Habit Persistence and International Co-movements

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Cross-country correlations of investment and labor are positive in the data.

Two-country real business cycle models with time-separable preferences and complete markets predict the opposite.

This phenomenon has been coined by Backus, Kehoe and Kydland (1995) as the *quantity anomaly*. 
The Quantity Anomaly

”It has proved particularly difficult to write down plausibly parameterized models which can generate positive comovement of labor and investment across countries. . . Thus a major challenge to the theory is to develop a model which can explain international comovement in labor input and investment”


- exogenous incomplete financial markets (Kollmann 1996; Baxter and Crucini 1995),
- variable factor utilization (Baxter and Farr 2005),
- labor market frictions (Yakhin 2007; Hairault 2002),
- limited enforcement of international borrowing contracts (Kehoe and Perri 2002).
Contribution

This paper develops a two-country one-good international real business cycles model in which we:

▶ Depart from the assumption of separability of preferences over time by introducing internal habits in consumption;
▶ Maintain the assumption of complete markets.

The Result:

▶ The model predicts empirically plausible value of cross-country investment correlation without deteriorating other business cycle statistics.
Why Habits?

- Some empirical evidence suggests that habit formation characterizes consumption behavior among most of the G-7 countries (Fuhrer and Klein 2006)
- Habits enjoyed some degree of success in asset pricing, monetary, and economic growth literature
- The notion of habits has been embraced by behavioral sciences.

"Habit formation captures a fundamental feature of psychology: repetition of a stimulus diminishes the perception of the stimulus and responses to it" (Campbell and Cochrane 1999:208).
The Model: Preferences

Consumers maximize expected lifetime utility:

\[ U = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t) u(c_{jt}(s^t), h_{jt}(s^{t-1}), l_{jt}(s^t)) \]

The instantaneous utility function:

\[ u(c, h, l) = \frac{[(c - bh)^\gamma (1 - l)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} \]

The law of motion for the stock of habits:

\[ h_{jt+1}(s^t) = \lambda c_{jt}(s^t) + (1 - \lambda) h_{jt}(s^{t-1}) \]
The Model: Production

Output in country $j$ after history $s^t$ is given by

$$y_{jt}(s^t) = z_{jt}(s^t) k_{jt}(s^{t-1})^\alpha l_{jt}(s^t)^{1-\alpha}$$

Productivity shocks follow a stationary VAR(1) process in logs:

$$\begin{bmatrix} \log z_{1t} \\ \log z_{2t} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{11} \end{bmatrix} \begin{bmatrix} \log z_{1t-1} \\ \log z_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

Innovations to the productivity process are serially independent normal random variables

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \sim NID \left( \mathbf{0}, \sigma^2_{\varepsilon} \cdot \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$
The Model: Capital Accumulation

The law of motion for the capital stock:

\[ k_{jt+1} (s^t) = (1 - \delta) k_{jt} (s^{t-1}) + \phi \left( \frac{i_{jt} (s^t)}{k_{jt} (s^{t-1})} \right) k_{jt} (s^{t-1}) \]

Capital adjustment cost function \( \phi \) follows Hayashi (1982):

\[ \phi(\cdot) \geq 0; \phi'(\cdot) > 0; \phi''(\cdot) < 0; \phi(\delta) = \delta; \phi'(\delta) = 1 \]

Functional form from Boldrin, Christiano and Fisher (2001):

\[ \phi \left( \frac{i_{jt}}{k_{jt}} \right) = \frac{a_1}{1 - 1/\zeta} \left( \frac{i_{jt}}{k_{jt}} \right)^{1-1/\zeta} + a_2 \]

- \( \zeta \) represents elasticity of investment with respect to Tobin’s \( q \).
- \( a_1 \) and \( a_2 \) are set to make deterministic steady state invariant to changes in \( \zeta \).
Optimality Conditions

