A Mechanism of Collapsing Bubble

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The Aim

- To propose a behavioral model of bubble and crash.
- To give a theoretical interpretation why bubbles is born and is unavoidably collapsed.
- To give a possible solution on Risk Premium Puzzle.
Internet Bubble and Crash in 1998-2002

Period II: 2000/3/10-2002/12/31

Internet Stocks:
- 10 (1998/1/2)
- 140 (2000/3/6): Peak
- 3 (2002/10/9): Bottom
- 14
- 1/50

Non-Internet Stocks:
- 10 (1998/1/2)
- 9.8 (2002/10/9)
- 1.4
- 0.7

List of internet companies
Ofek and Richardson (2003)
### Price Changes

#### Internet Stocks:

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I</td>
<td>0.2</td>
<td>4.1</td>
</tr>
<tr>
<td>Period II</td>
<td>-0.2</td>
<td>2.5</td>
</tr>
</tbody>
</table>

#### Non-Internet Stocks:

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I</td>
<td>0.007</td>
<td>0.01</td>
</tr>
<tr>
<td>Period II</td>
<td>-0.004</td>
<td>0.02</td>
</tr>
</tbody>
</table>

#### Covariance:

<table>
<thead>
<tr>
<th>Period</th>
<th>Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I</td>
<td>0.09</td>
</tr>
<tr>
<td>Period II</td>
<td>0.13</td>
</tr>
</tbody>
</table>

**Period I:** 1998/1/2-2000/3/9  
**Period II:** 2000/3/10-2002/12/31
Probability Distributions of Price-Changes

Abnormality of Internet stocks

- leptokurtic
- fat-tailed
Who Invested in Internet Stocks? 
Empirical Evidence (1)

• **Hedge funds:**

Brunnermeier and Nagel (2004)

“Hedge Funds and the Technology Bubble,”

*Journal of Finance* LIX, 5.
“Hedge Funds captured the upturn, but, by reducing their positions in stocks that were about to decline, avoided much of the downturn.”

Brunnermeier and Nagel (2004)
Who Invested in Internet Stocks? 
Empirical Evidence (2)

Inexperienced young fund manager

“Inexperienced Investors and Bubbles,”
by Greenwood and Nagel (2008),
Who invested in internet stocks? 
Empirical Evidence (2)

- Younger managers are more heavily invested in technology stocks than older managers.

- Younger Managers increase their technology holdings during the run-up, and decrease them during the downturn.

- Young managers, but not old managers, exhibit trend-chasing behavior in their technology stock investments.

Robin Greenwood and Stefan Nagel (2008)
Abnormal inflows to Internet stocks

Panel B. Abnormal inflows, as a fraction of total net assets, equal-weighted average for each age group

Net returns of internet stocks

Panel B. Value-weighted holdings-based returns, net of benchmark
Who invested in internet stocks?
Experimental Evidence

Traders’ Expectations in Asset Markets: Experimental Evidence

Haruvy, E., Lahav, Y., and C. Noussair,
Forthcoming in the American Economic Review (2009)
Bubbles and Crashes in Experimental Markets

(a) The bubble/crash pattern is observed when traders are inexperienced.

(b) The magnitude of bubbles decreases with repetition of the market, converging to close to fundamental values in market 4.

Experimental Evidence

Participants’ beliefs about prices are adaptive.

Table 5: Relationship between predicted and actual price change between periods $t-1$ and $t$.

$$P_t - P_{t-1} = \alpha + \beta (B_t' - B_{t-1})$$

<table>
<thead>
<tr>
<th>Market</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market 1</td>
<td>-15.769* (5.115)</td>
<td>1.566* (0.190)</td>
<td>0.453</td>
</tr>
<tr>
<td>Market 2</td>
<td>-10.297 (3.981)</td>
<td>1.550* (0.180)</td>
<td>0.476</td>
</tr>
<tr>
<td>Market 3</td>
<td>-7.963 (3.915)</td>
<td>1.066 (0.220)</td>
<td>0.223</td>
</tr>
<tr>
<td>Market 4</td>
<td>-6.584 (2.901)</td>
<td>0.921 (0.228)</td>
<td>0.193</td>
</tr>
</tbody>
</table>

* indicates $\alpha$ is significantly different from 0, and $\beta$ is significantly different from 1 at the 1% level.

Experimental Evidence

The existence of adaptive dynamics suggests the mechanism whereby convergence toward fundamental values occurs.

