To Scrap or Not to Scrap: A Dynamic Discrete Choice Model of Vehicle Scrappage

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Fluctuating Gasoline Prices
Goal

Construct a dynamic stochastic discrete choice model to

- Examine what factors determine vehicle scrappage, new vehicle sales, and fuel efficiency of new vehicles;
- Study the determination of the cross-sectional distribution of age and fuel efficiency of existing vehicles;
- Evaluate the effectiveness of scrappage subsidy policies in term of boosting vehicle sales and reducing gasoline consumption.
Relation to the Literature

- Durable Goods Purchases
  Adda and Cooper (2000, 2007)

- Gasoline Use and Vehicle Fuel Efficiency
  Wei (2010)
  Li, Timmins, and Von Haefen (2009)
Gas Price versus Fuel Efficiency of New Vehicles
Income, Vehicle Prices, Gasoline Prices and New Vehicle Sales
Number of Vehicles by Age
Average MPG by Age
The Model

- A dynamic stochastic discrete choice model (Rust 1987, Adda and Cooper 2000, 2007)

**Assumptions:**
- One vehicle per household at all times;
- No second-hand market for vehicles;
- New vehicle purchases paid out of current income.
The Household’s Problem

\[ V(n, x, z, S) = \max \left\{ V^k(n, x, S) + \alpha + z_k, V^r(n, x, S) + z_r \right\} \]

State Variables:

- **S**: household income, \( Y \); average prices of new vehicles, \( P \); gasoline prices, \( O \); aggregate taste shock, \( \xi \).
- **z**: a pair of household-specific taste shocks with regard to keeping or scrapping the vehicle (i.i.d. extreme value distribution)
- **n**: vehicle age
- **x**: vehicle fuel efficiency
If Keeping the Vehicle,

\[ V^k(n, x, S) = u(n, c_k) + \beta (1 - \delta) E_{z', s'|z, s} V(n + 1, x, z', S') \]
\[ + \beta \delta E_{z', s'|z, s} V^a(z', S') , \]

\[ c_k = Y - b \left( \frac{O}{x} \right)^{\eta+1} \]

- The elasticity of vehicle miles of travel to gasoline cost per mile \((O/x)\) is assumed to be \(\eta\).
- Each vehicle has \(\delta\) probability of being destroyed by accidents at the end of a period.
If Scrapping the Vehicle:

\[ V^r(n, x, S) = \max_{x' \in X} \left\{ u(1, c_r) + \beta (1 - \delta) E_{(z', s' | z, s)} V(2, x', z', S') + \beta \delta E_{(z', s' | z, s)} V^a(z', S') \right\} \]

\[ c_r = Y - P_{x'} - b \left( \frac{Q}{x'} \right)^{\eta+1} + \pi(n, x, x') \]

\[ P_{x,t} = P_t \left( \frac{x}{x} \right)^\theta \]
If Vehicle Destroyed by Accident

\[ V^\alpha(z, S) = \max_{x' \in X} \left\{ u(1, c_\alpha) + \beta (1 - \delta) E_{(z', s'|z, s)} V^\alpha(2, x', z', S') \right\} \]

\[ + \beta \delta E_{(z', s'|z, s)} V^\alpha(z', S') \right\} \]

\[ c_\alpha = Y - P_{x'} - b \left( \frac{O}{x'} \right)^{\eta+1} + \pi_\alpha \]
The Utility Function

\[ U(n, x, c) = -\exp\left[-\exp\left(\varphi_0 + \varphi_1 n\right)\right] + \xi \frac{c^{1-\gamma}}{1-\gamma}, \varphi_1 < 0. \]

The first term: auto service which declines with age

c: nondurable goods consumption

\( \xi \): aggregate taste shock
Optimal Fuel Efficiency
Decisions - I

- Marginal Cost:
  - Marginal increase in vehicle purchase prices;
  - Marginal increase in scrappage subsidies which reduces the cost.

