Averaging tests for jumps

Ana-Maria Dumitru

University of Surrey
School of Economics

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Abstract

We propose a new procedure to detect jumps in prices in the presence of microstructure noise. The procedure averages the results from existing tests across sampling frequencies. This approach overcomes the sub-sampling and low-power problems that plague existing tests for jumps. We use a modified version of Fisher’s method to combine p-values for the Barndorff-Nielsen and Shephard (2006) test applied at different sampling frequencies. We propose a double bootstrap procedure to obtain an empirical distribution for Fisher’s test statistic. As an additional contribution, we prove the bootstrap consistency of the Barndorff-Nielsen and Shephard (2006) test.

Keywords: jumps, nonparametric tests, combining p-values, double bootstrap
1 Introduction

The advances made during the past decade in the field of high frequency econometrics conferred both researchers and practitioners the possibility to detect and estimate jumps in asset prices through simple nonparametric techniques, as an alternative to simulation based estimation methods of stochastic volatility models with jumps. Several tests for jumps have been introduced in the literature by Andersen et al. (2007), Andersen et al. (2009), A"ıt-Sahalia and Jacod (2008), Barndorff-Nielsen and Shephard (2006), Corsi et al. (2010), Jiang and Oomen (2008), Lee and Mykland (2008), and Podolskij and Zagij (2010). All these procedures use high frequency data and work under very similar assumptions for the price process. They are based on asymptotic results that require that the number of intraday returns increases to infinity.

When applied to real or simulated price data, the above procedures tend to lead to different results. Dumitru and Urga (2012); Theodosiou and Žikeš (2010) perform thorough comparisons between various jump testing procedures and attempt to rank them considering both the power and size criteria. This inconsistency of results is mostly due to the fact that at very high frequencies, securities prices are contaminated with microstructure noise, which generates both size and power distortions for all jump tests. So far, the majority of the existing literature unequivocally proposed sub-sampling as a unique solution to overcome the problems generated by the presence of noise. As an exception, Dumitru and Urga (2012) point out that sub-sampling leads to loss of power and inefficiency, by “throwing away” a lot of data. They propose combining various tests and/ or sampling frequencies through both reunions and intersections in order to extract more information on jump occurrence. However, this multiple testing procedure, despite over-performing the existing single-testing procedures, it lacks rigor, by not having a clear size.

This paper proposes a new jump detection procedure, that manages to rigorously combine results for jump tests applied at different sampling frequencies. This new procedure is more efficient, by extracting information
from multiple sampling frequencies, and, at the same time, allows users to exert control over the size of the test. We use Fisher’s method to average p-values obtained from applying the same testing procedure at different frequencies. To fix the size of the tests at various frequencies, we use the empirical bootstrap distribution to extract the appropriate p-values. Moreover, as individual tests applied at different frequencies are not independent, Fisher’s statistic does not have a standard $\chi^2$ distribution. To obtain the empirical distribution of the latter statistic, a second round of bootstrap is applied. Here, we apply this double-bootstrap procedure for the classical Barndorff-Nielsen and Shephard (2006) (henceforth BNS) test. Consequently, an important co-product of this paper is the proof of bootstrap consistency for the BNS test. In future extensions to this paper, we will broaden our research to more jump tests.

This new procedure to apply tests for jumps brings an important contribution to the literature of jump identification in the presence of microstructure noise. While the literature on estimating volatility with noise is very rich (Zhang et al., 2005; Ait-Sahalia et al., 2005; Barndorff-Nielsen et al., 2008a,b), there are no significant contributions on testing for jumps when noise is added to the price process. The current paper, by proposing to use data sampled at different frequencies, is in line with similar works in volatility estimation, such as Zhang et al. (2005); Zhang (2006), that use information from different sampling scales to estimate volatility. Thus, the present contribution manages to build a bridge between the two significant threads of literature in high frequency econometrics, i.e. volatility estimation and jump identification.

