Risk Premiums, Curvature, and Technology Shocks

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Overview
Risk premiums are governed by curvature parameters and shock properties.

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[Related Literature] [Model] [Calibration] [Log-linear Model Results] [Non-linear Model Simulations] [Conclusion]
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- Temporary investment shock.

With CRRA = 9, the model can roughly match the means and volatilities of asset returns and the price-dividend ratio in long-run U.S. data.
Increasing the curvature of the capital production technology substantially raises the equity risk premium.

\[
k_{t+1} = B \left[ (1 - \lambda_t) k_t^{\psi_k} + \lambda_t i_t^{\psi_k} \right]^{1/\psi_k}, \quad \psi_k \equiv \frac{\sigma_k - 1}{\sigma_k}, \quad \sigma_k \in (0, \infty).
\]
Overview (continued)

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- Increasing the curvature of the output production technology has little effect on the equity risk premium.

\[ y_t = \left[ \theta \, k_t^\psi_y + (1 - \theta) \, \exp(z_t)^\psi_y \right] \frac{1}{\psi_y}, \quad \psi_y \equiv \frac{\sigma_y - 1}{\sigma_y}, \quad \sigma_y \in (0, \infty). \]
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Increasing the contribution from temporary vs. permanent productivity shocks serves to raise the mean and volatility of the equity risk premium.
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- Increasing the contribution from temporary vs. permanent productivity shocks serves to raise the mean and volatility of the equity risk premium.

- The temporary investment shock serves to raise the volatility of dividend growth, with little effect on return moments.
Related Literature (partial list)

- Explaining the Equity Premium in Endowment Economies
  - Habit or high risk version: Campbell & Cochrane (1999).
  - Prospect theory: Barberis, Huang, & Santos (2001).
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- **Asset Pricing in Production Economies**
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- **Asset Pricing in Production Economies**

- **Investment Shocks and Business Cycles**
  - Papanikolaou (2008).
Additive Habit Formation Implies High Risk Aversion

\[ U(\cdot) = \frac{(c_t - \gamma c_{t-1})^{1-\alpha} - 1}{1 - \alpha} \]

\[ \text{CRRA} \equiv -\frac{c_t U''(c_t)}{U'(c_t)} = \frac{\alpha}{1 - \gamma \exp(-\mu_{c,t})}, \]

where \( \mu_{c,t} = \text{consumption growth} \).
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where \(\mu_{c,t} = \) consumption growth.

Jermann (JME, 1998): \(\alpha = 5, \ \gamma = 0.82\)

U.S. consumption growth \(\approx 0.02 \Rightarrow \) CRRA = 25
RBC Model with Adjustment Costs and Investment Shocks

\[
\max_{c_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{c_t}{H_t} \right)^{1-\alpha} - 1 \right], \quad H_t = \exp \left( \gamma_h \bar{z}_t \right) \\
\text{CRRA} \equiv -\frac{c_t U''(c_t)}{U'(c_t)} = \alpha
\]

\[
c_t + i_t = \left\{ \theta k_t^{\psi_y} + (1 - \theta) \left[ \exp (z_t) \times 1 \right]^{\psi_y} \right\}^{\frac{1}{\psi_y}}, \quad \psi_y \equiv \frac{\sigma_y - 1}{\sigma_y}
\]

\[
z_t = \bar{z}_t + \varepsilon_t, \quad \Rightarrow \left\{ \begin{array}{l}
\bar{z}_t = \bar{z}_{t-1} + \mu + \tau_t, \quad \tau_t \sim N \left( 0, \sigma_\tau^2 \right) \\
\varepsilon_t = \rho \varepsilon_{t-1} + \eta_t, \quad \eta_t \sim N \left( 0, \sigma_\varepsilon^2 \right)
\end{array} \right.
\]
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\end{array} \right. \quad \tau_t \sim N(0, \sigma_\tau^2), \quad \eta_t \sim N(0, \sigma_\varepsilon^2)
\]

\[
k_{t+1} = B \left[ (1 - \lambda_t) k_t^\psi_k + \lambda_t i_t^\psi_k \right]^\frac{1}{\psi_k}, \quad \psi_k \equiv \frac{\sigma_k - 1}{\sigma_k},
\]

