A Forecasting Metric for Evaluating DSGE Models

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Introduction

- New Keynesian DSGE models are being used forecasting and policy analysis at Central banks
- e.g. Bank of England, Riksbank, US Federal Reserve, ECB, Bank of Canada among others have developed their own versions of DSGE models
- No clear consensus on what criteria should be used to select a particular model
- I propose a metric that evaluates the models suitability for policy analysis
This paper is divided in two parts:

First, I motivate the structure of one step ahead forecast errors as central to policy making
- Use Bayesian comparison analysis to check if model accounts for this structure of one step ahead forecast errors we observe in sample data.
- This is similar to a moment matching exercise.

Second, look at the structural interpretation the model puts on the one step ahead surprises or "News".
- Discuss the Kalman gains on the one step ahead forecast errors implied by the model and evaluate these in terms of our prior beliefs about the true structure.
Motivating the Metric

- **Policy Meetings**: At any meeting, policy makers:
  - Look at the policy rate they had expected to carry out, $i_{t|t-1}$
  - Observe new available information since last meeting, $\nu_t$.
  - Decide changes to policy in light of the "News"
Policy Function

**Optimal policy can be written as a function of state variables**
- [Svesson & Woodford, 2005]
  \[ i_t = \alpha X_{t|t} \]

**Example: Taylor Rule**
- \[ i_t = a + b\pi_t + cy_{t|t} \]
  - \( \pi \) is inflation gap-observed
  - \( y \) is output gap- not observed

**Update in policy rule:**
- \[ i_t - i_{t|t-1} = \alpha (X_{t|t} - X_{t|t-1}) \]
  - The state variable, output gap, is never observed
  - Use Kalman filter to obtain the output gap
Applying the Kalman filter we get:

\[ X_{t|t} - X_{t|t-1} = \gamma_x (Z_t - Z_{t|t-1}) \]
\[ = \gamma_x (\nu_t) \]

- Z is the vector of observed variables
- \( \nu_t \equiv Z_t - Z_{t|t-1} \equiv \text{News from observed variables} \)
- \( \gamma_x \): Kalman gains for the state variables, X.
- Tells us how surprises in different observed variables affect the state variable, output gap.
Comparing Structure of Forecast Errors

- The variance-covariance of 1 step ahead forecast errors:
  \[ \Omega = \text{vcov}(\nu_t) \]

- Compare \( \Omega \) implied by DSGE model to that observed in the sample data.

- I will use a data counterpart estimate of \( \Omega \) that is free from structural assumptions
  - unlike the identifying restrictions imposed on structural VARs for impulse response matching on data side.

- I will report VAR(1) results for both actual and simulated data for simplicity
DSGE Model

- I evaluate the Smets-Wouters Model for the US (AER 2007)- state of the art New Keynesian model with sticky prices and wages.

- Observed Variables: seven macro series
  - GDP growth, consumption growth, investment growth, inflation, real wage growth, labor hours and the nominal interest rate

- Time period: Quarterly data, 1966 to 2004 (156 observations)

- Estimation Method: Bayesian estimation

- Reason: This model has been shown to forecast as well as certain atheoretical benchmarks like BVAR’s
Reminder

- Approximating Model: VAR(1)
- DSGE model point estimates to follow are based on the posterior mode of the estimated parameters
- First, I will only talk about point estimates
## Results

### The Main Diagonal of $\Omega$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Actual Data</th>
<th>DSGE Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta GDP$</td>
<td>0.73</td>
<td>0.84</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>$\Delta I$</td>
<td>1.78</td>
<td>1.85</td>
</tr>
<tr>
<td>Hours</td>
<td>0.53</td>
<td>0.59</td>
</tr>
<tr>
<td>$\Delta W$</td>
<td>0.55</td>
<td>0.54</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>Interest-Rate</td>
<td>0.23</td>
<td>0.24</td>
</tr>
</tbody>
</table>

