Session 58
I6: Advanced Solution Techniques

Solving for a Cash-in-Advance economy using a Finite Element Method, and a Note on Velocity

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Summary of Talk

Review of Solution Methods
  Finite Element Method
  Parameterized Expectations Algorithm

Cash-in-Advance Model Economy
  implementation of PEA-FEM solution methodology

Velocity
  and High & Business Cycle Frequency

Possible Extensions: CIA with Information Frictions
Literature

FEM  - McGrattan (JEDC, 1996)
     - McGrattan & Marimon (Oxford Press, 1999)

PEA  - Christiano & Fisher (JEDC, 2000)

Speed and Accuracy
     - Aruoba, et.al. (JEDC, 1998)

CIA  - Lucas & Stockey (Econometrica, 1987)
     - Hodrick, et. al. (JPE, 1991)
Cash-in-Advance Model

Timing Digression

\{T_t, A_t\}

\{M_{t-1}, Z_{t-1}\}

\{c_t, n_t, Z_t/p_t, M_t/p_t, k_{t+1}\}

PERIOD t
Cash-in-Advance Model

Social Planner’s Problem

$$\max_{\{c_t, n_t, k_{t+1}, \frac{M_t}{p_t}, \frac{Z_t}{p_t}\}} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\tau} - 1}{1 - \tau} - \gamma n_t^{1+\gamma} \right\} \right\} \quad \beta \in (0, 1)$$

subject to the budget constraint

$$c_t + k_{t+1} + \frac{Z_t}{p_t} + \frac{M_t}{p_t} = Y_t + (1 - \delta)k_t + I_{t-1} \frac{Z_{t-1}}{p_t} + \frac{M_{t-1}}{p_t} + T_t$$

and a cash-in-advance constraint:

$$c_t \leq \frac{M_{t-1}}{p_t} + T_t$$
Cash-in-Advance Model

\[ Y_t = A_t h_\alpha^\gamma n_t^{1-\alpha}, \quad 0 < \alpha < 1. \]

\( A_t \) is a stochastic production technology

\( \theta_t \) is the growth rate of money supply

\[ \theta_t = M_t / M_{t-1} \]
Cash-in-Advance Model

$$\Pi_{wr} = \Pr[\Theta_{t+1} = \Theta(r)|\Theta_t = \Theta(w)], \text{ for } w \& r = 1, \ldots, Q$$

exogenous parameters state vector $\Theta = [\tilde{\theta}, \tilde{A}]$ $\Theta$ is of dimension $[Q, 2]$ $Q$ equals the product of the possible states of $\theta$ and $A$

$Q$ states of nature, $\sum_{z=1}^{Q} \Pi_{wz} = 1$

Markovian process with transitional probabilities $\Pi$
Cash-in-Advance Model

First Order Conditions

\[ \lambda_t = c_t^{\gamma} - \mu_t \]

\[ \gamma_n n_t^{\gamma} = \lambda_t (1 - \alpha) \frac{Y_t}{n_t} \]

\[ \lambda_t = \beta E_t \left\{ \lambda_{t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta) \right) \right\} \]

\[ \frac{\lambda_t}{p_t} = \beta E_t \left\{ I_t \frac{\lambda_{t+1}}{p_{t+1}} \right\} \]

\[ \frac{\lambda_t}{p_t} = \beta E_t \left\{ \frac{c_{t+1}^{\gamma}}{p_{t+1}} \right\} \]
Cash-in-Advance Model

Kuhn-Tucker Condition – CIA Constraint

\[ c_t \leq \left( \frac{M_{t-1}}{p_t} + T_t \right) \quad \text{and} \quad \mu_t \left[ c_t - \left( \frac{M_{t-1}}{p_t} + T_t \right) \right] = 0 \]
State space & functional forms of the economy

Partial state space $\Omega_t$ is composed of the possible realizations of the capital stock at time $t$ and the money supply at time $t - 1$.

$$\Omega = [k, \bar{k}] \times [M, \bar{M}]$$

Conditional on the current realization of the money growth parameter $\theta_t = \theta(w)$ and the technology parameter $A_t = A(w)$
State space & functional forms of the economy