The optimality requires that for all \( t \geq 0 \), all \( s^t \in S^t \), and \( j = 1, 2 \) the following conditions hold:

\[
\Lambda_{1t}(s^t) = \Lambda_{2t}(s^t),
\]

\[
\Lambda_{jt}(s^t) = \beta \sum_{s^{t+1} \in S} \pi(s_{t+1} \mid s^t) \Lambda_{jt+1}(s^t, s_{t+1}) R_{jt+1}(s^t, s_{t+1}),
\]

\[
-u_l(c_{jt}(s^t), h_{jt}(s^{t-1}), l_{jt}(s^t)) \frac{\Lambda_{jt}(s^t)}{\Lambda_{jt}(s^t)} = f_l(k_{jt}(s^{t-1}), l_{jt}(s^t), z_{jt}(s^t)),
\]

where

- \( R_{jt+1}(s^t, s_{t+1}) \) is one period real return in country \( j \);
- \( \Lambda_{jt}(s^t) \) is marginal utility of consumption after history \( s^t \).
Optimality Conditions: Habits and Adjustment Cost

Under habit formation preferences marginal utility of consumption is forward looking:

\[ \Lambda_{jt} = u_c(c_{jt}, h_{jt}, l_{jt}) \]

\[ + \lambda \beta E_t \left[ \sum_{\tau=0}^{\infty} (1 - \lambda)^\tau \beta^\tau u_h(c_{jt+\tau+1}, h_{jt+\tau+1}, l_{jt+\tau+1}) \right]. \]

One-period return takes into account costly capital adjustment:

\[ R_{jt+1} = \phi' \left( \frac{i_{jt+1}}{k_{jt+1}} \right) f_k(k_{jt+1}, l_{jt+1}, z_{jt+1}) \]

\[ + (1 - \delta) + \phi \left( \frac{i_{jt+1}}{k_{jt+1}} \right) - \phi' \left( \frac{i_{jt+1}}{k_{jt+1}} \right) \frac{i_{jt+1}}{k_{jt+1}}. \]
### Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences: Discount factor</td>
<td>$\beta$</td>
<td>0.989</td>
</tr>
<tr>
<td>Consumption share</td>
<td>$\gamma$</td>
<td>0.361</td>
</tr>
<tr>
<td>Utility curvature</td>
<td>$\sigma$</td>
<td>3.772</td>
</tr>
<tr>
<td>Habit intensity</td>
<td>$b$</td>
<td>0.73</td>
</tr>
<tr>
<td>Habit persistence</td>
<td>$\lambda$</td>
<td>0.75</td>
</tr>
<tr>
<td>Technology: Capital income share</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\sigma$</td>
<td>0.025</td>
</tr>
<tr>
<td>Productivity: Persistence of the productivity shocks</td>
<td>$A_{11}$</td>
<td>0.95</td>
</tr>
<tr>
<td>Spillover parameter</td>
<td>$A_{12}$</td>
<td>0</td>
</tr>
<tr>
<td>St. dev. of innovations to productivity</td>
<td>$\sigma^2_\varepsilon$</td>
<td>0.007</td>
</tr>
<tr>
<td>Correlation of innovations to productivity</td>
<td>$\rho$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Note: The time period is a quarter of a year. The adjustment cost parameter is set to match the relative standard deviation of investment in the data.
The social planner’s problem is solved with the Parameterized Expectations Approach (PEA).

PEA is not as vulnerable to the "curse of dimensionality" as state-space discretization methods due to its reliance on Monte-Carlo integration and endogenous oversampling.

