Equity Importance Modeling With Financial Network and Betweenness Centrality
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Summary

Motivation:
Financial network has been widely studied in terms of the structural information distribution, clustering phenomenon, dynamic diffusion, etc. However, rare work has been carried out on equity importance/impact modeling. In [1], the author consider degree of a vertex to be a measurement of stock’s importance. However, we found that betweenness centrality, a static measurement of the vertex’s impact in a graph, can be a better representative of the importance of an equity in a financial network.

Framework:
• Calculate correlation coefficients among the returns of all equities.
• Generate a financial network of the equities.
• Calculate the betweenness centrality of each vertex.
• Compare the result with the benchmark of the equities’ importance, market-cap.

Results:
We build a financial network using 473 stocks out of the SP-500 pool, during a one year period from June, 2008 to June, 2009. We found that the average market-cap of the stocks with 20 largest betweenness values is 50.5 billion dollars, more than twice of the SP-500’s average market-cap. The market-cap-betweenness plots also shows an upward tendency, meaning that betweenness values are positively correlated with the market-caps. It shows that betweenness centrality in the financial network context can be a good measurement of equities’ importance.


Betweenness Centrality

Degree:
The Degree of a vertex $v$ in a financial network is defined as the number of vertices connected to it. For example, in the right graph of Fig. 1, vertex $a$ has two vertices $b$ and $c$ connected to it, so the degree of $a$ is 2.

Shortest path:
A shortest path between a pair of vertices $u$ and $v$ is the path that the sum of its constituent edges is minimized. For example, in the right graph of Fig. 1, $(b, d), (b, e, d)$ and $(b, a, c, e, d)$ are all paths from vertex $b$ to $d$. However, since the length of $(b, d)$ is 1, which is the minimum among all three paths, it is the shortest path.

Betweenness Centrality:
The betweenness centrality $B_v(v)$ of a vertex $v$ is calculated in the following steps:
• Calculate $\sigma(s, t|v)$, the number of shortest path connecting any pairs of vertices $s$ and $t$, which passes $v$.
• Divide $\sigma(s, t|v)$ by $\sigma(s, t)$, the number of shortest path between $s$ and $t$.
• Sum up the quantities obtain $B_v(v) = \sum_{s \neq t} \frac{\sigma(s, t|v)}{\sigma(s, t)}$ for all pairs of vertices $v$.

The following equation also shows the above steps:

- Betweenness centrality measures the level to which a vertex is needed by others along shortest path, and can be considered an indicator of the importance of a vertex.
- We use the known fastest Brandes algorithm to calculate the betweenness centrality of each vertex.

Financial Network Model

Correlation coefficient among stocks:
The correlation coefficient is calculated based on the time series of the return of a stock. Considering a price series of stock $i$ within a period $T$, $P(i) = \{P(i, 1), P(i, 2), ..., P(i, T)\}$. The correlation coefficient of stock $i$ and $j$ is written as following:
$${\rho}_{ij} = \frac{\sum_{t=1}^{T} (P(i, t) - \mu_i)(P(j, t) - \mu_j)}{\sqrt{\sum_{t=1}^{T} (P(i, t) - \mu_i)^2 \sum_{t=1}^{T} (P(j, t) - \mu_j)^2}}$$

Financial network:
A natural financial network is that, for every pair of stocks, we assign an edge between them, and let the weight of the edge be the correlation coefficient between them, as shown as the left graph in the following plot:

![Figure 1: Example of financial network after removing the edges with the correlation coefficient less than 0.6.](image1)

We take two steps to simplify the graph:
• Remove the edges with weights smaller than a predefined threshold;
• Let all the weights of the remaining edges to be 1.

Experiments

Data:
The financial network is generated using 473 stocks out of the SP-500 pool, during a one year period from June, 2008 to June, 2009. Threshold is set to be 0.8.

Distributions:
Fig. 2 and Fig. 3 shows the distribution of the degree and betweenness centrality of the financial network, respectively. It shows that most of the vertices have relative low degree and betweenness, and both distributions present power-law.

Comparing with stocks’ market-cap:
It is found that the average market-cap of the 20 stocks with largest degree is 29.0 billion dollars, slightly larger than that of the SP-500 stocks, which is 23.5 billion dollars. However, the average market-cap of the stocks with 20 largest betweenness is 50.5 billion dollars, more than twice of the SP-500’s average market-cap. Fig. 5 is the plot for betweenness VS. market cap, which shows an upward tendency, meaning that betweenness values are positive correlated with the market-caps of the stocks. Fig. 4 is the plot for degree VS. market cap, which doesn’t show such phenomenon.

![Figure 2: Degree distribution](image2)

![Figure 3: Betweenness distribution](image3)

![Figure 4: Degree VS. Market-cap](image4)

![Figure 5: Betweenness VS. Market-cap](image5)