The rest of the paper is organized as follows. Section 2 explains the theoretical background and past relevant contributions in the field of high frequency testing for jumps. Section 3 covers all methodological issues concerning jump identification based on averaging p-values. Section 4 contains simulation results for the new procedure. Section 5 concludes the paper.
2 Setup, notation and existing theory

Following Barndorff-Nielsen and Shephard (2006), the logarithmic price process, $P_t$, is a Brownian semimartingale plus jump process:

$$dP_t = \mu_t dt + \sigma_t dW_t + dJ_t$$  \hspace{1cm} (1)

where the drift, $\mu_t$, and the volatility, $\sigma_t$, are assumed càdlàg, and $W_t$ a Brownian motion at time $t$. $J_t$ is the jump process at time $t$, defined as $J_t = \sum_{j=1}^{N_t} c_{t_j}$ where $c_{t_j}$ represents the size of the jump at time $t_j$ and $N_t$ is a counting process, representing the number of jumps up to time $t$ and assumed to be finite for all $t$.

The quadratic variation of the price process up to a certain point in time, $t$, ($QV_t$), usually a trading day, can be defined as follow:

$$QV_t = \int_0^t \sigma_s^2 ds + \sum_{j=1}^{N_t} c_{t_j}^2,$$ \hspace{1cm} (2)

where $\int_0^t \sigma_s^2 ds = IV_t$ is the integrated variance or volatility.

There are several estimators in the field of high frequency econometrics for both the quadratic variance and the integrated volatility of the price. Most of these estimators are based on equally spaced data. Thus, the interval $[0, t]$ is split into $n$ equal subintervals of length $\delta$. The j-th intraday return $r_j$ on day $t$ is defined as $r_j = p_{t-1+j\delta} - p_{t-1+(j-1)\delta}$.

Andersen and Bollerslev (1998) proposed $RV_t$ to estimate the quadratic variance:

$$RV_t = \sum_{j=1}^{n} r_j^2 \xrightarrow{p} QV_t, \quad \text{for } \delta \to 0$$ \hspace{1cm} (3)

where $\xrightarrow{p}$ stands for convergence in probability.

Barndorff-Nielsen and Shephard (2004) propose the first robust to jumps estimator of the integrated variance, the realized bipower variation ($BV_t$), constructed to reduce the impact of jump returns on the volatility estimate.
by multiplying them with adjacent jump-free returns:

\[ BV_t = \frac{\pi}{2} \sum_{j=2}^{n} |r_j||r_{j-1}| \]  

(4)

In the absence of jumps \((N_t = 0)\), both \(RV_t\) and \(BV_t\) consistently estimate the integrated variance. Barndorff-Nielsen and Shephard (2006) establish a CLT for \(RV_t\) and \(BV_t\), when \(N_t = 0\):

\[
\delta^{-1/2} \left[ \left( \frac{RV_t}{\pi BV_t} \right) - \left( \int_0^t \sigma_s^2 ds \right) \right] \xrightarrow{L} \mathcal{N} \left( \left( \begin{array}{c} 0 \\ 0 \end{array} \right), \int_0^t \sigma_s^4 ds \left( \frac{2}{\pi^2} \frac{2}{\pi^2} + \pi - 3 \right) \right)
\]

(5)

where \(\xrightarrow{L}\) stands for convergence in law.

We can test for jumps by comparing \(RV_t\) with \(BV_t\) under the null of no jumps. This leads to a Hausman-type test, where \(RV_t\) is more efficient, but consistent only under the null, whereas \(BV_t\) is consistent under both hypotheses. Barndorff-Nielsen and Shephard (2006) propose the difference statistics, whereas Huang and Tauchen (2005) point out that the use of the ratio statistic leads to a less oversized test. \(\int_0^t \sigma_s^4 ds\) is usually estimated by using the tripower quarticity, \(TQ_t\):

\[
TQ_t = n \cdot 1.74 \left( \frac{n}{n-2} \right) \sum_{j=3}^{n} |r_{j-2}|^{4/3} |r_{j-1}|^{4/3} |r_j|^{4/3}.
\]

