An Evolutionary Game Model of Financial Market with Momentum and Contrarian Players

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Abstract: Three types of trend traders, including momentum trader, contrarian trader and neutralist, are introduced to an evolutionary game model as market players, and their payoff structure is given. By a continuous replicator equation, a dynamic system is defined, and then the evolutionary stable states are presented, which respectively produce a stationary price oscillation and an exponential increasing (decreasing) price bubble respectively.

Keywords: evolutionary game, momentum strategy, contrarian strategy
1. Motivation

Try to use the evolutionary game framework to discuss financial market dynamics.
2. The Model

(1) Market participants

- Strategy traders
  - Momentum strategy: buying the past winners and shorting the past losers (Jegadeesh and Titman, 1993) → trend-following expectation
  - Contrarian strategy: buying the past losers and shorting the past winners → trend-reversal expectation

- Neutral traders: neutralists
  - No trend strategy, just randomly buy or sell
(2) Market population

The population consists of the three group players:

- Momentum traders: $x$
- Contrarian traders: $y$
- Neutralists: $1-x-y$.

All the traders can shift from one group to another
(3) Transaction rule

In every period, excess demand of each trader:

- Momentum trader: $e_M = e(\Delta p_t) = \text{sign}(\Delta p_t)$

- Contrarian trader: $e_C = -e(\Delta p_t) = -\text{sign}(\Delta p_t) = -e_M$

- Neutralist: buys or sells 1 unit asset with probability 1/2.

All the traders are matched randomly at each trade period.
(4) Market price dynamics

Market price change is linearly determined by the overall market excess demand:

\[ \Delta p_{t+1} = \alpha [x e_M + y e_C] = \alpha (x - y) \text{sign}(\Delta p_t) \]
(5) Payoff structure of heterogeneous traders

<table>
<thead>
<tr>
<th></th>
<th>M (x)</th>
<th>C (y)</th>
<th>N (1-x-y)</th>
</tr>
</thead>
<tbody>
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<td>M (x)</td>
<td>0</td>
<td>$e_M \Delta p_{t+1}$</td>
<td>$e_M \Delta p_{t+1}/2$</td>
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<tr>
<td>C (y)</td>
<td>$e_C \Delta p_{t+1}$</td>
<td>0</td>
<td>$e_C \Delta p_{t+1}/2$</td>
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<tr>
<td>N(1-x-y)</td>
<td>$-e_M \Delta p_{t+1}/2$</td>
<td>$-e_C \Delta p_{t+1}/2$</td>
<td>0</td>
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</tbody>
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Table 1. Payoff structure of the momentum trader, contrarian trader and neutralist.
Expected Payoff of heterogeneous traders and whole traders:

\[ \pi_M = yA_{MC} + (1 - x - y)A_{MN} \]
\[ = \alpha y(x - y) + \alpha(1 - x - y)(x - y) / 2 = \alpha(x - y)(1 - x + y) / 2 \]

\[ \pi_C = xA_{CM} + (1 - x - y)A_{CN} \]
\[ = -\alpha x(x - y) - \alpha(1 - x - y)(x - y) / 2 = -\alpha(x - y)(1 + x - y) / 2 \]

\[ \pi_N = xA_{NM} + yA_{NC} \]
\[ = -\alpha x(x - y) / 2 + \alpha y(x - y) / 2 = -\alpha(x - y)^2 / 2 \]

\[ \pi = x\pi_M + y\pi_C + (1 - x - y)\pi_N = 0 \]
(6) Market dynamics

We use the following continuous replicator equation (Weibull, 1995) to describe the dynamic process of the market:

\[
\begin{align*}
\dot{x} &= x(\pi_M - \pi) = \alpha x (x - y)(1 - x + y) / 2 \\
\dot{y} &= y(\pi_C - \pi) = -\alpha y (x - y)(1 + x - y) / 2
\end{align*}
\]
3. Dynamic System Analysis

(1) Equilibria and stability analysis

(2) Evolutionary stable state (ESS)

• Here the equilibria (0, 1) and (1, 0) can be regarded as the ESS.
  -- Momentum-trader-dominated market,
  -- Contrarian-trader-dominated market.

• If the system reaches the evolutionary stable state, there will be no real trade actually.
Figure 1. A flow diagram of the dynamic system

\[ x, y \geq 0, x + y \leq 1 \]
(3) Price dynamics

• Momentum-trader-dominated market: an exponential increasing (decreasing) price path, which corresponds to a rational asset price bubble.
• Contrarian-trader-dominated market: a stationary price oscillation.
4 Discussion

It is reasonable to argue whether the evolutionary stable state in our model reflects real asset market. At the first glance, the evolutionary stable state is far away from real market, because it means the market participants eventually converge at homogeneous trade strategy and the market will have no trade volume. This mainly relies on the assumption that the market is only occupied by trend traders, including the trend-following traders, trend-reversal traders and trend-neutral traders. Obviously, this assumption is too artificial. In fact, trend traders are just one type of market participants, which may correspond to so-called “technical traders”. In the general model framework, the financial market usually also involves fundamentalists, sometimes contains liquidity traders, and their population can not be wiped out. Thus, it is meaningful to understand our model conclusion just within a scope with technical traders, particularly with trend traders. In other words, the evolutionary stable state just corresponds to the evolutionary equilibrium of different groups of trend traders. It may be our further work to introduce fundamentalists to this game model.
Thank You For Your Attention!