Indeterminacy and E-stability in Real Business Cycle Models with Factor-Generated Externalities

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Introduction: E-stability puzzle and some relative literature;

Conditions for jointly indeterminacy and E-stability in model with separable utility function;

Conditions for IE in model with non-separable utility function;

Conclusion.
Introduction
E-stability puzzle (Evans and McGough, 2005)

Problem

The general reduced form of sunspot-driven RBC models can admit rational expectations that are both indeterminate and E-stable, while the calibrated structural models, such as Farmer and Guo (1994), generate equilibria which are indeterminate but always E-unstable.

Our aim

To find one-sector RBC model that can admit rational expectations that are both indeterminate and E-stable.
Indeterminacy: there exist a continuum of equilibria all converging to a common steady state;
Introduction

Two concepts

- **Indeterminacy**: there exist a continuum of equilibria all converging to a common steady state;

- **Expectational stability**: the forecast process under learning converges to the rational expectations equilibrium over time.
RBC model with production externalities and other type of nonconvexities may admit indeterminacy, see Benhabib and Farmer, 1994; Farmer and Guo, 1994; Wen, 1998; Schmitt-Grohè and Uribe, 1997.
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Critique 1: Calibrations of indeterminacy are empirically implausible, see Aiyagari (1995);
Introduction
Indeterminacy in one-sector RBC model

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- Critique 1: Calibrations of indeterminacy are empirically implausible, see Aiyagari (1995);

- Critique 2: Indeterminate results are not stable under adaptive learning dynamics, i.e. *E-stability puzzle*, see Evans and McGough (2005).
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- Agents know the correct reduced form equations of the model;
- Agents don’t know the true parameters;
- What they will do? – Agents act like econometricians and estimate unknown parameters in a correctly specified model using recursive least squares estimation.
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E-stability: The forecast process under learning converges to the rational expectations equilibrium over time.
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Bennett and Farmer (2000), Hintermaier (2003) show the possibility of indeterminacy under general felicity function.
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Seek regions of E-stability for sunspot solutions in discrete time RBC models with non-convexities as Benhabib-Farmer-Guo model (1994) and:

- Separable utility function, negative capital externalities with Cobb-Douglas technology;
- Non-separable felicity function and greater than one social returns to scale in capital;
Seek regions of E-stability for sunspot solutions in discrete time RBC models with non-convexities as Benhabib-Farmer-Guo model (1994) and:

- Separable utility function, negative capital externalities with Cobb-Douglas technology;
- Non-separable felicity function and greater than one social returns to scale in capital;

With general utility function, the necessary condition for joint indeterminacy and E-stability requires that the labour-demand curve is upward-sloping and steeper than Frisch labour-supply curve.
The model with separable utility function

In the Farmer-Guo model, the representative agent solves

\[
\max \sum_{t=0}^{\infty} E_t \rho_t \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{L_t^{1+\chi}}{1 + \chi} \right)
\]

subject to

\[
C_t + K_{t+1} \leq (1 - \delta) K_t + Y_t
\]

\[
Y_t = A_t K_t^a L_t^b
\]

where \(a + b = 1\), and

\[
A_t = \frac{\bar{K}_t^{a-\alpha} \bar{L}_t^{-\beta - b}}{}
\]

\(A_t\) : the factor-generated externalities

\(\bar{K}_t\) and \(\bar{L}_t\) : average levels of capital and labor.

