Rational Exuberance

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"Clearly, sustained low inflation implies less uncertainty about the future, and lower risk premiums imply higher prices of stocks and other earning assets. We can see that in the inverse relationship exhibited by price/earnings ratios and the rate of inflation in the past. But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade?"

[Alan Greenspan, 5 December 1996]
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  2. Exogenous coordination needed
  3. Stable sunspot exist with endogenous variables overreacting to expectations
  4. They are typically not learnable (they could provided a certain representation is adopted, but anyway never for a positive expectational feedback)
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1. based on imperfect knowledge of the fundamental (agents miss relevant information)...potentially open to learning
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3. exogenous correlation needed on individual signals
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They are learnable.
Consider a Lucas type aggregate supply:

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A Laboratory model

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where \( \beta < 1 \) and \( \alpha_{t-1} = \pi^*_t / (1 - \beta) \) if the objective of the PM is

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Two conjectures

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- **Evolution**: with the second specification every out-of-the-equilibrium path under adaptive learning dynamics is fated to diverge.

- *The two conjectures are wrong when agents have noisy perceptions of others’ expectations.*
A "sunspot-like" forecasting problem

Suppose the individual information set \( \Omega^i_{t-1} \equiv (\Omega_{t-1}, E_{t-1}y_t + \nu_{i,t-1}) \)
where \( \nu_{i,t-1} = \varepsilon_{t-1} + \eta_{i,t-1} \).
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- Agents respond to their incentive to refine their forecasts

$$E^i_{t-1} \Delta \pi_t = b (E_{t-1} \Delta \pi_t + \nu_{i,t-1})$$
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- A REX exists if $E^i_{t-1} \pi_t = E[\pi_t | \Omega^i_{t-1}]$ and $b \neq 0$. 
Two heterogeneous information REX exist in correspondence of

\[ b_{\pm} = 1 \pm \sqrt{\frac{\beta - 1}{\delta}} \]

whenever \( \beta > 1 \) with \( \delta = \delta_i / \delta_\varepsilon \) labeling the ratio between the variances of the individual and the common part of the expectational signals.
Figure 2. T-map obtained for different values of $\beta$. 
Figure: Figure 3. OLS convergence to the fundamental REE and to the high REX with $\alpha_t = 0$, $\beta = 1.1$, $(\delta_\varepsilon + \delta_i) = \delta_\eta = 1$. 
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- But **differently** from classical sunspots:
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  2. REX exist without forward expectations
  3. REX are learneable provided agents know the target
Similarly to classical Sunspot equilibria
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1. Correlation across all individual signals is exogenously imposed
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The paper shows how to go beyond these two points, obtaining

1. endogenous correlation across all individuals as an equilibrium property → informational islands
2. REX may exist also with $\beta < 1$ → target imperfectly announced
Consider agents’ structural forecasting relations

\[
\begin{align*}
E_{t-1}^1 \Delta \pi_t &= b \left( E_{t-1}^2 \Delta \pi_t + \eta_{1,t-1} \right), \\
E_{t-1}^2 \Delta \pi_t &= c \left( E_{t-1}^1 \Delta \pi_t + \eta_{2,t-1} \right).
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where 1 and 2 label two symmetrical informational islands.
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Covariance between the signals is *endogenous* to agents’ forecasting rule

\[ \mathbb{E} \left( E_{t-1}^{2} \Delta \pi_t + \eta_{1,t-1}, E_{t-1}^{1} \Delta \pi_t + \eta_{2,t-1} \right) = \frac{b + c}{(1 - bc)^2} \delta_t \]

where, we assumed \( \mathbb{E} \left( \eta_{l,j,t-1} \eta_{j,l,t-1} \right) = 0. \)
Recursive OLS learning

Figure: Figure 5. OLS convergence to the fundamental REE and to the high REX with $\alpha_t = 0$, $\beta = 1.1$, $\delta_\varepsilon = \delta_\eta = 1$. 
Agents forecasting rule is

\[
E_{t-1}^i \pi_t = a(\alpha_{t-1} + \kappa_{t-1}) + b(E_{t-1} \pi_t - a(\alpha_{t-1} + \kappa_{t-1}) + \nu_{i,t-1})
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where \(\kappa_{t-1}\) is a noise in the public announcement possibly correlated with \(\varepsilon_{t-1}\).
Imperfect announcements of inflation targets

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- Notice now the fundamental REE cannot be any longer an equilibrium if the noises are correlated.
Figure 7. $T(b)$-map obtained fixing $\delta_\epsilon, \delta_k$ and $\delta$ and for different values of $\beta, \alpha$ and $\rho$. 
Numerical analysis
Recursive OLS learning

a) Convergence to low REX

b) Convergence to high REX
Conclusions

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