On the nature of the financial system in the Euro Area: a Bayesian DSGE approach *

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Abstract

This paper compares from a Bayesian perspective three dynamic stochastic general equilibrium models in order to analyse whether financial frictions empirically relevant in the Euro Area (EA) and, if so, which type of financial frictions is preferred by the data. The models are: (i) Smets and Wouters (2007) (SW); (ii) a SW model with financial frictions originating in non-financial firms à la Bernanke et al. (1999), (SWBGG); and (iii) a SW model with financial frictions originating in financial intermediaries, à la Gertler and Karadi (2011), (SWGK). The comparison between the three estimated models is made along different dimensions: (i) the Bayes factor; (ii) business cycle moments; and (iii) impulse response functions. The analysis of the Bayes factor and of simulated moments provides evidence in favour of the SWGK model. This result holds for different robustness checks. This paper also finds that the SWGK model outperforms the SWBGG model in forecasting EA inflationary pressures in a Phillips curve specification with either the series of the output gap or the spread generated by the two estimated models.

Keywords: Financial frictions, DSGE models, Bayesian estimation.

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1 Introduction

The Great Recession has drawn the attention on the effects that financial frictions have on business cycle fluctuations. And the dynamic stochastic general equilibrium (DSGE) literature on the financial system has been expanding in recent times. Some contributions emphasise the role of financial frictions in affecting the persistence and the magnitude of the shocks hitting the economy; other contributions focus on the source of financial frictions; others discuss the role of liquidity (see Brunnermeier et al., 2011, for a survey). A common features of DSGE models with financial frictions is the existence of a spread between lending rates and risk-free rates due to the presence of asymmetric information between lenders and borrowers, where borrowers can either be firms or financial intermediaries or households. The attention here is restricted to firms and financial intermediaries.

In the literature the micro-foundation of financial frictions can be presented in different ways. The influential model of Bernanke et al. (1999) (BGG) is considered as a workhorse for the analysis of financial frictions. The BGG model features constrained firms who are the source of frictions in the form of a costly state verification problem (Townsend, 1979). Many papers have adopted the BGG approach (Christensen and Dib, 2008; De Graeve, 2008; De Fiore and Tristani, 2009; von Heideken, 2009; Carrillo and Poilly, 2010; Fernández-Villaverde, 2010; Gelain, 2010, among many others). Much of the macroeconomic literature stemming from BGG emphasises credit market constraints on non-financial borrowers and treats financial intermediaries largely as a veil. The model of Christiano et al. (2010) features both agency problems originating in non-financial firms à la BGG and liquidity constraints on banks which are in involved in commercial bank activities as well as intermediation through securities markets. Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) (GK) explicitly model the banking sector as a source of financial frictions due to the presence of a moral hazard problem. Another approach is offered by the seminal paper of Stiglitz and Weiss (1981), who focus on adverse selection as a source of financial frictions (see also Christiano and Ikeda, 2011).

Given such a variety of approaches, the main questions of this paper are: (i) whether financial frictions are empirically relevant in DSGE models for the Euro Area and (ii) which type of financial frictions is favoured by the data. In order to examine these issues, this paper compares three DSGE models: (i) the Smets and Wouters (2007)(SW) model which features a perfect financial market; (ii) the SWBGG model; and (iii) the SWGK model. The SWBGG model incorporates financial frictions à la Bernanke et al. (1999). In this model financial frictions arise because monitoring the loan applicant is costly and this drives an endogenous wedge between the cost of external and internal funds, the external finance premium (i.e. the credit spread). Other papers presenting a SW economy with BGG type of financial frictions are Carrillo and Poilly (2010), Gelain (2010) and Del Negro and Schorfheide (2012). The first
investigates the effects of a fiscal stimulus in a zero lower bound setting while the second obtains a time series for the external finance premium. Del Negro and Schorfheide (2012) analyse the ... The SWGK model incorporates in a SW economy financial frictions à la Gertler and Karadi (2011) (GK). The source of financial frictions is in the financial intermediary (FI henceforth), facing endogenously determined balance sheet constraints and an endogenously determined leverage (the ratio between total assets and net worth). Another contribution employing the GK model is Gertler et al. (2012) who analyse the role of macroprudential policies. The choice of these two modelling strategies for micro-founding financial frictions is explained by: (i) the established importance of the BGG approach in the mainstream DSGE literature on financial frictions; (ii) the important role assigned to financial intermediaries in the GK model and its growing popularity in academia and central banking; (iii) their relative analytical tractability. These two models also share a common feature, i.e. financial frictions originate in the group of agents that need to borrow and borrowing capacity is linked to own net worth.

From an empirical point of view, the three models (SW, SWBGG, and SWGK) are estimated with EA data for the period 1980Q1-2008Q3 using output, consumption, investment, wage, employment, inflation and the nominal interest rate as observables. The comparison between the three estimated models is made along different dimensions: (i) the Bayes factor; (ii) business cycle moments; and (iii) impulse response function analysis. The main results are that: (i) the introduction of financial frictions either à la BGG or à la GK improves the models’ fit, suggesting that these frictions are empirically relevant for the EA; and (ii) the SWGK model outperforms the SWBGG model. The first result is confirmed by other studies, such as Christiano et al. (2010) who find that financial factors, such as agency problems in financial contracts and shocks that hit financial intermediation, are prime determinants of economic fluctuations in the EA and US. The second result is a novel contribution of the paper. Robustness analysis of the main result stemming from the Bayes factor is then presented by examining: (i) different calibrations of the leverage ratio of the SWBGG and SWGK models; and (ii) different models specifications.

From a theoretical point of view, the presence of asymmetric information affects the propagation mechanism of the contractionary shocks hitting the economy. All models deliver plausible impulse response functions (IRFs). However, since the financial sectors differ across the three models, so do the internal propagation mechanisms. For example, a monetary policy shock causes the standard transmission mechanism (Smets and Wouters, 2007), plus the financial accelerator effect stemming in the SWBGG model from the decline in the net worth of firms. This implies that the potential divergence of interests between firms and lenders (the suppliers of external funds) is greater and, therefore, agency costs increase. In equilibrium lenders must be compensated for higher agency costs by a larger spread, since the spread depends inversely on the borrowers’ net worth. A rise in the spread causes a fall in investment and, therefore, output. This effect further reinforces the simulated contraction. In the SWGK
model the monetary policy shock determines a reduction in investment and, therefore, in the demand for loans. This implies a deterioration in the balance sheet of financial intermediaries which leads to a rise in the spread in order to restore profits. The increase in financing costs makes lending more expensive and reduces the demand for loans, further squeezing investment. Financial frictions, therefore, exacerbate the simulated contraction. But it can also be the case that financial frictions leads to an attenuator effect. For example, a contractionary investment-specific technology shock causes in the SW model a fall in investment due to the change in the price of capital. In the SWBGG model, the financial accelerator effect embedded in the model is attenuated with this shock, due to the rise in the price of capital which, on one hand, leads to a fall in investment and, on the other, implies an increase in the net worth of firms. As a result, the spread decreases mitigating the impact of the contractionary shock. In the SWGK, an investment-specific technology shock has to three main effects: (i) the price of capital rises, causing a fall in investment and output; (ii) the retrenchment in investment leads to a lower demand for lending, affecting in turn FI’s profits; and (iii) the net worth of FI rises because of the higher return on capital. The first two effects act in the direction of reducing investment while the latter effect attenuates the fall in investment. The presence of financial frictions attenuates the transmission mechanism of this shock.

Finally the SWBGG and SWGK models are compared in their ability to forecast EA inflation. Both the flexible-price output gap and the credit spreads are found to be good predictors of inflation (Coenen et al., 2009; Gilchrist and Zakrajšek, 2011a). The comparison between the two models is based on a Phillips curve specification with either the series of the output gap or the spread generated by the two estimated models. And the SWGK model outperforms the SWBGG model in gauging EA inflationary pressures.

The structure of the paper is as follows. Section 2 illustrates the three models. Section 3 describes the data and discusses the estimation strategy. Section 4 compares the three estimated models. Section 5 presents robustness checks. Section 6 investigates the predictive power of the SWBGG and SWGK models in gauging EA inflationary pressures. Finally, Section 7 briefly concludes.

2 The Models

This section presents the three DSGE models. Compared to the standard SW economy, the different features are: (i) a utility function comparable with Smets and Wouters (2003) and Gertler and Karadi (2011); (ii) the Dixit-Stiglitz aggregator for final output and composite labour, as in Galí et al. (2011); (iii) the price mark-up, wage mark-up and government shocks are modelled as in Smets and Wouters (2003).
profit maximisation activity. In order to simplify the optimisation problems of intermediate goods firms, retailers are the source of price stickiness similarly to Bernanke et al. (1999) and Gertler and Karadi (2011).

In all models the economy is populated by: households; labour unions; labour packers; final good firms; retailers; intermediate goods firms; and the policymaker. In the SWBGG and SWGK models the economy is also populated by capital producers, while the SWGK model incorporates FI.

Households consume, save, and supply labour. A labour union differentiates labour and sets wages in a monopolistically competitive market. Competitive labour packers buy labour service from the union, package and sell it to intermediate goods firms. The good market has a similar structure: retailers buy goods from intermediate goods firms, differentiate them and sell them in a monopolistically competitive market. The aggregate final good is produced by perfectly competitive firms assembling a continuum of intermediate goods. The policymaker sets the nominal interest rate following a Taylor rule.

In the SWBGG model, intermediate goods firms maximize the flow of discounted profits by choosing the quantity of factors for production and stipulate a financial contracts to obtain funds from lenders. For the latter decision there is a costly state verification problem (Townsend, 1979) and lenders might have to pay a fixed auditing cost to observe an individual borrower’s return. FI are just a “veil” in the model (Gilchrist and Zakrajšek, 2011b). Capital producers purchase investment and depreciated capital to transform them into capital sold to intermediate goods firms and used for production. They face adjustment costs for investment.

In the SWGK model, the production sector is also made of intermediate goods firms and capital producers. The optimal choice of the quantity of factors for production from intermediate goods firms and the optimisation problem of capital producers are the same as in the SWBGG model. The intermediate goods firms finance their capital acquisitions each period by obtaining funds from the FI. While there are no financial frictions in this activity, there is a problem of moral hazard between FI and households.