Table 4b: Stated beliefs as a function of fundamental value

\[ B_{n+1} = C_i + \gamma f_{i+1} \]

<table>
<thead>
<tr>
<th>Market</th>
<th>Fundamental value (( \gamma ))</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market 1</td>
<td>-0.896* (.028)</td>
<td>.33</td>
</tr>
<tr>
<td>Market 2</td>
<td>0.377* (.033)</td>
<td>.20</td>
</tr>
<tr>
<td>Market 3</td>
<td>1.196* (.027)</td>
<td>.38</td>
</tr>
<tr>
<td>Market 4</td>
<td>1.049 (.019)</td>
<td>.50</td>
</tr>
</tbody>
</table>

* indicates coefficient is significantly different from 1 at the 1% level

Participants’ beliefs about prices are adaptive.

Figure 3: Average prediction for each period in each market, session 6

The Setting of the Model

Assets traded

• **Bubble asset**: $x_1$
  
ex. Internet stocks

• **Non-bubble asset**: $x_2$
  
ex. Large stocks like utility stocks

• **Risk-free asset**: $x_f$
  
ex. Government bonds, fixed time deposits
The Setting of the Model

Agents

• Rational traders (Experienced managers)
  (i) They hold a portfolio of three assets.
  (ii) Capital Asset Pricing Model (CAPM).
    (Mossin(1966), Lintner (1969))

• Noise traders (Inexperienced managers)
  (i) They hold either risk-free asset or bubble asset.
  (ii) Maximization of random utility function of discrete choice  (MacFadden (1974))
The structure of the model

- **Rational traders**
  - bubble asset
  - Non-bubble asset
  - risk-free asset

- **Noise-traders**
  - bubble asset
  - risk-free asset

- **Risky asset prices**
  - buy (sell)
  - sell (buy)

**CAPM**

**Discrete Choice model**
Rational traders

\[
\max \left\{ E(W) - \frac{\gamma}{2} V(W) \right\} \quad \text{subject to} \quad p_1 (x_1 - \bar{x}_1) + p_2 (x_2 - \bar{x}_2) + q (x_f - \bar{x}_f) = 0
\]

\[
x_1 : \text{ Demand for bubble asset} \quad \sigma_{ii} : \text{ Variance of the asset price change}
\]

\[
x_2 : \text{ Demand for non-bubble asset} \quad \sigma_{ij} : \text{ Covariance of the asset price change}
\]

\[
p_1 : \text{ Price of bubble asset} \quad E(p_i) : \text{ Expected Price of bubble asset}
\]

\[
p_2 : \text{ Price of non-bubble asset}
\]

\[
q : \text{ Price of risk-free asset}
\]
The rational investor’s demands

\[
\begin{align*}
    x_1 &= \frac{1}{\gamma|A|} \left\{ \left( E(p_1) - \frac{p_1}{q} \right) \sigma_{22} - \left( E(p_2) - \frac{p_2}{q} \right) \sigma_{12} \right\} \\
    x_2 &= \frac{1}{\gamma|A|} \left\{ \left( E(p_2) - \frac{p_2}{q} \right) \sigma_{11} - \left( E(p_1) - \frac{p_1}{q} \right) \sigma_{21} \right\} \\
    x_f &= \bar{x}_f - \frac{p_1}{q} (x_1 - \bar{x}_1) - \frac{p_2}{q} (x_2 - \bar{x}_2)
\end{align*}
\]

where \( |A| = \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} \)
The rational investors’ aggregate excess demands

\[
\begin{align*}
M \Delta x_{1t} &= \frac{M}{\gamma |A|} \left[ \sigma_2^2 (E(\Delta p_{1t+1}) - \Delta p_{1t} / q) - \rho^{12} (E(\Delta p_{2t+1}) - \Delta p_{2t} / q) \right] \\
M \Delta x_{2t} &= \frac{M}{\gamma |A|} \left[ \sigma_1^2 (E(\Delta p_{2t+1}) - \Delta p_{2t} / q) - \rho^{21} (E(\Delta p_{1t+1}) - \Delta p_{1t} / q) \right]
\end{align*}
\]

where

\[
\Delta x_{it} = x_{it} - x_{it-1}
\]

\[
\Delta p_{jt+1} = p_{jt+1} - p_{jt}
\]

\[
E(\Delta p_{jt+1}) = E(p_{jt+1}) - E(p_{jt})
\]
Noise Traders

Their preference:

\[ s_j = \begin{cases} +1, & \text{for holding bubble asset} \\ -1, & \text{for holding risk free asset} \end{cases} \quad (j = 1, 2, \ldots, N) \]

