fall in IP during the meltdown. The response analysis shows very strong effects during low growth, where positive shocks decrease output markedly. Positive shocks in a high-growth regime also reduce output sharply. U.S. output responds rather sensitively to financial shocks and does so especially to stress reduction in low-growth states. The link between the banking sector and output seems to be rather strong in the U.S.

Overall, the empirical analysis may suggest that the stronger the position of an economy in the world economy in terms of its GDP and share of world trade, the more autonomous are the financial stress effects, having direct impact on their economies. Moreover, in these economies, a large stress reduction in a low growth regimes have a relatively more sizeable improvement on output. Yet the smaller the economies are, the more they are dependent on the external dynamics and the spillover effects, which seem to significantly affect the consequences of their own financial stress shocks. Moreover, as in the case of France, a larger public sector seems to act as a buffer against (positive and negative) stress shocks—possibly, as compared to Germany, at the expense of long-term growth.

6 Conclusions

Although, historically, most severe economic crises have led to a meltdown of the banking sector, the banking sector has usually exacerbated and amplified crises, regardless of their origin. To investigate those feedback effects, we have studied the linkage between financial stress, credit spreads and economic growth. In particular, we studied the question of the banking sector’s instability, being exposed to asset-price shocks, credit-spread shocks and financial stress in general. We modeled financial intermediaries as a wealth fund, which accumulates capital assets, heavily borrow and grants generous payouts. When the banking sector is exposed to a deterioration of its balances sheets, it turns out that varying credit spreads and payouts play an

59 UK’s industrial production is roughly 11% of GDP.
important role for the dynamics of the leverage ratio, the financial stress and the domain of attraction of the stable steady state. Greater credit spreads, larger payout and larger adverse shocks beyond some threshold, may induce instabilities, whereas smaller adverse shocks may not affect mean-reversion.

In contrast to previous studies that use the financial accelerator—which is locally amplifying but globally stable and mean reverting—our model admits downward instability as predicted in BS (2010). Whereas the financial accelerator leads, in terms of econometrics, to a single-regime VAR specification, the multi-regime dynamics studied here requires a multi-regime VAR (MRVAR) approach. Using the IMF (2011) financial stress index and growth regimes, our method of a MRVAR-based study enables us to conduct growth-regime specific response analysis.

Empirically, we find that financial-stress shocks have asymmetric effects, depending on the regime the economy is in. We also show that the effects of the shocks are dependent on the size of the shocks. Although there is quite some heterogeneity between countries in the sense that output responses to stress-shocks are larger in the bigger economies (for example Germany and the U.S.), there are also common features: Large positive financial stress shocks in a high growth regime tend to have less of a contractionary effect than in a recession; and large reductions tends to induce stronger expansionary effects in low-rather than in high-growth regimes.

The latter seems important when evaluating “unconventional” monetary policy. The empirical analysis presented here strongly suggests that both timing and the intensity of policy actions matter—findings that cannot be obtained by conventional, linear, single-regime analysis.

References


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60 In earlier literature on Keynesian macrodynamics this has been called “corridor stability”: small shocks have only small effects but larger shocks could be destabilizing and have large effects.


Appendix 1: Numerical Solution of the Model

We have used the dynamic programming method by Gruene and Semmler (2004) to study the dynamics of the stochastic version of debt-asset ratio with consumption and growth rate of assets as controls. The dynamic programming method can explore the local and global dynamics by using a coarse grid for a larger region and then employing grid refinement for a smaller region. As BS (2010) correctly state, the dynamics should not be studied by first or second order Taylor approximations to solve for the local dynamics, since this will not capture the global instabilities of the model, in particular below the steady state. Instead we use dynamic programming, which can provide us with the truly global dynamics in a larger region of the state space without losing much accuracy (see Becker et al., 2007). In contrast, local linearization, as has been argued there, does not give sufficient information on the global dynamics.

Hence, before going into the model discussion, we start by briefly describing this dynamic programming algorithm and the mechanism by which it enables us to numerically solve our dynamic model variants. The adaptive discretization of the state space feature of the dynamic programming algorithm leads to high numerical accuracy with moderate use of memory. In particular,
the algorithm is applied to discounted infinite horizon dynamic decision problems of the type introduced for the study of our search and matching models. In our model variants we have to numerically compute \( V(x) \):

\[
V(x) = \max_u \int_0^\infty e^{-rt} f(x,u) dt \\
\text{s.t.} \quad \dot{x} = g(x,u), x(0) = x_0 \in \mathbb{R}^1,
\]

where \( u \) represents the decision variable, and \( x \) a vector of state variables. Note that one of the components of the vector of state variables, for example \( x_i \), could represent the time index \( x \), as used in sect 2.2.

In the first step, the continuous time optimal decision problem has to be replaced by a first order discrete time approximation given by

\[
V_h(x) = \max_{u \in U} J_h(x,u),
\]

where \( J_h(x,u) = h \sum_{i=0}^{\infty} (1 - \theta h) f(x_h(i), u_i) \), and \( x_h \) is defined by the discrete dynamics

\[
x_h(0) = x, \quad x_h(i + 1) = x_h(i) + hg(x_h(i), u_i)
\]

and \( h > 0 \) is the discretization time step. Note that \( U \) denotes the set of discrete control sequences \( u = (u_1, u_2, ...) \) for \( u_i \in U \).

