Competition, Cascades and Connectivity:  
A Multi-Market Model of Endogenous Mergers

Camillia Zedan  
Institute for Complex Systems Simulation  
University of Southampton, UK  
Email: c.zedan@soton.ac.uk

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Abstract

There is an increasing trend towards global financial consolidation. Empirical evidence has shown that though consolidation can promote market efficiency, it can also increase systemic risk. Therefore, understanding the effects of company mergers on the financial markets is an increasingly pertinent issue.

This paper presents a multi-market agent-based model of endogenous merger formation, consistent with empirical analyses of UK merger data. The conditions under which market competition is sufficiently disrupted to prompt endogenously-created merger waves are identified. It is also shown how dependencies between markets can cause destabilising effects to propagate throughout the entire multi-market world.¹

Keywords—Cournot competition, agent-based simulation, endogenous mergers, emergent behaviour, merger waves, systemic risk, cascade behaviour, economic networks.

JEL Classification Codes—C63, D21, G34.

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1 Introduction

In response to the damage caused by the recent global economic crisis, the UK government announced plans to restructure the way in which the financial markets are regulated (HMTreasury, 2010). It is believed that suitable regulation could enable the early detection and possible prevention of future crises by, for example, encouraging market stability or reducing systemic risk (FSA, 2009). Overall, this move has highlighted the importance of understanding how complex economic systems form, interact and evolve over time.

In 2002, the the Competition Commission, an independent public body, was made responsible for regulating mergers, markets and industries with an aim of ‘...ensuring healthy competition between companies in the UK for the benefit of companies, customers and the economy.’ (Commission, 2011) This was done in an attempt to address the problem of market failure\(^2\): the situation in which a free market is not economically efficient.

Competitive markets are considered beneficial because they have the potential to promote fair prices for consumers, encourage firms to innovate, and increase efficiency by putting downward pressure on costs. Therefore, increases in competition should lead to increases in economic welfare.

Mergers are among the most direct causes of changes to competition in a market. Therefore, their careful regulation is important. However, the effect of regulation at the firm or market level is not always predictable or desirable (Gual, 2007). Due to the complex interdependencies that connect companies and markets, a seemingly small policy change or ruling can potentially lead to negative effects on the consumer (Norback and Persson, 2007) and propagate to other industries.

Research has found that though consolidation can lead to an increase in market efficiency, it also increases systemic risk (Mishkin, 1999; De Nicolo and Kwast, 2002). The trend towards global financial integration is increasing the strength and number of market interdependencies and, according to Stephanou (2009), leading to a ‘shrinking role of the state in financial systems’ as cross-border ownership increases. In the past fifteen years, emerging market cross border M&A deals have nearly quadrupled in number and increased more than five-fold in value. Over the past 40 years, global outward foreign direct investment has gone from roughly $14bn to over $1tr.\(^3\)

\(^2\)Though established in 1999, the Competition Commission was not given remedial powers until the Enterprise Act (2002). Up until this time, its role was limited to providing the government with recommendations.

\(^3\)In Bator (1958), Bator determines that markets become efficient if they remain competitive.
Therefore, understanding the causes and effects of merger waves has become increasingly important in the regulation of an increasingly global market.

Similarly, understanding the potential cascade effects of mergers enables firms to better evaluate the risk associated with developing inter- and intra-market dependencies (Pergler and Lamarre, 2009). It could also potentially lead to better long-term regulation by providing policy makers with constraints that are less likely to cause disruptive chain effects. Modelling the origin of these effects could also be used later for the testing of new regulations.

In addition to this, the markets themselves are affected by the phenomenon of merger waves. It is well documented that merger activity follows a wave-like pattern; that is, a period of high merger activity is followed by a period of low merger activity (Town, 1992; Lipton, 2006; Gugler et al., 2008). However, econometric studies have shown that the precise behaviour of these waves is both highly country and market dependent (Resende, 1996; Maksimovic and Yang, 2011). For example, Figure 1 shows merger activity in the UK between 1987 and 2010. As can be seen, both outward and inward mergers and acquisitions (M&A) follow a sinusoidal wave pattern. However, UK domestic M&A appears to follow a two-phase regime-switching pattern in which high merger activity steps to low merger activity rather than transitioning more gently between the two (Gartner and Halbheer, 2009; Resende, 2008).

In attempting to identify the causes of such merger waves, a number of interesting traits have been noted. For instance, the peaks of merger waves approximately coincide with the peaks of stock market booms (Gugler et al., 2008) and some waves can only be seen to affect a subset of markets (Ahern and Harford, 2010). Broadly speaking, the literature provides three potential theory groups for these surges in merger activity: neoclassical, behavioural and random.

Neoclassical theories, such as the Q-Theory of Mergers (Jovanovic and Rousseau, 2002) and the Industry Shocks Theory (Harford, 2005), make use of standard neoclassical assumptions to explain waves. For instance, an industry might receive a shock such as the introduction of a new technology, which enables them to produce goods at a lower cost. This may then provide new merger opportunities resulting in a flurry of merger activity. Aggregate merger waves are then caused when multiple, simultaneous shocks affect a number of industries.

Behavioural theories, such as the Managerial Discretion Theory (Shleifer and Vishny, 2003) and the Overvalued Shares Theory (Rhodes-Kropf et al., 2005), assume non-rational behavioural traits.