- Marginal Benefit:
  - Marginal increase in present discounted value of current and future gasoline savings.
Optimal Fuel Efficiency Decisions - II

\[
U_c (1, c_{r,t}) \left( \frac{\partial P_{x,t}}{\partial x'} - \frac{\partial \pi (n, x, x')}{\partial x'} \right)
= U_c (1, c_{r,t}) b (\eta + 1) \left( \frac{O_t}{x'} \right)^{\eta + 1} \frac{1}{x'}
\]

\[
+ \sum_{j=2}^{N} E_t \left\{ [\beta (1 - \delta)]^{j-1} \Pi_{m=2}^{j} \left[ \mu (m, x', S_{t+m-1}) \right] U_c (j, c_{k,t+j-1}) b (\eta + 1) \left( \frac{O_{t+j-1}}{x'} \right)^{\eta + 1} \frac{1}{x'} \right\}
\]

\[
+ \sum_{j=2}^{N} E_t \left\{ [\beta (1 - \delta)]^{j-1} \Pi_{m=3}^{j} \left[ \mu (m, x', S_{t+m-1}) \right] \left[ 1 - \mu (j, x', S_{t+j-1}) \right] \times U_c (j, c_{r,t+j-1}) \frac{\partial \pi (j, x', x'')}{\partial x'} \right\}.
\]
Stochastic Discount Factor

- Stochastic discount factor is affected by aggregate taste shocks and nondurable goods consumption.
- If low household income often coincides with high gasoline prices, gasoline savings from high fuel efficiency vehicles are even more valuable.
- Scrappage subsidies are valued more at times of low household income.
Optimal Scrappage Decisions

\[ \mu(n, x, S) = \frac{\exp[V^k(n, x, S) + \alpha]}{\exp[V^k(n, x, S) + \alpha] + \exp[V^r(n, x, S)]} \]

- A decision on whether to scrap the vehicle this period or wait till next period to decide (option value of waiting)
- The scrappage decision depends upon aggregate state variables, and the age and fuel efficiency of the vehicle under consideration.
Aggregate Implications

- Given
  - the initial cross-sectional distribution of vehicles by age and fuel efficiency
  - the scrappage rates of vehicles by age and fuel efficiency,

- the model can generate
  - new vehicle sales
  - average fuel efficiency of new vehicles
  - cross-sectional distribution of vehicles by age and fuel efficiency over time.
Calibration and Specification - I

\[
\begin{bmatrix}
Y_t \\
O_t \\
P_t \\
\xi_t
\end{bmatrix} = \begin{bmatrix}
\mu_Y \\
\mu_o \\
\mu_p \\
\mu_\xi
\end{bmatrix} + \begin{bmatrix}
\rho_{yy} & \rho_{yo} & \rho_{yp} \\
\rho_{oy} & \rho_{oo} & \rho_{op} \\
\rho_{py} & \rho_{po} & \rho_{pp} \\
\rho_{\xi y} & \rho_{\xi o} & \rho_{\xi p}
\end{bmatrix} \begin{bmatrix}
Y_{t-1} \\
O_{t-1} \\
P_{t-1}
\end{bmatrix} + \begin{bmatrix}
e_{Y_t} \\
e_{ot} \\
e_{pt} \\
e_{\xi t}
\end{bmatrix}
\]

\[
\Omega = \begin{bmatrix}
\omega_{yy} & \omega_{yo} & \omega_{yp} & 0 \\
\omega_{oy} & \omega_{oo} & \omega_{op} & 0 \\
\omega_{py} & \omega_{po} & \omega_{pp} & 0 \\
0 & 0 & 0 & \omega_{\xi \xi}
\end{bmatrix}
\]
Calibration and Specification - II

Subjective time preference $\beta$ 0.97
Relative risk aversion $\gamma$ 1.8
Probability of accidents $\delta$ 0.01
Maximum age of vehicles $N$ 20
Elasticity of mileage to cost $\eta$ 0.022
Constant in mileage regression $b$ 1.2
Price elasticity of fuel efficiency $\theta$ 0.128
Constant utility gain from keeping the vehicle $\alpha$ 4
Age-related utility decline ($n<10$) $[\varphi_0, \varphi_1]$ [1.65, −0.05]
Age-related utility decline ($n>10$) $[\varphi_0, \varphi_1]$ [1.65, −0.02]
Scrap value if scrapped $\overline{\pi_r}$ 1000
Scrap value if destroyed by accident $\overline{\pi_a}$ 500
## Quantitative Analysis

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<th></th>
<th>Aggregate</th>
<th>Gasoline Price</th>
<th>Vehicle Price</th>
<th>Household Income</th>
<th>Taste Shock</th>
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<tr>
<td>No. of Grids Range</td>
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<td>Individual No. of Grids</td>
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<td>Vehicle age</td>
<td>Fuel Efficiency</td>
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<tr>
<td>Range</td>
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<td>[15, 40]</td>
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Age-Specific Survival Rates
Sensitivity of Scrappage Rates
Choices of Fuel Efficiency of New Vehicles
Ongoing Work

- Data Matching and Estimation
- Policy Experiments