Equilibrium Pricing of Contingent Claims in Tradable Permit Markets

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Plan of Talk

1. Introduction
2. The Model
3. Equilibrium Prices without Banking
4. The Effect of Banking
5. Discussions
6. Conclusions
Introduction
Kyoto Protocol

- The first five-year commitment period started on January 1, 2008.
- Ratifying countries are required to meet the cap of pollution emission amount.
- Each country also imposes the emission limitation to its industries.
- How to meet the regulation:
  - Reduce the emission amount by self-efforts.
  - Buy pollution rights (pollution permit).
### Table 3.1: Reported volumes and value 2005, 2006, forecast 2007

Reported and estimated volumes 2005 and 2006, together with forecasted volumes for 2007, in Mt CO2e and million €. 7% discount rate employed for CDM and JI where price is at point of delivery. Prevailing carbon prices at time of writing for 2007 forecast.

<table>
<thead>
<tr>
<th></th>
<th>Final figures</th>
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<th>Forecast</th>
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<tbody>
<tr>
<td></td>
<td>[Mt]</td>
<td>[€ million]</td>
<td>[Mt]</td>
<td>[€ million]</td>
<td>[Mt]</td>
<td>[€ million]</td>
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<tr>
<td><strong>EU ETS total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>- OTC + exch.</td>
<td>362</td>
<td>7,218</td>
<td>1,017</td>
<td>18,143</td>
<td>1,750</td>
<td>18,503</td>
</tr>
<tr>
<td>- Bilateral</td>
<td>262</td>
<td>5,400</td>
<td>817</td>
<td>14,575</td>
<td>1,550</td>
<td>15,903</td>
</tr>
<tr>
<td><strong>Other ETSs</strong></td>
<td>100</td>
<td>1,818</td>
<td>200</td>
<td>3,568</td>
<td>200</td>
<td>2600</td>
</tr>
<tr>
<td><strong>CDM</strong></td>
<td>7.8</td>
<td>52</td>
<td>31</td>
<td>300</td>
<td>50</td>
<td>500</td>
</tr>
<tr>
<td><strong>CDM 2nd</strong></td>
<td>397</td>
<td>1,985</td>
<td>523</td>
<td>3,349</td>
<td>456</td>
<td>3,260</td>
</tr>
<tr>
<td><strong>JI</strong></td>
<td>4</td>
<td>50</td>
<td>40</td>
<td>571</td>
<td>96</td>
<td>1,061</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>28</td>
<td>96</td>
<td>21</td>
<td>95</td>
<td>45</td>
<td>277</td>
</tr>
</tbody>
</table>

Statistics of carbon market (2)

ECX CFI Options Volume

Volume (tonnes CO2)

Date

The purpose of the paper is to propose a pricing formula that evaluates any contingent claim in the pollution permit market.

Literature review:
- Maeda (2004): only the forward price is examined.
- Daskalakis et al. (2007), Chesney and Taschini (2008): the spot price (underlying) is exogenously given.

To our best knowledge, this paper is the first to study the pricing of any contingent claim of permit markets in a general equilibrium framework.
Pricing Functional

- Let $\tilde{Y}$ be the payoff of any contingent claim.
- It is a well-known result in the finance literature that the price of $\tilde{Y}$, denoted by $\pi(\tilde{Y})$, can be expressed with some random variable $\tilde{\phi}$ as

$$\pi(\tilde{Y}) = \mathbb{E}[\tilde{\phi}\tilde{Y}].$$

- The random variable $\tilde{\phi}$ is the so-called state price density.
The Model
2-period economy: time 0 (present) and time 1 (future).

Market participants (all price takers)
- regulated emitters who need to meet the cap of emission.
- unregulated financial traders who are not obliged to follow the regulation.

At time 0, there are two markets:
- Spot market: spot permits of time 0 are traded.
- Contingent claims market: contingent claims written on the permits of time 1 are traded.