\[
\lambda_t = \lambda \exp (v_t), \quad v_t = \rho_v v_{t-1} + u_t, \quad u_t \sim N(0, \sigma_u^2)
\]
Consumption Euler Equation and Model Solution

\[ i_t f \left( \frac{k_{t+1}}{k_t}, v_t \right) = E_t M_{t+1} \left[ \frac{s_{k,t+1} y_{t+1} - i_{t+1}}{d_{t+1}} \right] + i_{t+1} f \left( \frac{k_{t+2}}{k_{t+1}}, v_{t+1} \right) \]

\[ M_{t+1} \equiv \beta \left[ \frac{c_{t+1}}{c_t} \right]^{-\alpha} \left[ \frac{H_{t+1}}{H_t} \right]^{-(1-\alpha)} \]

\[ f \left( \frac{k_{t+1}}{k_t}, v_t \right) \equiv \left( \frac{k_{t+1}}{B k_t} \right)^{\psi_k} \left[ \left( \frac{k_{t+1}}{B k_t} \right)^{\psi_k} - 1 + \lambda \exp(v_t) \right]^{-1} \]

\[ s_{k,t+1} \equiv \frac{\theta k_{t+1}^{\psi_y}}{\theta k_{t+1}^{\psi_y} + (1-\theta) \exp(z_{t+1})^{\psi_y}} \quad (s_{k,t+1} = \theta \text{ for Cobb-Douglas}) \]
Consumption Euler Equation and Model Solution

\[
i_t f \left( \frac{k_{t+1}}{k_t}, \nu_t \right) = E_t M_{t+1} \left[ s_{k,t+1} y_{t+1} - i_{t+1} \right] \underbrace{d_{t+1}}_{p_t} + i_{t+1} f \left( \frac{k_{t+2}}{k_{t+1}}, \nu_{t+1} \right) \underbrace{p_{t+1}}_{p_t}
\]

\[
M_{t+1} \equiv \beta \frac{c_{t+1}}{c_t}^{-\alpha} \left[ H_{t+1} / H_t \right]^{-(1-\alpha)}
\]

\[
f \left( \frac{k_{t+1}}{k_t}, \nu_t \right) \equiv \left( \frac{k_{t+1}}{B k_t} \right)^{\psi_k} \left[ \left( \frac{k_{t+1}}{B k_t} \right)^{\psi_k} - 1 + \lambda \exp(\nu_t) \right]^{-1}
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\]

\[
x_t \equiv i_t / c_t, \quad k_{n,t} \equiv k_t / \exp(z_t), \quad \text{(Stationary)}
\]

\[
x_t / \tilde{x} = \left[ k_{n,t} / \tilde{k}_n \right]^{\gamma_k} \exp(\gamma_\varepsilon \varepsilon_t + \gamma_\nu \nu_t), \quad \text{(Approx. solution)}
\]

\[
\tilde{x} \equiv \exp \left[ E \log (x_t) \right], \quad \tilde{k}_n \equiv \exp \left[ E \log (k_{n,t}) \right], \quad \text{(Approx. point)}
\]
Assess Pricing Variables

\[ \frac{p_t}{d_t} = \left[ \frac{x_t}{s_{k,t} - (1 - s_{k,t}) x_t} \right] f \left( \frac{k_{t+1}}{k_t}, v_t \right) \]

\[ R_{t+1}^e = \frac{p_{t+1} + d_{t+1}}{p_t}, \quad \left\{ \begin{array}{l} p_t = i_t f \left( \frac{k_{t+1}}{k_t}, v_t \right) \quad \text{(Equity)} \\ d_t = s_{k,t} y_t - i_t \end{array} \right. \]
Asset Pricing Variables

\[
p_t \frac{d_t}{s_{k,t} - (1 - s_{k,t}) x_t} = f\left(\frac{k_{t+1}}{k_t}, v_t\right)
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\[
R_{t+1}^e = \frac{p_{t+1} + d_{t+1}}{p_t}, \quad \begin{cases} p_t = i_t f\left(\frac{k_{t+1}}{k_t}, v_t\right) & \text{(Equity)} \\ d_t = s_{k,t} y_t - i_t \end{cases}
\]

\[
R_{t+1}^f = \frac{1}{p_{t+1}^{b,1}} = \frac{1}{E_t M_{t+1}}, \quad \text{(1-period bond)}
\]
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\]