(6)

In this paper, we will consider the following feasible difference and ratio statistics:

\[
\frac{\delta^{-1/2}(RV_t - BPV_t)}{\sqrt{0.61 TQ_t}}
\]

(7)

\[
\frac{1 - \frac{BV_t}{RV_t}}{\sqrt{0.61 \delta \max \left( 1, \frac{TQ_t}{BV_t^2} \right)}}
\]

(8)
3 Averaging tests for jumps

Similarly to the literature on volatility estimation (see Zhang et al., 2005; Zhang, 2006), we propose using data from different time scales to identify jumps when prices are contaminated with microstructure noise. Our procedure consists of applying the BNS test at different sampling frequencies, obtain the corresponding p-values and then average them by using Fisher’s method. Simulation studies looking at the size and power properties of the BNS test (see, for instance Dumitru and Urga, 2012) show that its size tends to grow as the sampling frequency becomes lower (i.e. as $\delta$ becomes higher). However, to be able to combine the results of the test applied at different frequencies, it must have the same size for all frequencies. Thus, instead of using the asymptotic distribution to obtain p-values, we propose using the empirical distribution of the bootstrapped statistics.

The combination of test statistics with simple null hypotheses has been long used in the statistics and econometrics literature, with the oldest and most famous including Tippett (1931)’s min($p_i$) statistics, Fisher (1932)’s $X^2$ statistic and Liptak (1958)’s $\sum_{j=1}^{p} \Phi^{-1}(1 - p_i)$ statistic, where $p_i$ represents the i-th p-value, $i = 1 \ldots p$. Here, as we attempt to extract information from more frequencies, we focus on Fisher (1932)’s $X^2$, defined as:

$$X^2 = -2 \sum_{j=1}^{p} \log p_i \quad (9)$$

When the combined test statistics are independent, $X^2 \xrightarrow{L} \chi^2(2p)$. However, this is not the case here, as the test is applied to the same data, but sampled at different frequencies. Brown (1975) and Kost and McDermott (2002) proposed approximating the distribution of the $X^2$ with that of a scaled $\chi^2$ variable, but assuming certain approximations for the covariances between the p-values. The advances in both computational technology and power allow us to easily simulate the distribution of the $X^2$ statistic.

There are several important contributions in the field of econometrics.
that use meta-analysis methods like Tippett (1931), Fisher (1932) or Liptak (1958) to combine test results and simulate the distributions of the “combined” statistic. Maddala and Wu (1999) apply Fisher (1932)’s method for unit root tests for panel data; Smeekes and Taylor (2012) use unions of rejections of unit root tests; Dufour et al. (2004) use combined procedures to test for heteroskedasticity when there are unknown breakpoints in the variance.

A very relevant contribution is the one by Godfrey (2005), who suggests applying double bootstrap methods to control the overall significance level of several diagnostic tests applied for an ordinary least squares regression model.

In this paper, even if the test statistic is asymptotically pivotal, the asymptotic size depends on the sampling frequency. Thus, following Godfrey (2005), we apply a double-bootstrap procedure to control the overall significance level of our test averaging procedure.

### 3.1 The double bootstrap procedure

Let us assume we are interested in finding out whether jumps occurred during the interval $[0,t]$, which could be, for instance, a trading day. Moreover, let the data be sampled at sampling intervals of the form $k_i \cdot \delta$, where $k_i > 0$ multiplies $\delta$, $i = 1, \ldots, p$, where $p$ is the number of frequencies we choose to combine. In applying the double bootstrap, we take the following steps:

1. compute the BNS statistic on the original data for different sampling frequencies $(k_i)$, $bns_{t,k_i}$

2. for each day and each frequency, re-sample the returns under the null and obtain $B$ replicates for the BNS statistics, $bns^*_{t,k_i}$