Algorithms that rely on value function iteration can not rule out ex ante the values of decision variables that the agent would try very hard to avoid. (Agents might end up consuming negative habit adjusted consumption!).
The Results: International Co-movements

Getting investment co-movements right

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Time Separable Preferences (λ=0)</th>
<th>Non-Persistent Habits (b=0.73, λ=1)</th>
<th>Persistent Habits (b=0.73, λ=0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.56</td>
<td>0.06</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.46</td>
<td>0.72</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>Investment</td>
<td>0.43</td>
<td>-0.20</td>
<td>0.29</td>
<td>0.33</td>
</tr>
<tr>
<td>Employment</td>
<td>0.31</td>
<td>-0.39</td>
<td>-0.62</td>
<td>-0.68</td>
</tr>
</tbody>
</table>

Note: Domestic statistics of the Data column (Panel A,B, and D) correspond to the U.S. quarterly time series sample 1970:1-2008:2. International co-movements statistics are calculated from U.S. data and aggregated data of 15 European countries. The model statistics are computed from a single simulation on a 100,000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with the smoothing parameter of 1600.
The Results: Within Country Business Cycles

<table>
<thead>
<tr>
<th>Panel</th>
<th>Data</th>
<th>Time Separable Preferences (λ=0)</th>
<th>Non-Persistent Habits (b=0.73, λ=1)</th>
<th>Persistent Habits (b=0.73, λ=0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A - Volatilities - Standard deviation (in %)</td>
<td>Output</td>
<td>1.51</td>
<td>0.80</td>
<td>0.78</td>
</tr>
<tr>
<td>Standard deviations relative to output</td>
<td>Consumption</td>
<td>0.81</td>
<td>0.41</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
<td>2.88</td>
<td>2.88</td>
<td>2.88</td>
</tr>
<tr>
<td></td>
<td>Employment</td>
<td>0.84</td>
<td>0.43</td>
<td>0.40</td>
</tr>
<tr>
<td>Panel B - Correlations with output</td>
<td>Consumption</td>
<td>0.86</td>
<td>0.93</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
<td>0.94</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Employment</td>
<td>0.88</td>
<td>0.97</td>
<td>0.94</td>
</tr>
<tr>
<td>Panel C - Autocorrelations</td>
<td>Output</td>
<td>0.87</td>
<td>0.73</td>
<td>0.73</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.88</td>
<td>0.73</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
<td>0.90</td>
<td>0.71</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>Employment</td>
<td>0.92</td>
<td>0.73</td>
<td>0.74</td>
</tr>
</tbody>
</table>
The Persistence of Memory

Corr(I₁, I₂)

Persistence parameter, λ

Corr(l₁, l₂)

Persistence parameter, λ

Corr(cₜ, cₜ₋₁)

Persistence parameter, λ

St.dev(c)/St.dev(y)

Persistence parameter, λ
Responses to a positive productivity shock in Country 1

Time-Separable Preferences

Investment in Country 1

Consumption in Country 1

Investment in Country 2

Consumption in Country 2
Responses to a positive productivity shock in Country 1
Internal Habits vs. Time-Separable Preferences

Investment in Country 1
Consumption in Country 1

Investment in Country 2
Consumption in Country 2
Productivity shocks follow a stationary VAR(1) process in logs:

\[
\begin{bmatrix}
\log z_{1t} \\
\log z_{2t}
\end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{11} \end{bmatrix} \begin{bmatrix}
\log z_{1t-1} \\
\log z_{2t-1}
\end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}
\]

Prediction of the International RBC models are sensitive to the specification of the forcing process Baxter and Crucini (1995). This is especially important for the models with restricted international markets.

For instance, a model with endogenously incomplete financial markets of Kehoe and Perri (2002) predicts positive co-movements for investment for the process we use here, but negative for BKK process.
Parameterization of the Forcing Process

Persistence of the Technology Shocks, $A_{11}$

International Spillover of the Technology Shocks, $A_{12}$

Corr($y_1,y_2$)
Corr($c_1,c_2$) − Corr($y_1,y_2$)
Corr($I_1,I_2$)
Corr($l_1,l_2$)
Concluding Remarks

- Standard international RBC models generate negative cross-country correlations of labor and investment. The opposite is true in the data.
- Introduction of internal habits in consumption helps to resolve this anomaly.
- Habit formation and costly capital adjustment complement each other in explaining investment comovements. Habits provide a channel through which the capital adjustment costs become larger than the opportunity costs of not investing in a more productive country.
- The other discrepancy is still present: predicted labor co-movements remain at odds with the data.