Predicting the Term Structure of Interest Rates in Australia

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Motivations

- Debt market participants need models of yield dynamics.
- Some models explain yields, others predict.
- Compare predictive power of Cox, Ingersoll and Ross (1985) [“CIR”] model and Diebold and Li (2006) [“DL”] model:
  - Formulate new CIR forecasting methodology
  - Provide first test of DL with Australian data
- Findings may help Australian debt market participants understand nature of their exposures to changes in yields.
CIR Model

- CIR formulate a model of the real term structure—the real yields of real bonds, as opposed to nominal yields of nominal bonds.
- Agents have state independent, logarithmic preferences in economy with single state variable, $X$, which follows an Itô process.
- Moments and cross moments of return on $n$ production technologies proportional to level of $X$.
- Markets exist for
  - Instantaneous riskfree borrowing and lending; and
  - Contingent claims on the $n$ production technologies.
Derive FOC for maximisation of utility, and impose market clearing conditions. Short rate inherits properties of $X$, so that

$$dr = \kappa(\theta - r)dt + \sigma \sqrt{r}dz,$$

(1)

also, $r$ has conditional non-central $\chi^2$ distribution, hence

$$E[r(s)|r(t)] = r(t)e^{\kappa(t-s)} + \theta(1 - e^{\kappa(t-s)}), \quad s > t. \quad (2)$$

Derive equilibrium return on contingent claims [“CC”] from FOC, thereby make explicit the form of the factor risk premium. Derive valuation PDE by equating drift of equilibrium return on CC with alternative representation implied by Itô’s lemma.
CIR Model (ctd)

Solution to valuation PDE for pure discount real bond is

\[ P_{CIR}(t, \tau) = A(t, \tau)e^{-B(t, \tau)r(t)}, \quad (3) \]

where

\[
A(t, \tau) \equiv \left[ \frac{2\gamma e^{\frac{1}{2}\tau(\kappa+\lambda+\gamma)}}{(\gamma + \kappa + \lambda)(e^{\gamma\tau} - 1) + 2\gamma} \right]^{\frac{2\kappa \theta}{\sigma^2}}
\]

\[
B(t, \tau) \equiv \frac{2(e^{\gamma\tau} - 1)}{(\gamma + \kappa + \lambda)(e^{\gamma\tau} - 1) + 2\gamma}
\]

\[
\gamma \equiv \sqrt{(\kappa + \lambda)^2 + 2\sigma^2}.
\]
CIR Model (ctd)

- CIR real yield can be written

\[ r_{CIR}(t, \tau) = \frac{B(t, \tau)r(t) - \ln A(t, \tau)}{\tau}. \]  

(4)

- Note that \( \lambda \) parameter determines market risk premium; and money is neutral, so Fisher relation holds in CIR model.
DL Model

- DL utilise a three factor functional form for the static nominal term structure, namely,

\[ y_{DL}(t,\tau) = \beta_1 t + \beta_2 t \left( \frac{1-e^{-\tau/\gamma}}{\tau/\gamma} \right) + \beta_3 t \left( \frac{1-e^{-\tau/\gamma}}{\tau/\gamma} - e^{-\tau/\gamma} \right). \]  

\[ (5) \]

- \( \gamma \) is fixed as constant; \( \beta \)'s are estimated with OLS, and postulated to follow AR(1) processes. AR(1) models on the factors allow for computation of forecasts.

- Nelson and Siegel (1987): instantaneous forward rate follows second-order ODE with equal roots, and solution equation equivalent to (5).
Which Model Should Perform Better?

- Single factor CIR model, augmented by inflation model that incorporates info from macro variables, should account for multiple sources of uncertainty that determine changes in nominal yields of nominal bonds.
- DL model attempts to use perceived geometric patterns in interpolated static nominal term structures to predict nominal term structure.
- Hence, we expect CIR to predict more accurately than DL.
Details: Use BBSW rates and swap rates.
Daily set of 3764 observations of rates at eleven maturities from 1 month to 10 years over the period 28 March 1991 to 31 March 2006.
Extract month end observations to get 181 monthly observations over the same period.
Convert all rates to continuous compounding, treat this as a proxy for the swap zero curve.
Inflation and macroeconomic data

- Price level datum: quarterly CPI values over the period December 1990 to December 2005.
- Macroeconomic real activity data:
  - Real GDP (quarterly)
  - Non farm labour share (quarterly)
  - WMI consumer sentiment index (monthly)
  - NAB business confidence index (monthly)
  - Unemployment rate (monthly)
- Intermediate observations are deleted, so that all macroeconomic real activity data is quarterly.
Partitioning the data

- Halfway point observation is at 30 September 1998.
- Designate observations up to this point as the in-sample partition of the data.
- Designate observations subsequent to halfway point as out-of-sample partition of the data.
- Corresponding partitions of inflation and macro data are Dec 1990 to Jun 1998 and Sep 1998 to Dec 2005.
CIR Estimation

1. Compute in-sample real term structure by subtracting inflation rate from (continuously compounded) nominal yields.

2. Estimate $\sigma$, $\kappa$ and $\theta$ with GMM; calibrate $\kappa$ to make estimation errors unbiased; extract in-sample $\lambda$ estimates using CIR real yield formula; estimate best fit ARMA model for in-sample $\lambda$ using BIC; estimate best fit ARMAX inflation model with macro data using BIC.

3. Predict out-of-sample inflation, $\lambda$ and short rate.


5. Evaluate how well the forecasts performed, look to forecast errors and root mean squared errors.
Find optimal $\gamma$ parameter by minimising sum of squared errors over an increasingly fine sequence of possible $\gamma$ values.

2. Find in-sample estimates of the DL $\beta$ factors, using regressions of the form of (5).

3. Model the in-sample timeseries of $\beta$ estimates with best fit ARMA processes using BIC.

4. Predict the out-of-sample DL nominal term structure using the time series models for the $\beta$ factors.

5. Evaluate how well the forecasts performed, look to forecast errors and root mean squared errors.
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<th>DLBEST</th>
<th>DLBARE</th>
<th>RW</th>
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- **CIRARMAX**: CIR forecasts of real yields augmented by an ARMAX inflation model.
- **CIRARMA**: CIR forecasts of real yields augmented by an ARMA inflation model.
- **DLBEST**: Optimal $\gamma$ parameter and best fit ARMA models chosen for the three in-sample $\beta$ factor timeseries with BIC.
- **DLBARE**: Set $\gamma = 0.0609$ and in-sample $\beta$ factor timeseries each follow AR(1) process.
- **RW**: Random walk on yield levels.
- **YLAR1**: Univariate AR(1) on yield levels.
Can We Improve on the CIR Forecasts?

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- **CIRARMAX**: CIR forecasts of real yields augmented by an ARMAX inflation model.
- **CIRARMA**: CIR forecasts of real yields augmented by an ARMA inflation model.
- **CIRALT**: CIR forecasts of real yields with best fit ARMA models for all parameters (including short rate) augmented by ARMAX inflation model.
- **CIRNOM**: CIR model fit directly onto nominal yields, and best fit ARMA models for all parameters used for forecasts.
- **RW**: Random walk on yield levels.
Final Remarks

- For daily term structure data, the random walk was the best model, but given recent volatility in markets, DLBEST seems preferable.
- For monthly term structure data, the DLBEST model produced the most accurate forecasts. Suggests economic forces which drive evolution of term structure may transcend theoretical models.
- CIR model performed poorly due to inflation data limitations and the fact that it only had two state variables (short rate and inflation).
- While the former concern cannot be addressed without more frequent CPI releases, the latter concern diminished in importance with the CIRNOM model.