**Table**: One period ahead forecast error standard deviations
## Results

### Table: One period ahead forecast error correlation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Actual Data</th>
<th>DSGE Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta GDP, \Delta C)</td>
<td>0.63</td>
<td>0.56</td>
</tr>
<tr>
<td>(\Delta GDP, \Delta I)</td>
<td>0.51</td>
<td>0.53</td>
</tr>
<tr>
<td>(\Delta GDP, \text{Hours})</td>
<td>0.53</td>
<td>0.61</td>
</tr>
<tr>
<td>(\Delta GDP, \Delta W)</td>
<td>0.06</td>
<td>0.19</td>
</tr>
<tr>
<td>(\Delta GDP, \text{Inflation})</td>
<td>-0.25</td>
<td>-0.08</td>
</tr>
<tr>
<td>(\Delta GDP, \text{Interest-Rate})</td>
<td>0.19</td>
<td>0.03</td>
</tr>
<tr>
<td>(\Delta C, \Delta I)</td>
<td></td>
<td>0.48</td>
</tr>
<tr>
<td>(\Delta C, \text{Hours})</td>
<td></td>
<td>0.38</td>
</tr>
<tr>
<td>(\Delta C, \Delta W)</td>
<td></td>
<td>0.23</td>
</tr>
<tr>
<td>(\Delta C, \text{Inflation})</td>
<td></td>
<td>-0.28</td>
</tr>
<tr>
<td>(\Delta C, \text{Interest-Rate})</td>
<td></td>
<td>0.27</td>
</tr>
<tr>
<td>(\Delta I, \text{Hours})</td>
<td></td>
<td>0.48</td>
</tr>
<tr>
<td>(\Delta I, \Delta W)</td>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td>(\Delta I, \text{Inflation})</td>
<td></td>
<td>-0.05</td>
</tr>
<tr>
<td>(\Delta I, \text{Interest-Rate})</td>
<td></td>
<td>0.19</td>
</tr>
<tr>
<td>(\text{Hours}, \Delta W)</td>
<td>-0.27</td>
<td>-0.02</td>
</tr>
<tr>
<td>(\text{Hours, Inflation})</td>
<td>0.01</td>
<td>0.08</td>
</tr>
<tr>
<td>(\text{Hours, Interest-Rate})</td>
<td>0.35</td>
<td>0.27</td>
</tr>
<tr>
<td>(\Delta W, \text{Inflation})</td>
<td>-0.21</td>
<td>-0.11</td>
</tr>
<tr>
<td>(\Delta W, \text{Interest-Rate})</td>
<td>-0.06</td>
<td>-0.07</td>
</tr>
<tr>
<td>(\text{Inflation, Interest-Rate})</td>
<td>0.11</td>
<td>0.31</td>
</tr>
</tbody>
</table>

- **Entries in green** are a close match
  - 1 step ahead FE \(\text{Corr}(\Delta GDP, \Delta I)\) and \((\Delta C, \text{Hours})\) is well matched by the model

- **Entries in red** are problematic
  - The model is unable to match the 1 step ahead FE \(\text{Corr}(\Delta C, \Delta I)\) and \((\text{Hours}, \Delta W)\)
Bayesian Inference: Prior and Posterior Predictive Analysis

- Bayesian Estimation:
  - Posterior density for structural parameters is estimated as a product of a fairly arbitrary prior and the likelihood function for observed data

- Prior and Posterior Predictive Analysis
  - The objective of prior and posterior predictive analysis is to ascertain the prior and posterior distribution of certain observable implications of the model that are of interest [John Geweke, 2005]

  - I am interested in the prior and posterior predictive distribution of the observed feature $\hat{\Omega}$ (the one step ahead vcov matrix of forecast errors) implied by the model
Semi Formal Metrics

- Where does $\hat{\Omega}_{data}$ lie in the prior predictive density
  - if lies in tail; says the sample is a freak from the standpoint of the prior

- Where does $\hat{\Omega}_{data}$ lie in the posterior predictive density
  - if lies in tail; says next sample is very unlikely to give values like $\hat{\Omega}_{data}$
Example: FE Corr($\Delta C, \Delta I$)

- **Vertical line:** 1 step ahead FE Corr($\Delta C, \Delta I$) in actual data
- **Green line:** The prior predictive density
- **Blue line:** The posterior predictive density
- The data feature is captured neither by the prior nor the posterior