3. compute the p-value corresponding to the original $bns_{t,k_i}$ statistics,
based on the bootstrapped distribution:

$$p_{t,k_i}^* = \frac{\sum_{j=1}^{B} bns_{t,k_i} > bns_{t,k_i}^* (j)}{B}$$  \quad (10)

4. for each new sample from 1 to B, generate $B_1$ sub-samples and compute the BNS statistic $bns_{t,k_i,b}^*$, where $b = 1, \ldots, B$

5. for each new sample from 1 to B, compute corresponding p-value:

$$p_{t,k_i,b}^{**} = \frac{\sum_{j=1}^{B_1} bns_{t,k_i} > bns_{t,k_i}^{**} (j)}{B_1}$$  \quad (11)

Thus, this procedure generates for each $k_i$, one p-value from the first round of bootstrap and B p-values from the second round of bootstrap. The Fisher $X^2$ statistic is computed as:

$$X_t^2 = -2 \sum_{i=1}^{p} \log p_{t,k_i}^*$$  \quad (12)

The distribution of $X_t^2$ can be obtained as:

$$X_t^{2**} = -2 \sum_{i=1}^{p} \log p_{t,k_i}^{**},$$  \quad (13)

where $X_t^{2**}$ is a vector with B elements.

To re-sample under the null, if a jump is identified on that day, we remove the corresponding return from the data and sample from the remaining returns. To identify the jump return on a certain day, we take the maximum standardized return of that day, where the standardization is performed as in Andersen et al. (2007) and Lee and Mykland (2008).
3.2 Bootstrap consistency

We propose the i.i.d bootstrap method for the BNS test, which we will extend to wild bootstrap in future developments of this paper. As Gonçalves and Meddahi (2009) explains, intraday returns are independent, but usually heteroskedastic, making the wild bootstrap the appropriate bootstrap method. However, at the same time, i.i.d. bootstrap is valuable as a benchmark and for the case in which intraday volatility does not vary substantially.

We denote the bootstrap intraday return $r^*_j$, where $r^*_j$ is i.i.d. from $\{r_j : j = 1, \ldots, n\}$. $P^*$ denotes the probability measure under bootstrap re-sampling, conditional on the original sample. Let $E^*$ and $Var^*$ denote the expected value and the variance under the $P^*$ probability measure. In order to be able to bootstrap one of the test statistics in 7 and 8, we prove a CLT-type result similar to the one in 5.

The bootstrap realized variance and realized bipower variation are defined as follows:

$$RV^*_t = \sum_{j=1}^{n} (r^*_j)^2$$  \hspace{1cm} (14)

$$BV^*_t = \frac{\pi}{2} \sum_{j=2}^{n} |r^*_j||r^*_{j-1}|$$  \hspace{1cm} (15)

Gonçalves and Meddahi (2009) derive a CLT result for the bootstrapped realized variance, whereas Podolskij and Ziggel (2007) derive a similar result for the bootstrapped bipower variation. Following their steps, we first compute the expected values and variances of $RV^*_t$ and $BV^*_t$, as well as the
covariances between the two, under the bootstrap measure, $P^*$.

\[
E^*(RV_t^*) = RV_t
\]

\[
E^*(BV_t^*) = \frac{n-1}{2n^2} \left( \sum_{i=1}^{n} |r_i| \right)^2
\]

\[
Var^*(RV_t^*) = \sum_{i=1}^{n} r_i^4 - \frac{1}{n} RV_t^2
\]

\[
Var^*(BV_t^*) = \frac{n^2}{4} \left[ \frac{n-1}{n^2} RV_t^2 + \frac{2(n-2)}{n^3} RV_t \left( \sum_{i=1}^{n} |r_i| \right)^2 - \frac{3n-5}{n^4} (\sum_{i=1}^{n} |r_i|)^4 \right]
\]

\[
cov^*(RV_t^*, BV_t^*) = \frac{n-1}{2n^3} \sum_{i=1}^{n} |r_i|^3 |r_i| \sum_{i=1}^{n} |r_i| - \frac{n-1}{n} RV_t^2 (\sum_{i=1}^{n} |r_i|)^2
\]