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4The time series UK merger data was taken from datasets provided by the Office for National Statistics
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(a) Domestic UK M&A Activity

(b) Outward UK M&A Activity

(c) Inward UK M&A Activity

Figure 1: UK M&A activity. Data is provided by the Office for National Statistics, www.ons.gov.uk.

of market players. For instance, an overconfidence in the market may lead to an overvaluation of stock. This may then encourage managers to make more merger bids than usual, either due to the increased perceived value of their own firm or a potential target’s.

Despite using similar datasets, empirical investigations have found evidence to support each of these theories dependent on the filtering techniques applied (Resende, 1999, 2008, 1996; Harford, 2005; Gugler et al., 2008). Lipton (2006) concludes that discrepancies between the findings are caused by the models themselves: ‘The overriding problem with these models is that none of them work very well outside the market or timeframe over which they were created’. He argues that this is because there is in fact no single factor that stimulates a wave, but instead a complex combination of economic, social and legal conditions that make mergers more appealing during certain periods. It is this complexity that encourages a bottom-up, agent-based approach to modelling this problem in which the aggregate effect of individual merger decisions can be considered. This approach is adopted in this paper.

According to a 2007 study by the Hay Group (HayGroup), despite motives to the contrary, 91% of corporate mergers and acquisitions fall short of their objectives. Of the British companies considered, this rose to 97%. It was also found that on average mergers and acquisitions disrupt
company operations for over two years, integration takes 19 months, and newly merged businesses are leaderless for two and a half months. Therefore, the effect of a single merger on a company is often considered unpredictable.

One way of gaining a greater understanding of the outcome of a potential merger is through the construction of dynamic, simulation-based models. Such models have the potential to investigate why mergers take place, test how relevant merger behaviour can arise through different theoretical explanations, and suggest ways of reducing the negative effects of phenomena such as merger waves.

This paper demonstrates some of the effects that mergers can have on interconnected markets. This is done by developing a multi-market agent-based model of endogenous merger formation. Section 2 presents a single market model of competition and endogenous merging; Section 3 develops this model into a multi-market network; Section 4 concludes.
2 Competition in an Agent-Based Market

There are several economically incentivised models of merging that lend themselves well to agent-based modelling (see (Toxvaerd, 2008) (Matsushima, 2001) (Gowrisankaran, 1999)). However, for simplification purposes and ease of simulation, the single market model presented is a dynamic adaptation of Qiu and Zhou (2005).

The single market model acts as a repeated, two-stage game. In the first stage, an agent is given the opportunity to merge; in the second, all firms engage in a Cournot competition. This is then repeated. The outcome of the second stage Cournot competition incentivises the merger behaviour in the first stage.

2.1 The Single Market Model

A market is initialised with $N$ firms ($i \in 1, 2, 3 \cdots n$).

Each firm $i$ is assigned a fixed marginal cost $c_i$. This is the total cost of producing a unit of an homogenous good. This cost is publicly known.

The firms are indexed so that $c_1 \leq c_2 \leq \cdots \leq c_n$.

Market demand is given by:

$$P = \alpha - Q$$  \hspace{1cm} (1)

where $P$ is the market price, $\alpha$ is the market size and $Q$ is the total output of the $N$ firms.

The market size is such that at initialisation:

$$\alpha > \alpha_0 \equiv (n + 1)c_n - \sum_{i=1}^{n} c_i$$  \hspace{1cm} (2)

This ensures that all active firms in the market are producing some positive quantity of the good.

The set of firms in a market is given by $M$. $|M| = m$ is the number of firms in the market.

The Cournot quantity produced by each firm in a market $M$ is given by:
2.1 The Single Market Model

\[ q_i^M = \alpha + \frac{\sum_{j \in M, j \neq i} c_j}{m + 1} - c_i \]  

(3)

The Cournot profit is:

\[ \pi_i^M = (q_i^M)^2 \]  

(4)

Therefore, a firm can benefit from the reduced competition caused by a merger among its competitors. This is sometimes called the ‘free-riding’ effect of mergers and is well documented in the literature (Bernile et al., 2011).

2.1.1 Evaluating Merger Profitability

If firm \( i \) merges with firm \( j \), the newly formed firm adopts the lowest production cost of the two. That is, \( c_i \) becomes \( \min(c_i, c_j) \) where firm \( i \) is the proposing firm.

Let \( v_i^M \) be firm \( i \)'s expected payoff in \( M \).

If there is no merger:

\[ v_i^M = \pi_i^M \]  

(5)

If \( i \) acquires \( j \) at price \( T \):

\[ v_i^M = v_i^{M \setminus j} - T \]  

(6)

\[ v_j^M = T \]  

(7)

\[ v_k^M = v_k^{M \setminus j} \quad \forall k \in M \setminus \{i, j\} \]  

(8)

Therefore, to ensure that firm \( j \) accepts a proposal from firm \( i \), the following condition must hold:

\[ T \geq \pi_j^M \]  

(9)
That is, if firm $j$’s production cost is greater than $i$’s, the price offered must be at least as much as $j$ would make without the merger. Therefore, $i$’s optimal acquisition price is:

$$T = \pi_j^M$$

Therefore, the incentive to merge is both technological (since merging may provide a firm with a lower production cost) and competitive (since merging removes a competing firm from the market).