Subsection 2.1 presents the core SW model. Subsection 2.2 presents the optimisation problems in the SWBGG that are different from the ones in the SW model. Subsection 2.3 shows the analytical aspects which are peculiar to the SWG model.

2.1 The core SW model

2.1.1 Households

The economy is populated by a continuum of households indexed by $j \in (0, 1)$. Following Gertler and Karadi (2011), each household’s preferences are represented by the following intertemporal utility function:\footnote{All households choose the same allocation in equilibrium; hence, for sake of notation, the $j$ index is dropped.}
\[ U_t (\cdot) = \ln (C_t - hC_{t-1}) - \frac{L_t^{1+\phi}}{1 + \phi} \]  

where \( h \) measures the degree of superficial external habits in consumption, \( L_t \) is labour supply in terms of hours worked and \( \phi \) is the inverse of the Frisch elasticity of labour supply. The representative household enters period \( t \) with real government bonds, that pay the gross real interest rate, \( R_t \), between \( t-1 \) and \( t \). During period \( t \), each household chooses to consume \( C_t \); supplies \( L_t \) hours of work; and allocates savings in government bonds, \( B_{t+1} \). Each household gains an hourly real wage, \( W_t^h/P_t \); and dividend payments, \( \Pi_t \), from firms. The government grants transfers \( TR_t \) and imposes real lump-sum taxes \( T_t \). In addition, each household owns the capital stock which she rents to intermediate goods firms at a real gross rental rate \( R_t^H \). As explained by Smets and Wouters (2003), the supply of rental services from capital can be risen either by investing, \( I_t \), or by changing the utilization rate of installed capital, \( U_t \). There are adjustment costs for investment as in Christiano et al. (2005). The law of motion of capital, \( K_t \), is equal to:

\[ K_{t+1} = (1 - \delta)K_t + x_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \]  

where \( \delta \) stands for depreciation. The adjustment cost function \( F \) satisfies the following properties: \( F(1) = F'(1) = 0 \), and \( F''(1) = \xi > 0 \). The exogenous shock \( x_t \) follows an AR(1) process, \( \rho_x \) is an autoregressive coefficient and \( \varepsilon_t^x \) is a serially uncorrelated, normally distributed shock with zero mean and standard deviation \( \sigma_x \). The shock to the marginal efficiency of investment varies the efficiency with which the final good can be transformed into physical capital.

The budget constraint is as follows:

\[ C_t + B_{t+1} \leq \frac{W_t^h}{P_t} L_t + b_t R_t B_t + R_t^H U_t K_{t-1} - \Psi(U_t) K_{t-1} + \Pi_t + TR_t - T_t \]  

where \( b_t \) is a risk premium shock which follows an AR(1) process, with the autoregressive coefficient \( \rho_b \) and standard deviation \( \sigma_b \). The term \( \Psi(U_t) \) represents the costs of changing capital utilization, with \( \zeta = \Psi''(U_t)/\Psi'(U_t) \). Maximization of equation (1) subject to (2) and (3) yields the following first-order conditions with respect to \( C_t, B_{t+1}, L_t, I_t, K_t \) and \( U_t \):

\[ U_{Ct} = mu_t \]  

\[ \beta E_t [R_{t+1} b_{t+1} mu_{t+1}] = mu_t \]  

\[ -U_{Lt} = mu_t \frac{W_t^h(j)}{P_t} \Leftrightarrow \frac{U_{Lt}}{U_{Ct}} = -MRS_t \equiv -\frac{W_t^h}{P_t} \]
\[ mu_t = mu_t^k x_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) - F' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left[ mu_{t+1}^k x_{t+1} F' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \]

\[ mu_t^k = \beta E_t \left[ mu_{t+1} \left( R_{t+1}^H U_{t+1} - \Psi(U_t) \right) + (1 - \delta) mu_{t+1}^k \right] \tag{8} \]

\[ R_{t+1}^H = \Psi'(U_t) \tag{9} \]

where \( \beta \in (0, 1) \) is the discount factor, \( mu_t \) is the Lagrange multiplier associated with the budget constraint and let \( \Lambda_{t,t+1} \equiv \frac{mu_{t+1}}{mu_t} \). And \( mu_t^k \) is the Lagrange multiplier associated with capital accumulation equation. The Tobin’s \( Q \) is the ratio of the two multipliers, i.e. \( Q_t = \frac{mu_t^k}{mu_t} \).

### 2.1.2 Wage stickiness

Households supply homogeneous labour to monopolistic labour unions which differentiate it. Labour service used by intermediate goods firms is a composite of differentiated types of labour indexed by \( l \in (0, 1) \):

\[ L_t = \left[ \int_0^1 L_t \left( \frac{\varepsilon_w^{-1}}{\varepsilon_w} \right) dl \right]^{\varepsilon_w} \tag{10} \]

where \( \varepsilon_w \) is the elasticity of substitution across different types of labour. Labour packers solve the problem of choosing the varieties of labour to minimise the cost of producing a given amount of the aggregate labour index, taking each nominal wage rate \( W_t(l) \) as given:

\[ \min_{L_t(l)} \int_0^1 W_t(l) L_t(l) dl \tag{11} \]

s.t. \[ \left[ \int_0^1 L_t \left( \frac{\varepsilon_w^{-1}}{\varepsilon_w} \right) dl \right]^{\varepsilon_w} \geq \bar{L} \tag{12} \]

The demand for labour is given by:

\[ L_t(l) = \left( \frac{W_t(l)}{W_t} \right)^{-\varepsilon_w} L_t \tag{13} \]

where \( W_t \) is the aggregate wage index. Equations (13) and (10) imply:

\[ W_t = \left[ \int_0^1 W_t(l)^{1-\varepsilon_w} dl \right]^{\frac{1}{1-\varepsilon_w}} \tag{14} \]
Labour unions adjust wages infrequently following the Calvo scheme. Let $\sigma_w$ be the probability of keeping wages constant and $(1 - \sigma_w)$ the probability of changing wages. In other words, each period there is a constant probability $(1 - \sigma_w)$ that the union is able to adjust the wage, independently of past history. This implies that the fraction of unions setting wages at $t$ is $(1 - \sigma_w)$. For the other fraction that cannot adjust, the wage is automatically increased at the aggregate inflation rate. As explained by Cantore et al. (2010), the wage for non-optimising unions evolves according to the following trajectory $W_t^*(l)$, $W_t^*(l)\left(\frac{P_t}{P_{t-1}}\right)^{\sigma_{wi}}$, $W_t^*(l)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\sigma_{wi}}$, ..., where $\sigma_{wi}$ denotes the degree of wage indexation.

The union chooses $W_t^*$ to maximise:

$$
E_t\sum_{s=0}^{\infty} \Lambda_{t,s} (\beta \sigma_w)^s L_{t+s}(l) \left[ \frac{W_t^*(l)}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\sigma_{wi}} - \frac{W_{t+s}^h}{P_{t+s}} \right] = 0
$$

subject to the labour demand (13), and the indexation scheme so that $L_{t+s}(l) = \left[\frac{W_t^*(l)}{W_{t+s}^*(l)}\left(\frac{P_{t+s}}{P_{t+s-1}}\right)^{\sigma_{wi}}\right]^{-\varepsilon_w}$.

The first order condition is:

$$
E_t\sum_{s=0}^{\infty} \Lambda_{t,s} (\beta \sigma_w)^s L_{t+s}(l) \left[ \frac{W_t^*(l)}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\sigma_{wi}} - \frac{W_{t+s}^h}{P_{t+s}} M_{w,t} \right] = 0
$$

where $M_{w,t} = \frac{\varepsilon_w}{\varepsilon_w-1} u_{t}^w$ is the time varying gross wage mark-up and $u_t^w$ is the wage mark-up shock which follows an AR (1) process, $\rho_w$ is an autoregressive coefficient and $\varepsilon_{wm}$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_{wm}$. The dynamics of the aggregate wage index is:

$$
W_{t+1} = \left[ (1 - \sigma_w) (W_{t+1}^*(l))^{1-\varepsilon_w} + \sigma_w \left( W_t \left(\frac{P_t}{P_{t-1}}\right)^{\sigma_{wi}} \right)^{1-\varepsilon_w} \right]^{1/(1-\varepsilon_w)}
$$

### 2.1.3 Goods market

Competitive final goods firms buy intermediate goods from the retailers and assemble them. Final output is a composite of intermediate goods indexed by $f \in (0,1)$ differentiated by retailers:

$$
Y_t = \left[ \int_0^1 Y_t(f) \frac{f^{\varepsilon-1}}{\varepsilon-1} df \right]^{\frac{1}{\varepsilon-1}}
$$
where $\varepsilon$ is the elasticity of substitution across varieties of goods. Final goods firms solve the problem of choosing $Y_t(f)$ to minimise the cost of production:

$$\min_{Y_t(f)} \int_0^1 P_t(f) Y_t(f) \, df$$  \hspace{1cm} (19)

subject to

$$\left[ \int_0^1 Y_t(f)^{\varepsilon-1} \, df \right]^{\frac{1}{\varepsilon-1}} \geq \bar{Y}$$  \hspace{1cm} (20)

The demand function for intermediate good $f$ is given by:

$$Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\varepsilon} Y_t$$  \hspace{1cm} (21)

where $P_t$ is the aggregate wage index. Equations (21) and (18) imply:

$$P_t = \left[ \int_0^1 P_t(f)^{1-\varepsilon} \, df \right]^{\frac{1}{1-\varepsilon}}$$  \hspace{1cm} (22)

Retailers simply purchase intermediate goods at a price equal to the marginal cost and differentiate them in a monopolistically competitive market, similarly to labour unions in the labour market. Retailers set nominal prices in a staggered fashion à la Calvo (1983). Each retailer resets its price with probability $(1 - \sigma_p)$. For the fraction of retailers that cannot adjust, the price is automatically increased at the aggregate inflation rate. As explained by Cantore et al. (2010) the price for non-optimising retailers evolves according to the following trajectory $P_t^*(f), P_t^*(f) \left( \frac{P_t}{P_{t-1}} \right)^{\sigma_{pi}}, P_t^*(f) \left( \frac{P_{t+1}}{P_{t-1}} \right)^{\sigma_{pi}}, \ldots$, where $\sigma_{pi}$ denotes the degree of price indexation. The real price $\Phi_t$ charged by intermediate goods firms in the competitive market represents also the real marginal cost common to all final goods firms, i.e. $MC_t = \Phi_t$.