(16)

As seen above, in the case of i.i.d. bootstrap, the $BV_t^*$ statistic is not centered in $BV_t$. This is contrary to the findings in Podolskij and Ziggel (2007), where the products $b_j = |r_j^*||r_{j-1}^*|$ are i.i.d. re-sampled. Moreover, the variances of both bootstrapped estimators are different from the variances of the original estimators. As explained also in Gonçalves and Meddahi (2009), these discrepancies occur because for the original estimators, the asymptotics are based on a local Gaussian assumption, that does not longer hold in the case of i.i.d. bootstrap.

To estimate the expected values, variances and covariances of $RV_t^*$ and $BV_t^*$, we define the following consistent estimators:

\[
E^{*\prime}(RV_t^{*\prime}) = \sum_{i=1}^{n} r_i^{2m}
\]

\[
E^{*\prime}(BV_t^{*\prime}) = \frac{n-1}{2n^2} \left( \sum_{i=1}^{n} |r_i|^i \right)^2
\]

\[
Var^{*\prime}(RV_t^{*\prime}) = \sum_{i=1}^{n} r_i^{4m} - \frac{1}{n} \sum_{i=1}^{n} r_i^{2m}
\]

\[
Var^{*\prime}(BV_t^{*\prime}) = \frac{n^2}{4} \left[ \frac{n-1}{n^2} RV_t^{2m} + \frac{2(n-2)}{n^3} RV_t \left( \sum_{i=1}^{n} |r_i|^i \right)^2 - \frac{3n-5}{n^4} (\sum_{i=1}^{n} |r_i|^i)^4 \right]
\]

\[
cov^{*\prime}(RV_t^{*\prime}, BV_t^{*\prime}) = \frac{n-1}{2n^3} \sum_{i=1}^{n} |r_i|^i |r_i| \sum_{i=1}^{n} |r_i| - \frac{n-1}{n} RV_t^{2m} (\sum_{i=1}^{n} |r_i|^i)^2
\]

(17)

In the above equations, each power variation -type of sum, $\sum_{i=1}^{n} |r_i|^m$, $m \in \{1, 2, 3, 4\}$, is estimated using the bootstrapped counterpart, $\sum_{i=1}^{n} |r_i|^m$. This estimation procedure will be jump robust, as bootstrapped returns are sampled under the null.

**Theorem 1. CLT of the bootstrapped vector $(RV_t^*; BV_t^*)'$ (Consistency of the i.i.d. bootstrap)** Suppose the price process can be de-
scribed as in 1. Let \((RV_t^*, BV_t^*)'\) be the vector of bootstrapped statistics. As \(n \to \infty (\delta \to 0)\),

\[
\begin{pmatrix}
    (RV_t^*) \\
    (BV_t^*)
\end{pmatrix}
\xrightarrow{L} \mathcal{N}(O, \Omega^*),
\]

where

\[
O = \begin{pmatrix}
    0 \\
    0
\end{pmatrix}
\quad \text{and} \quad
\Omega^* = \begin{pmatrix}
    \text{Var}^*(RV_t^*) & \text{cov}^*(RV_t^*, BV_t^*) \\
    \text{cov}^*(RV_t^*, BV_t^*) & \text{Var}^*(BV_t^*)
\end{pmatrix}
\]

(18)

Given that the original centered vector \((RV_t; BV_t)'\) is also multivariate normal (see 5), the consistency of the bootstrap follows.