For a merger between firms $i$ and $j$ ($i + j$), the payoff can then be given by:

$$u_{i}^{M, i+j} = v_{\min\{i,j\}}^M - T$$

Merger profitability becomes:

$$\Delta \pi_{i,j}^M \equiv (v_{\min\{i,j\}}^M - T) - \pi_i^M$$

Therefore, a merger is only profitable when $\Delta \pi_{i,j}^M \geq 0$.

The sale price of the acquisition $T$ does not exist in the newly created firm - it is essentially taken by the previous owner and is no longer part of the system.

## 2.1.2 Timestep Evaluation

At initialisation, there are $N$ firms. Dependent on the simulation, market entrants may be modelled by the introduction of new agents, and agents unable to produce any good ($q_i^M \leq 0$) are forced to leave the market.

Since only active firms (i.e. those producing some positive quantity of goods) should be present at initialisation, the value of $\alpha$ should satisfy:

$$\alpha > \alpha_0$$

In Qiu and Zhou (2005), an upper bound of $\alpha$ is also defined for which a merger $i + j$ (where $m > 2$ and $c_i < c_j$) is profitable:
\[ \alpha \leq \frac{(m + 1)}{(m - 1)^2} - \frac{2}{2}((m^2 - 1)c_j - 2mc_i) - \sum_{k=1}^{m} c_k \equiv \alpha_{i,j}^M \] (14)

Therefore, \( \alpha_{i,j}^M \) is largest between firms with the greatest difference in cost.

If \( \alpha_{i,j}^M \geq \alpha_t \geq \alpha_0 \) for some \( i, j \) such that \( i \neq j \), there is at least one profitable merger in the system.

- When \( \alpha > \alpha_{i,j}^M \), the market size is too great to make any strategic merger profitable.
- When \( \alpha < \alpha_0 \), there is not a minimum amount of demand to keep all firms producing.

Therefore, we define \( \alpha_{\text{max}} \) and \( \alpha_{\text{min}} \) as the maximum and minimum values of \( \alpha \) for which mergers are profitable.

\[ \alpha_{\text{max}} = \max(\alpha_{i,j}^M) \quad \forall i, j \neq j \] (15)

In each round:

1. A firm \( i \) is randomly selected to act.
2. \( i \) generates a payoff matrix representing all possible pair-wise mergers it could propose and including the option not merging. \( i \) may merge with at most one other firm.
3. \( i \) selects the action that gives the largest payoff.
4. Once any mergers have taken place, the remaining firms take part in a Cournot competition.

A merger is possible if it is acceptable to the acquired firm. Agents are therefore myopic in their decision-making by:

1. Considering only direct mergers (and not considering the indirect benefit of two other firms merging). For instance, an agent might receive a larger payoff by strategically waiting for two other firms to merge, but it could end up waiting indefinitely. Therefore, there is a need for the agent to be miopic.
2. Considering payoffs only from immediate actions in the next time step, and not evaluating multiple timesteps ahead. However, this may be argued to be more realistic as real life mergers are a potentially infinite timestep game, and so it would be impossible for humans to consider the full set of possible outcomes before choosing a course of action.
A firm reaches *stability* when, if selected to act, it would not choose to merge with another firm. A market reaches stability when all firms are in this state.

The simulation terminates when a market reaches stability or after a finite number of timesteps.

### 2.1.3 Additional Notes

Since an agent may receive the same payoff for a number of market configurations, a preferential default is required. Agents in this model prefer inertia (i.e. an agent would prefer to perform the least number of mergers). The effect of this assumption is that the system promotes fewer mergers. Therefore, if a merger wave occurs it will be more transparent. From the agent’s perspective, it also leads to less chance of inadvertently benefitting competitors by reducing their competition (the free-riding effect discussed above).

The specific outcome of each simulation is highly path dependent, though general trends in behaviour may be the same. This is due to the stochasticity introduced when randomly selecting which agent may act.

### 2.2 Market Dynamics

Given the complexity of the model, it is useful to begin by considering the effect of introducing a single entrant to a stable market.

A market is stable when no agents wish to merge (i.e. when $\alpha_{i,j}^{M} < \alpha$ or $\alpha < \alpha_{0}$). Therefore, the effect of a new firm $i$ on the market depends on $c_{i}$ and the existing configuration of the market. If $c_{i}$ significantly changes the current value of $\alpha$, one or more mergers may become profitable.

To exemplify how this might happen, consider the following model: $n = 5$, $\forall i \ c_{i} = 10$ (i.e. firms have identical costs), and $\alpha = \alpha_{0} + 1$ (i.e. all firms are producing at $t_{0}$).

At initialisation, there are 5 equally producing firms (see Figure 2).

The market is stable, as can be seen in Figure 3. $\alpha_{\text{max}}$ is the same as $\alpha$; the maximum benefit to be gained comes from the current configuration of firms. This is further supported by Figure 4, which shows that each firm currently receives the same payoff, which is more than any payoff from merging.

