A Unified Solution to Inventory Puzzles

Louis J. Maccini
Johns Hopkins University

Bartholomew Moore
Fordham University

Huntley Schaller
Carleton University
**Inventories are Important**

The fall in inventory investment accounts for 80% of drop in Y in post-war US recessions [Blinder-Maccini, 1991]. Large % in most G7 countries [Ramey-West, 1999].

**Several Puzzles Remain**

- **Traditional Puzzles**
  - Variance Ratio Puzzle
  - Wen (2005) Puzzle
  - Slow-Adjustment Puzzle
  - Input Cost Puzzle

- **Monetary Policy Puzzles**
  - Sign Puzzle
  - Mechanism Puzzle
  - Timing Puzzle

**A Unified Explanation**

Model incorporates several economic mechanisms but is tractable enough to be linearized. Use non-stationary econometrics to abstract from the high-frequency noise in the data. Calibrate structural parameters using the cointegrating regression. Use calibrated structural parameters in decision rule to provide explanation of the puzzles.
Traditional Puzzles

**Variance Ratio Puzzle:** If production costs are convex, then firms should smooth production in response to demand shocks. Production should vary less than sales. It doesn't.

**Wen (2005) Puzzle:** At medium horizons (8-40 quarters), production varies more than sales. Surprisingly, at short horizons (less than three quarters) he finds that production is less volatile than sales.

**Slow-Adjustment Puzzle:** Estimated adjustment speeds are extremely low, often less than 10 percent per month. "Theory strains to explain low adjustment speeds unless the incentive to smooth production is extremely strong, which is hard to reconcile with the fact that production is more variable than sales." Blinder and Maccini (1991)

**Input cost puzzle:** When costs are low, firms have an incentive to produce more and build up their inventories. It has been difficult, however, to find statistically significant relationship between observable costs and inventories.
The Model (briefly):

\( N_t = \) Inventories, \( Y_t = \) Output, \( X_t = \) Sales, \( W_t = \) Real Input Prices,

\( \beta_t = \frac{1}{1 + r_t} = \) Real Discount Factor, \( r_t = \) Real Interest Rate.

The firm minimizes

\[
E_0 \sum_{t=0}^{\infty} \left[ \prod_{j=0}^{t} \beta_j \right] C_t,
\]

\[
C_t = Y_t^{\theta_1} W_t^{\theta_2} + \delta_1 \left( \frac{N_{t-1}}{X_t} \right)^{\delta_2} X_t + \delta_3 N_{t-1}
\]

s.t. \( N_t - N_{t-1} = Y_t - X_t. \)

\( \theta_1 > 1 \) governs the convexity of production costs
\( \theta_2 > 0 \) Input costs
\( \delta_2 < 0 \) governs the convexity of stockout avoidance costs
\( \delta_1 > 0 \) stockout avoidance costs, \( \delta_3 > 0 \) Inventory Holding Costs
We derive the cointegrating regression from the Euler equation:

The log-linear Euler can be written as

\[
E_{t-1} \left\{ \chi_t + \bar{\beta} (\delta_2 - 1) \delta_2 \psi \left[ \ln N_t - b_X \ln X_t - b_W \ln W_t - b_{\pi_1} \pi_{1,t-1} - b_{\pi_3} \pi_{3,t-1} \right] \right\} = 0
\]

(23)

where \( \chi_t \) is stationary, \( \pi_1 = \text{Prob\{low-interest-rate state\}} \), and \( \pi_3 = \text{Prob\{high-interest-rate state\}} \).

\( \ln N_t, \ln X_t, \ln W_t, \pi_{1,t-1} \) and \( \pi_{3,t-1} \) are I(1). Eqn (23) implies that they are cointegrated, with cointegrating regression

\[
\ln N_t = b_0 + b_X \ln X_t + b_W W_t + b_{\pi_1} \pi_{1,t-1} + b_{\pi_3} \pi_{3,t-1} + \nu_t.
\]

\[
b_X = 1 - \frac{\bar{r} (\theta_1 - 1) \theta_1 \bar{J}}{(\delta_2 - 1) \delta_2 \psi} > 0
\]

\[
b_W = -\frac{\bar{r} \theta_2 \theta_1 \bar{J}}{(\delta_2 - 1) \delta_2 \psi} < 0
\]

\[
b_{\pi_1} = -(\gamma_1 - \gamma_2) \left( \frac{1 + \bar{r}}{\delta_2 - 1} \right) \theta_1 \bar{J} > 0
\]

\[
b_{\pi_3} = -(\gamma_3 - \gamma_2) \left( \frac{1 + \bar{r}}{\delta_2 - 1} \right) \theta_1 \bar{J} < 0.
\]
\[ \ln N_t = b_0 + b_X \ln X_t + b_W \ln W_t + b_{\pi_1} \pi_{1,t-1} + b_{\pi_3} \pi_{3,t-1} + \nu_t \]