A retailer resetting its price in period $t$ maximises the following flow of discounted profits with respect to $P_t^*$:

$$E_t \sum_{s=0}^{\infty} (\sigma_p \beta)^s A_{t,t+s} Y_{t+s}(f) \left[ \frac{P_t^*(f)}{P_{t+s}} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{pi}} - MC_{t+s} \right]$$  \hspace{1cm} (23)

subject to the demand function (21), and the indexation scheme so that $Y_{t+s}(f) = \left[ \frac{P_t^*(f)}{P_{t+s}} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{pi}} \right]^{-\varepsilon} Y_{t+s}$. Let $MC_t^n$ denote the nominal marginal cost. The gross mark-up charged by final good firm $f$ can be defined as $M_t(f) \equiv P_t(f)/MC_t^n = P_t(f) / \frac{MC_t^n}{P_t} = p_t(f)/MC_t$. In the symmetric equilibrium all final good firms charge the same price, $P_t(f) = P_t$, hence the relative price is unity. It follows that, in the symmetric equilibrium, the mark-up is simply the inverse of the marginal cost.
The first order condition for this problem is:

$$E_t \sum_{s=0}^{\infty} (\sigma_p \beta)^s \Lambda_{t,t+s} Y_{t+s}(f) \left[ \frac{P_t^*(f)}{P_{t+s}} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{pi}} - M_{p,t} MC_{t+s} \right] = 0$$

(24)

Similarly to the labour market, the gross time varying price mark up is $M_{p,t} = \frac{\varepsilon}{1-\varepsilon} u^p_t$ and $u^p_t$ is the price mark-up shock, which follows an AR(1) process, $\rho_p$ is an autoregressive coefficient and $\varepsilon_{pm}^t$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_{pm}$.

The equation describing the dynamics for the aggregate price level is given by:

$$P_{t+1} = \left[ (1 - \sigma_p)(P_{t+1}^*(f))^{1-\varepsilon} + \sigma_p \left( P_t \left( \frac{P_t}{P_{t-1}} \right)^{\sigma_{pi}} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}$$

Intermediate goods firms produce goods in a perfectly competitive market. They maximize the flow of discounted profits by choosing the quantity of factors for production:

$$E_t \beta \Lambda_{t,t+1} \left[ \Phi_{t+1} Y_{t+1} - R^H_{t+1} K_{t+1} - \frac{W_{t+1}}{P_{t+1}} L_{t+1} \right]$$

(25)

where $\Phi_t$ is the competitive real price at which intermediate good is sold and $R^H_t$ is the real rental price of capital.

The production function follows a Cobb-Douglas technology:

$$Y_t = A_t (U_t K_t)^{\alpha} L_t^{1-\alpha} - \Theta$$

(26)

where $\Theta$ represents fixed costs in production (Smets and Wouters, 2007). $A_t$ is the transitory technology shock following an AR(1) process, $\rho_a$ is an autoregressive coefficient and $\varepsilon_{a}^t$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_{a}$.

Maximisation yields the following first order conditions with respect to capital and labour:

$$R^H_t = MC_t M_{P^K_t}$$

(27)

$$\frac{W_t}{P_t} = MC_t M_{P^L_t}$$

(28)

where $M_{P^K_t}$ is the marginal product of capital and $M_{P^L_t}$ is the marginal product of labour. The real price $\Phi_t$ represents the shadow value of output and hence, given perfect competition in the market, it also represents its real marginal cost, $MC_t$.  

11
2.1.4 The policymaker and aggregation

The policymaker sets the nominal interest rate according to the following Taylor rule:

\[
\ln \left( \frac{R^*_{t+1}}{R^*_{t}} \right) = \rho_i \ln \left( \frac{R^*_{t}}{R^*_{t-1}} \right) + (1 - \rho_i) \left[ \rho_\pi \ln \left( \frac{\Pi_t}{\Pi_{t-1}} \right) + \rho_y \ln \left( \frac{Y_t}{Y^*_t} \right) \right] \\
+ \rho_{\Delta \pi} \ln \left( \frac{\Pi_t}{\Pi_{t-1}} \right) + \rho_{\Delta y} \ln \left( \frac{Y_t}{Y^*_t} \right) + \varepsilon^r_t
\]

(29)

and:

\[
R_{t+1} = E_t \left[ \frac{R^*_{t}}{\Pi_{t+1}} \right]
\]

(30)

where \(R^*_{t}\) is the nominal gross interest rate, \(\Pi\) is the steady state inflation rate, \(Y^*_t\) is the level of output that would prevail under flexible prices and wages without the two mark-up shocks, and \(\varepsilon^r_t\) is the monetary policy shock. The meanings of the parameters of equation (29) are standard (Smets and Wouters, 2003).

The resource constraint completes the model:

\[
Y_t = C_t + I_t + G_t + \Psi(U_t)K_{t-1}
\]

(31)

There are seven orthogonal structural shocks: the risk premium, \(\varepsilon^h_t\); the investment-specific technology, \(\varepsilon^x_t\); the monetary policy, \(\varepsilon^r_t\); the technology, \(\varepsilon^a_t\); the government, \(\varepsilon^g_t\); the price mark-up, \(\varepsilon^{pm}_t\); and the wage mark-up, \(\varepsilon^{wm}_t\), shocks. In each model, the shocks follows an AR(1) process, but the monetary policy shock.

2.2 The SWBGG model

The presence of financial frictions originating in the demand side of the credit market alters the set-up of intermediate goods firms compared to the SW economy. This section also presents the set-up of capital producers which determine the price of capital; this simplifies the optimisation problem of households.

2.2.1 Households

In the SWBGG model capital producers purchase investment and depreciated capital to transform them into capital sold to firms and intermediate goods firms choose the optimal utilization rate of capital. Hence the household’s intertemporal budget constraint simply reads as follows:

\[
C_t + B_{t+1} \leq \frac{W^h_t}{L_t}L_t + b_tR_tB_t + \Pi_t + TR_t - T_t
\]

(32)

where \(B_t\) represents real deposits in the FI as well as real government bonds. Both intermediary deposits and government debt are one period real bonds that pay the gross real interest rate,
\( R_t \) between \( t - 1 \) and \( t \). Both instruments are riskless and are thus perfect substitutes. Maximization with respect to \( C \), \( B_t+1 \) and \( L_t \) yields the first-order conditions (4), (5) and (6) respectively.

2.2.2 Capital producers

Following Gelain (2010), capital producers purchase at time \( t \) investment and depreciated capital to transform them into capital sold to firms and used for production at time \( t + 1 \). Capital producers also face adjustment costs for investment as in Christiano et al. (2005). The law of motion of capital is then equal to equation (2).

The profits are given by the difference between the revenue from selling capital at the relative price \( Q_t \) and the costs of buying capital from intermediate goods firms and the investment needed to build new capital. The optimality condition is a Tobin’s \( Q \) equation, which relates the price of capital to the marginal adjustment cost:

\[
1 = Q_t x_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) - F' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left[ A_{t,t+1} Q_{t+1} x_{t+1} F' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right]
\]

(33)

2.2.3 Intermediate goods firms

Intermediate goods firms produce goods in a perfectly competitive market and they borrow in order to finance the acquisition of capital. They maximize the flow of discounted profits by choosing the quantity of factors for production. This problem is identical to that in the SW economy, described by equations (25), (27) and (28). In addition, following Gelain (2010), firms also decide the optimal capital utilization rate solving the following maximisation problem:

\[
\max_{U_t} Z_t^k U_t K_{t-1} - \Psi(U_t) K_{t-1}
\]

(34)

This optimisation problem is summarized by the following equilibrium condition:

\[
Z_t^k = \Psi'(U_t)
\]

(35)

Intermediate goods firms face also the problem of stipulating the financial contract. In order to ensure that entrepreneurial net worth will never be enough to fully finance capital acquisitions, it is assumed that each firm survives until the next period with probability \( \theta \) and her expected lifetime is consequently \( 1/(1 - \theta) \). At the same time, the new firms entering receive a transfer, \( N_t^e \), from firms who die and depart from the scene.\(^2\) At the end of period \( t \),

\(^2\)Following Christensen and Dib (2008) consumption of exiting firms, a small fraction of total consumption,
firms buy capital $K_{t+1}$ that will be used throughout time $t + 1$ at the real price $Q_t$. The cost of purchased capital is then $Q_t K_{t+1}$. A fraction of capital acquisition is financed by their net worth, $N_{t+1}$, and the remainder by borrowing from a FI that obtains funds from household deposits and faces an opportunity cost equal to the risk-free rate, $R_t$. In equilibrium the optimal capital demand is:

$$E_t \left[ R_{t+1}^k \right] = E_t \left[ \frac{R_{t+1}^H + (1 - \delta)Q_{t+1}}{Q_t} \right]$$

(36)

where $E_t \left[ R_{t+1}^k \right]$ is the expected marginal external financing cost.