Though a firm would receive no reduced production cost from merging, the payoff from a merger
would come from reduced competition in the market. Therefore, it is interesting to ask if there is a size of market for which a same-cost market would be unstable? There is. However, this occurs when the number of agents is very small in the market (≈ 2). This is because, for any two agents to merge, the payoff from merging must be greater than the payoff from not merging:

\[ \pi_i^M < \pi_i^{M-1} \]
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Figure 4: Merger Payoffs for $N = 5$, $c_i = 10$ at Initialisation. A red cross represents the payoff from a particular merger; a blue cross represents the payoff from not merging.

\[
\left( \frac{\alpha + \sum_{i=1}^{m} c_i}{m+1} - c_i \right)^2 < \left( \frac{\alpha + \sum_{i=1}^{m-1} c_i}{m} - c_i \right)^2 - \left( \frac{\alpha + \sum_{i=1}^{m} c_i}{m+1} - c_i \right)^2 - \left( \frac{\alpha + \sum_{i=1}^{m} c_i}{m+1} - c_i \right)
\]

(16)

In the case where $c_i = c$ for all $i$, this becomes:

\[
\left( \frac{\alpha + cm}{m+1} - c \right)^2 < \left( \frac{\alpha + c(m-1)}{m} - c \right)^2 - \left( \frac{\alpha + cm}{m+1} - c \right)
\]

(17)

Figure 5 shows the payoff to a firm in such a market from merging or not merging. As can be seen, the payoff from merging is only more beneficial than not merging for very small $N$. The point at which this occurs depends on the market configuration. Therefore, in such markets with a large number of firms, the market is stable.

2.2.1 Introducing Same-Cost Firms

Consider the effect of introducing additional same-cost firms to this stable environment ($n = 5$, $c_i = 10$). After initialisation, at every second timestep a new firm with cost $c_i = 10$ will be introduced to the system.

As expected, the introduction of same cost firms has little effect on the stability of the system. As new firms enter the system, existing firms produce less in order to share the market equally between themselves (see Figure 6).
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Figure 5: Payoff from Merging vs. Not Merging for \( c_i = 10 \) and Varying \( N \)

Figure 6: Market production by firm for \( N = 5 \), \( \forall i \) \( c_i = 10 \) and same-cost firm entry every 2 timesteps.

Though firms are still producing, this production is now less due to the increased competition. This can be seen clearly in Figure 7, which shows the quantity produced by each firm in each timestep. As \( n \) increases, the production (and profit) of each firm is reduced.

Despite this decline in profit, there is still no incentive to merge (as can be seen in Figure 8).
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Figure 7: Quantity produced by firms for \( N = 5 \), \( \forall i \ c_i = 10 \) and same-cost firm entry every 2 timesteps

2.2.2 Introducing Higher-Cost Firms

Consider now the introduction of higher-cost firms to this stable environment \((n = 5, \forall i \ c_i = 10)\).

Due to the distribution of the market, any firm with a higher production cost than any of the other firms finds it difficult to enter the market. The higher cost of production means that they are unable to produce any good. They effectually go out of production before they are able to produce anything; they become instantly extinct. Since dead firms are removed from the market, these firms never get a chance to compete.

This principle continues to be hold for markets initialised with firms of non-equal production cost (see Section 2.2.5).

2.2.3 Introducing Lower-Cost Firms

Consider now the introduction of lower-cost firms to this stable environment \((n = 5, \forall i \ c_i = 10)\). Every two timesteps, a firm is introduced with \( c_k = 9 \).

At initialisation, there are 5 equally producing firms (see Figure 9). At the end of the simulation, there are still 5 equally producing firms. However, there has been a change in the production costs of these firms.

Initially, no firms wanted to merge. However, after the introduction of a single new firm with a
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Figure 8: Merger profitability for $N = 5$, $c_i = 10$ and same-cost firm entry every 2 timesteps.
2.2 Market Dynamics

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Figure 9: Market production by firm for \( N = 5 \), \( c_i = 10 \) and lower-cost firm entry \( c_k = 9 \) every 2 timesteps.

...suitable production cost, all firms in the market favour merging.

To understand the sequence of events, consider the \( \alpha \) values in Figure 10. For the first two timesteps (0 and 1), there is no additional benefit for firms who merge. In timestep 2, the introduction of a lower cost firm causes \( \alpha_{\text{max}} \) to become greater than \( \alpha_{\text{min}} \). In other words, it becomes beneficial to merge: the lower production cost provided by the new firm becomes a target for all other firms, and through merging the new firm may derive a slight profit through reduced competition.

Figure 10: Alpha for \( N = 5 \), \( c_i = 10 \) and Lower-Cost Firm Entry Every 2 Timesteps

Immediately after the introduction of the new firm, it merges with an existing firm, sometimes as the acquirer and sometimes as the acquiree. This affects the production quantities of all the firms (see Figure 11). As long as the unstable divide in production costs between firms in the market...
exists, mergers will be profitable. As can be seen by Figure 12, the cost distribution of firms is shifting to the lower cost of production over time until eventually all firms have the same lower cost of production. After this point, entrants to the market all incur the same production cost and so the market behaves as in the same-cost entrant case.

Dependent on the production cost distribution of agents present in the market, some of the higher production cost firms may still be able to produce goods up until the point at which they merge with a lower production cost firm. However, as the number of higher production cost firms decreases, the ability for those firms to produce also decreases. Therefore, they become more likely to be unable to compete in the market and die out.

2.2.4 Introducing Multiple Lower-Cost Firms

When multiple lower-cost firms are introduced in each timestep, the ‘bias’ of the market towards those firms occurs earlier. The higher production cost firms struggle sooner to produce goods, and a higher proportion of them are forced to exit the market, rather than remain competitive by merging with a lower-cost firm. This is because only one firm gets to act per timestep. Therefore, by the time a particular higher-cost firm would have been selected to act in the previous example, it may already have been forced to exit the market.