The contingent claims market is complete in the sense that any contingent claim is tradable.
### Glossary

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\mathcal{M}_E$</td>
<td>set of emitters</td>
</tr>
<tr>
<td>$\mathcal{M}_S$</td>
<td>set of financial traders</td>
</tr>
<tr>
<td>$E_{it}$</td>
<td>the net abatement target of emitter $i \in \mathcal{M}_E$ at time $t$</td>
</tr>
<tr>
<td>$c_{it}(X_{it})$</td>
<td>the cost function to reduce the amount of emission $X_{it}$ by emitter $i$ at time $t$,</td>
</tr>
<tr>
<td>$S_t$</td>
<td>spot permit price at time $t$,</td>
</tr>
<tr>
<td>$R_{kt}$</td>
<td>exogenous income of agent $k \in \mathcal{M}_E \cup \mathcal{M}_S$,</td>
</tr>
<tr>
<td>$r$</td>
<td>risk-free rate determined in the financial market.</td>
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</tbody>
</table>
The preference of emitter $i$ is represented by an CARA utility with absolute risk-aversion coefficient $\gamma_i$:

$$\mathbb{E}[-\exp\{-\gamma_i \tilde{W}_i\}].$$

To match the regulation, emitter $i$ can reduce emission by her own effort and by buying the spot permits in the market.

The final wealth of emitter $i$ is described as

$$W_i(\omega) = (1 + r)[R_{i0} - c_{i0}(X_{i0}) - S_0 \cdot (E_{i0} - X_{i0}) - \pi(\tilde{H}_i)]$$

$$+ \quad R_{i1}(\omega) - c_{i1}(X_{i1}(\omega)) - S_1(\omega)\{E_{i1}(\omega) - X_{i1}(\omega)\} + H_i(\omega),$$

where $\tilde{H}_i$ is the contingent claim that emitter $i$ purchases.
Financial traders

- The preference of financial trader $j$ is also represented by an CARA utility:

$$
\mathbb{E}[-\exp\{-\gamma_j \tilde{W}_j\}].
$$

- Financial traders have no incentive to enter the spot market, and only trade in the contingent claims market to hedge the risk of their exogenous income.

- The final wealth of financial trader $j$ is written as

$$
W_j(\omega) = (1 + r)[R_{j0} - \pi(\tilde{H}_j)] + R_{j1}(\omega) + H_j(\omega), \quad j \in \mathcal{M}_S.
$$
Equilibrium Prices without Banking
Market clearing

- Spot markets:

\[
\sum_{i \in M_E} (E_{i0} - X_{i0}) = 0,
\]

\[
\sum_{i \in M_E} (E_{i1}(\omega) - X_{i1}(\omega)) = 0
\]

- Contingent claims market:

\[
\sum_{k \in M_E \cup M_S} H_k(\omega) = 0
\]

for almost all \( \omega \).
Cost function

Assumption

The cost function $c_{it}(\cdot)$ is strictly increasing, continuously differentiable, strictly convex, and satisfies

$$c'_{it}(0) = 0, \quad c'_{it}(\infty) = \infty.$$

- The assumption guarantees the existence and uniqueness of the equilibrium.
Some notations

- Let $c_t := \sum_{i \in M_E} c_{it}$. Then, we verify that $c_t$ is a function of $E_t$ in equilibrium, where $E_t := \sum_{i \in M_E} E_{it}$, i.e., we have an expression

$$c_t = c_t(E_t).$$

- Define

$$R_t := \sum_{k \in M_{E \cup M_S}} R_{kt}, \quad \gamma := \frac{1}{\sum_{k \in M_{E \cup M_S}} \frac{1}{\gamma k}}.$$
Pricing formula in the case of no banking

Proposition

*The state price density \( \phi \) is given by*

\[
\phi(\omega) = \frac{1}{1 + r} \frac{e^{\gamma c_1(E_1(\omega)) - R_1(\omega)}}{\mathbb{E}\left[e^{\gamma c_1(\tilde{E}_1) - \tilde{R}_1}\right]}
\]

Proof.

Apply the result of Bühlmann (1980).
Forward price

Corollary

The price of forward contract is derived as

\[ F = \frac{\mathbb{E} \left[ c_1' (\tilde{E}_1) e^{\gamma \{ c_1 (\tilde{E}_1) - \tilde{R}_1 \}} \right]}{\mathbb{E} \left[ e^{\gamma \{ c_1 (\tilde{E}_1) - \tilde{R}_1 \}} \right]} \]

Proof.