BGG assume that an agency problem makes external finance more expensive than internal funds and solve a financial contract that maximises the payoff to the firms subject to the lender earning the required rate of return. Following Townsend (1979), there is a problem of asymmetric information about the project’s ex-post return. While the borrower can costlessly observe the realisation of the project ex-post, the lender has to pay a fixed auditing cost to observe borrower’s return. If the borrower pays in full there is no need to verify the project’s return; but in the case of default the lender verifies the return and pays the cost. As a consequence, the financial contract implies an external finance premium $EP(\cdot)$, i.e. the difference between the cost of external and internal funds, that depends on the inverse of the firm’s leverage ratio. $^3$ Hence, in equilibrium, the marginal external financing cost must equate the external finance premium gross of the riskless real interest rate:

$$R_{t+1}^k = EP \left( \frac{N_{t+1}}{Q_t K_{t+1}} \right) R_{t+1}$$

(37)

with $EP(\cdot) < 0$ and $EP(1) = 1$. As the borrower’s equity stake in a project $N_{t+1}/Q_t K_{t+1}$ falls, i.e. the leverage ratio rises, the loan becomes riskier and the cost of borrowing rises. Linearisation of equation (37) yields: $^4$

$$\dot{R}_{t+1}^k = \dot{R}_{t+1} + \kappa [\dot{Q}_t + \dot{K}_{t+1} - \dot{N}_{t+1}]$$

(38)

where $\kappa \equiv -\frac{\partial R_{t+1}^k N/K}{\partial R_{t+1}^k} = -\frac{EP(\cdot) N/K}{R_{t+1}^k} R$ measures the elasticity of the external finance premium with respect to the leverage position of intermediate goods firms.

Aggregate entrepreneurial net worth evolves according to the following law of motion:

$$N_{t+1} = \theta \left[ R_t^k Q_{t-1} K_t - E_{t-1} \left[ R_t^k (Q_{t-1} K_t - N_t) \right] \right] + (1 - \theta) N_t^e$$

(39)

where the first component of the right-hand-side represents the net worth of the $\theta$ fraction of

is ignored in the general equilibrium.

$^3$See BGG for the derivation of the financial contract and for the aggregation.

$^4$A variable with a ‘hat’ denotes a percentage deviation from steady state.
surviving entrepreneurs net of borrowing costs carried over from the previous period, and \( N_t^f \) is the transfer that newly entering entrepreneurs receive.

Following BGG and Gabriel et al. (2010), monitoring costs are ignored in the resource constraint since, under reasonable parameterisations, they have negligible impact on model’s dynamics.

2.3 The SWGK model

The presence of financial frictions à la Gertler and Karadi does not affect the optimisation problem of households, which is the same as in SWBGG, although their structure is slightly different. This subsection then presents the features of financial intermediaries and intermediate goods firms.

2.3.1 Households

The optimisation problem of households in the SWGK model is analogous to that in the SWBGG model. However, in the former model within each household there are two types of members at any point in time: the fraction \( g \) of the household members are workers and the fraction \( (1 - g) \) are bankers. The FI have a finite horizon in order to avoid the possibility that they can reach the point where they can fund all investment from their own capital. Every banker stays banker next period with a probability \( \theta \), which is independent of history. Therefore, every period \( (1 - \theta) \) bankers exit and become workers. Similarly, a number of workers become bankers, keeping the relative proportion of each type of agents constant. The household provides her new banker with a start-up transfer, which is a small fraction of total assets, \( \chi \). Each banker manages a financial intermediary.

2.3.2 Financial intermediaries

The FI’s balance sheet is:

\[
Q_t S_t = N_t + B_{t+1}
\]

(40)

where \( B_t \) stands for deposits, \( N_t \) is FI capital (or net worth), \( S_t \) is the quantity of financial claims on intermediate goods firms and \( Q_t \) is the price of each claim.

The problem of moral hazard consists in the fact that the banker can choose to divert the fraction \( \lambda \) of available funds from the project and transfer them back to her household. The depositors require to be willing to supply funds to the banker that the gains from diverting assets should be less or equal than the costs of doing so:

\[
\Upsilon_t \geq \lambda Q_t S_t
\]

(41)
where $\Upsilon_t$ is the expected terminal wealth, defined as:

$$\Upsilon_t = E_t \sum_{s=0}^{\infty} (1 - \theta)^s \beta^{s+1} \Lambda_{t,t+1+s} \left( \left( R_{t+1+s}^k - R_{t+1+s} \right) Q_{t+s} S_{t+s} + R_{t+1+s} N_{t+s} \right)$$  \hspace{1cm} (42)

Equation (41) translates in the following constraint for the FI:

$$Q_t S_t = lev_t N_t$$  \hspace{1cm} (43)

where $lev_t$ stands for the FI leverage ratio. The agency problem introduces an endogenous capital constraint on the FI’s ability to lend to firms.

Total net worth is the sum of net worth of existing bankers, $N^e_t$, and net worth of new bankers, $N^n_t$. As far as the first is concerned, net worth evolves as:

$$N^e_{t+1} = \theta [(R^k_{t+1} - R_{t+1}) levt + R_{t+1}] N_t$$  \hspace{1cm} (44)

where $\varepsilon^n_t$ is a shock to FI net worth. Net worth of new bankers is a small fraction of total assets:

$$N^n_t = \chi Q_t S_t$$  \hspace{1cm} (45)

### 2.3.3 Intermediate goods firms

Intermediate goods firms maximize profits in a perfectly competitive market and borrow from FI. In order to make a meaningful comparison, the three models are as closer as possible, and the optimisation problems of intermediate goods firms follow SWBGG, i.e. equations (27), (28), (35) and (36).

Each intermediate goods firm finances the acquisition of capital, $K_{t+1}$, by obtaining funds from the FI. The firm issues $S_t$ state-contingent claims equal to the number of units of capital acquired and prices each claim at the price of a unit of capital $Q_t$:

$$Q_t K_{t+1} = Q_t S_t$$  \hspace{1cm} (46)

### 3 Data and estimation strategy

The three models – SW, SWBGG and SWGK – are estimated with quarterly EA data for the period 1980Q1-2008Q3 using as observables real GDP, real investment, real private consumption, employment, GDP deflator inflation, real wage and the nominal interest rate. The final quarter corresponds to the pre-crisis period: the collapse of the Lehman Brothers in September 2008 has been used as characterizing the crisis period, e.g. Lenza et al. (2010) and Giannone et al. (2011). The purpose of this paper is to make a comparison between the
three models in normal times. Data come from the Area Wide Model database (Fagan et al., 2005, see). Following Smets and Wouters (2003), all variables are logged and detrended by a linear trend. The inflation rate is measured as a quarterly log-difference of GDP deflator. Data on the nominal interest rate are detrended by the same linear trend of inflation. Data on employment are used since there are no data available for hours worked in the Euro Area. Similarly to Smets and Wouters (2003) a Calvo-type of adjustment is assumed for employment and hours worked:

\[
\hat{E}_t = \frac{1}{1 + \beta} \hat{E}_{t-1} + \frac{\beta}{1 + \beta} E_t \left[ \hat{E}_{t+1} - \frac{(1 - \beta \sigma_E)(1 - \sigma_E)}{(1 + \beta \sigma_E)} \left( \hat{L}_t - \hat{E}_t \right) \right]
\]

where \( E_t \) is employment and \( 1 - \sigma_E \) represents the fraction of firms that can adjust the level of employment to the preferred amount of total labor input. Data on employment are also logged and detrended since there is an upward trend in the employment series for the EA and hours worked and employment are stationary variables in the model.

The solution of the rational expectations system takes the form:

\[
s_t = A s_{t-1} + B \eta_t \quad (47)
\]

\[
a_t = C s_t + D u_t \quad (48)
\]

\[
\eta_t \sim N(0, \Omega) \quad \text{and} \quad u_t \sim N(0, \Phi)
\]

where \( s_t \) is a vector containing the model’s variables expressed as log-deviation from their steady-state values. It includes not only endogenous variables but also the exogenous processes. Vector \( \eta_t \) contains white noise innovations to the shocks. Matrices \( A \) and \( B \) are functions of the structural parameters of the DSGE model; \( a_t \) is the vector of observables and \( u_t \) is a set of shocks to the observables.

As far as the Bayesian estimation procedure is concerned, the likelihood function and the prior distributions are combined to approximate a posterior mode, which is used as the starting value of a Random Walk Metropolis algorithm (RWMA). This Markov Chain Monte Carlo (MCMC) method generates draws from the posterior density and updates the candidate parameter after each draw (see An and Schorfheide, 2007; Fernández-Villaverde, 2009, for details).

### 3.1 Calibration and priors

The parameters which cannot be identified in the dataset and/or are related to steady state values of the variables are calibrated, following a standard procedure (Christiano et al., 2010). The time period in the model corresponds to one quarter in the data.

---

5Version 4.2.4 of the Dynare toolbox for Matlab is used for the computations.
Table 1: Calibration of parameters common to both models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$, discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha$, capital income share</td>
<td>0.3</td>
</tr>
<tr>
<td>$\delta$, depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\frac{G}{Y}$, government spending to GDP ratio</td>
<td>0.22</td>
</tr>
<tr>
<td>$\varepsilon$, elasticity of substitution in good market</td>
<td>set to target $M = 1.20$</td>
</tr>
<tr>
<td>$\varepsilon_w$, elasticity of substitution in labour market</td>
<td>set to target $M^w = 1.20$</td>
</tr>
</tbody>
</table>

Table 2: Calibration of model-specific parameters

<table>
<thead>
<tr>
<th>Financial Parameters</th>
<th>SWBGG Model</th>
<th>SWGK Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$, survival rate</td>
<td>0.9715</td>
<td>0.9715</td>
</tr>
<tr>
<td>$S$, steady state spread</td>
<td>150 basis point py</td>
<td>150 basis point py</td>
</tr>
<tr>
<td>$\frac{K}{N}$, leverage ratio</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\chi$, fraction of assets given to the new bankers</td>
<td>–</td>
<td>0.001</td>
</tr>
<tr>
<td>$\lambda$, fraction of divertable assets</td>
<td>–</td>
<td>0.515</td>
</tr>
</tbody>
</table>

Table 1 shows the calibration of the parameters common to both models. The discount factor, $\beta$, is equal to 0.99, implying a quarterly steady state real interest rate of 1%; the capital income share, $\alpha$, is equal to 0.33, implying a steady state labour income share of two third. The depreciation rate is equal to 0.025, corresponding to an annual depreciation rate of 10%. The ratio of government spending to GDP is equal to 0.22. The elasticities of substitution in goods and labour markets are equal to 6 in order to target a gross steady state mark up of 1.20, as in Christiano et al. (2010) and Gelain (2010), among others. The steady state inflation rate is calibrated at 1.