2.2.5 Is it Possible to Successfully Introduce a Higher Cost Firm to any Market?

Given the above findings, it is interesting to consider whether all higher production cost entrants to the market will be unable to produce any good. Suppose we introduce a firm which would have a
higher production cost, $c_n$, to all existing firms in the market. For it to be able to produce:

$$q_i^M > 0$$

$$\frac{\alpha + \sum_{i=1}^{m-1} c_i}{m+1} - c_i > 0$$

$$\Rightarrow c_n < \frac{\alpha + (c_1 + c_2 + \ldots + c_{n-1})}{m}$$

(18)
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That is, the production cost of the new firm must be within some bound defined by the existing market for it to be able to compete. This is to be expected of real-world markets and provides us with an upper bound on the production cost of a viable entrant.

2.2.6 Complex Market Behaviour: Waves

Until now, the behaviour of the system has been predictable: given a particular type of entrant and market, the behaviour of the whole system can be determined. However, when we randomise the introduction of agents whose costs are drawn randomly from a given range, the aggregate behaviour of agents over time becomes much more interesting.

![Figure 13: Merger desirability with 2-Step random entrants (100 timesteps)](image)

Figure 13 shows the number of active firms in a market over time, along with ‘merger desirability’: the number of firms that would merge if given the opportunity. The market is initialised with 5 firms and every second timestep a new agent, whose production cost is drawn at random from a given range, attempts to enter the market.

As can be seen, there are two distinct regimes in merger desirability: periods in which a large number of agents wish to merge, and periods in which none do. Sharp spikes in this value indicate the temporary attractiveness of a new firm. Of more interest to us, is the appearance of sustained peaks, which exhibit similar characteristics to the UK merger data. This could arguably be termed a ‘merger wave’ in the model. However, as at most one merger may take place in a given timestep, the two trends cannot directly be compared.

The regime-switching behaviour of aggregate merger desirability is a consequence of the $\alpha$ limits. It is only possible for the market to be in one of two states: one in which at least one merger is desirable, and one in which no mergers are. In addition to this, it is more likely for a market to be stable when agent costs are closely clustered.
Entrants to the market must have a relatively low cost of production in order to create a peak in merger desirability. Since costs are drawn randomly from a given range, the time between waves increases as the simulation progresses. Eventually, an equilibrium is reached in which even the introduction of the lowest possible cost agent will not disrupt the system enough to bring about another wave of merger desirability. This point can be seen in Figure 14 at around 415 timesteps. During the rest of the simulation, successful entrants to the market result in an increase in the total number of firms, and therefore decrease the market share of each firm.

![Figure 14: Merger desirability with 2-Step random entrants (1000 timesteps)](image)

2.3 Examining Theories of Wave Generation

This single market dynamic merger model can be used to demonstrate how periods of high merger interest can be generated by each of the three theories of merger waves outlined in the introduction: neoclassical, behavioural and randomised. Though there are several ways in which each may be simulated, the following sections provide a brief implementation and demonstration.

2.3.1 Neoclassical Theories

The neoclassical industry shocks theory suggests that merger waves are caused by the introduction of a new technology to a particular industry, which enables competing firms to produce goods at a lower cost if they can acquire the technology. This can be generated by lowering the range of costs assigned to entrants. The lower costs could represent the introduction of a new technology to the market that might reduce production costs. Market incumbents are then incentivised to merge in order to obtain this reduced production cost, or are forced to leave the market if they are unable to continue production.
For example, during the simulation in Figure 15 the market reaches a stable state before 250 timesteps. At 500 timesteps, we exogenously introduce a ‘technology shift’ as described above. The effect is almost immediate as entrants spark another set of periods of merger activity. At around 750 timesteps, this market reaches a new stability in which the average production cost of firms is much lower than before.

![Figure 15: Industry shock at 500 timesteps](image)

### 2.3.2 Behavioural Theories

Behavioural theories assume that agents are not always rational when making decisions. An example of this, such as market optimism, can be generated by making agents estimate (rather than know) the value of $\alpha$. Since $\alpha$ is used by agents when determining how much to produce, this results in some agents overproducing and some underproducing. When $\alpha$ is made low, demand is overestimated and agents underproduce so that some needlessly exit the market; when $\alpha$ is high, they overproduce and it becomes easier to enter and compete in the market.

For example, during the simulation in Figure 16 the market again reaches a stable state before 250 timesteps. At 500 timesteps, we exogenously create a bubble in which all agents believe that demand is higher than it really is. This is done by setting $\alpha$ an arbitrary number of units lower than it should be (in this case it is 20). As can be seen, there is a significant increase in the number of agents present in the market, higher-cost entrants believe themselves able to compete. Sustained peaks in merger desirability can also be seen.

At 800 timesteps, $\alpha$ is reset to its correct value, resulting in a large culling of all inefficient firms.
2.3 Examining Theories of Wave Generation

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#### 2.3.3 Random Theories

The theory that merger waves occur purely at random may also be simulated, for example by introducing noise to agent decision-making. One way of doing this is during the evaluation of payoffs for potential mergers. Therefore, some agents will think some mergers are more profitable than they actually are, and others less so (type I and type II errors).