Random Utility Function:

\[ \begin{align*}
U_1 &= \bar{U}_1 + \varepsilon_1 \\
U_2 &= \bar{U}_2 + \varepsilon_2
\end{align*} \]

\( \varepsilon_i : \text{Random variable} \)

\( \bar{U}_i : \text{Deterministic part} \)
Noise-Trader’s Random Utility Function:

\[
\begin{align*}
U_+ &= \lambda s + H + \varepsilon_+ \\
U_- &= -(\lambda s + H) + \varepsilon_-
\end{align*}
\]

- Average preference: \( s = \frac{1}{N} \sum_{j=1}^{N} s_j \) \((-1 \leq s \leq +1)\)

- Strength of Herding: \( \lambda = \tilde{\lambda} N \) (An increasing function on \( N \))

- Return Momentum: \( H \)

\[
H_t = \theta H_{t-1} + (1-\theta)(r_{t-1} - r_f), \quad H_{t=0} = H_0
\]

(Adaptive expectation)
Probabilities

The probability that a utility-maximizing noise trader will choose each alternative:

\[
\begin{align*}
P_+ &= \frac{\exp[\kappa \bar{U}_+]}{\exp[\kappa \bar{U}_+] + \exp[\kappa \bar{U}_-]} \\
P_- &= \frac{\exp[\kappa \bar{U}_-]}{\exp[\kappa \bar{U}_+] + \exp[\kappa \bar{U}_-]}
\end{align*}
\]

under the Weibull distribution:

\[
F(x) \equiv \Pr[\varepsilon_i \leq x] = \exp[-\exp[-\kappa x]]
\]

McFadden, Daniel (1974)
Transition Probabilities:

\[
\begin{align*}
\begin{cases}
 p_\downarrow(s) = \nu \cdot \frac{\exp[-(\lambda s + H)]}{\exp[\lambda s + H] + \exp[-(\lambda s + H)]} \\
 p_\uparrow(s) = \nu \cdot \frac{\exp[\lambda s + H]}{\exp[\lambda s + H] + \exp[-(\lambda s + H)]}
\end{cases}
\end{align*}
\]

\( \nu \) : probability that one noise-trader attempts to trade
The Master Equation:

\[ p(s; t + 1) - p(s; t) = [w_\downarrow(s + \Delta s; t)p(s + \Delta s; t) - w_\downarrow(s; t)p(s; t))] \\
+ [w_\uparrow(s - \Delta s; t)p(s - \Delta s; t) - w_\uparrow(s; t)p(s; t))]
\]

\[
\begin{align*}
w[(s_t - N/2) \leftarrow s_t] &\equiv w_\downarrow(s_t) = N_+p_\downarrow(s_t) = \frac{N}{2}(1+s_t)p_\downarrow(s_t) \\
w[(s_t + N/2) \leftarrow s_t] &\equiv w_\uparrow(s_t) = N_-p_\uparrow(s_t) = \frac{N}{2}(1-s_t)p_\uparrow(s_t) \\
w(s'_t \leftarrow s_t) &= 0 \quad \text{for} \quad s'_t \neq s_t \pm \frac{N}{2}
\end{align*}
\]

\[ \Delta s = \frac{2}{N} \]
Representative Noise-Trader’s Behavior:

\[
\Delta \langle s \rangle_{t+1} \equiv \langle s \rangle_{t+1} - \langle s \rangle_t = \nu [ \tanh(\lambda \langle s \rangle_t + H_t) - \langle s \rangle_t ]
\]

(Keynesian Beauty Contest Theory)

The noise trader’s Aggregate Excess demands

\[
Q \left[ \langle n_+ \rangle_t - \langle n_+ \rangle_{t-1} \right] = \frac{QN}{2} \left( \langle s \rangle_t - \langle s \rangle_{t-1} \right)
\]

If the return momentum \( H \) increases, then \( s \) increases.
Market-clearing Conditions:

\[
\begin{aligned}
M \Delta x_{1t} + \frac{QN}{2} \Delta \langle s \rangle_t &= \frac{M}{\gamma |A|} \left[ \sigma_2^2 (E(\Delta p_{1t+1}) - \Delta p_{1t} / q) - \rho_{12}^2 (E(\Delta p_{2t+1}) - \Delta p_{2t} / q) \right] \\
+ \frac{QN}{2} \Delta \langle s \rangle_t &= 0 \\
M \Delta x_{2t} &= \frac{M}{\gamma |A|} \left[ -\rho_{12}^2 (E(\Delta p_{1t+1}) - \Delta p_{1t} / q) + \sigma_1^2 (E(\Delta p_{2t+1}) - \Delta p_{2t} / q) \right] = 0
\end{aligned}
\]
Market-clearing prices:

\[
\begin{align*}
\Delta p_{1t} &= q[\gamma \sigma^2_1 \kappa \Delta \langle s \rangle_t + E(\Delta p_{1t+1})] \\
\Delta p_{2t} &= q[\gamma \rho^{12} \kappa \Delta \langle s \rangle_t + E(\Delta p_{2t+1})] \\
\Delta \langle s \rangle_t &= \nu[\tanh(\lambda \langle s \rangle_t + H_t) - \langle s \rangle_t] \\
H_t &= \theta H_{t-1} + (1-\theta)(r_{1t-1} - r_f)
\end{align*}
\]

If \( \langle s \rangle_t \) increases, then price \( p_{1t} \) increases.
Proposition 1

If $0 < \lambda < 1$, then the dynamics converge to the unique equilibrium, $\bar{s}_t = 0, \Delta p_{1t} = \Delta p_{2t} = 0$, provided that the expected (fundamental) prices are constant,

$E(\Delta p_{1t+1}) = E(\Delta p_{2t+1}) = 0$.
Bimodal: \( \lambda > 1, H = 0 \)  \hspace{1cm} \text{Unimodal:} \( \lambda < 1, H = 0 \) 

\[
\langle s \rangle = \tanh(\lambda \langle s \rangle + H)
\]

\( A \text{Aline;} (\lambda, H) = (0.9,0) \)

\( B \text{Bline;} (\lambda, H) = (2.0,0) \)
**Unimodal:** $(\lambda < 1, H \neq 0)$

\[
\langle s \rangle = \tanh(\lambda \langle s \rangle + H)
\]

*AA line: $(\lambda, H) = (0.8, 0.3)$

*BB line: $(\lambda, H) = (0.8, -0.3)$*
Proposition 2

If $\lambda > 1$, then the dynamics are unstable, and cycles of bubble and crash are repeated, provided that are constant, $E(\Delta p_{1_{t+1}}) = E(\Delta p_{2_{t+1}}) = 0$. 
Phase Transition

\[ \langle X \rangle_{c} \]

\[ \langle X \rangle_{c+1} \]

\[ A \]

\[ B \]

\[ \lambda \]

\[ \lambda \]

\[ \lambda \]

\[ \lambda \]

\[ = - \pm \]

\[ = \]

\[ = \]

\[ A \text{Line}; (\lambda, H) = (1.8, 0.4) \]

\[ B \text{Line}; (\lambda, H) = (1.8, -0.4) \]

\[ \cosh^2[\lambda s \pm \sqrt{\lambda(\lambda - 1)}] = \lambda \]

\[ s_c = \sqrt{\frac{\lambda - 1}{\lambda}} \]
Mechanism of bubble and crash

\[ \lambda = 0.9; \ H = 0.0 \]

\[ \lambda = 1.3; \ H = 0.1 \]

\[ \lambda = 1.2; \ H = 0.0 \]

Bubble birth

\[ \lambda = 1.3; \ H = 0.05 \]
Mechanism of bubble and crash (continued)

\[ \lambda = 1.3; \ H = 0.1 \]

\[ \lambda = 1.3; \ H = 0.01 \]

Crash!!

\[ \lambda = 1.3; \ H = -0.1 \]

\[ \lambda = 1.3; \ H = -0.05 \]
\( \lambda = 1.1; \ \theta = 0.9 \)
\[ \sigma_{11} = 4.1; \]
\[ \sigma_{12} = 0.09 \]
\[ E(p_1) = 10; E(p_2) = 5 \]

Return Moment H
Price on bubble asset \( p_{1r} \)

\[ \lambda = 1.1; \theta = 0.9 \]
\[ \sigma_{11} = 4.1; \]
\[ \sigma_{12} = 0.09 \]
\[ E(p_1) = 10; E(p_2) = 5 \]

Price on non-bubble asset
Risk Premium Puzzle:

\[ r_1(t) - r_f > E[r_1(t)] - r_f \]

Expected risk premium:

\[ E[r_1] - r_f = \left( \frac{E(p_1) - p_1 / q}{p_1} \right) \]

\[ E[r] = \text{Risk Premium} + \text{Risk Free Rate} \]
A Model Simulation: Risk Premium

Realized Return: $r_1(t) - r_f$

Expected Discount Rate: $E[r_1(t)] - r_f$

The period of bubble: $r_1(t) - r_f > E[r_1(t)] - r_f$

After burst of bubble: $r_1(t) - r_f < E[r_1(t)] - r_f$