**Proof:** To prove Theorem 1, two steps must be followed:

1. Show that:

\[
\begin{pmatrix}
    (RV_t^*) \\
    (BV_t^*)
\end{pmatrix}
\xrightarrow{L} \mathcal{N}(O, \Omega^*),
\]

where

\[
\Omega^* = \begin{pmatrix}
    \text{Var}^*(RV_t^*) & \text{cov}^*(RV_t^*, BV_t^*) \\
    \text{cov}^*(RV_t^*, BV_t^*) & \text{Var}^*(BV_t^*)
\end{pmatrix}
\]

(19)

2. Show that \(\widehat{\Omega}^* \xrightarrow{p} \Omega^*\).

We show here the proof for 1. We need to prove that

\[
\begin{pmatrix}
    \sum_{j=2}^n r_j^2 \\
    \sum_{j=2}^n |r_{j-1}^*|
\end{pmatrix}
\xrightarrow{L} \mathcal{N}(O, \Omega^*),
\]

This is equivalent to proving that:

\[
(c_1 \ c_2)\begin{pmatrix}
    \sum_{j=2}^n r_j^2 \\
    \sum_{j=2}^n |r_{j-1}^*|
\end{pmatrix}
\xrightarrow{L} \mathcal{N}(0, c'\Omega^* c),
\]

where \(c = (c_1 \ c_2)'\) is a vector of constants.
Thus, we have:

\[
(c_1, c_2) \left[ \left( \sum_{j=2}^{n} r_j^* \right) - \left( E^*(RV^*_j) \right) \right] = \\
\sum_{j=2}^{n} \left[ c_1 r_j^* + c_2 \frac{\pi}{2} |r_j^*||r_{j-1}^*| - c_1 r_j^2 - c_2 \frac{\pi}{2} \frac{1}{n} (r_j^2 + |r_{j-1}^*| \sum_{i \neq j} |r_i|) \right] = \\
\sum_{j=2}^{n} z_j^*
\]

Given that \( r_j^* \)'s are i.i.d., \( z_j^* \) is a 1-dependent process, with dependence disappearing after the first period, a general CLT result applies.

What remains to show is that \( \text{Var}^*(\sqrt{n} \sum_{j=2}^{n} z_j^*) \overset{p}{\to} \sigma_z^2 = O(1) \), \( \sigma_z^2 \) finite and positive. Barndorff-Nielsen et al. (2006) define realized power variations as \( n^{q/2-1} \sum_{j=1}^{n} |r_j^q| \) and prove that, in the absence of jumps, they consistently estimate \( \mu_q \int_0^t \sigma_q^2 ds \), where \( \mu_q = E|u|^q \), where \( u \) is a standard normal. Given that in our case the bootstrap is applied under the null of no jumps, we have:

\[
\text{Var}^*(\sqrt{n} \sum_{j=2}^{n} z_j^*) \overset{p}{\to} \\
c_1^2 \left[ \mu_4^{-1} \int_0^t \sigma_4^2 ds - \left( \int_0^t \sigma_2^2 ds \right)^2 \right] + \\
4c_1c_2\mu_2^{-2} \left[ \mu_3^{-1} \mu_1^{-1} \int_0^t \sigma_3^2 ds \int_0^t \sigma_2^2 ds + \mu_1^{-2} \int_0^t \sigma_2^2 ds \left( \int_0^t \sigma_3^2 ds \right)^2 \right] + \\
c_2^2 \mu_1^{-4} \left[ \left( \int_0^t \sigma_2^2 ds \right)^2 + 2\mu_1^{-2} \int_0^t \sigma_2^2 ds \left( \int_0^t \sigma_2^2 ds \right)^2 - 3\mu_4^{-4} \left( \int_0^t \sigma_4^2 ds \right)^4 \right] = \\
= O(1)
\]

11
4 Simulation study

To assess the effectiveness of the proposed jump detection procedure, we simulate a stochastic volatility process with finite jumps. We use a similar simulation setup as in Huang and Tauchen (2005) and Dumitru and Urga (2012). The stochastic volatility model for the log of the price process follows the subsequent dynamics:

\[
\begin{align*}
    dp_t &= 0.03dt + \exp[0.125 \nu_t]dW_{p_t}, \\
    d\nu_t &= -0.1\nu_t dt + dW_{\nu_t}, \quad \text{corr}(dW_{p_t}, dW_{\nu_t}) = -0.62
\end{align*}
\]

(20)

where \( p_t \) is the log-price process, the \( W \)'s are standard Brownian motions, \( \nu_t \) the volatility factor. This is the process that we simulate under the null hypothesis of no jumps.