In the example shown in Figure 17, the market reaches a stable state before 250 timesteps. At 500 timesteps, we inject random noise to all agent decision making (in this case, merger desirability is 50% random). As soon as a merger takes place, the actual payoff from it is realised. As can be seen, immediately there is a much higher level of merger desirability. However, since only successfully producing firms may enter the market, the increased number of erroneous mergers does not have as severe an effect on the number of active firms in the market as an industry shock. This is due to the approximate production cost distribution remaining more or less the same throughout the simulation. At 600 timesteps, this stops.

![Figure 17: Random noise introduced at 500 timesteps and removed at 600 timesteps.](image)

Figure 17: Random noise introduced at 500 timesteps and removed at 600 timesteps.
3 Connecting a Multi-Market Economic Network

Markets do not exist in isolation; they are connected through a series of complex dependencies such as supply chains, policy agreements and geographical proximity (see Global Dependency Explorer, 2011\(^5\)). Therefore, the single market model is developed into a global model: multiple single markets are now connected through a given network dependency structure.

The most prominent cause of interdependencies between real markets are supply chains (Ahern and Harford, 2010) (Resende, 1999). For example, a market of competing car manufacturers receives some parts from a market of competing tyre manufacturers. However, there are many ways in which such a dependency may be implemented in the existing model.

This paper considers the effect of two types of supply-chain dependency on market and network stability under different topological configurations.

3.1 The Multi-Market Model

Consider two markets of competing firms connected to each through a supply chain: a supplier market \( S \) and a distributor market \( D \). Rather than direct purchase agreements being formed between individual firms in each supply chain market, the homogeneous goods produced by one market are pooled and used to supply firms in another. This abstraction greatly simplifies the model but enables a cleaner interpretation of simulation results at this stage.

The product created by \( S \) is used by \( D \) to create another product. There are two ways this relationship may be modelled: price dependency and quantity dependency.

Consider **price dependency**. The price of goods produced by \( S \) directly affect the cost of production in \( D \). This may be modelled by an extension of the production cost for each agent defined in the single market model.

Let \( c_{D,i} \) be the cost of production for agent \( i \) in market \( D \).

This cost can now be divided into two parts: the cost dependent on the price of \( S \)'s product (\( P_S \)) and the cost independent of that price (\( c_i \)). Let \( \theta \) be the proportional importance of each of these subcosts.

\(^5\)http://cephea.de/gde
\[ c_{D,i} = (1 - \theta)c_i + (\theta)P_S \]  

Therefore, a market with no supplier has \( \theta = 0 \).

This way of modelling supply-chain dependency allows price changes in an upstream market to affect production costs in a downstream market (i.e. supply affects price and price affects production). However, it does not include the demand for a good generated by changes in production level. Therefore, an additional way of linking markets needs to be modelled.

As a brief example of the behaviour of this type of market dependency, consider a stable \( S \) and \( D \) at initialisation:

Let \( N = 5 \) for each market, \( c_i = 5 \), and \( \theta = 0 \) for \( S \) and \( \theta = 0.8 \) for \( D \).

This sets up two stable markets in which production and price are slightly higher in \( D \). Figure 19 shows \( \alpha \) values along with the quantity and price of goods produced in each market.

At timestep 7, firms with costs drawn at random begin to enter market \( S \). As shown in Figure ??, the introduction of agents instantly disrupts the quantity of goods produced and therefore the price of goods in \( S \). There are also two distinct periods during which merging becomes desirable (\( T_7 \) to \( T_{11} \) and \( T_{14} \) to \( T_{17} \)).

Due to the connection between \( S \) and \( D \), the effect of entrants in \( S \) has on the price of \( S \) affects the quantity produced in \( D \). The increase in production in \( S \) leads to an increase in production and reduction in price in \( D \).

Interestingly, the affect on \( \alpha \) is not sufficient enough to promote merger activity. This is because each firm in \( D \) is equally effected by the price change: all firms are equal. Also, increasing competition in \( S \) increases the quantity produced, which has a positive effect on the productivity of \( D \). From this we learn that \( \alpha \) is no longer the sole indicator of merger desirability.

Consider quantity dependency. Suppose that one good produced by a firm in \( S \) is required for a firm to produce one unit of good in \( D \): a 1-to-1 production line. It follows that if \( Q_S \) goods are produced by \( S \), a maximum of \( Q_S \) goods may be produced by \( D \). Suppose \( Q'_D \) goods are requested by \( D \). If \( Q_S < Q'_D \), demand is greater than supply, \( S \) should increase production. Therefore, there is a feedback between demand and supply that now also dynamically affects production.

As a brief example of quantity dependency, consider a three market model (\( M_1 \), \( M_2 \) and \( M_3 \)) with supply-demand chain dependencies as shown in Figure 20.
As head and tail markets, $M_1$’s supply and $M_3$’s demand are self-determined (however, they may be exogenously defined). Supply and demand for other markets are endogenously generated by the model. $M_1$’s demand is determined by how much $M_2$ wants to produce. $M_2$’s demand is determined by $M_3$. $M_2$’s supply is provided by $M_1$ and $M_3$’s supply is determined by $M_2$. Suppose this is a 1-to-1 production line. Therefore, if $M_2$ wants to produce a smaller quantity than that supplied by $M_1$, $M_1$ will reduce its production to adjust to that demand.