Table 1
Estimated Cointegrating Regression

<table>
<thead>
<tr>
<th>Constant</th>
<th>Time</th>
<th>ln X</th>
<th>ln W</th>
<th>(\pi_1)</th>
<th>(\pi_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.573</td>
<td>0.000</td>
<td>0.381</td>
<td>-0.897</td>
<td>0.100</td>
<td>-0.036</td>
</tr>
<tr>
<td>(18.200)</td>
<td>(-39.170)</td>
<td>(4.530)</td>
<td>(-5.781)</td>
<td>(11.267)</td>
<td>(-5.377)</td>
</tr>
</tbody>
</table>

DOLS estimates of the cointegrating vector with (t-statistic).

**Input cost puzzle:** When costs are low, firms have an incentive to produce more and build up their inventories. It has been difficult, however, to find statistically significant relationship between observable costs and inventories.

**The mechanism puzzle:** Monetary policy changes the interest rate and should affect inventories, since the interest rate represents the opportunity cost of holding inventories. But empirical research has generally failed to find any significant effect of the interest rate on inventories. [MMS 2004].
**Decision Rule**

\[
\ln N_t = \Gamma_0 + \lambda_1 \ln N_{t-1} + \Gamma_X \ln X_{t-1} + \Gamma_W \ln W_{t-1} + \Gamma_{\pi_1} \pi_{1t-1} + \Gamma_{\pi_3} \pi_{3t-1} + u_t
\]  \hspace{1cm} (17)

Inventories buffer shocks to production and sales:

\[
u_t = (\bar{R}_Y / \bar{R}_N)(u_t^Y - u_t^X) \text{ where } u_t^Y = \ln Y_t - E_{t-1} \ln Y_t, \text{ and, } u_t^X = \ln X_t - E_{t-1} \ln X_t.
\]

(Our model incorporates several economic mechanisms: convex production costs, stockout avoidance, inventories buffer sales shocks)

The decision-rule coefficients \((\lambda_1, \Gamma_X, \Gamma_W, \Gamma_{\pi_1}, \Gamma_{\pi_3})\) are known functions of the model’s structural parameters \((\bar{\beta}, P, r_v; \theta_1, \theta_2, \delta_2; \bar{R}_N, \bar{R}_Y, \bar{x}, \bar{J})\). Using the values of \(\bar{\beta}, P, \) and \(r_v\) from our estimation of the Markov-switching model, the values of \(\bar{R}_N, \bar{R}_Y, \bar{x}, \) and \(\bar{J}\) from the data, and the calibrated values of the structural parameters \(\theta_1, \theta_2, \) and \(\delta_2\) the decision rule coefficients can be calculated.
Table 2: Structural Parameters and Decision Rule Coefficients

<table>
<thead>
<tr>
<th>Panel A: Cost Function Parameters</th>
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</thead>
<tbody>
<tr>
<td>θ₁</td>
</tr>
<tr>
<td>41.146</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Decision Rule Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ₁</td>
</tr>
<tr>
<td>0.944</td>
</tr>
</tbody>
</table>

\[
\ln N_t = \Gamma_0 + \lambda_1 \ln N_{t-1} + \Gamma_X \ln X_{t-1} + \Gamma_W \ln W_{t-1} + \Gamma_{\pi_1} \pi_{1t-1} + \Gamma_{\pi_3} \pi_{3t-1} + u_t
\]
**Variance Ratio Puzzle:** If production costs are convex, then firms should smooth production in response to demand shocks. Production should vary less than sales. It doesn't.

**Wen (2005) Puzzle:** At medium horizons (8-40 quarters), production varies more than sales. Surprisingly, at short horizons (less than three quarters) production is less volatile than sales.

From the decision rule we derive the **conditional variance ratio**: 

$$\frac{\text{Var}[\ln Y_t]}{\text{Var}[\ln X_t]} = \frac{1}{1+\frac{1}{n}} + \frac{(1-\lambda_1 + \tilde{\Gamma}_X)}{(1+n)(1-\lambda_1)} \left[ \left(1-\lambda_1 + \tilde{\Gamma}_X\right) \left(1-\lambda_1^{2^n}\right) \right]$$

$$+ \frac{\sigma_w^2}{\sigma_X^2} \left[ \frac{\tilde{\Gamma}_w^2}{(1+n)(1-\lambda_1)} \right] \left(1-\lambda_1^{2^n}\right)$$

