Asset Return Dynamics under Bad Environment-Good Environment Fundamentals

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Motivation: bridge existing literatures

Consumption-based asset pricing literature

- mostly focuses on matching unconditional stock and bond statistics
  - e.g. mean and volatility of equity returns, risk free rate
- most models totally fail to produce realistic options prices
  - but options price data and volatility dynamics are likely very informative about underlying economics in financial markets
Motivation: bridge existing literatures

Option pricing literature

- Chernov and Ghysels (2000), Pan (2002), etc.
- focuses on matching price data and volatility dynamics
  - e.g. the “variance premium”
- but usually takes stock return process as exogenous
  - jumps
  - stochastic volatility
  - stochastic volatility jumps
  - stochastic volatility stochastic volatility
Motivation: bridge existing literatures

We seek to integrate these two literatures

- introduce a new consumption-based asset pricing framework
  - novel non-Gaussian data generating process
  - closed-form asset price solutions
  - fits unconditional asset price statistics
  - fits options price statistics

- related to concurrent work by Drechsler and Yaron (2008), Bollerslev, Tauchen and Zhou (2008)
Our point of departure: consumption shocks are non-Gaussian

- two shocks: one is positively skewed, one is negatively skewed
- both are fat-tailed and have time-varying volatility and skewness
- “BEGE”
  - $BE = \text{bad environment: negative skewness dominates}$
  - $GE = \text{good environment: positive skewness dominates}$
Formally, consumption growth, $\Delta c_t$, follows

$$\Delta c_{t+1} = g + \sigma_{cp} \omega_{p,t} - \sigma_{cn} \omega_{n,t}$$

- $\omega_{p,t}$ and $\omega_{n,t}$ are gamma distributed with time-varying parameters
  - $n_t$ determines shape of the negative tail
  - $p_t$ determines shape of the positive tail
Examples of the BEGE density
BEGE densities have simple expressions for higher moments

\[ E_t \left[ (\Delta c_{t+1} - \bar{g})^2 \right] = \sigma_{cp}^2 p_t + \sigma_{cn}^2 n_t \]

\[ E_t \left[ (\Delta c_{t+1} - \bar{g})^3 \right] = 2\sigma_{cp}^3 p_t - 2\sigma_{cn}^3 n_t \]
We assume $p_t$ and $n_t$ follow persistent, square-root volatility processes

\[
p_t = \bar{p} + \rho_p (p_t - \bar{p}) + \sigma_{pp} \omega_{p,t}
\]
\[
n_t = \bar{n} + \rho_n (n_t - \bar{n}) + \sigma_{nn} \omega_{n,t}
\]

- $p_t$ and $n_t$ innovations are the same as those for consumption growth
Does consumption data exhibit non-Gaussian behavior?

- yes! (especially if one considers a long sample)

- estimated conditional variance (top) and 3\textsuperscript{rd} moment (bottom)

uses a projection of squared and cubed realized consumption growth onto a vector of lagged instruments
Our preference specification follows Campbell and Cochrane (1999)

- expected utility takes the form

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1 - \gamma} \right] \]

- \( H_t \) is an exogenous “external habit stock” with \( C_t > H_t \)
- (log) risk aversion, \( q_t \), rises as \( C_t - H_t \) falls
The pricing kernel has two factors

\[ m_{t+1} = \ln (\beta) - \gamma \Delta c_{t+1} + \gamma \Delta q_{t+1} \]

We model risk aversion as a persistent latent factor that is also subject to the consumption shocks

\[ q_{t+1} = \bar{q} + \rho_q (q_t - \bar{q}) + \sigma_{qp} \omega_{p,t+1} + \sigma_{qn} \omega_{n,t+1} \]

- “habit” requires that bad consumption outcomes raise risk aversion
Under no-arbitrage conditions, the short rate, $rrf_t$, is given by

$$\exp( rrf_t ) = E_t \left[ \exp( m_{t+1} ) \right]^{-1}$$

The solution for $rrf_t$ is affine in $p_t$, $n_t$ and $q_t$

- higher risk aversion, $q_t$, raises the short rate
  - investors desire to borrow to smooth utility

- higher $n_t$ or higher $p_t$ lowers the short rate
  - higher uncertainty increases precautionary desire to save
  - $n_t$ has a stronger precautionary effect than $p_t$
Under no-arbitrage conditions, and assuming that dividends equal consumption, the equity price-dividend ratio is

$$\frac{P_t}{D_t} = E_t \left[ \sum_{i=1}^{\infty} \exp \left( \sum_{j=1}^{i} (m_{t+j} + \Delta d_{t+j}) \right) \right]$$

- the solution for $P_t/D_t$ is exponential-affine in $q_t$, $n_t$ and $p_t$
- higher $q_t$ lowers $P_t/D_t$
  - → because long-lived risky assets are less desirable
- higher $n_t$ and $p_t$ raise $P_t/D_t$
  - → this is a term structure effect
We define the physical and risk-neutral equity return variance measures as

\[ pvar_t = E_t \left[ (\text{return}_{t+1} - E_t [\text{return}_{t+1}])^2 \right] \]

\[ qvar_t = E_t^Q \left[ (\text{return}_{t+1} - E_t^Q [\text{return}_{t+1}])^2 \right] \]

- both \( pvar_t \) and \( qvar_t \) are affine and increasing \( p_t \) and \( n_t^3 \)

\[ \text{under a linear approximation of equity returns} \]
The variance premium, $vprem_t$, is defined as

$$vprem_t = qvar_t - pvar_t$$

- at reasonable parameter values, $vprem_t$ is
  - positive
  - increasing in $n_t$
  - decreasing in $p_t$
Formal estimation strategy