As before, suppose each market is intialised with 10 agents. Let $\theta_{M_2} = \theta_{M_3} = 0.9$. At timestep 5, let agents with randomly assigned $c_i$ values enter $M_1$ every timestep. The effect
3.1 The Multi-Market Model  

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of this on other markets is caused solely by supply chains.

Figure 21 shows the behaviour of the system during one simulation.

![Figure 21: Quantity dependency example.](image)

Initially, all markets are stable. As firms begin to enter $M1$, the effect on competition encourages mergers. A few timesteps later, this has an effect on competition in $M2$ and some mergers become desirable. Unlike in previous models, this desirability is not shown in the $\alpha$-values. This is because an agent’s production cost can change across a simulation, and the quantity it can produce is
determined by its supply and demand. Therefore, the profitability of a merger is no longer solely indicated by $\alpha$-values.

Stability eventually returns to both markets for a while. However, at timestep 13, the entry of an agent to $M_1$ results stream of merger desirability, not just for $M_1$ and $M_2$, but $M_3$ as well.

In the last simulation timestep, production in $M_2$ becomes impossible for all firms. All firms exit the market. Due to the delayed reaction there is very little effect on $M_3$. However, were the simulation continued, all firms in $M_3$ would be unable to produce as well.

### 3.2 Global Dynamics

It is interesting to consider how *worlds* of agent markets connected in this way behave given this added complexity. Though a direction of flow is always required by the simulation, there is no constraint on the network topologies that may be investigated. Three small-scale (chain, tree and lattice) and three large-scale (random, ‘small-world’ and scale-free) network topologies are considered. Each configuration begins by considering combinations of markets that would be stable in the single market model.

Due to the scale of the three large-scale world, graphical visualisations and simulation results for those networks are omitted from this paper.

#### 3.2.1 A Chain World

Consider a simple chain network of 7 markets (A, B, C, D, E, F, G), in which periphery markets (A and G) have either self-referencing supply or demand. All intermediate markets receive their supply from the preceeding market and demand from the market below (see Figure 22).

The chain is a fairly stable configuration: without explicit interference, the production chain of markets can function steadily. However, the flaw of this topology is that as soon as any market in the chain is severely disrupted, this disruption can spread to other markets since there is no alternative path of production.

An example of this can be seen in Figure 23. All markets in the chain world are stable at initialisation: the markets easily adjust their production dependent on the quantity dependency (1-to-1 production) defined. After 5 timesteps, market D begins to receive random-cost market entrants. Some of these entrants promote merger behaviour and some do not. The effect of entrants has an
immediate but fairly subtle effect on quantity produced by the market. Though this does not have much of an effect downstream, the increased demand begins to affect upstream markets (C, B and A).

Price dependency plays an important role in the stability of markets in this topology. In the example simulation, $\theta = 0.5 \forall$ markets. This is a conservative amount for this topology: as $\theta$ increases, the effect of upstream disruption is felt more keenly by downstream markets.

In the simulation, the effect on quantity becomes more erratic and soon spreads to price. Downstream markets are more dependent on price fluctuations, and therefore they are most affected by this change.

Eventually, the agents in markets G and F ‘crash’ (i.e. all firms become unable to produce). Due to the chain topology, a dead market affects all other markets. Within a few timesteps, all other markets inevitably reach the same fate.

Merger waves in one market can incite mergers in other markets. This is due to the ‘affordability’ of removing competition brought about by changes in the supply chain. However, merger waves cannot be sparked since waves are the result of changes in cost distribution.

### 3.2.2 A Tree World

Consider a simple tree network of 7 markets (A, B, C, D, E, F, G), in which periphery markets (A, D, E, F and G) have either self-referencing supply or demand. All intermediate markets receive their supply from the preceding market and demand from the market below (see Figure 24).
Due to the new dependency configuration, the market struggles to reach an immediate stable configuration as in the simple chain world. Only the lowest level nodes (i.e. D, E, F and G) are stable. This can be seen in Figure 25, which shows the price and quantity output for each market in a single simulation. No agents enter any market in this simulation. Rather, the feedback in the market structure attempts to find an equilibrium. Some mergers may be required to do this.

The structure of the simulated world is still strongly dependent on all markets. For instance, if all production in market B is stopped, D and E can no longer produce. Market A then suffers a reduction in demand. Market C reacts sharply to this. Within a few timesteps, this effect filters down to prevent all other market production (see Figure 26).

Suppose instead that market G was ‘killed’ at timestep 25. Figure 27 shows this.

As can be seen, both parent and parallel markets (C and F) are eventually brought down by the terminating production. The prices in market A are also sharply affected by G’s demise. However, A is able to absorb the shock and no spread it to markets B, D and E.

Suppose an industry shock hit market B (i.e. there were a number of entrants to market B with a lower production cost than the current average). Figure 28 shows how this affects the other markets.

As can be seen, market B is clearly affected by the introduction. In fact, through mergers and ‘deaths’ caused in part by the shock it reaches a monopoly. The effect on B has a knock on effect to its supplier: A. The effect on A naturally affects C, though only slightly. All other markets remain constant. This suggests that the upstream effect of an industry shock is mostly absorbed by the parent market.

Figure 29 shows what happens when all markets suffered a single, equal industry shock at the same time. Interestingly in this brief simulation, all markets reach a monopoly.

In this configuration, terminal markets (D, E, F and G) are able to maintain stability throughout simulations as long as there is no direct change to the make up of firms in their markets, or parallel or parent markets are drastically affected (as in the case of killing market G).