Under the alternative, we add rare compound Poisson jumps, arriving with intensity \( \lambda = 0.5 \) and having normally distributed sizes with mean 0 and standard deviation \( \sigma_{\text{jump}} = 1.5 \).

To the simulated stochastic volatility plus jump model, we add i.i.d. microstructure noise normally distributed with mean 0 and \( \sigma_{\text{noise}} = 0.04 \).

Here, we report results only for the difference BNS statistic (see 7), but results for the ratio statistic will also be included in future extensions to this work.

Table 1 reports the size and power of the i.i.d. bootstrapped statistic in comparison to the asymptotic one. The nominal significance level is 5%.

The bootstrap version of the test displays a higher power than the asymptotic version, but is also more oversized.

Table 2 reports the results from using the test averaging procedure proposed by us, as well as the asymptotic benchmark for comparison purposes. The nominal significance level is 5%.

The test combinations manage to outperform the asymptotic results in almost all cases. This is because in the presence of i.i.d. microstructure noise, the BNS asymptotic test becomes severely undersized at high frequencies. However, at lower frequencies, where size is getting close to the nominal
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<th>5 min</th>
<th>10 min</th>
<th>15 min</th>
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<td>Size</td>
<td>Power</td>
<td>Size</td>
<td>Power</td>
<td>Size</td>
</tr>
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Our procedure, by combining higher with lower frequencies, manages to maintain a high power, combined with a manageable size. For instance, combining tests applied on data sampled every 2 and 5 minutes renders a size very close to the nominal one (6%), coupled with a very high power (78%). This power is higher than the one obtained for the asymptotic test at either 2 or 5 minutes. Moreover, it is higher than the power obtained for all frequencies for the asymptotic test.

Further developments of this paper include applying our procedure to infrequent trading data, in order to assess the impact of infrequent trading on jump detection.

5 Conclusion

This paper proposes a new procedure to detect jumps based on high frequency data that uses Fisher (1932)’s method to average p-values from tests applied at different sampling frequencies. The procedure is probably the first contribution in the high frequency econometrics literature that manages to rigorously detect jumps when prices are contaminated with microstructure noise. Moreover, relying on more than one time scales in detecting jumps is more efficient, as we discard less data, and in line with the corresponding
Table 2: Size and power from averaging p-values over frequencies. The BNS difference statistic

<table>
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<tr>
<th>Freq</th>
<th>'Size'</th>
<th>'Power'</th>
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<tbody>
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<td>'1-2'</td>
<td>0.019</td>
<td>0.783</td>
</tr>
<tr>
<td>'1-5'</td>
<td>0.028</td>
<td>0.761</td>
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<td>'1-10'</td>
<td>0.032</td>
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<td>0.677</td>
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<tr>
<td>'15 min'</td>
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literature on volatility estimation in the presence of microstructure noise.

In applying the procedure, we make use of Barndorff-Nielsen and Shephard (2006)’s nonparametric test for jumps. To control the overall size of the test, we apply a double bootstrap procedure. A by-product of this paper is the proof of consistency of the i.i.d. bootstrap for the Barndorff-Nielsen and Shephard (2006) test statistic. We prove the effectiveness of our procedure in a Monte Carlo simulation study. We show that averaging p-values of tests applied at different frequencies outperforms the simple asymptotic test, by delivering a higher power, combined with a manageable size.

We plan to extend the results in this paper in various ways. First, we will also include the Wild bootstrap method to account for varying intraday volatility. Second, we will extend the simulation setup to various types of microstructure noise, including infrequent trading. Third, we plan to consider other jump detection procedures in addition to the classic Barndorff-Nielsen and Shephard (2006) test.

References