The calibration of the financial parameters is shown in Table 2. The parameter $\theta$ represents the survival rate of intermediate goods firms in the SWBGG model and of FI in the SWGK model. This parameter is set equal to 0.9715 implying an expected working life for bankers and firms of almost a decade; this value is consistent with both BGG and Gertler and Karadi (2011). In the SWBGG model, the parameter pinning down the steady state spread, $S$, is set to match the steady state spread in the dataset of 150 basis points. Following BGG, Christensen and Dib (2008) and Gelain (2010), the ratio of capital to net worth is set to 2, implying that 50% of firm’s capital expenditures are externally financed. As long as the calibration of the SWGK model is concerned, the fraction of assets given to new bankers, $\chi$, and the fraction of assets that can be diverted, $\lambda$, are equal to 0.0007 and 0.4255, respectively, to target the same steady state spread in the SWBGG model and a steady state leverage ratio of 4, the value used by Gertler and Karadi (2011). Section 5 investigates the robustness of the main results to the calibration of the financial parameters.

Table 3 shows the assumptions for the prior distributions of the estimated parameters.
for both models. The choice of the functional forms of parameters and the location of the
prior mean correspond to a large extent to those in Smets and Wouters (2003, 2007) where
applicable. In general, the Beta distribution is used for all parameters bounded between 0
and 1, the Normal distribution is used for the unbounded parameters and the Inverse Gamma
(IG) distribution for the standard deviation of the shocks. The prior of some model-specific
parameters are as follows. The parameter measuring the inverse of the Frisch elasticity of
labour supply follows a Normal distribution with a prior mean of 0.33, the value used by
Gertler and Karadi (2011). Following De Graeve (2008), the elasticity of external finance
premium with respect to leverage is assumed to follow a Uniform distribution, with values in
the interval (0, 0.3).

4 Model comparison

The comparison between the three models is made first by looking at the estimated parameters
and the Bayes factor. Second, the forecasting performance is discussed. Finally, impulse
response functions are presented.

4.1 Estimated parameters and the Bayes factor

The mean of the estimated parameters for each model are computed with the Metropolis-
Hastings algorithm with a sample of 250,000 draws (see Smets and Wouters, 2003 for further
details). For each model Table 3 reports the posterior mean with 95% probability intervals in
parentheses. Most parameters are remarkably similar across the two models. As in Smets and
Wouters (2005), the fact that in almost all the cases the posterior estimate of a parameter in
one model falls in the estimated confidence band for the same parameter of the other model
can be considered as a rough measure of similarity. Nevertheless, the posterior mean of few
parameters differs.

Concerning the set of parameters similar across the two models, the main findings are as
follows. The degree of price stickiness reveals that firms adjust prices almost every year (see
also Gelain, 2010), with a higher degree of stickiness in SWKG model. The Calvo parameter
for wage stickiness reveals that the average duration of wage contracts is about three-quarters,
lower than the degree of price stickiness, as in Smets and Wouters (2003). There is a moderate
degree of price indexation and a higher degree of wage indexation similarly to previous esti-
mates for the EA. The mean of the parameter measuring the elasticity of capital utilisation is
higher than its prior mean, revealing that capital utilisation is more costly than assumed a-
priori. There is evidence of external superficial habit in consumption, with a relatively higher
value in the SWKG model.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior distribution</th>
<th>Mean</th>
<th>Std./df</th>
<th>SW model</th>
<th>SWBGG model</th>
<th>SWGK model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_p$, Calvo prices</td>
<td>Beta</td>
<td>0.75</td>
<td>0.05</td>
<td>0.743 [0.696:0.793]</td>
<td>0.712 [0.673:0.759]</td>
<td>0.775 [0.735:0.815]</td>
</tr>
<tr>
<td>$\sigma_w$, Calvo wages</td>
<td>Beta</td>
<td>0.75</td>
<td>0.05</td>
<td>0.678 [0.628:0.730]</td>
<td>0.653 [0.605:0.708]</td>
<td>0.666 [0.627:0.711]</td>
</tr>
<tr>
<td>$\sigma_{pi}$, price indexation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.156 [0.104:0.252]</td>
<td>0.139 [0.057:0.219]</td>
<td>0.142 [0.062:0.227]</td>
</tr>
<tr>
<td>$\sigma_{wi}$, wage indexation</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.493 [0.245:0.738]</td>
<td>0.431 [0.222:0.668]</td>
<td>0.457 [0.191:0.748]</td>
</tr>
<tr>
<td>$\sigma_E$, Calvo employment</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.727 [0.684:0.771]</td>
<td>0.714 [0.675:0.754]</td>
<td>0.720 [0.677:0.762]</td>
</tr>
<tr>
<td>$\zeta$, elasticity of capital util</td>
<td>Normal</td>
<td>0.25</td>
<td>0.1</td>
<td>0.730 [0.686:0.771]</td>
<td>0.733 [0.692:0.771]</td>
<td>0.698 [0.634:0.771]</td>
</tr>
<tr>
<td>$h$, habit parameter</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.593 [0.523:0.663]</td>
<td>0.547 [0.469:0.631]</td>
<td>0.565 [0.589:0.728]</td>
</tr>
<tr>
<td>$\phi$, inverse of Frisch elasticity</td>
<td>Normal</td>
<td>0.33</td>
<td>0.1</td>
<td>0.222 [0.100:0.325]</td>
<td>0.281 [0.103:0.445]</td>
<td>0.313 [0.168:0.447]</td>
</tr>
<tr>
<td>$\kappa$, elast. of external finance</td>
<td>Uniform</td>
<td>0</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{\pi}$, Taylor rule</td>
<td>Normal</td>
<td>1.7</td>
<td>0.1</td>
<td>1.708 [1.560:1.848]</td>
<td>1.892 [1.743:2.037]</td>
<td>1.768 [1.604:1.929]</td>
</tr>
<tr>
<td>$\rho_{\Delta \pi}$, Taylor rule changes in $\pi$</td>
<td>Normal</td>
<td>0.3</td>
<td>0.1</td>
<td>0.150 [0.060:0.238]</td>
<td>0.194 [0.107:0.279]</td>
<td>0.118 [0.045:0.187]</td>
</tr>
<tr>
<td>$\rho_y$, Taylor rule</td>
<td>Normal</td>
<td>0.125</td>
<td>0.05</td>
<td>-0.015 [-0.761:0.030]</td>
<td>-0.051 [-0.793:0.024]</td>
<td>0.087 [0.041:0.129]</td>
</tr>
<tr>
<td>$\rho_{\Delta y}$, Taylor rule changes in $y$</td>
<td>Normal</td>
<td>0.0625</td>
<td>0.05</td>
<td>0.101 [0.063:0.138]</td>
<td>0.113 [0.077:0.151]</td>
<td>0.090 [0.063:0.117]</td>
</tr>
<tr>
<td>$\rho_s$, Taylor rule smoothing</td>
<td>Beta</td>
<td>0.80</td>
<td>0.2</td>
<td>0.905 [0.888:0.923]</td>
<td>0.901 [0.878:0.923]</td>
<td>0.929 [0.915:0.943]</td>
</tr>
<tr>
<td>$\rho_a$, persistence of tech shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.1</td>
<td>0.973 [0.956:0.991]</td>
<td>0.970 [0.951:0.990]</td>
<td>0.973 [0.958:0.989]</td>
</tr>
<tr>
<td>$\rho_x$, persistence of investment shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.1</td>
<td>0.968 [0.976:0.999]</td>
<td>0.989 [0.975:0.985]</td>
<td>0.972 [0.959:0.984]</td>
</tr>
<tr>
<td>$\rho_g$, persistence of gov shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.1</td>
<td>0.969 [0.947:0.994]</td>
<td>0.975 [0.897:0.971]</td>
<td>0.973 [0.952:0.993]</td>
</tr>
<tr>
<td>$\rho_p$, persistence of price mark-up shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.1</td>
<td>0.931 [0.874:0.991]</td>
<td>0.956 [0.920:0.997]</td>
<td>0.877 [0.796:0.956]</td>
</tr>
<tr>
<td>$\rho_w$, persistence of wage mark-up shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.1</td>
<td>0.652 [0.549:0.757]</td>
<td>0.641 [0.551:0.738]</td>
<td>0.668 [0.547:0.789]</td>
</tr>
<tr>
<td>$\rho_b$, persistence of risk premium shock</td>
<td>Beta</td>
<td>0.85</td>
<td>0.1</td>
<td>0.891 [0.848:0.936]</td>
<td>0.986 [0.974:0.999]</td>
<td>0.878 [0.831:0.926]</td>
</tr>
<tr>
<td>$\sigma_{\alpha}$, std of tech shock</td>
<td>IG</td>
<td>0.1</td>
<td>2</td>
<td>0.516 [0.413:0.622]</td>
<td>0.436 [0.359:0.513]</td>
<td>0.528 [0.410:0.639]</td>
</tr>
<tr>
<td>$\sigma_i$, std of monetary shock</td>
<td>IG</td>
<td>0.1</td>
<td>2</td>
<td>0.155 [0.132:0.177]</td>
<td>0.157 [0.132:0.183]</td>
<td>0.138 [0.117:0.159]</td>
</tr>
<tr>
<td>$\sigma_{b}$, std of risk premium shock</td>
<td>IG</td>
<td>0.1</td>
<td>2</td>
<td>0.171 [0.106:0.236]</td>
<td>0.089 [0.066:0.109]</td>
<td>0.322 [0.188:0.452]</td>
</tr>
<tr>
<td>$\sigma_g$, std of gov shock</td>
<td>IG</td>
<td>0.1</td>
<td>2</td>
<td>1.344 [1.192:1.491]</td>
<td>1.317 [1.166:1.469]</td>
<td>1.345 [1.193:1.498]</td>
</tr>
<tr>
<td>$\sigma_{pm}$, std of price mark-up shock</td>
<td>IG</td>
<td>0.1</td>
<td>2</td>
<td>0.983 [0.677:1.295]</td>
<td>0.802 [0.620:0.971]</td>
<td>1.188 [0.687:1.649]</td>
</tr>
<tr>
<td>$\sigma_{wm}$, std of wage mark-up shock</td>
<td>IG</td>
<td>0.1</td>
<td>2</td>
<td>4.381 [3.672:5.000]</td>
<td>4.345 [3.623:4.999]</td>
<td>4.365 [4.073:5.000]</td>
</tr>
</tbody>
</table>
The estimates of the parameter measuring the Taylor rule reaction to inflation are also in line with previous estimates for the EA, with a higher value in the SWBGGG model. There is also evidence of short-term reaction to the current change in inflation and in the output gap. Turning to the exogenous shock processes, all shocks are quite persistent but the wage mark-up shock. The mean of the standard errors of the shocks is in line with the studies of Smets and Wouters (2003) and Gelain (2010), but the standard deviation of the investment-specific technology shock and the wage mark-up shock which are higher.