\[
R_{t+1}^f = \frac{1}{p_{t+1}^{b,1}} = \frac{1}{E_t M_{t+1}}, \quad \text{(1-period bond)}
\]

\[
p_t^c = E_t M_{t+1} (1 + \delta p_{t+1}^c), \quad \delta \in (0, 1], \quad \text{(Consol with decaying coupon)}
\]

\[
R_{t+1}^c = \frac{1 + \delta p_{t+1}^c}{p_t^c}, \quad \Rightarrow \quad \text{Duration} = \frac{1}{1 - \delta \exp \left[ E \log \left( M_{t+1} \right) \right]}
\]
## Baseline Parameter Values (Non-linear model)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description/Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Coefficient of relative risk aversion.</td>
<td>9</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>Capital-investment substitution elasticity.</td>
<td>0.333</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Capital-labor substitution elasticity.</td>
<td>0.6</td>
</tr>
<tr>
<td>$\rho_{\varepsilon}$, $\rho_{\nu}$</td>
<td>Temporary shock persistence.</td>
<td>0.9</td>
</tr>
<tr>
<td>$\sigma_{\eta}$</td>
<td>Std. dev. consumption growth = 3.99 %.</td>
<td>0.0742</td>
</tr>
<tr>
<td>$\sigma_{\upsilon}$</td>
<td>Std. dev. dividend growth = 12.2 %.</td>
<td>0.1329</td>
</tr>
<tr>
<td>$\sigma_\tau$</td>
<td>Permanent shock volatility.</td>
<td>0</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Mean $s_{k,t} = 0.4 =$ capital share of income.</td>
<td>0.832</td>
</tr>
<tr>
<td>$B$</td>
<td>Mean $k_{t}/y_{t} = 3$.</td>
<td>1.06</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Mean $i_{t}/y_{t} = 0.25$.</td>
<td>0.00058</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean consumption growth = 1.98 %</td>
<td>0.0198</td>
</tr>
<tr>
<td>$\gamma_h$</td>
<td>Exogenous habit trend.</td>
<td>1.0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Mean $p_{t}/d_{t} = 26.6$.</td>
<td>0.943</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Consol duration = 10 years.</td>
<td>0.973</td>
</tr>
</tbody>
</table>
Impulse Response Functions

Output

Consumption

Investment

Capital Stock

Dividends

1-Period Bond Price

Equity Price

Consol Bond Price
Effect of Increasing Risk Aversion
Other parameters are recalibrated to match targets.
Effect of Less Curvature in Capital Technology

Other parameters are recalibrated to match targets.

![Graph showing the effect of capital technology curvature on returns and volatilities.](image)

- **Effect of Capital Technology Curvature on Returns**
  - Equity
  - Consol
  - 1-Period Bond

- **Effect of Capital Technology Curvature on Volatilities**
  - Equity
  - Consol
  - 1-Period Bond
Effect of Less Curvature in Output Technology
Other parameters are recalibrated to match targets.
Effect of Temporary Productivity Shock
Model matches U.S. consumption growth volatility only at baseline.

Effect of Temporary Productivity Shock on Returns

Effect of Temporary Productivity Shock on Volatilities
Effect of Permanent Productivity Shock
Other parameters are recalibrated to match targets.
Effect of Temporary Investment Shock

Model matches U.S. dividend growth volatility only at baseline.
Comparing Log-linear and Non-linear Model Solutions

Forecast variable is an estimated power function of state variables (similar to PEA).

**Investment–Consumption Ratio**

**Normalized Capital Stock**

**Price–Dividend Ratio**

**1–Period Bond Return**
Nonlinear Model Simulations
Rational price-dividend ratio exhibits high volatility and persistence.
Non-linear Model Simulations
U.S. and model returns exhibit time-varying moments.
## Asset Pricing Moments