$\sigma_X^2$ is the variance of the sales shock and $\sigma_w^2$ is the variance of the cost shock.
The solid line shows the conditional variance ratio (variance (\(\ln Y\)_n)/variance (\(\ln X\)_n), conditional on \(\ln Y_0\) and \(\ln X_0\), where \(Y\) is output and \(X\) is sales) calculated from equation (38), where the structural parameters are calibrated using the cointegrating regression. The dashed line shows the conditional variance ratio for \(\theta_1\) equal to 2.5 times the calibrated value. A larger value of \(\theta_1\) implies greater convexity of the production cost function. The horizontal axis shows the horizon (\(n\)) in months.
The solid line shows the path of sales in the wake of a one-time permanent sales shock. The line with open circles shows the response of output in the model when the structural parameters are calibrated using the cointegrating regression for inventories. The line with solid triangles shows what the response of output would be if $\theta_1$ were 2.5 times the calibrated value. (A larger value of $\theta_1$ implies greater convexity of the production cost function.) The horizontal axis shows time in months.
Slow-Adjustment Puzzle: Estimated adjustment speeds are extremely low, often less than 10 percent per month. "Theory strains to explain low adjustment speeds unless the incentive to smooth production is extremely strong, which is hard to reconcile with the fact that production is more variable than sales." Blinder and Maccini (1991)

From the decision rule we derive

$$\ln N_t - \ln N_{t-1} = (1 - \lambda_1) \left[ \ln N_t^* - \ln N_{t-1} \right] + u_t \quad (40)$$

where $\ln N_t^*$ is the “desired” or target stock of (log) inventories.

$$\ln N_t^* \equiv \frac{1}{1 - \lambda_1} \left[ \Gamma_x \ln X_{t-1} + \Gamma_w \ln W_{t-1} + \Gamma_{\pi_1} \pi_{1t-1} + \Gamma_{\pi_3} \pi_{3t-1} + \Gamma_o \right]$$

We see in (40) that $(1 - \lambda_1)$ measures the speed of adjustment.

For our calibration $\lambda_1 = 0.944$ so the speed of adjustment is $(1 - \lambda_1) = 0.056$, about 6% per month.
Monetary Policy Puzzles

Sign Puzzle: Expansionary monetary policy lowers the interest rate and should increase inventories (lower interest rate reduces the opportunity cost of holding inventories). VAR studies find that the short-term effect of expansionary monetary policy is the opposite -- inventories decrease.

Timing Puzzle: A monetary expansion induces a transitory decline in the interest rate. The effect of monetary policy on the interest rate largely disappears within one year. Inventories begin to rise only after the transitory shock to the interest rate has dissipated.

Mechanism Puzzle: Monetary policy changes the interest rate and should affect inventories, since the interest rate represents the opportunity cost of holding inventories. In fact, VAR studies find that monetary policy affects inventories. But 40 years of empirical research on inventories has generally failed to find any significant effect of the interest rate on inventories.
Estimate a VAR

Characterize the empirical response of inventories to a monetary policy shock.

Find the response of the driving variables, \( \ln X_t, \ln W_t, \pi_{1t}, \) and \( \pi_{3t} \), to a monetary policy shock. Use those responses in the decision rule to characterize the theoretical response of inventories to a monetary policy shock.

\[
\ln N_t = \Gamma_0 + \lambda_1 \ln N_{t-1} + \Gamma_X \ln X_{t-1} + \Gamma_W \ln W_{t-1} + \Gamma_{\pi_1} \pi_{1t-1} + \Gamma_{\pi_3} \pi_{3t-1} + u_t
\]
Figure 7
Empirical Response of the Fed Funds Rate to a Stimulative Monetary Policy Shock

The solid line presents the empirical impulse response function of the Fed funds rate to a one-standard-deviation stimulative monetary policy shock. The horizontal axis shows time in months.
The solid line presents the empirical impulse response function of inventories to a one-standard-deviation stimulative monetary policy shock. The horizontal axis shows time in months.
The solid line displays the theoretical response of inventories to a one-standard-deviation stimulative monetary policy shock, based on the model presented in Section II, calibrating the structural parameters using the cointegrating regression, as described in Section III. The horizontal axis shows time in months.
Summary

Traditional Puzzles
Variance Ratio Puzzle
Wen (2005) Puzzle
Slow-Adjustment Puzzle
Input Cost Puzzle

Monetary Policy Puzzles
Sign Puzzle
Mechanism Puzzle
Timing Puzzle

Our model incorporates several economic mechanisms – convex production costs, stockout avoidance, inventories buffer sales shocks – but is tractable enough to be linearized.

Driving variables are highly persistent. To abstract from the high-frequency noise in the data we calibrate structural parameters using the cointegrating regression.

Using the calibrated parameters in the decision rule we can explain the seven puzzles.