Forecasting with DSGE Models

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*The views expressed here do not necessarily reflect those of the European Central Bank.
Introduction: Background

*Structural econometric forecasting, because it is based on explicit theory, rises and falls with the theory, typically with a lag.*
*Francis X. Diebold (1998, p. 175).*

- Recent success of estimating medium-size dynamic stochastic general equilibrium (DSGE) models:

- Bayesian methods help keep models manageable even though scope and coverage of sectors/variables have grown.

- Increased use of DSGE models for policy analysis and in the forecasting environment of policy institutions.
Introduction: Forecasting Evidence

- Results in Smets and Wouters (2004) suggest that closed-economy DSGE models compare well with VAR and BVAR models.

- See also Rubaszek and Skrzypczyński (2008) and Edge, Kiley and Laforte (2010) for studies using real time data; Wang (2009) for a forecast comparison with Stock and Watson (2002a,b) type factor models.

- Adolfson, Lindé and Villani (2007) show that open-economy DSGE models can also compete with reduced-form forecasting models. See also Lees, Matheson and Smith (2010).
Introduction: Objectives

1. Review methodology of forecasting with log-linearised DSGE models using Bayesian methods:
   - estimation of the predictive distribution through simulation methods (cf. Adolfson, Lindé and Villani, 2007),
   - out-of-sample evaluation of point and density forecasts.

2. Illustrate methodology by applying it to the New Area-Wide Model (NAWM; Christoffel, Coenen and Warne, 2008):
   - micro-founded small open-economy model of the euro area for forecasting and policy analysis.
   - designed for use in the ECB/Eurosystem staff projection exercises, and estimated on 18 key projection variables.

3. Consider alternative benchmark forecasting models.
Outline

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Predictive Distribution

Evaluating Forecast Accuracy
  Point Forecasts
  Density Forecasts

Conclusions

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State-Space Representation

- The solution (reduced-form) of a log-linearised DSGE model can be represented by:

\[ \xi_t = F\xi_{t-1} + B\eta_t, \quad t = 1, \ldots, T, \]

where \( F \) and \( B \) are uniquely determined by the structural parameters \( \theta \), while \( \eta_t \sim \text{iid} N(0, I_q) \).

- The observed variables, \( y_t \), are linked with the state variables, \( \xi_t \), through the measurement equation:

\[ y_t = A'x_t + H'\xi_t + w_t, \quad t = 1, \ldots, T, \]

where \( x_t \) is deterministic, \( w_t \sim \text{iid} N(0, R) \) are measurement errors, and the matrices \( A, H, \) and \( R \) are uniquely determined by \( \theta \).
The Predictive Density

- The predictive density of $y_{T+1}, \ldots, y_{T+H}$ can be expressed as

$$p(y_{T+1}, \ldots, y_{T+H}|\mathcal{Y}_T) = \int p(y_{T+1}, \ldots, y_{T+H}|\mathcal{Y}_T, \theta)p(\theta|\mathcal{Y}_T)d\theta,$$

where $p(\theta|\mathcal{Y}_T)$ is the posterior density of $\theta$ based on $\mathcal{Y}_T$.

- If we wish to estimate quantiles, confidence regions or the probability that the variables reach some barrier, then we need a numerical algorithm since the above integral cannot be solved analytically.

- If the forecast evaluation only requires moments from the predictive distribution, such an algorithm is not needed.
The forecasting performance of the NAWM is assessed relative to a number of reference models:

1. Small VAR model: 7 variables.
3. Large BVARs with shrinkage (Bańbura, Giannone and Reichlin, 2010): 18 variables.
4. Random walk and sample mean.

Another interesting reference model would be a DSGE-VAR (Del Negro and Schorfheide, 2004, 2006; Del Negro, Schorfheide, Smets and Wouters 2007), or some other BVAR whose prior is based on the DSGE model.
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Rolling Forecast Evaluation Scheme


- 1 through 8-quarters ahead forecasts are computed; this yields 32 1-quarter ahead forecast periods and 25 8-quarter ahead forecast periods.

- Models estimated with Bayesian methods are re-estimated annually, while the other models are re-estimated every period.

- We forecast both quarterly and annual changes of variables that appear in first differences.
  - Point forecast: univariate and multivariate MSE-based measures.
  - Density forecasts: log predictive score.
Point Forecasts: RMSEs – Some Observations

- NAWM fares quite well, especially when forecasting quarterly changes of variables in first differences.

- Compares favourably for: real GDP, real exports and imports, import price deflator, employment, short-term nominal interest rate, and real exchange rate.

