Changing central bank mandates and escapes from Nash inflation

Alina Barnett (Bank of England) and Martin Ellison (Oxford University)
Changing central bank mandates and escapes from Nash inflation

- Motivation
- Related Literature
- Model
- Results
"... not until Volcker took office did controlling inflation become the organising focus of monetary policy" - Clarida, Gali, Gertler (2000)
Q1: Why did inflation in the US, UK and other countries decrease in the late 1980s?

Q2: Why has it remained low and stable?
Reasons for which inflation fell and rose after World War II:

- Kydland and Prescott (1977): time consistency problem
- Cogley and Sargent (2003), Sargent and Zha (2004): higher volatility of non-policy shocks during 1960s and 1970s vs 1980
- Clarida, Gali, Gertler (2000): policy mistakes
- Sargent (1999): changes in government beliefs are the driving factors
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- Sargent (1999)
- Cho, Williams and Sargent (2002)
- Williams (2004)

○ Motivation

- Time variant inflation aversion
- Adaptive private agents expectations

○ Model

○ Results
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- Results
Central Bank:
- forms an opinion on the relation $U_t - x_t$
- optimise objective function
- sets intended inflation rate $x_t$

Private agents:
- make expectations of inflation

The economy

\[
\begin{align*}
U_t \\
\pi_t
\end{align*}
\]
Cho, Williams and Sargent (2002)

Actual law of motion

\[ U_t = U^* - \theta(\pi_t - \pi^e_t) + \nu_{1,t} \]

Perceived law of motion

\[ U_t = \gamma_0 t + \gamma_1 t \pi_t + \xi_t \]

\[ \pi_t = x_t + \nu_{2,t} \]

Natural rate of unemployment
Realised inflation
Expected inflation
Real shock
Beliefs of central bank
Inflation set by central bank
Nominal shock
Beliefs are updated according to:

\[
\begin{pmatrix}
\gamma_{0,t+1} \\
\gamma_{1,t+1}
\end{pmatrix} =
\begin{pmatrix}
\gamma_{0,t} \\
\gamma_{1,t}
\end{pmatrix} + a_g R^{-1} \begin{pmatrix} 1 \\
\pi_t
\end{pmatrix} (U_t - \gamma_{0,t} - \gamma_{1,t} \pi_t)
\]

Gain coefficient

\[
R_{t+1} = R_t + a_g (M_t - R_t)
\]

Measures the precision of current estimates

\[
M_t = \begin{pmatrix} 1 \\
\pi_t
\end{pmatrix} (1 \quad \pi_t)
\]
Central bank solves the Phelps problem:

\[
\Omega = -E \sum_{t=0}^{\infty} (U_t^2 + \beta_t \pi_t^2)
\]

\[st.
U_t = \gamma_0 + \gamma_1 \pi_t + \xi_t
\]

\[\pi_t = x_t + \nu_{2,t}
\]

with \( \gamma_{0,t}, \gamma_{1,t} \) and \( \beta_{0,t} \) given

\[x_t = \frac{\gamma_0 \gamma_1}{\beta_t + \gamma_1^2}
\]
Inflation aversion

\[ \beta_t = l + \frac{u - l}{1 + e^{-b(t-s)}} \]

\[ \beta_t = 1 + \frac{1 - 1.5}{1 + e^{-10(t-4)}} \]
Private agents:

$$\pi_{t+1}^e = \pi_t^e + \alpha_p (\pi_t - \pi_t^e)$$

private gain coefficient
In Equilibrium

**Mean Dynamics**

\[
\begin{align*}
\dot{\gamma}_t &= \bar{g}(\gamma_t) \\
\dot{R} &= \bar{M}(\gamma_t) - R \\
\dot{\pi}_t^e &= x_t - \pi_t^e
\end{align*}
\]

= 0

**Nash Equilibrium**

\[
\begin{align*}
\bar{\gamma}_0 &= U^* (1 + \frac{\theta^2}{\beta_t}) \\
\bar{\gamma}_1 &= -\theta \\
\bar{\pi}^e &= \bar{x} = \frac{\theta U^*}{\beta_t}
\end{align*}
\]
Out of Equilibrium