Time invariant policy functions $\Upsilon_w$ & $P_w$, for all $w = [1, \ldots, Q]$

$$\Upsilon_w(\Omega_t) \equiv \Upsilon_t|\Theta_t = \Theta(w) = \beta E_t \left\{ \lambda_{t+1} \alpha \frac{Y_{t+1}}{k_{w,t+1}} + (1 - \delta) \right\}.$$  

$$P_w(\Omega_t) \equiv p_t|\Theta_t = \Theta(w) = P(k_t, M_{t-1}|\Theta_t = \Theta(w)).$$

Define $\bar{\Upsilon}$ and $\bar{P}$ as column vectors containing the policy functions $\Upsilon_w$ and $P_w$ for all $w \in \{1, \ldots, Q\}$, i.e. $\bar{\Upsilon} = [\Upsilon_1, \ldots, \Upsilon_Q]'$ $\bar{P} = [P_1, \ldots, P_Q]'$. 
State space & functional forms of the economy

Given the policy functions $\gamma_w(k_t, M_{t-1})$ and $P_w(k_t, M_{t-1})$

$$R^K_w(k, M_{-1}; \bar{\gamma}, \bar{P}) = \gamma_w(k, M_{-1}) - \beta \sum_{z=1}^{Q} \Pi_{wz}\{\gamma_z(\bar{k}_w, M)[\alpha A\bar{k}_w^{\alpha-1}\tilde{n}_z^{1-\alpha} + (1 - \delta)]\}$$

$$R^M_w(k, M_{-1}; \bar{\gamma}, \bar{P}) = \frac{\gamma_w(k, M_{-1})}{P_w(k, M_{-1})} - \beta \sum_{z=1}^{Q} \Pi_{wz}\left\{\frac{\tilde{c}_z^{-\tau}}{P_z(\bar{k}_w, M)}\right\}$$

for all $w = \{1, ..., Q\}$. 
State space & functional forms of the economy

Where the real variables are defined by

\[ \tilde{k}_w = A k^\alpha n_w^{1-\alpha} + (1 - \delta) k - c_w \]

\[ n_w = \left[ \frac{(1 - \alpha)}{\gamma_n} \Upsilon_w(k, M_{-1}) \right]^{\frac{1-\alpha}{\gamma+1}} \]

\[ c_w = \begin{cases} \Upsilon_w(k, M_{-1})^{-1} & \text{if } \mu_w = 0 \\ M/P_w(k, M_{-1}) & \text{if } \mu_w > 0 \end{cases} \]

and next period’s money supply is \( M = \theta(w)M_{-1} \)
Equilibrium of the economy (using functional forms)

The policy functions in \( \bar{\bar{Y}} \) and \( \bar{P} \), in combination with \( \bar{k}_w, n_w, c_w \& M \), are the solutions to \( R^K_w(k, M_{-1}; \bar{\bar{Y}}, \bar{P}) = 0 \) and \( R^M_w(k, M_{-1}; \bar{\bar{Y}}, \bar{P}) = 0 \), for all \( w \), along the space \( \Omega \), for each \( \Theta(w) \).
Finite Element Method

The true decision rules $\Upsilon_w(k, M-1)$ & $P_w(k, M-1)$ are replaced by the parametric approximations $\psi^h_w(k, M-1)$ & $p^h_w(k, M-1)$, for $w = \{1, ..., Q\}$.

$\psi^h_w$ & $p^h_w$ are approximated using an implementation of the finite element method, McGrattan (1996).
Finite Element Method

To create the approximate time invariant functions $\psi^h_w$ & $p^h_w$, the space $\Omega = [k, \bar{k}] \times [M, \bar{M}]$ is divided in $n_e$ nonoverlapping rectangular subdomains called "elements".

Parameterizations of the functions for each element, at each realization of $w$, are constructed using linear combinations of low order polynomials or "basis functions"; creating local approximations for each function.
Finite Element Method

The parametrized functions $v^h_w(k, M_{-1})$ & $p^h_w(k, M_{-1})$ are built as follows:

\[
v^h_w(k, M_{-1}) = \sum_{ij}^I^J v^w_{ij} W_{ij}(k, M_{-1})
\]

\[
p^h_w(k, M_{-1}) = \sum_{ij}^I^J p^w_{ij} W_{ij}(k, M_{-1})
\]

where $i = \{1, \ldots, I\}$ indicate capital nodes,

$j = \{1, \ldots, J\}$ indicate money supply nodes.
Finite Element Method

$W_{ij} (k, M_{-1})$ is a set of linear basis functions dependant on the element $[k_i, k_{i+1}] \times [M_j, M_{j+1}]$, for all $i, j$, over which the local approximations are performed.