Industry shocks have been seen to prompt mergers in other markets, sparking short periods of increased merger desirability.

However, it is interesting to note that upstream markets play an important role in absorbing any shocks created by downstream markets. As was seen, the greater the number of connections
between two markets, the less one impacts the other.

### 3.2.3 A Lattice World

Consider a simple lattice network of 8 markets (A, B, C, D, E, F, G, H), in which periphery markets (A, B, C, F, G and H) have either self-referencing supply or demand. All intermediate markets receive their supply from the preceding market and demand from the market below (see Figure 30).

![A lattice world market structure](image)

Any form of market stability in this world is very difficult to achieve. Using previously stable markets, the quantity dependency feedback in this configuration results in total market collapse within a few timesteps.

### 3.2.4 A Random World

Consider a world of 100 identically initialised markets randomly connected. During any simulation, there are many tangled levels of dependency (on average more than 30). However, it is important still to impose the strict hierarchy of directional supply and demand in order to create a closed world. For this rule to hold, it is interesting to note that several market ‘levels’ contain only one market. Additionally, random networks rarely form with one component as in the previous examples. Each simulation often has a small number of unconnected nodes (i.e. single markets that self-regulate supply and demand).

It is interesting to note that this configuration often manages to reach some kind of stability. Though at initialisation several markets destabilise, ‘die’ and cease production, this does not tend to spread across the whole world. In fact, it is unusual to reach total collapse. This is due to there being enough levels and connections between other markets that prevent this risk from spreading across the whole network. Using the small-scale examples from above, this can be understood as tree and chain networks being more likely to randomly form than lattices.
3.2 Global Dynamics

3.2.5 A Small World

Consider a world of 100 identically initialised markets connected as a small-world network. In small-world networks, any node is connected to another node through at most a small number of links. This results in very tightly connected networks. When a direction is imposed on these connections, lattice-like worlds form even when the minimum distance between any two nodes is increased.

Just as in the lattice example, the small world is very unstable. This is due to the high level of feedback that comes from each dependency. The larger number of markets than before emphasises this instability, leading to total collapse within just a few timesteps.

3.2.6 A Scale-Free World

Consider a world of 100 identically initialised markets connected to form a scale-free network. A scale-free network is one in which the degree distribution follows a power law. Once a directional hierarchy is applied, this results in something akin to a multi-branching tree.

Just as in the tree example, the scale-free world requires several timesteps at initialisation to stabilise. Due to the scale of the network, not all branches are able to reach stability and some markets do collapse at this stage of the simulation. However, the structure of the rest of the world can prevent these collapses from propagating.

Not all worlds are able to stabilise or absorb the shock of collapsing markets. Total collapse can occur, and does more refrequently than in the random world.
4 Conclusion

This paper has highlighted the continued importance of understanding aggregate merger phenomena and the destabilising effect it can have on connected markets. A dynamic model of endogenous merging was presented and it was shown how and why more complex aggregate behaviour from merging can arise. The single market model was able to capture sustained increases in merger desirability by theories widely supported in empirical literature: neoclassical theories such as industry shocks, behavioural theories such as overconfidence, and the belief that they are random.

A multi-market model was then developed using the single market model. The effect of market dependencies and topologies on market stability was studied and the cases for more stable configurations were identified. Simulations suggested that fewer dependencies between markets resulted in an increased stability, such as in the chain world, but such configurations were less successful at absorbing shocks than more distributed configurations, such as the tree world. Through the supply chain, destabilising behaviour was able to be propagated upwards as well as downwards from markets. Overall, scale-free and random networks were found to be the most stable to perturbations caused by merger behaviour in markets, and small world networks were the least stable.

However, there are several limitations to the model. The waves that were generated in the single market model show only how a simple regime-switching model can create periods of opportunity in which mergers become desirable to a large number of agents. This was due to the limit on the number of firms that may merge in a single timestep. However, it is believed that sustained periods of merger desirability may abstractly be considered representative of periods of merger activity.

Similarly, the two types of dependency considered in the multi-market model were an abstract implementation of one type of market relationship. For example, firms were not penalised for overproduction and did not develop individual relationships with suppliers in other markets. Additionally, relationships between markets were fixed and, once a market had collapsed, surrounding markets could not develop new connections.

Unlike most contagion models or models of network dependencies, the competitive behaviour within each node created an added layer of complexity to this investigation. The preliminary results found in these simulations have highlighted several interesting areas to continue as further work. For example, the effect of different types of market inter-relationship, the dynamic formation and removal of dependencies by the markets themselves, and the creation of new markets over the course
of the simulation. It is hoped that such extensions may help bring about a greater understanding of the potential effects of mergers, which could enable firms to better evaluate the risk associated with developing inter- and intra-market dependencies, potentially lead to better long-term regulation by providing policy makers with constraints that are less likely to cause disruptive chain effects, and even be useful later for the testing of new regulations.

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References


HayGroup. Dangerous liaisons: M&a - the integration game.


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Figure 23: A chain world simulation showing the quantity produced and price of a good for each market ($\theta = 0.5 \forall \text{markets}$).
Figure 25: Simulation output.
Figure 26: Simulation output: no production in B.
Figure 27: Simulation output: kill production in G in timestep 25.
Figure 28: Simulation output: industry shock to B.
Figure 29: Simulation output: industry shocks to all markets.