Escape Dynamics

\[ \Psi = \inf_{\dot{v}} \int_0^t \dot{v}(\varphi)' Q(\Pi(\varphi), R(\varphi), \beta_t)^{-1} \dot{v}(\varphi) d(\varphi) \]

\[ s.t. \]

\[ \dot{\Pi} = \bar{g}(\Pi) + \bar{\dot{v}} \]

\[ \dot{R} = \bar{M}(\gamma) - R \]

\[ \dot{\beta} = \frac{2.5e^{-b(t-s)}}{(1 + e^{-b(t-s)})^2} \]

Williams (2004)- Dominant escape path:
“least cost path of perturbations that push believes away from Nash Equilibrium”
Motivation

Related Literature

Model

Results
Numerical Results

- Motivation
- Related Literature
- Model
- Numerical Results

![Graph showing numerical results](graph.png)

- Graph 1: Intended inflation (gamma 0) vs. Expected inflation (gamma 1)
- Graph 2: Another numerical result comparison

Legend:
- Blue line: intended inflation
- Red dashed line: expected inflation
- Green dotted line: gamma 0
- Black dotted line: gamma 1
Numerical Results

Motivation

Model

Related Literature

- gamma 0- Rational expectations
- gamma 0- Adaptive expectations
- gamma 1- Adaptive expectations
- gamma 1- Rational expectations
- Expected inflation- Adaptive expectations
- Realised inflation- Adaptive expectations
- Realised inflation- Rational expectations
Starting values for simulation- same as Cho, Williams and Sargent (2002)

\[
\begin{align*}
\pi_0^e &= 5 \\
\pi_0 &= 5 \\
a_p &= 0.0275 = a_g \\
\beta_r &= 1 \\
\gamma_0 &= 10 \\
\gamma_1 &= -1 \\
U^* &= 5 \\
\theta &= 1
\end{align*}
\]
Simulation Results

Motivation

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Related Literature

expected inflation - adaptive expectations

intended inflation - rational expectations
Simulation Results

- Motivation
- Related Literature
- Model
- Rational expectations
- Adaptive expectations
Simulation Results

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- Simulation Results
### Simulation Results

#### Motivation

#### Related Literature

#### Model

<table>
<thead>
<tr>
<th>Model/Escape statistics</th>
<th>Number</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational expectations &amp; $\beta = 1$</td>
<td>15(0.75)</td>
<td>218 (78)</td>
</tr>
<tr>
<td>Rational expectations &amp; $\beta = {1, 1.5}$</td>
<td>15 (0.63)</td>
<td>145 (70)</td>
</tr>
<tr>
<td>Adaptive expectations &amp; $\beta = 1$</td>
<td>7(1.22)</td>
<td>100 (29)</td>
</tr>
<tr>
<td>Adaptive expectations &amp; $\beta = {1, 1.5}$</td>
<td>6 (0.89)</td>
<td>101 (30)</td>
</tr>
</tbody>
</table>

\[
U_t = U^* - \theta(\pi_t - \pi_t^e) + \nu_{1,t}
\]

\[
\pi_t = x_t + \nu_{2,t}
\]

- $U_t$: Natural rate of unemployment
- $\pi_t$: Inflation
- $\pi_t^e$: Expected inflation
- $\nu_{1,t}$: Real shock
- $\nu_{2,t}$: Nominal shock
- $x_t$: Inflation set by central bank
Related Literature

- Sargent (1999)
- Cho, Williams and Sargent (2002)
- Williams (2004)

Model

- Time variant inflation aversion
- Adaptive private agents expectations
Conclusion

- we find that an increase in inflation aversion ($\beta_i$) can act as an activation mechanism for an escape route, thus reduce inflation

- simulations indicate that once inflation aversion is higher, average inflation is lower and more stable.
Q1: Why did inflation in the US, UK and other countries decreased in the late 1980s?

A1: Either because of a temporary loss of trust in the Phillips curve or because of a more permanent change in beliefs about the use of inflation for stabilisation purposes.

Q2: Why has it remained low and stable?

A1: A more conservative central bank would be less likely to use inflation for stabilisation purposes.
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Graph showing simulation results with labels for gamma0- adaptive expectations, gamma0- rational expectations, gamma1- rational expectations, and gamma1- adaptive expectations.
Simulation Results

Motivation

Model

Related Literature

- gamma0- adaptive expectations
- gamma0- rational expectations
- gamma1- rational expectations
- gamma1- adaptive expectations