$v_{ij}^w$ & $p_{ij}^w$ are vectors of coefficients to be determined.
Finite Element Method

The approximate solutions for $v_w^h (k, M_{-1})$ and $p_w^h (k, M_{-1})$, over the complete space $\Omega$ and for each $w$, are obtained by piecing together all the local approximations; therefore "piecewise linear functions".
Finite Element Method

\[ W_{ij} (k, M_{-1}) = \Psi_i (k) \Phi_j (M_{-1}) \]

The basis functions \( W_{ij} (k, M_{-1}) \) are constructed such that they take a value of zero for most of the space \( \Omega \), except for a small interval where they take a simple linear form.
Finite Element Method

\[ \Psi_i (k) = \begin{cases} \frac{k-k_i}{k_{i+1}-k_i} & \text{if } k \in [k_{i-1}, k_i] \\ \frac{k-k_i}{k_{i+1}-k_i} & \text{if } k \in [k_i, k_{i+1}] \\ 0 & \text{elsewhere} \end{cases} \]

\[ \Phi_j (M) = \begin{cases} \frac{M_{j-1}-M_j}{M_j+1-M_j} & \text{if } M_{j-1} \in [M_j, M_{j+1}] \\ \frac{M_{j+1}-M_j}{M_{j+1}-M_j} & \text{if } M_{j+1} \in [M_j, M_{j+1}] \\ 0 & \text{elsewhere} \end{cases} \]
Finite Element Method

\( \Psi_i (k) \) & \( \Phi_j (M_{-1}) \) have the shape of a continuous pyramid which peaks at nodal points \( k = k_i \) & \( M_{-1} = M_j \), respectively, and are only non-zero on the surrounding elements of these nodes.
Finite Element Method

The approximations $\psi^h_k (k, M_{-1}) \& p^h_k (k, M_{-1})$, for all $w$, are chosen to simultaneously satisfy the equations:

$$
\int_{M_{-1}}^{M} \int_{k}^{K} \omega (k, M_{-1}) R^K_w (k, M_{-1}; \bar{\psi}^h, \bar{p}^h) \, dk \, dM = 0,
$$

for $w = \{1,...,Q\}$

$$
\int_{M_{-1}}^{M} \int_{k}^{K} \omega (k, M_{-1}) R^M_w (k, M_{-1}; \bar{\psi}^h, \bar{p}^h) \, dk \, dM = 0,
$$

for $w = \{1,...,Q\}$
Finite Element Method

A Galerkin scheme employs the basis functions \( W_{ij}(k, M_{-1}) \) as weights for the residual equations and since the basis functions are only nonzero surrounding their nodes, \( R^K_w \) & \( R^M_w \) can be rewritten in terms of the individual elements:

\[
\sum_{e=1}^{n_e} \int_{\Omega_e} W_{ij}(k, M_{-1}) \cdot R^S_w(k, M_{-1}; \bar{u}^h, \bar{p}^h) \, dk \, dM = 0
\]

\( \forall \ i, j \ \& \ all \ w \ and \ S = \{K, M\} \)

where \( n_e \) is the total number of elements and \( \Omega_e \) is the capital and money stock domain covered by the element \( e \).
Finite Element Method

A Newton algorithm is used to find the coefficients for $[\bar{v}_s, \bar{p}_s]$ which solve for the nonlinear system of equations $H$:

$$H ([\bar{v}_s, \bar{p}_s]) = 0$$

where $H ([\bar{v}_s, \bar{p}_s])$ is denoted by

residual equations $R^K_w (k, M_{-1}; \bar{v}^h, \bar{p}^h) \& R^M_w (k, M_{-1}; \bar{v}^h, \bar{p}^h)$
Parameterized Expectations Algorithm

1. Create the parameterization $v^h_w(k, M_{-1})$ to approximate the value of $\Upsilon_w$, and $p^h_w(k, M_{-1})$ to approximate the value of $p_w$, using a Finite Element Method.

2. Initiate a recursive solution procedure by creating the conjecture that the cash-in-advance constraint for this economy, does not bind, i.e. $\mu_w = 0$, & $c_w \leq M/p_w$. This implies $c^{-\tau}_w = \lambda_w \equiv v^h_w(k, M_{-1})$.