The second set of parameters is made of those for which the posterior means differ. The elasticity of the cost of changing investment is higher in the SW model compared to the SWBGGG and SWG models, suggesting a slower response of investment to changes in the value of capital in the former model. Another friction, the share of fixed costs in production, is estimated to be higher in the SW model. The absence of financial frictions appears to generate a higher degree of real frictions in the SW model. The response to the output gap level is low and negative in the SW and SWBGG models, while it is low and positive in the SWG model.

The AR coefficients of the shock processes are generally similar among the models, but in the SWBGG model the investment-specific technology shock has a lower persistence while the risk premium shock is more persistent compared to the other two models.

A third set of parameters includes the parameter which differ among the two models, i.e. the elasticity of the external finance premium with respect to the leverage position. This parameter is estimated in the SWBGG model with a posterior mean of 0.069, revealing an external premium reactive to the firms’ leverage position.

One-step ahead forecasts are computed in order to evaluate the forecasting performance of alternative models, as Adolphson et al. (2007) and Kirchner and Rieth (2010) among others. The forecasts are the estimates of the observed variables, \( o_t \), conditional on period \( t \) information: \( o_{t+1|t} = C{s_{t+1|t}} \), where \( s_{t+1|t} \), containing the model’s variables, is computed as \( s_{t+1|t} = A{s_{t|t}} \) and \( s_{t|t} \) is the updated variables obtained from the application of the Kalman filter. Figure 1 shows that the three models fit the data well; however, the graphical inspection makes it difficult to assess the comparative measure of fit (Gelain, 2010).

Hence, the Bayes factor is used to judge the relative fit of the models, as in An and Schorfheide (2007) and Levine et al. (2010), among many others. Such a comparison is based on the marginal likelihood of alternative models. Let \( m_i \) be a given model, with \( m_i \in M, \theta \) the parameter vector and \( p_i(\theta|m_i) \) the prior density for model \( m_i \). The marginal likelihood for a given model \( m_i \) and common dataset \( Y \) is:

\[
L(Y|m_i) = \int_\theta L(Y|\theta,m_i)p_i(\theta|m_i)d\theta
\]

where \( L(Y|\theta,m_i) \) is the likelihood function for the observed data \( Y \) conditional on the param-
Table 4: Log data density and Bayes factor

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>SWBGG</th>
<th>SWGK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Log data density</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harmonic mean</td>
<td>-378.32</td>
<td>-359.50</td>
<td>-351.54</td>
</tr>
<tr>
<td>Bayes factor (_{i,SW}) (=) (\frac{\exp(LL(Y</td>
<td>m_{SWBGG}))}{\exp(LL(Y</td>
<td>m_{SW}))})</td>
<td>(1.5 \times 10^8)</td>
</tr>
<tr>
<td>Bayes factor (_{i,SWBGG})</td>
<td>(-)</td>
<td>(-)</td>
<td>(2.9 \times 10^3)</td>
</tr>
</tbody>
</table>

Parameter vector and on the model; and \(L(Y|m_i)\) is the marginal data density. The Bayes factor is calculated as follows:

\[
BF = \frac{L(Y|m_i)}{L(Y|m_j)} = \frac{\exp(LL(Y|m_i))}{\exp(LL(Y|m_j))}
\]

where \(LL\) stands for log-likelihood. The log data density of the three models is computed with the Geweke (1999)’s modied harmonic mean estimator (based on 250,000 draws from two chains of the MH algorithm).

Table 4 shows the log data density and the Bayes factor for the three models. The main results are as follows. First, the introduction of financial frictions either \(\text{à la BGG}\) or \(\text{à la GK}\) improves the marginal likelihood, suggesting that these frictions are empirically relevant. The value of the Bayes factor between the SWBGG or the SWGK and the SW models is high. Second, the comparison between the two models with financial frictions provides clear evidence in favour of the SWGK model. Therefore, the SWGK model outperforms the other models.

4.2 Business cycle moments

The moments generated by the models are compared to those in the data to assess the conformity between the data and the models and to compare the three alternative models, as in Gabriel et al. (2010) among many others. Table 5 shows some selected second moments of output, consumption, investment, ination and nominal interest rate.

The comparison of the relative standard deviations (with respect to output) shows that the SW model fits the data better in terms of implied relative volatility of consumption, although the difference with the SWGK model is negligible. The three models generate the same relative standard deviation of ination, while the SWGK model clearly outperforms the other two models in capturing the relative standard deviations of investment and the nominal interest rate.

The comparison of the cross-correlation with output reveals that the SWBGG and the SWGK models reproduce the same values of cross-correlation of investment and interest rate, which are preferred to the SW model when compared to the data. And the SWGK model fits
Table 5: Selected second moments

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Inflation</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Relative standard deviations to output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>1.00</td>
<td>1.17</td>
<td>2.86</td>
<td>0.24</td>
<td>0.49</td>
</tr>
<tr>
<td>SW</td>
<td>1.00</td>
<td><strong>1.13</strong></td>
<td>2.72</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>SWBGG</td>
<td>1.00</td>
<td>1.27</td>
<td>2.99</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>SWGK</td>
<td>1.00</td>
<td>1.12</td>
<td><strong>2.96</strong></td>
<td>0.07</td>
<td><strong>0.07</strong></td>
</tr>
</tbody>
</table>

| **Cross-correlations with output** |        |             |            |           |               |
| Data           | 1.00   | 0.93        | 0.86       | 0.06      | 0.21          |
| SW             | 1.00   | 0.64        | 0.93       | 0.26      | 0.04          |
| SWBGG          | 1.00   | 0.60        | **0.84**   | -0.05     | **0.25**      |
| SWGK           | 1.00   | **0.70**    | **0.84**   | -0.03     | **0.25**      |

| **Autocorrelations (order=1,2)** |        |             |            |           |               |
| Data           | 0.935,0.864 | 0.942,0.888 | 0.934,0.861 | 0.728,0.690 | 0.962,0.895  |
| SW             | 0.994,0.983 | 0.997,0.989 | 0.994,0.983 | 0.899,0.763 | 0.827,0.663  |
| SWBGG          | 0.991,0.978 | 0.995,0.987 | 0.992,0.976 | 0.836,0.655 | 0.955,0.889  |
| SWGK           | **0.989,0.970** | **0.994,0.981** | **0.989,0.968** | **0.817,0.611** | **0.960,0.900** |

the data better than the other two models in terms of cross-correlations of consumption and inflation. Overall the SWGK model gets closer to the data in this dimension.

Table 5 also reports the autocorrelation coefficients up to order 2. The SWGK model clearly outperforms the other two models in capturing the positive autocorrelations over a short horizon. Variables are more autocorrelated in all models than in the data, as in Gabriel et al. (2010). This is also evident from Figure 2, which shows the autocorrelation of output, consumption, investment, inflation and interest rate when the number of lags is equal to 10. All series are positively correlated. As far as the interest rate is concerned, both the SWBGG and SWGK models do extremely well at matching the autocorrelation observed in the data. When it comes to matching inflation, the three models are not able to replicate the dynamics in the data at all horizons: the SWGK model is preferable at lag one, the SWBGG model at lag two, and from lag three onwards the SW model gets closer to the data.

The three models fail in replicating the relative standard deviations of inflation and interest rate. This result could cast some doubt on the overall ability of the model to replicate the data. As a robustness test, the three models are then estimated allowing for measurements errors for inflation and wages, as well as for a moving-average component in the price and wage mark up shocks. Table 6 shows an improvement in the ability of the models in replicating the relative standard deviation of inflation and, to a minor extent, the nominal interest rate. The log data density reveals that the ranking of the models is not affected and the SWGK model is still the preferred one.
Table 6: Relative standard deviations to output and log data density when allowing for measurements errors.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>SW</th>
<th>SWBGG</th>
<th>SWGK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.24</td>
<td>0.08</td>
<td>0.20</td>
<td>0.16</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.49</td>
<td>0.09</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>Log data density</td>
<td>-416.62</td>
<td>-395.40</td>
<td>-381.39</td>
<td>-381.39</td>
</tr>
</tbody>
</table>

Overall the presence of financial frictions helps in fitting some selected EA variables. In turn, the presence of financial frictions originating in FI is preferable in the data compared to a model where financial frictions originate in non-financial firms.

4.3 Impulse response functions

This section presents the impulse response functions. Figures 3–6 examine four shocks, two demand, the monetary policy and investment-specific technology shocks, and two supply, the technology and wage mark-up, shocks in order to highlight how the presence of financial frictions affects the transmission mechanism of these shocks. All the shocks are set to produce a downturn. The first row of each chart of each figure shows the responses of output, investment, inflation and the interest rate in the SW model, the second row shows the responses of the same variables in the SWBGG model and the last row refers to the SWGK model. In all the figures, the solid lines represent the estimated median and the dotted lines represent the 95% highest posterior density confidence intervals.