Verify that \( S_1 = c_1' (E_1) \) in equilibrium.
Normal risks (1)

• Suppose

\[
(\tilde{E}_1, \tilde{R}_1) \sim \mathcal{N}\left[ (\mu_E, \mu_R), \begin{pmatrix} \sigma^2_E & \rho \sigma_E \sigma_R \\ \rho \sigma_E \sigma_R & \sigma^2_R \end{pmatrix} \right]
\]

• The price whose payoff at time 1 is \(g(S_1)\) is given by

\[
\pi(g(\tilde{S}_1)) = \frac{\mathbb{E}[h(\tilde{Z}^*) e^{\gamma c_1(\tilde{Z}^*)}]}{(1 + r)\mathbb{E}[e^{\gamma c_1(\tilde{Z}^*)}]},
\]

where \(h(\cdot) = g(c'_1(\cdot))\), and \(\tilde{Z}^*\) is a normal with mean \(\mu_E - \gamma \rho \sigma_E \sigma_R\) and variance \(\sigma^2_E\).
Normal risks (2): comparative statics

Corollary

Let

\[ \Delta_1 = \mathbb{E}_\phi [h'(\tilde{Z}^*)] - \gamma \mathbb{C}_\phi [h(\tilde{Z}^*), c'_1(\tilde{Z}^*)], \]

where \( \mathbb{E}_\phi \) and \( \mathbb{C}_\phi \) are the expectation and covariance operators under the risk-neutral measure, respectively. Then,

\[ \frac{\partial \pi(g)}{\partial \mu_E} = \frac{1}{1 + r} \Delta_1, \quad \frac{\partial \pi(g)}{\partial \rho} = -\frac{\gamma \sigma_E \sigma_R}{1 + r} \Delta_1. \]
**Quadratic cost function**

- Suppose furthermore that the cost function of each emitter is quadratic as \( c_{it}(x) = \frac{\hat{c}_{it}}{2} x^2 \). Then, the social cost function is given by

\[
c_t(E_t) = \frac{1}{2} \hat{c}_t E_t^2,
\]

where \( \hat{c}_t = \frac{1}{\sum_{i \in \mathcal{M}_E} \frac{1}{\hat{c}_{it}}} \).

- The forward price is derived as

\[
F = \frac{\hat{c}_1 \left( \mu_E - \gamma \rho \sigma_E \sigma_R \right)}{1 - \gamma \hat{c}_1 \sigma_E^2} = \frac{\hat{c}_1 \mu_Z}{1 - \alpha},
\]

where \( \mu_Z := \mu_E - \gamma \rho \sigma_E \sigma_R \) and \( \alpha := \gamma \hat{c}_1 \sigma_E^2 \).

- This is the case that Maeda (2004) considered.
The Effect of Banking
Banking (and borrowing)

- Banking: additional reduction amount in the current period can be used in later periods.
- When banking and borrowing are allowed, the current and future spot markets are connected.
- The market clearing condition of the permit:

\[
\sum_{t=0}^{1} \sum_{i \in \mathcal{M}_E} (E_{it} - X_{it}) = 0
\]

or equivalently

\[
\sum_{i \in \mathcal{M}_E} (E_{i0} + B_{i0} - X_{i0}) = 0 \quad \text{and} \quad \sum_{i \in \mathcal{M}_E} (E_{i1}(\omega) - B_{i0} - X_{i1}(\omega)) = 0.
\]

where \( B_{i0} \) denotes the banking of emitter \( i \) at time 0.
No arbitrage condition

- No arbitrage condition implies

\[
(1 + r) c_0'(E_0 + B_0) = \frac{\mathbb{E} \left[ c_1' (\tilde{E}_1 - B_0) e^{\gamma \{ c_1 (\tilde{E}_1 - B_0) + \tilde{R}_1 \}} \right]}{\mathbb{E} \left[ e^{\gamma \{ c_1 (\tilde{E}_1 - B_0) + \tilde{R}_1 \}} \right]},
\]

where \( B_0 = \sum_{i \in \mathcal{M}_E} B_{i0} \).