The value function is the unique solution of a discrete Hamilton-Jacobi-Bellman equation such as

\[
V_h(x) = \max_{u \in U} \{ hf(x,u) + (1 - \theta h)V_h(x_h(1)) \} \tag{14}
\]

where \( x_h(1) = x + hg(x,u) \) denotes the discrete solution corresponding to the control and initial value \( x \) after one time step \( h \). Using the operator

\[
T_h(V_h)(x) = \max_{u \in U} \{ hf(x,u_o) + (1 - \theta h)V_h(x_h(1)) \}
\]

the second step of the algorithm now approximates the solution on a grid \( \Gamma \) covering a compact subset of the state space, i.e. a compact interval \([0, K]\) in our setup. Denoting the nodes of \( \Gamma \) by \( x^i \) with \( i = 1, ..., P \), we are now looking for an approximation \( V_h^\Gamma \) satisfying

\[
V_h^\Gamma(x^i) = T_h(V_h^\Gamma)(x^i) \tag{15}
\]

for each node \( x^i \) of the grid, where the value of \( V_h^\Gamma \) for points \( x \) which are not grid points (these are needed for the evaluation of \( T_h \)) is determined by linear interpolation. We refer to Gruene and Semmler (2004) for the description of iterative methods for the solution of (15). This procedure
allows then the numerical computation of approximately optimal trajectories.

In order to distribute the nodes of the grid efficiently, we make use of an *a posteriori* error estimation. For each cell $C_l$ of the grid $\Gamma$ we compute

$$\eta_l := \max_{k \in C_l} | T_h(V_{h}^{\Gamma})(k) - V_{h}^{\Gamma}(k) |$$

More precisely, we approximate this value by evaluating the right hand side in a number of test points. It can be shown that the error estimators $\eta_l$ give upper and lower bounds for the real error (i.e., the difference between $V_j$ and $V_{h}^{\Gamma}$) and hence serve as an indicator for a possible local refinement of the grid $\Gamma$. It should be noted that this adaptive refinement of the grid is particularly effective for computing steep value functions, non-differential value functions and models with multiple equilibria, see Gruene *et al.* (2004) and Gruene and Semmler (2004). These are all cases where local linearizations are not sufficiently informative.

### Appendix 2: Estimating Movements of Credit Spreads

We take the IMF’s (2011) FSI as a proxy of the time varying default risk and credit cost. We apply the *Fast Fourier Transformation* (FFT) to the IMF (2011) FSI for Germany.\(^{61}\) The time period is from February 1980 to October 2011 at monthly frequency.

When we apply the FFT for Germany we estimate the periodic components of the actual time series. The estimates of the coefficients are reported in table 3, and the results are then illustrated in Figure A1. We estimate a linear combination of sine-cos functions, representing the low frequency components of the actual time series, which then can be used as input into our DP described in Appendix 1. The form of our estimate is

$$x_t = \sum_{i=1}^{n} \left( a_i \sin \left( \frac{2\pi \tau_i}{t - t_0} \right) + b_i \cos \left( \frac{2\pi \tau_i}{t - t_0} \right) \right). \quad (16)$$

with the coefficients given in Table 3 below. Figure A1 plots the actual and fitted series.

<table>
<thead>
<tr>
<th>$\tau_i$ (month)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
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<td>$a_i$</td>
<td>-0.263</td>
<td>0.0854</td>
<td>1.1592</td>
<td>0.1483</td>
<td>1.9493</td>
<td>0.1523</td>
</tr>
<tr>
<td>$b_i$</td>
<td>0.0445</td>
<td>-0.0837</td>
<td>1.2705</td>
<td>0.6065</td>
<td>1.6699</td>
<td>1.2001</td>
</tr>
</tbody>
</table>

Table 3: Coefficients of the harmonic fit of the real bond yield in the equation (16)

\(^{61}\)For details see Hsiao and Semmler (2009)
Appendix 3: Figures of MRVAR Responses for Selected EU Countries and the U.S.

Information on the VAR and MRVAR estimations for Germany, France, Italy, Spain, UK, and the U.S. are provided in Table 2 of Section 5, reporting for each country the estimated VAR and MRVAR orders, AIC values, threshold values, and the number of observations in the high- and low-growth regimes. This Appendix shows graphs of smoothed FSI and industrial production (IP) and the MRVAR responses for each of these countries (except for Germany, which is treated in Section 5).
Figure A5: Italy: smoothed FSI and IP

Figure A6: Italy: response to negative shocks

Figure A7: Italy: response to positive shocks
Figure A8: Spain: smoothed FSI and IP

Figure A9: Spain: responses to negative shocks

Figure A10: Spain: responses to positive shocks
Figure A11: UK: smoothed FSI and IP

Figure A12: UK: responses to negative shocks

Figure A13: UK: responses to positive shocks
Figure A14: US: smoothed FSI and IP

Figure A15: U.S.: responses to negative shocks

Figure A16: U.S.: responses to positive shocks