- Has problems with: nominal wages, consumption deflator, real consumption.
Log-determinant Statistics: Quarterly Changes

12 variables

7 variables

3 variables

- Mean
- Random–walk
- Small VAR
- Small BVAR
- Large BVAR – mixed prior
- Large BVAR – white–noise prior
- DSGE
Density Forecasts: Background

- Forecasts are both probabilistic and sequential ("prequential"), taking the form of probability distributions over a sequence of future values (cf. Dawid, 1984).

- Forecast uncertainty has since been given an increasingly important role:

- The use of uncertainty bands in inflation reports of several central banks have become critical to forecast analyses.
Log Predictive Score: Quarterly Changes

12 variables

7 variables

3 variables

Mean
Random–walk
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Conclusions

- Results support earlier studies of forecasting ability of the new generation of DSGE models.
  - The NAWM compares favourably when forecasting real GDP growth, the trade variables, employment, the nominal interest rate, and the real effective exchange rate.
  - However, the model is less successful when forecasting nominal wage growth, consumption deflator inflation (real wages), and real consumption growth.
- Mean forecast errors explain a large fraction of the MSEs for the NAWM, but we should not forget that misspecified models can have good forecasting properties.
- The large BVAR with a mixed prior seems to have the overall best forecasting performance.
Appendix I: The New Area-Wide Model
NAWM: Agents

- There are four types of agents:
  
  1. **Households**: consume, accumulate physical capital, supply differentiated labour services, set wages in monopolistically competitive markets, trade in domestic and foreign bonds.
  
  2. **Firms**: produce either tradable intermediate goods or non-tradable final goods (further details provided below).
  
  3. **Fiscal authority**: purchases public consumption goods, issues bonds, levies distortionary as well as lump-sum taxes, the latter closing the budget constraint.
  
  4. **Monetary authority**: sets the short-term nominal interest rate by following a Taylor-type interest-rate rule.

- Households and firms make optimal choices and form expectations in a forward-looking manner.
NAWM: Firms

There are four types of firms:

1. **Domestic intermediate-good firms**: use labour and capital services as inputs, produce tradable differentiated goods, set prices in *producer* currency in monopolistically competitive markets at home and abroad.

2. **Final-good firms**: combine domestic and foreign intermediate goods into three non-tradable goods: a private consumption good, a private investment good, a public consumption good.

3. **Foreign intermediate-good firms**: sell differentiated goods in domestic markets, set prices in *local* currency in monopolistically competitive markets.

4. **Foreign retailer**: combines the exported domestic intermediate goods.
The model contains a relatively large number of frictions to improve its empirical fit:

- External habit formation in consumption.
- Generalised adjustment cost in investment, imports and exports.
- Fixed cost in intermediate-good production.
- Monopolistic competition in intermediate-good and labour markets.
- Sticky prices and wages à la Calvo, with dynamic indexation.

Similarly, the model contains a set of 18 shocks (ensuring that the 1-step ahead covariance matrix in the likelihood function is non-singular).
Appendix II: The Predictive Density
The predictive density can be obtained by simulation (see Adolfson, Lindé and Villani, 2007):

1. Draw $\theta$ from $p(\theta|Y_T)$.
2. Draw the state variables at time $T$ from $\xi_T \sim N(\xi_T|T, P_T|T)$.
3. Simulate a path for the state variables via the state equation using the drawn value for $\xi_T$ as initial value and a sequence of structural shocks $\eta_{T+1}, \ldots, \eta_{T+H}$ drawn from $N(0, I_q)$.
4. Draw a sequence of measurement errors $w_{T+1}, \ldots, w_{T+H}$ from $N(0, R)$ and compute the path for the observed variables $y_{T+1}, \ldots, y_{T+H}$ using the measurement equation.
5. Repeat steps 2-4 $M_1$ times for the same $\theta$.
6. Repeat steps 1-5 $M_2$ times.
To estimate the mean of the predictive distribution we may proceed as follows:

$$E[y_{T+h}|Y_T] \approx \frac{1}{M} \sum_{i=1}^{M} E[y_{T+h}|Y_T, \theta^{(i)}],$$

where $\theta^{(i)}$ is drawn from $p(\theta|Y_T)$, and

$$E[y_{T+h}|Y_T, \theta] = A'x_{T+h} + H'F^h\xi_T|T.$$
The covariance matrix of $y_{T+h}$ conditional on $\mathcal{Y}_T$ and $\theta$ is:

$$
C[y_{T+h}|\mathcal{Y}_T, \theta] = H'F^hP_{T|T}(F^h)'H
$$

$$
+ H' \left( \sum_{j=1}^{h} F^{j-1}BB'(F^{j-1})' \right) H + R.
$$

The first term represents state-variable uncertainty conditional on $\theta$, the second uncertainty due to the structural shocks, and the third measurement error uncertainty.
The $h$-steps ahead prediction covariance matrix is now:

\[
C[y_{T+h}|\mathcal{Y}_T] = E_T[C[y_{T+h}|\mathcal{Y}_T, \theta]] + C_T[E[y_{T+h}|\mathcal{Y}_T, \theta]],
\]

where $E_T$ and $C_T$ denote the expectation and covariance wrt the posterior of $\theta$.