- The aggregated amount of banking in equilibrium is determined by (NAC).
Normal risks (1)

Consider the case of Example 1 and suppose that the system of banking and borrowing is introduced.

The aggregated amount of banking is derived as

\[ B_0 = \frac{\hat{c}_1 (\mu_E - \gamma \rho \sigma_E \sigma_R)}{1 - \gamma \hat{c}_1 \sigma_E^2} - (1 + r) \hat{c}_0 E_0 \]

The forward price is given by

\[ F_{WB} = \frac{\hat{c}_1 (E_0 + \mu_E - \gamma \rho \sigma_E \sigma_R)}{1 - \gamma \hat{c}_1 \sigma_E^2 + \frac{\hat{c}_1}{(1+r)\hat{c}_0}} = \frac{\hat{c}_1 (E_0 + \mu_Z)}{1 - \alpha + \frac{\hat{c}_1}{(1+r)\hat{c}_0}}. \]
Normal risks (2)

Comparing $F$ and $F_{WB}$, we have

$$\frac{\partial F}{\partial \mu_E} \geq \frac{\partial F_{WB}}{\partial \mu_E} > 0, \quad \frac{\partial F}{\partial \rho} \leq \frac{\partial F_{WB}}{\partial \rho} < 0$$

The introduction of banking and borrowing lowers the sensitivity of exogenous parameters on the forward price (smoothing effect).

The sensitivity of the forward price is partly absorbed by the change of the current spot price.
Discussions
Price spike: cost function and spot price

- Suppose the normality of $\tilde{E}_1$ and $\tilde{R}_1$, and that the social abatement cost function is given by

$$c_t(x) = \hat{c}_L \times \begin{cases} 
0 & x \leq 0, \\
\frac{1}{2} x^2 & x \leq \bar{X}_B, \\
\frac{\kappa}{2} x^2 - (\kappa - 1) \bar{X}_B x + \frac{\kappa - 1}{2} \bar{X}_B^2 & x > \bar{X}_B,
\end{cases}$$

where $\kappa \geq 1$.

- The spot price of time 1 is derived as

$$S_1 = \hat{c}_L \begin{cases} 
0 & \tilde{E}_1 \leq 0, \\
\tilde{E}_1 & \tilde{E}_1 \leq \bar{X}_B, \\
\kappa \tilde{E}_1 - (\kappa - 1) \bar{X}_B \tilde{E}_1 & \tilde{E}_1 > \bar{X}_B.
\end{cases}$$

- The spot price kinks at $x = \bar{X}_B$, and the parameter $\kappa$ measures the magnitude of the kink.
Price spike: no banking case

Forward price with different $\kappa$:

The spike level is lower in the forward price.
Price spike: introduction of banking

Forward price with different $\kappa$:

The price spike is significantly mitigated by the introduction of banking and borrowing.
Comparative statics

- By simply differentiating the forward price formula, we have

\[
\begin{array}{c|c}
\sigma_E & \Rightarrow \text{indeterminate} \\
\gamma & \Rightarrow \text{indeterminate} \\
\rho & \Rightarrow F \\
\sigma_R & \Rightarrow F
\end{array}
\]

- The impact on the risk-premium and the correlation with the exogenous income should be considered.
Conclusions
In this paper,

- we have proposed a formula that prices any contingent claim in the tradable permit market.
- we have analyzed the equilibrium price in the permit market with the case of normal distributions.
- we have shown that the prices in permit market have some specific properties that are not observed in ordinary financial markets.
Summary

Key determinants of the pricing:

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<tbody>
<tr>
<td>(i)</td>
<td>Social cost function to meet the abatement target</td>
</tr>
<tr>
<td>(ii)</td>
<td>Uncertainty of the required abatement level</td>
</tr>
<tr>
<td>(iii)</td>
<td>Risk-aversion of market participants</td>
</tr>
<tr>
<td>(iv)</td>
<td>Correlation of the abatement level with exogenous income.</td>
</tr>
</tbody>
</table>

- System design:
  - Banking and borrowing,
  - Limitation of market participants.
Thank you for your attention