A contractionary monetary policy shock is shown in Figure 3. While the sign of the impact responses are similar between the three models, the transmission mechanism is different. In all models an increase in the nominal interest rate reduces investment and, therefore, output. Demand downward pressures feed through changes in the output gap to inflation. This causes a downward shift in aggregate demand, which reduces inflation on impact. This is the standard interest rate channel of monetary policy transmission. In the SWBGG and SWGK models, the transmission mechanism of the policy shock is enhanced through its impact on credit markets. In the SWBGG model the decline in the price of capital due to the tightening of monetary policy causes a fall in net worth of intermediate goods firms, and the spread rises. This mechanism further reinforces the contraction in capital and investment. In the SWGK model, due to the retrenchment in investment, loans decrease as well. At the same time the fall in asset prices worsen FI’s balance sheet. As explained by Villa and Yang (2011), three factors affect the profits of financial intermediaries: the amount of loans, the lending rate and the leverage. The fall in profits makes financial intermediaries willing to increase the lending rate more than the increase in the deposit rate, in order to restore profits. Hence the spread rises. The increase in financing costs causes a further decline in loans and investment as shown.
in Figure 3.

Figure 4 shows the effects of the investment-specific technology shock in the models. In the SW model this demand shock causes a rise in the price of capital, $Q_t$, which leads to a fall in investment and, hence, output. On the nominal side the contraction in aggregate demand leads to a decline in inflation. In the SWBGG model, the shock also implies a rise in the price of capital, $Q_t$. But a change in the price of capital has two effects: (i) investment falls as well as output; and (ii) net worth of firms increases due to the higher return on capital, equation (39). The latter effect causes a fall in the spread. This should cause an increase in investment. However, the first effect dominates and investment decreases. The presence of financial frictions, therefore, attenuates the fall in investment and output, as also shown by Christensen and Dib (2008) and Gelain (2010). In the SWGK model, an investment-specific technology shock has to three main effects: (i) the price of capital rises, causing a fall in investment and output; (ii) the retrenchment in investment leads to a lower demand for lending, affecting in turn FI’s profits; and (iii) net worth of FI rises because of the higher return on capital, equation (44). The first two effects acts in the direction of reducing investment while the latter effect attenuates the fall in investment. The presence of financial frictions attenuates the transmission mechanism of this shock.

A technology shock has a direct impact on output by making factors less productive, and leads to an increase in prices due to the contraction in supply. Since the Taylor rule is operating, the nominal interest rate rises as shown in Figure 5. Consistently with the literature stemming from Gali (1999) this delivers an increase in labor hours. Since the capital stock is fixed, the desire for more input is achieved with an increase in the utilization rate. Investment and consumption decline due to the contraction in output. In the SWBGG model the decline in investment reduces the borrowing needs of firms, improving their leverage position. This affects the external finance premium and the fall in investment is smaller when the financial accelerator is present. In the SWGK model this shock implies also a decrease in asset prices, which worsens the FI’s balance sheet. Such a deterioration makes FI willing to push up the premium to increase profit, but this further reduces the amount of lending. Hence the presence of financial frictions originating in financial intermediaries exerts a financial accelerator effect.

In the SW model the wage mark-up shock leads to an increase in the prices of factors for production, causing a fall in their equilibrium quantity. This exerts a contractionary effect on output. Figure 6 shows that the rise in prices is accompanied by a rise in the nominal interest rate. In the SWBGG model the same mechanism applies. In addition the fall in capital improves the leverage position of firms, which in turn leads to a fall in the spread. This second effect acts in the direction of attenuating the impact of the wage mark-up shock on the real variables. In the SWGK model the increase in prices also leads to a fall in the demand of factors for production, which leads to a decrease in the leverage of FI. This in turn has the effect of alleviating the problem of asymmetric information between FI and household.
Hence an attenuator effect takes place.

5 Robustness analysis

This section illustrates: (i) whether the results of the Bayes factor are robust to the calibration of the steady state leverage ratio of the SWBGG and SWGK models; (ii) whether those results are also robust to the models’ specification; and (iii) the series of the credit spread generated by the SWBGG and SWGK model and it compares the estimated series with the actual ones.⁶

5.1 Sensitivity to the leverage ratio

The importance of the value of the leverage ratio is stressed by several studies, such as Peersman and Smets (2005) and Carlstrom et al. (2011). In the SWBGG model a change in the steady state leverage ratio has a direct impact on equation (39). Any change in the leverage ratio clearly influences the financial accelerator effect. In the SWGK model a change in the steady state leverage ratio affects the evolution of net worth of FI, equation (44). Similarly to the SWBGG model, a change in the steady state leverage ratio of FI affects the spread, and therefore total output, as explained in Subsection 4.3.

The leverage ratio is equal to 2 in the SWBGG model and 4 in the SWGK model as shown in the baseline calibration, Table 2. Table 7 reports the baseline calibration in bold and shows how the Bayes factor is affected by changes in the leverage ratio of the two models one at a time. In the SWBGG model the leverage ratio of firms changes from 1.5 to 4.5, implying that from 33% to 78% of firms’ capital expenditure are externally financed. The second column of Table 7 reports the Bayes factor between the log data density of the SWGG model, computed with the modified harmonic mean estimator (based on 100,000 draws from the random walk MH algorithm for computational tractability) for each value of the leverage ratio, and the log data density of the SW model. Similarly, the third column reports the the Bayes factor between the log data density of the SWGG model and that of the SWGK model. For sake of brevity, parameters estimates are not reported. The comparison between the SWBGG and SW models provides mixed results: for values of the leverage ratio lower or equal to 2.5 the SWBGG is preferred in the data, while for a leverage ratio equal to 3 the SW model is preferred. When the leverage ratio of firms becomes higher than 3, the Bayes factor is close to one; so there is no strong evidence in favour of a model versus the other one. The comparison between the SWBGG and SWGK models shows that the SWGK model is always favoured.

⁶As a further robustness check the SWBGG and SWGK models are estimated using as observable a series on the spread instead of the nominal interest rate. The available series is the spread between the yield on corporate financial BBB bonds and government AAA bonds. The series cover the period 1996Q1-2008Q3. The log data density computed with the modified harmonic mean estimator is equal to -209.17 in the SWBGG model and -201.73 in the SWGK. Hence, data still favour the SWGK model with a different dataset.
Table 7: Sensitivity of the Bayes factor to the steady state leverage ratio.

<table>
<thead>
<tr>
<th>Leverage, $\kappa_N$, in the SWBGG model</th>
<th>Bayes factor</th>
<th>Bayes factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWBGG vs SW = &amp; $\exp(LL(Y</td>
<td>m_{SWBGG}))$ &amp; $\exp(LL(Y</td>
<td>m_{SW}))$</td>
</tr>
<tr>
<td>1.5</td>
<td>$2.10 \times 10^{11}$</td>
<td>$4.92 \times 10^{-1}$</td>
</tr>
<tr>
<td>2</td>
<td>$1.49 \times 10^{8}$</td>
<td>$3.49 \times 10^{-4}$</td>
</tr>
<tr>
<td>2.5</td>
<td>$9.95 \times 10^{5}$</td>
<td>$2.33 \times 10^{-6}$</td>
</tr>
<tr>
<td>3</td>
<td>$2.72 \times 10^{-4}$</td>
<td>$6.37 \times 10^{-16}$</td>
</tr>
<tr>
<td>3.5</td>
<td>$0.09 \times 10^{-1}$</td>
<td>$2.11 \times 10^{-14}$</td>
</tr>
<tr>
<td>4</td>
<td>$0.50 \times 10^{-1}$</td>
<td>$1.18 \times 10^{-13}$</td>
</tr>
<tr>
<td>4.5</td>
<td>$0.38 \times 10$</td>
<td>$7.32 \times 10^{-12}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Leverage, $\kappa_N$, in the SWGK model</th>
<th>Bayes factor</th>
<th>Bayes factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWGK vs SW = &amp; $\exp(LL(Y</td>
<td>m_{SWGK}))$ &amp; $\exp(LL(Y</td>
<td>m_{SW}))$</td>
</tr>
<tr>
<td>3</td>
<td>$4.01 \times 10^{12}$</td>
<td>$2.69 \times 10^{4}$</td>
</tr>
<tr>
<td>3.5</td>
<td>$5.88 \times 10^{11}$</td>
<td>$3.94 \times 10^{3}$</td>
</tr>
<tr>
<td>4</td>
<td>$4.27 \times 10^{11}$</td>
<td>$2.86 \times 10^{3}$</td>
</tr>
<tr>
<td>4.5</td>
<td>$4.50 \times 10^{10}$</td>
<td>$3.02 \times 10^{2}$</td>
</tr>
<tr>
<td>5</td>
<td>$1.84 \times 10^{12}$</td>
<td>$1.23 \times 10^{4}$</td>
</tr>
<tr>
<td>5.5</td>
<td>$5.95 \times 10^{10}$</td>
<td>$3.99 \times 10^{2}$</td>
</tr>
</tbody>
</table>

by the data independently of the value of the leverage ratio in the SWBGG model, being the Bayes factor always smaller than 1.