Similar to that of Christiano & Fisher (2000)
3. Compute the values of $n_w$, of actual output $Y_w$, and of real money holdings $M/p^h_w(k, M_{-1})$.

4. Check whether the initial conjecture was correct. If "yes", go to next step.

   If not, the constraint binds: $\mu_w > 0$ & $c_w = M/p^h_w(k, M_{-1})$. The value of the CIA multiplier becomes $\mu_w = [c_w^{-\tau} - v^h_w(k, M_{-1})]$.

5. Obtain next period’s capital $\tilde{k}_w$, and next period’s technology and stock of money.
Parameterized Expectations Algorithm

6. Create the parameterizations $v^h_w(\tilde{k}, M) \& p^h_w(\tilde{k}, M)$.

7. Create the conjecture that $\mu_\gamma = 0, \& c_\gamma \leq \tilde{M}/p$. Consumption is automatically solved, i.e. $\tilde{c}_\gamma = \tilde{\lambda}_\gamma^{-1}$.

8. Compute the value of each possible $\tilde{n}_\gamma, \tilde{Y}_\gamma$, and $\tilde{M}/p^h_\gamma(\tilde{k}, M)$. 
Parameterized Expectations Algorithm

9. Check whether the conjecture in Step 9 was correct. If "yes", go to next step.

If not, the constraint binds: $\tilde{\mu}_z > 0$, & $\tilde{c}_z = \tilde{M}/p^h_z(\tilde{k}, M)$. The value of the CIA multiplier becomes $\tilde{\mu}_z = [\tilde{c}^{-\tau} - \nu^h_z(k, M)]$.

10. Construct the residual functions $R_K (k, M; \bar{\nu}^h, \bar{p}^h)$ & $R_M (k, M; \bar{\nu}^h, \bar{p}^h)$.

11. If the weighted approximations of the residual functions for each element are sufficiently close to zero then "stop", else update the vectors of coefficients $v^w_{ij}$ & $p^w_{ij}$, and go to Step 1.
Real Business Cycle Properties - Calibration

Table 2: Benchmark Economy

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$A$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\gamma$</th>
<th>$\tau$</th>
<th>$\theta^{ss}$</th>
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<tr>
<td></td>
<td>1.00</td>
<td>0.33</td>
<td>0.99</td>
<td>0.02</td>
<td>0.00</td>
<td>3.00</td>
<td>1.01727</td>
</tr>
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</table>

Real Business Cycle Properties - Calibration

\[
\theta_{t+1} = (1 - \rho_\theta)\theta^{ss} + \rho_\theta \theta_t + \gamma_\theta \ln(A_t) + \varepsilon^{\theta}_{t+1} \quad \text{where } \varepsilon^{\theta} \sim N(0, \sigma^{\theta^2}_{\varepsilon})
\]

\[
\ln(A_{t+1}) = (1 - \rho_A)\ln(A^{ss}) + \rho_A \ln(A_t) + \varepsilon^{A}_{t+1} \quad \text{where } \varepsilon^{A} \sim N(0, \sigma^{A^2}_{\varepsilon})
\]

\[
\rho_\theta = 0.727 \text{ and } \sigma_{\varepsilon^\theta} = 0.01.
\]

\[
\rho_A = 0.90 \text{ and } \sigma_{\varepsilon^A} = 0.05.
\]

\[
\gamma_\theta = -0.15
\]

Real Business Cycle Properties - Calibration

Case 1: constant technology

AR(1) Money growth
Floden (2008)

Case 2: with serially correlated technology

VAR
Discrete Markov – Tauchen (1986)
Mertens (WP2008)
Velocity: case of constant technology