\(a, b, \alpha, \beta > 0\) without additional restrictions. In particular, if \(a > \alpha\) we have negative capital externalities.
Corollary

Corresponding to each case in Proposition 3.1, respectively, the following are necessary conditions for indeterminacy:

(i) \(1 - \beta \rho (1 - \delta) + \chi < 0\), and \(\frac{\alpha}{a} > 1\) (Farmer and Guo, 1994);

(ii) \(1 - \beta + \chi > 0\), \(\sigma < 1\), and \(\frac{\alpha}{a} < 1\) (This paper);

(iii) \(\beta \rho (1 - \delta) < 1 + \chi < \beta\), \(\sigma < 1\), and \(\frac{\alpha}{a} < 1\) (Meng and Yip, 2008).
According to the conditions for E-stability, $b_c < 0$ plays a critical role, where

$$b_c = \frac{1 - \beta \rho (1 - \delta) + \chi}{1 - \beta + \chi} \quad (5)$$
The model with separable utility function

Numerical examples

\[ a = 0.3, \ b = 0.7, \ \delta = 0.025, \ \rho = 0.99, \ \chi = 0.08. \]
\[ a = 0.33, \ b = 0.67, \ \delta = 0, \ \rho = 0.99, \ \chi = 0, \ \beta = 1.005 \]
The model with general utility function

General utility function

\(u(C_t, L_t)\) satisfies the general concavity and normality assumptions

\[u_C > 0, u_L < 0,\]
\[u_{CC} \leq 0, u_{LL} \leq 0,\]
\[u_C u_{CL} - u_L u_{CC} < 0, u_L u_{CL} - u_C u_{LL} > 0,\]
\[u_{CC} u_{LL} - u_{CL}^2 \geq 0.\]
The model with general utility function

General utility function

Proposition

The necessary condition for both indeterminacy and E-stability are

\[
\left[ 1 - \left( 1 - \frac{\delta_{lc}}{\delta_{cc}} \right) (1 - \rho) \right] \beta < 1 + \delta_{ll} \delta_{lc} \frac{\delta_{lc}}{\delta_{cc}} < \beta
\]

which implies that the labor demand curve and the Frisch labor supply curve should cross with wrong slopes.
We assume that the felicity function is of a particular type (King, Plosser and Rebelo (1988)),

\[ u(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} v(L_t), \quad \text{for } \sigma > 0, \sigma \neq 1, \]

\[ = \log C_t - v(L_t), \quad \text{for } \sigma = 1, \]

Here \( v > 0 \). For \( \sigma \neq 1 \), \( (1 - \sigma)v' < 0 \), \( u \) is strictly concave and

\[ \sigma vv'' / (\sigma - 1) - (v')^2 \geq 0. \]
Proposition

The necessary conditions for joint indeterminacy and E-stability with KPR utility function are: when $\delta = 0$ and $\sigma \neq 1$,

$$
\left(1 - \frac{1-\rho}{\sigma}\right) \beta < 1 + \chi_1 < \beta
$$

$$
\left(\frac{\alpha}{a} - 1\right) \frac{1+\chi_2}{1+\beta} < \frac{1-\sigma}{\sigma}
$$

$$
\alpha - 1 > 0
$$

where

$$
\chi_1 = \left(v''v^{-1} - v'v^{-1} + v'v^{-1}\frac{1}{\sigma}\right) l^* \geq 0
$$

$$
\chi_2 = (v''v^{-1} - v'v^{-1}) l^* > 0
$$
Corollary

Corresponding to Proposition 4.2 of KPR utility function, the following are necessary conditions for indeterminacy and E-stability:

\[ \left(1 - \frac{1 - \rho}{\sigma}\right) \beta < 1 + \chi_1 < \beta, \; \alpha - 1 > 0 \; \text{and} \; \sigma < 1. \]
\[ a = 0.8, b = 0.2, \delta = 0, \rho = 0.99, \beta = 1.1 \]
We find a region of joint indeterminacy and E-stability in the one-sector model with factor-generated externalities when

- the felicity function is separable in consumption and leisure, there are negative capital externalities and the labor-demand curve is upward-sloping,
- the felicity function is non-separable, the social returns to scale in capital is greater than one and the slope of labor-demand curve is also larger than the one of labor-supply curve.

We give out a solution for the "stability puzzle".
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We find a region of joint indeterminacy and E-stability in the one-sector model with factor-generated externalities when

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We find a region of joint indeterminacy and E-stability in the one-sector model with factor-generated externalities when

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