The second part of the table shows how the Bayes factor varies when the leverage ratio of financial intermediaries changes from 3 to 5.5 in the SWGK model. The second and third columns of Table 7 report the Bayes factor between the SWGK and SW models and between the SWGK and SWBGG models for different values of the leverage ratio of financial intermediaries. The comparison between the SW and SWGK models shows that the latter is favoured by the data. Moreover, there is clear evidence in favour of the SWGK model also in comparison to the SWBGG model. While the Bayes factor between the SWBGG and SW models is affected by the value of the leverage ratio in the SWBGG model, there is always evidence in favour of the SWGK model for different values of the steady state leverage ratio of firms and financial intermediaries. It is worth noting that when firms and financial intermediaries have the same leverage ratio – 3, 3.5, 4, and 4.5 – the SWGK model is always the preferred one.
Table 8: Log data density for different models’ specifications

<table>
<thead>
<tr>
<th>Friction</th>
<th>SW</th>
<th>SWBGG</th>
<th>SWGK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-378.32</td>
<td>-359.50</td>
<td>-351.54</td>
</tr>
<tr>
<td>$\sigma_p = 0.1$, Calvo prices</td>
<td>-524.80</td>
<td>-425.17</td>
<td>-420.45</td>
</tr>
<tr>
<td>$\sigma_w = 0.1$, Calvo wages</td>
<td>-396.37</td>
<td>-390.67</td>
<td>-375.76</td>
</tr>
<tr>
<td>$h = 0.1$, habit parameter</td>
<td>-405.43</td>
<td>-404.97</td>
<td>-385.63</td>
</tr>
<tr>
<td>$\xi = 0.1$, inv. adj. costs</td>
<td>-544.45</td>
<td>-472.44</td>
<td>-466.39</td>
</tr>
<tr>
<td>$\zeta = 0.99$, elasticity of capital util</td>
<td>-347.34</td>
<td>-331.13</td>
<td>-318.43</td>
</tr>
<tr>
<td>$\Theta = 1.1$, fixed costs in production</td>
<td>-397.01</td>
<td>-362.71</td>
<td>-359.82</td>
</tr>
</tbody>
</table>

5.2 Sensitivity to models’ specification

The three models embed the following same types of frictions: price stickiness, price indexation, wage stickiness, wage indexation, investment adjustment costs, variable capital utilization, habit in consumption and fixed costs in production. As a further robustness check, each of the main common frictions is turned off one at a time in the spirit of SW. Table 8 reports the log data density, computed with the modified harmonic mean estimator. This experiment makes also it possible to analyse which frictions are important to account for the dynamics of each model. For each row the log data density of the model most preferred by the data is shown in bold. The first row reports the Bayes factor of the baseline estimates in Subsection 4.1. The comparison between the two models with financial frictions – SWBGG and SWGK – and the model with perfect capital markets – the SW model – is always in favour of the former. And there is evidence in favour of the SWGK model compared to the SWBGG model, independently of which friction is turned off. The removal of each friction at a time has a similar effects in the three models. On the side of nominal frictions, removing price stickiness implies a considerable deterioration in terms of the log data density. On the side of real frictions, the most important in terms of the log data density is investment adjustment costs. Reducing habit formation in consumption and fixed costs in production is also costly in terms of the log data density. A larger value of the capital utilization elasticity implies higher marginal depreciation cost, and therefore less variation in capital utilization. Removing this friction does not imply a deterioration of the log data density; its value is even higher in all models.

5.3 External validation of the time series of the spread

TBD
6 Forecasting evaluation

The estimation results can be used to analyse the predictive power of some series such as the output gap and credit spreads in the two models. Coenen et al. (2009) find that flexible-price output gap performs relatively well in predicting EA inflation over medium-term horizons. Gilchrist and Zakrajšek (2011a) find that the credit spread is a more powerful predictor of economic activity compared to the standard default-risk indicators.

This section examines the predictive power of the SWBGG model versus the SWGK model in forecasting inflation in the EA. Following Coenen et al. (2009) and Gelain (2010), inflation forecasts are based on the basis of a traditional Phillips curve with the flexible-price output gap (output gap henceforth) generated by the two estimated models. Then a modified version of the Phillips curve (PC) replaces the output gap with the spread. As benchmarks, two control models, a random walk and an autoregressive process, are used.

The forecast of inflation is made using several vintages of data, i.e. for rolling samples in pseudo-real time, as described in Fischer et al. (2006), with the initial sample spanning 1980Q2-1998Q4 and the final sample covering 1980Q2-2008Q3. Similarly to Gelain (2010), the 4-quarter change in the private consumption deflator, $\pi_{t+4}^{4}$, is forecast:

$$\pi_{t+4}^{4} = 100 \left( \frac{P_{t+4}}{P_{t}} - 1 \right)$$

First, the following equation is estimated by OLS:

$$\pi_{t+4}^{4} = a_{v} + b_{v}(L)\pi_{v,t} + c_{v}(L)x_{v,t} + \varepsilon_{v,t+4}^{4}$$

where $\pi_{v,t} = 400 \left( \frac{P_{t+4}}{P_{t}} - 1 \right)$ is the annualised one-period change in the private consumption deflator, $x_{v,t}$ is either the output gap or the credit spreads generated by the estimated models, $b_{v}(L)$ and $c_{v}(L)$ are finite polynomials. The optimal number of lags is selected using the Schwartz information criterion.

Then, for each vintage a forecast of inflation is obtained:

$$\tilde{\pi}_{t+4}^{4} = a_{v}^{OLS} + b_{v}(L)^{OLS}\pi_{v,t} + c_{v}(L)^{OLS}x_{v,t} + \tilde{\varepsilon}_{v,t+4}^{4}$$

The autoregressive model of inflation is obtained following the same procedure described above. In the random walk model inflation forecast is given by the average rate of inflation over the previous four quarters available for a given data vintage:

$$\tilde{\pi}_{t+4}^{4,RW} = 100 \left( \frac{P_{t}}{P_{t-4}} - 1 \right)$$
Table 9: MSFE for 4 steps ahead inflation forecast

<table>
<thead>
<tr>
<th>Model</th>
<th>MSFE</th>
<th>MSFE_{RW}^M</th>
<th>σ^2</th>
<th>bias^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWBGG with output gap</td>
<td>0.061</td>
<td>0.244</td>
<td>0.023</td>
<td>0.038</td>
</tr>
<tr>
<td>SWBGG with spread</td>
<td>0.060</td>
<td>0.238</td>
<td>0.020</td>
<td>0.040</td>
</tr>
<tr>
<td>SWGK with output gap</td>
<td>0.056</td>
<td>0.224</td>
<td>0.021</td>
<td>0.035</td>
</tr>
<tr>
<td>SWGK with spread</td>
<td>0.057</td>
<td>0.225</td>
<td>0.020</td>
<td>0.037</td>
</tr>
<tr>
<td>RW</td>
<td>0.252</td>
<td>1.000</td>
<td>0.232</td>
<td>0.020</td>
</tr>
<tr>
<td>AR</td>
<td>0.063</td>
<td>0.250</td>
<td>0.022</td>
<td>0.042</td>
</tr>
</tbody>
</table>

For each model $\mathcal{M}$, forecast errors, $f_{t+4}$, are defined as:

$$fe_{t+4}^M = \pi_{t+4}^M - \pi_{t+4}^4$$

(54)

where $\pi_{t+4}^4$ is the realised inflation rate in the last available vintage of data.

Alternative models $\mathcal{M}$ are compared on the basis of the mean squared forecast error (MSFE), which is given by:

$$MSFE^M = (bias^M)^2 + (\sigma^M)^2$$

(55)

where

$$bias^M = \frac{1}{T} \sum_{t=1}^{T} fe_{t+4}^M$$

and

$$\sigma^M = \frac{1}{T} \sum_{t=1}^{T} \left( fe_{t+4}^M - \frac{1}{T} \sum_{t=1}^{T} fe_{t+4}^M \right)^2$$

(56)

Table 9 shows the MSFE for 6 different models: the SWBGG model with the output gap in the PC, the SWBGG model with the spread in the PC, the SWGK model with output gap in the PC, the SWGK model with the spread in the PC, the random walk, and the AR model. The third column of Table 9 shows the ratio between the MSFE of model $\mathcal{M}$ and the MSFE of the random walk model; the last two columns shows the variance and the bias of each model.

The results of this forecasting exercise are as follows. First, the RW model shows the worst performance in terms of the MSFE criterion, as shown by the third column, and the highest forecast error variance. Second, the comparison between the SWBGG model and the SWGK model provides evidence in favour of the SWGK model no matter whether the PC is estimated with the output gap or the spread. Third, the flexible-price output gap adds more predictive power compared to the spread in the SWGK model while the contrary happens in the SWBGG model. Finally, the AR model outperforms only the RW model. These findings need to be interpreted cautiously given the short forecast interval used for this exercise.
7 Conclusion

This paper compares three DSGE models which have a Smets and Wouters (2007) economy in common but feature different types of financial frictions: (i) the SW model with perfect capital markets; (ii) the SWBGG model with financial frictions originating in intermediate goods firms due to a costly state verification problem à la Bernanke et al. (1999); and (iii) the SWGK model with financial frictions embedded in financial intermediaries due to a moral hazard problem à la Gertler and Karadi (2011). The three models are estimated with Bayesian techniques for the period 1980Q1-2008Q3 with Euro Area data. The main results are: first, the presence of financial frictions, either à la BGG or à la GK, improves the model’s fit. And second, the SWGK model is always the model favoured by the data according to the analysis of the Bayes factor and the comparison of simulated versus actual second moments. This result is robust to series of models’ calibration and specifications. The main intuition behind the second result is that the EA is a bank-based financial system, as opposed to the market-based financial system in the United States, e.g. Trichet (2009). Hence the SWGK model is able to capture the nature of the financial system in the EA since it explicitly provides a microfoundation of the banking sector, while in the SWBGG model intermediation is channeled through securities markets and banks are treated largely as a veil.

All models deliver plausible impulse response functions (IRFs). However, the internal propagation mechanisms of the shocks differ between the three models, with financial frictions leading to an accelerator or attenuator effect depending on which shock is analysed.

Finally, the SWGK model outperforms the SWBGG model in forecasting Euro Area inflation in a Phillips curve specification with the measure of output gap or of the credit spread generated by the two estimated models.

The results presented in this paper can offer some avenues for future research: (i) it would be interesting to analyse a model featuring both types of financial frictions, at firms level and in the banking sector, in order to examine the transmission mechanism of the shocks and the accelerator/attenuator effects in line with the recent contribution by Rannenberg (2012); and (ii) such a model could also incorporate the same form of financial friction (costly state verification or moral hazard at both levels) in order to empirically verify which modelling device is preferred by the data. DSGE models with a comprehensive structure of financial markets would improve our understanding of business cycle fluctuations.
References


Trichet, J. (2009). The ECB’s enhanced credit support. In *Speech at the University of Munich*.


Figure 1: Data versus fitted values in the three models.
Figure 2: Autocorrelations in the data versus the models.
Figure 3: Monetary policy shock. Solid lines represent mean IRF and dashed line represent the 95% highest posterior density confidence intervals.

Figure 4: Investment-specific technology shock. Solid lines represent mean IRF and dashed line represent the 95% highest posterior density confidence intervals.
Figure 5: Technology shock. Solid lines represent mean IRF and dashed line represent the 95% highest posterior density confidence intervals.

Figure 6: Wage mark-up shock. Solid lines represent mean IRF and dashed line represent the 95% highest posterior density confidence intervals.