Case of serially correlated money growth rate and constant technology

Figure 1: Cash-in-Advance Constraint
Velocity: case of constant technology

<table>
<thead>
<tr>
<th></th>
<th>$\tau = 1.0$</th>
<th>$\tau = 2.5$</th>
<th>$\tau = 3.0$</th>
<th>$\tau = 3.5$</th>
<th>Data</th>
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<tbody>
<tr>
<td>$cv(V)$</td>
<td>0</td>
<td>0.0210</td>
<td>0.0306</td>
<td>0.0385</td>
<td>1.9719$^2$</td>
</tr>
<tr>
<td></td>
<td>n.a.</td>
<td>(0.0081)</td>
<td>(0.0127)</td>
<td>(0.0161)</td>
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</tr>
<tr>
<td>$corr\left(V, \frac{C_t}{C_{t-1}}\right)$</td>
<td>0</td>
<td>-0.1634</td>
<td>-0.1534</td>
<td>-0.1442</td>
<td>-0.3537$^3$</td>
</tr>
<tr>
<td></td>
<td>n.a.</td>
<td>(0.0672)</td>
<td>(0.0663)</td>
<td>(0.0637)</td>
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</tr>
<tr>
<td>$corr\left(V, I\right)$</td>
<td>0</td>
<td>0.4319</td>
<td>0.4243</td>
<td>0.4167</td>
<td>0.5165$^3$</td>
</tr>
<tr>
<td></td>
<td>n.a.</td>
<td>(0.1472)</td>
<td>(0.1535)</td>
<td>(0.1540)</td>
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<tr>
<td>$corr\left(\pi, I\right)$</td>
<td>0.7059</td>
<td>0.7277</td>
<td>0.7300</td>
<td>0.7309</td>
<td>0.5135$^3$</td>
</tr>
<tr>
<td></td>
<td>(0.0334)</td>
<td>(0.0399)</td>
<td>(0.0413)</td>
<td>(0.0407)</td>
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</tr>
<tr>
<td>$corr\left(\pi, r\right)$</td>
<td>-0.5166</td>
<td>-0.5691</td>
<td>-0.5768</td>
<td>-0.5839</td>
<td>-0.4940$^3$</td>
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<tr>
<td></td>
<td>(0.0326)</td>
<td>(0.0375)</td>
<td>(0.0371)</td>
<td>(0.0364)</td>
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</tr>
<tr>
<td>$corr\left(\pi, \theta\right)$</td>
<td>0.1506</td>
<td>0.2535</td>
<td>0.2626</td>
<td>0.2816</td>
<td>0.1844$^2$</td>
</tr>
<tr>
<td></td>
<td>(0.1192)</td>
<td>(0.1264)</td>
<td>(0.1226)</td>
<td>(0.1287)</td>
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</table>

Numbers in parenthesis are standard deviations over 500 simulations.
Velocity: Case of serially correlated technology

Table 5: Simulated Moments

<table>
<thead>
<tr>
<th>$\rho = 0.73$</th>
<th>$\rho = 0$</th>
<th>$\rho = 0.73$</th>
<th>$\rho = 0.73$</th>
<th>Data</th>
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<td>$\gamma = 0$</td>
<td>$\gamma = 0.90$</td>
<td>$\gamma = 0.90$</td>
<td>$\gamma = 0.90$</td>
<td></td>
</tr>
<tr>
<td>$\phi = \phi^*$</td>
<td>$\theta = \theta^*$</td>
<td>$\phi = 0$</td>
<td>$\theta = -0.15$</td>
<td></td>
</tr>
<tr>
<td>$cv(V)$</td>
<td>0.0506</td>
<td>0</td>
<td>1.48E-06</td>
<td>0.0189</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>n.a.</td>
<td>(1.98E-05)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>$cv(Y)$</td>
<td>-0.10889</td>
<td>3.2691</td>
<td>2.2337</td>
<td>2.3432</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.301)</td>
<td>(0.281)</td>
<td>(0.311)</td>
</tr>
<tr>
<td>$corr(V, \frac{\pi_t}{\pi_{t-1}})$</td>
<td>-0.1584</td>
<td>0</td>
<td>0.0003</td>
<td>-0.1456</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>n.a.</td>
<td>(0.004)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>$corr(V, I)$</td>
<td>0.4543</td>
<td>0</td>
<td>0.0009</td>
<td>0.39205</td>
</tr>
<tr>
<td></td>
<td>(0.154)</td>
<td>n.a.</td>
<td>(0.013)</td>
<td>(0.138)</td>
</tr>
<tr>
<td>$corr(\pi, I)$</td>
<td>0.7500</td>
<td>-0.9684</td>
<td>0.7232</td>
<td>0.72808</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.039)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>$corr(\pi, r)$</td>
<td>-0.5768</td>
<td>-0.9891</td>
<td>-0.0419</td>
<td>-0.028016</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.005)</td>
<td>(0.141)</td>
<td>(0.138)</td>
</tr>
<tr>
<td>$corr(\pi, \theta)$</td>
<td>0.2626</td>
<td>-0.0070</td>
<td>0.0524</td>
<td>0.057785</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.028)</td>
<td>(0.029)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>$\sigma_{Yf}^H/\sigma_{Yf}^H$</td>
<td>48.286</td>
<td>0.0788</td>
<td>3.2387</td>
<td>3.1617</td>
</tr>
<tr>
<td></td>
<td>(1.230)</td>
<td>(0.003)</td>
<td>(0.822)</td>
<td>(0.944)</td>
</tr>
<tr>
<td>$\sigma_{BCf}/\sigma_{BCf}$</td>
<td>10.74</td>
<td>0.0260</td>
<td>0.6080</td>
<td>0.60346</td>
</tr>
<tr>
<td></td>
<td>(0.539)</td>
<td>(0.001)</td>
<td>(0.184)</td>
<td>(0.179)</td>
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</tbody>
</table>
Possible Extension:
Cash-in-Advance Model with Information Frictions