We use classical minimum distance estimation to fit reduced-form statistics

- our basic data is monthly from December 1990-March 2009
- we use **means**, **volatilities** and **autocorrelations** of

<table>
<thead>
<tr>
<th>cons grow</th>
<th>cons grow variance</th>
<th>cons grow 3rd moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>short rate</td>
<td>div yld</td>
<td>return</td>
</tr>
<tr>
<td>pvar</td>
<td>vprem</td>
<td></td>
</tr>
</tbody>
</table>

- we also use some unconditional higher order consumption growth stats and some asset price correlations
Model performance: selected statistics

- model-implied moments in “[ ]”
- sample statistics with standard error in “( )”

<table>
<thead>
<tr>
<th></th>
<th>cons</th>
<th>grw</th>
<th>short rate</th>
<th>div yld</th>
<th>return</th>
<th>pvar</th>
<th>vprem</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>[0.0026]</td>
<td>[0.0009]</td>
<td>[-6.4227]</td>
<td>[0.0042]</td>
<td></td>
<td>[0.0016]</td>
<td>[0.0017]</td>
</tr>
<tr>
<td></td>
<td>0.0025</td>
<td>0.0010</td>
<td>-6.3948</td>
<td>0.0037</td>
<td></td>
<td>0.0021</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0003)</td>
<td>(0.0941)</td>
<td>(0.0042)</td>
<td></td>
<td>(0.0005)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>std</td>
<td>[0.0028]</td>
<td></td>
<td>0.0013</td>
<td>[0.3594]</td>
<td>[0.0398]</td>
<td></td>
<td>[0.0010]</td>
</tr>
<tr>
<td></td>
<td>0.0028</td>
<td>0.0012</td>
<td>0.3375</td>
<td>0.0433</td>
<td></td>
<td>0.0028</td>
<td>0.0013</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0001)</td>
<td>(0.0410)</td>
<td>(0.0045)</td>
<td></td>
<td>(0.0008)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>ac(1)</td>
<td>[0.0000]</td>
<td>0.9726</td>
<td>[0.9944]</td>
<td>[-0.0029]</td>
<td></td>
<td>[0.9954]</td>
<td>[0.6508]</td>
</tr>
<tr>
<td></td>
<td>-0.1947</td>
<td>0.9839</td>
<td>0.9830</td>
<td>0.0612</td>
<td></td>
<td>0.7584</td>
<td>0.6986</td>
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<tr>
<td></td>
<td>(0.0941)</td>
<td>(0.1666)</td>
<td>(0.2214)</td>
<td>(0.0976)</td>
<td></td>
<td>(0.0869)</td>
<td>(0.1644)</td>
</tr>
<tr>
<td>cons</td>
<td>[0.1254]</td>
<td></td>
<td>cons</td>
<td>[3.9318]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>grow</td>
<td>-0.1101</td>
<td></td>
<td>grow</td>
<td>3.7293</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>skew</td>
<td>(0.1924)</td>
<td></td>
<td>kurt</td>
<td>(0.2964)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The model matches the properties of conditional consumption variance well, but generates too little volatility in the conditional third moment.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std</th>
<th>ac(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>cons grow variance(_t)</strong></td>
<td>[0.0774]</td>
<td>[0.0408]</td>
<td>[0.6508]</td>
</tr>
<tr>
<td></td>
<td>0.0784</td>
<td>0.0330</td>
<td>0.7791</td>
</tr>
<tr>
<td></td>
<td>(0.0062)</td>
<td>(0.0063)</td>
<td>(0.0971)</td>
</tr>
<tr>
<td><strong>cons grow third moment(_t)</strong></td>
<td>[0.2704]</td>
<td>[0.5476]</td>
<td>[0.7922]</td>
</tr>
<tr>
<td></td>
<td>-0.1851</td>
<td>3.1936</td>
<td>0.7599</td>
</tr>
<tr>
<td></td>
<td>(0.5420)</td>
<td>(0.6286)</td>
<td>(0.0537)</td>
</tr>
</tbody>
</table>
The model matches the mean level of risk-neutral return variance near-perfectly, but generates too much conditional skewness and kurtosis.

<table>
<thead>
<tr>
<th>means</th>
<th>qvar^{1/2}_t</th>
<th>qskew_t</th>
<th>qkurt_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.20]</td>
<td>[-6.6]</td>
<td>[78.1]</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>-2.4</td>
<td>20.5</td>
<td></td>
</tr>
</tbody>
</table>

The model matches the strong negative contemporaneous correlation between stock returns and persistent changes in risk-neutral volatility.

<table>
<thead>
<tr>
<th>corr with Δqvar_t</th>
<th>return_t</th>
<th>return_{t-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-0.5141]</td>
<td>[0.0027]</td>
<td></td>
</tr>
<tr>
<td>-0.6291</td>
<td>0.0748</td>
<td></td>
</tr>
</tbody>
</table>
Model performance: summary

Overall, the model fit is very good, but there are some key misses

- volatility of return variance too low
- not enough volatility in the conditional third moment of consumption
- too much risk-neutral skewness and kurtosis of returns

This could owe to our sample period

- very benign consumption data, but fairly dramatic asset price data
We calculate some longer monthly consumption data from January 1929, and perform an alternative estimation

- recalculate the consumption statistics over the longer time period
- let model fit the long-run consumption statistics
  
  → the model misses largely disappear
  
  → suggests Great Depression may have left lasting imprint on asset prices
Loose ends and future work

Some remaining counterfactual BEGE model implications suggest extensions of the framework

- more realistic term structure dynamics
  - time-variation in the conditional mean of growth
- “flight-to-quality” (stock-bond correlation) effects
  - dividend process that is distinct from consumption

Further applications of the BEGE framework are planned

- term structure
- reduced-form return dynamics