The first term on the rhs includes the share of the forecast uncertainty due to the unobserved state variables, shock uncertainty, and measurement error uncertainty once the dependence on $\theta$ has been integrated out; the second term reflects parameter uncertainty.
The predictive density makes it possible to take forecast uncertainty into account:

\[ p(y_{T+1}, \ldots, y_{T+H} | \mathcal{Y}_T, m) = \frac{p(\mathcal{Y}_{T+H} | m)}{p(\mathcal{Y}_T | m)} \]

with the height of the predictive density being equal to the improvement in the marginal likelihood after having observed \( y_{T+1}, \ldots, y_{T+H} \).

The log predictive (density) score function is:

\[ S(m) = \frac{1}{N_H} \sum_{t=T}^{T+N_H-1} \log p(y_{t+1}, \ldots, y_{t+H} | \mathcal{Y}_t, m) \]

and depends only on the data and the predictive density.
We focus on the $h$-steps ahead forecasts, i.e., on log predictive scores for the marginal densities:

$$S_h(m) = \frac{1}{N_h} \sum_{t=T}^{T+N_h-1} \log p(y_{t+h}|Y_t, m).$$

The relationship between marginal likelihood and the log score holds for $h = 1$ but not for $h > 1$.

Computing $S_h(m)$ for $h > 1$ is difficult since $p(y_{t+h}|Y_t, m)$ does not have a closed form expression.

We follow Adolfson, Lindé and Villani (2007) and approximate the predictive density with a multivariate normal.
Appendix IV: The Data
We utilise data on 18 key macroeconomic times series:

- Real GDP
- Private consumption
- Government consumption
- Total investment
- Extra-euro area exports
- Extra-euro area imports
- GDP deflator
- Consumption deflator
- Import deflator
- Employment
- Nominal wages
- Nominal interest rate
- Nominal effective exchange rate
- Competitors’ export prices
- Foreign demand
- Foreign GDP deflator
- Foreign nominal interest rate
- Oil price

covering the period 1985Q1-2006Q4 for the euro area (using 1980Q2-1984Q4 as training sample).

The series with a dagger (‘†’) are modelled using a structural VAR, while government consumption is modelled as an AR(2).
The Data – 2 (3)
Appendix V: Mean Predictions
Point Forecasts: Quarterly Real GDP Growth

Random-walk

Mean

Large BVAR – white-noise prior

Large BVAR – mixed prior

DSGE

Small BVAR

Small VAR
Point Forecasts: Quarterly Nominal Wage Growth

Random-walk

Mean

Large BVAR – white-noise prior

Large BVAR – mixed prior

DSGE

Small BVAR

Small VAR
Point Forecasts: Quarterly Consumption Deflator Inflation

Random–walk

Mean

Large BVAR – white–noise prior

Large BVAR – mixed prior

DSGE
## Point Forecasts: Mean Predictions

Percentage share for squared mean errors of mean squared errors.

<table>
<thead>
<tr>
<th>Variable</th>
<th>NAWM</th>
<th>BVAR - mixed prior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 1$</td>
<td>$h = 4$</td>
</tr>
<tr>
<td>Real GDP</td>
<td>0.0</td>
<td>28.9</td>
</tr>
<tr>
<td>Private consumption</td>
<td>12.6</td>
<td>62.9</td>
</tr>
<tr>
<td>Total investment</td>
<td>39.1</td>
<td>34.1</td>
</tr>
<tr>
<td>Exports</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Imports</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>GDP deflator</td>
<td>8.3</td>
<td>43.6</td>
</tr>
<tr>
<td>Consumption deflator</td>
<td>21.9</td>
<td>65.1</td>
</tr>
<tr>
<td>Import deflator</td>
<td>1.8</td>
<td>3.8</td>
</tr>
<tr>
<td>Employment</td>
<td>7.5</td>
<td>28.4</td>
</tr>
<tr>
<td>Nominal wages</td>
<td>45.1</td>
<td>79.2</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>25.7</td>
<td>2.3</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>1.2</td>
<td>5.5</td>
</tr>
</tbody>
</table>
The NAWM overestimates nominal wage growth and underestimates consumption deflator inflation; it therefore also overestimates real wage growth.

Quarterly real wage growth is 0.3 percent in the steady state of the NAWM, while the forecast sample mean is 0.08 percent ("wage moderation").

Consumers consistently overestimate their expected real income, and thereby they overestimate consumption.