Timing Digression

\{T_t, A_t\}

\{M_{t-1}, Z_{t-1}\} 

\{M_{t-1}/p_t\} \rightarrow \{c_t, n_t, Z_t/p_t, k_{t+1}\}

PERIOD t

Similar to that of Svensson (1985) and Christiano & Eichenbaum (1992)
Cash-in-Advance Model with Information Frictions

$$\max_{\left\{ \frac{M_{t-1}}{p_{t-1}}, \frac{Z_{t-1}}{p_{t-1}} \right\}} E_{t-1} \left\{ \max_{\{c_t, n_t, k_{t+1}\}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\tau} - 1}{1 - \tau} - \gamma n_t \frac{n_t^{1+\gamma}}{1 + \gamma} \right\} \right\} \right\} \beta \in (0, 1)$$

subject to the budget constraint

$$c_t + k_{t+1} + \frac{Z_t}{p_{t-1}} + \frac{M_t}{p_{t-1}} = Y_t + (1 - \delta)k_t + I_{t-1} \frac{Z_{t-1}}{p_{t-1}} + \frac{M_{t-1}}{p_{t-1}} + T_t$$

and a cash-in-advance constraint with ex-ante prices:

$$c_t \leq \frac{M_{t-1}}{p_{t-1}} + T_t$$
## Cash-in-Advance Model with Information Frictions

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Information Friction</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c v (V)$</td>
<td>0.0306</td>
<td>0.4600</td>
<td>1.9719</td>
</tr>
<tr>
<td></td>
<td>(0.0127)</td>
<td>(0.0480)</td>
<td></td>
</tr>
<tr>
<td>$corr \left( V, \frac{C_t}{C_{t-1}} \right)$</td>
<td>-0.1534</td>
<td>-0.4072</td>
<td>-0.3537</td>
</tr>
<tr>
<td></td>
<td>(0.0663)</td>
<td>(0.073)</td>
<td></td>
</tr>
<tr>
<td>$corr (V, I)$</td>
<td>0.4243</td>
<td>-0.0482</td>
<td>0.5165</td>
</tr>
<tr>
<td></td>
<td>(0.1535)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>$corr (\pi, I)$</td>
<td>0.7300</td>
<td>-0.0018</td>
<td>0.5135</td>
</tr>
<tr>
<td></td>
<td>(0.0413)</td>
<td>(0.025)</td>
<td></td>
</tr>
<tr>
<td>$corr (\pi, r)$</td>
<td>-0.5768</td>
<td>-0.0564</td>
<td>-0.4940</td>
</tr>
<tr>
<td></td>
<td>(0.0371)</td>
<td>(0.189)</td>
<td></td>
</tr>
<tr>
<td>$corr (\pi, \theta)$</td>
<td>0.2626</td>
<td>0.9995</td>
<td>0.1844</td>
</tr>
<tr>
<td></td>
<td>(0.1226)</td>
<td>(0.0001)</td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parenthesis are standard deviations over 500 simulations.
Concluding Remarks

Fast and Accurate algorithm

Many possible applications –

  Occasionally binding constraints: Irreversible Investment, CIA, Inventory Investment, Credit restrictions –

Information Frictions

Labor Hoarding
Thanks