<table>
<thead>
<tr>
<th>Variable</th>
<th>Statistic</th>
<th>U.S. Data</th>
<th>Nonlinear Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t / d_t$</td>
<td>Mean</td>
<td>26.6</td>
<td>26.7</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>13.8</td>
<td>12.6</td>
</tr>
<tr>
<td></td>
<td>Skew.</td>
<td>2.20</td>
<td>2.03</td>
</tr>
<tr>
<td></td>
<td>Kurt.</td>
<td>8.2</td>
<td>13.2</td>
</tr>
<tr>
<td></td>
<td>Corr. Lag 1</td>
<td>0.93</td>
<td>0.97</td>
</tr>
<tr>
<td>$R_t^e - 1$</td>
<td>Mean</td>
<td>8.0%</td>
<td>8.5%</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>20.4%</td>
<td>20.6%</td>
</tr>
<tr>
<td>$R_t^c - 1$</td>
<td>Mean</td>
<td>2.6%</td>
<td>6.6%</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>10.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>$R_t^f - 1$</td>
<td>Mean</td>
<td>1.1%</td>
<td>2.1%</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>4.7%</td>
<td>7.6%</td>
</tr>
</tbody>
</table>

Model results computed from 20,000 period simulation
## Business Cycle Moments: Standard Deviations

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ log (y&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>1871-2008</td>
<td>5.28%</td>
<td>4.47%</td>
</tr>
<tr>
<td>Δ log (c&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>1890-2008</td>
<td>3.99%</td>
<td>3.97%</td>
</tr>
<tr>
<td>Δ log (i&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>1930-2008</td>
<td>16.2%</td>
<td>7.14%</td>
</tr>
<tr>
<td>Δ log (d&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>1872-2008</td>
<td>12.2%</td>
<td>12.3%</td>
</tr>
<tr>
<td>Δ log (p&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>1872-2008</td>
<td>17.9%</td>
<td>18.9%</td>
</tr>
<tr>
<td>R&lt;sub&gt;e&lt;/sub&gt; - R&lt;sub&gt;c&lt;/sub&gt;</td>
<td>1900-2008</td>
<td>21.2%</td>
<td>5.99%</td>
</tr>
<tr>
<td>R&lt;sub&gt;e&lt;/sub&gt; - R&lt;sub&gt;f&lt;/sub&gt;</td>
<td>1900-2008</td>
<td>20.2%</td>
<td>22.3%</td>
</tr>
</tbody>
</table>

Model results computed from 20,000 period simulation.
Business Cycle Moments: Correlations with Output Growth

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dates</th>
<th>U.S. Data Corr. w/ $\Delta \log(y_t)$</th>
<th>Non-linear Model Corr. w/ $\Delta \log(y_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \log(y_t)$</td>
<td>1871-2008</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\Delta \log(c_t)$</td>
<td>1890-2008</td>
<td>0.48</td>
<td>0.96</td>
</tr>
<tr>
<td>$\Delta \log(i_t)$</td>
<td>1930-2008</td>
<td>0.23</td>
<td>0.89</td>
</tr>
<tr>
<td>$\Delta \log(d_t)$</td>
<td>1872-2008</td>
<td>0.35</td>
<td>0.79</td>
</tr>
<tr>
<td>$\Delta \log(p_t)$</td>
<td>1872-2008</td>
<td>0.14</td>
<td>0.98</td>
</tr>
<tr>
<td>$R^e_t - R^c_t$</td>
<td>1900-2008</td>
<td>0.21</td>
<td>0.93</td>
</tr>
<tr>
<td>$R^e_t - R^f_t$</td>
<td>1900-2008</td>
<td>0.19</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Model results computed from 20,000 period simulation.
Capital Share of GDP
Capital share is pro-cyclical in U.S. data.

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<thead>
<tr>
<th>Statistic</th>
<th>U.S Data 1947-2008</th>
<th>Non-linear Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $[s_{k,t}]$</td>
<td>0.360</td>
<td>0.407</td>
</tr>
<tr>
<td>Std. Dev. $[s_{k,t}]$</td>
<td>0.014</td>
<td>0.039</td>
</tr>
<tr>
<td>Corr $[s_{k,t}, s_{k,t-1}]$</td>
<td>0.80</td>
<td>0.95</td>
</tr>
<tr>
<td>Corr $[s_{k,t}, \Delta \log(y_t)]$</td>
<td><strong>0.28</strong></td>
<td><strong>0.29</strong></td>
</tr>
</tbody>
</table>

Model results computed from 20,000 period simulation.
Adding curvature to the capital production technology can help account for the behavior of asset prices in long-run U.S. data.
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Adding curvature to the output production technology improves business cycle moments (output growth and capital share).
Conclusion

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Model can match many quantitative features of the data under rational expectations, including high volatility of price-dividend ratio.