Figure: 1 Period Ahead FE Correlation($\Delta C, \Delta I$)
**Figure:** Prior predictive graphs for 1 period ahead FE moments from a VAR(1). Green line is prior predictive, and red line is actual US data 1966-2004
Figure: Prior and Posterior predictive graphs. Green line is prior predictive, blue line is posterior predictive and red line is actual US data 1966-2004
Summary: Forecast Errors- Priors

- There is a disconnect between the priors on the structural parameters and our actual priors over the features of interest. For instance:
  - Actual Prior: Investment growth more volatile than Output Growth
  - Structural Parameters Prior: Investment and Output growth have similar volatility
  - Off diagonal elements of $\hat{\Omega}$ also show a similar disconnect

- Prior Predictive analysis of $\hat{\Omega}$ indicates potential problems with:
  - Prior specification over the structural parameters and/or
  - The structure of the DSGE model
Summary: Forecast Errors- Posteriors

- Diagonal elements of $\Omega$:
  - Smets- Wouters DSGE model does a good job

- Off-Diagonal elements:
  - Matches some off-diagonal elements of $\Omega$
  - Fails with respect to some key one step ahead FE Correlations such as:
    - $(\Delta I, \text{Interest Rates})$, $(\Delta C, \Delta I)$, $(\text{Hours}, \Delta W)$

- Not a co-incidence !!!

- The DSGE model has been fine tuned to match the variance of 1 step ahead forecast error since that has been the focus of the literature

- This moment matching exercise for the 1 step ahead forecast errors can be seen as an off-diagonal test of the model
  - The model fails this test.
The Kalman gains give a structural interpretation of the DSGE model.
- They tell us what $\nu_t$(News) implies for output gap and structural shocks in the model.

I calculate the prior and posterior distributions for Kalman gains.

Can not ask if they conform to data, since these are not a data feature.

I do ask if the prior and the posterior distributions for Kalman gains conform to our actual priors regarding the true structure.
Kalman Gains for Output Gap

- $\gamma_x$: Kalman gains for the unobserved state variables, $X$

  $$X_{t|t} - X_{t|t-1} = \gamma_x(Z_t - Z_{t|t-1})$$
  $$= \gamma_x(News)$$

- Example:
  - When consumption is surprisingly high, what does the model imply for the output gap

- The model says: higher than expected consumption growth is associated with an increase in output gap (actual minus potential).
  - output more or less unaffected
  - decline in potential output
### Results

<table>
<thead>
<tr>
<th>State Variable\FE</th>
<th>$\Delta y$</th>
<th>$\Delta c$</th>
<th>$\Delta i$</th>
<th>hours</th>
<th>$\Delta w$</th>
<th>$\pi$</th>
<th>int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>0.53</td>
<td>0.06</td>
<td>-0.13</td>
<td>0.46</td>
<td>0.05</td>
<td>-0.07</td>
<td>-0.43</td>
</tr>
<tr>
<td>pot. output</td>
<td>0.55</td>
<td>-0.24</td>
<td>-0.07</td>
<td>-0.09</td>
<td>-0.03</td>
<td>-0.08</td>
<td>-0.30</td>
</tr>
<tr>
<td>output gap (difference)</td>
<td>-0.02</td>
<td>0.30</td>
<td>-0.06</td>
<td>0.55</td>
<td>0.08</td>
<td>0.02</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

**Table:** Kalman Gains- Regression coefficients from running the update in the state variables on the one step ahead forecast errors in observed variables.
Figure: Prior and Posterior graphs for Kalman gains on output gap. Green line is prior, blue line is posterior, and red line is population posterior mode.
Kalman Gains for Structural Errors

- $\gamma_e$: Similarly we can obtain Kalman gains for the structural errors, $\epsilon$

\[
\begin{align*}
\epsilon_{t|t} - \epsilon_{t|t-1} &= \gamma_e(Z_t - Z_{t|t-1}) \\
\epsilon_{t|t} &= \gamma_e(News)
\end{align*}
\]

- Example:
  - When output comes in surprisingly high, what does the model say about the shocks that are responsible

- The Kalman gains tell us that surprisingly high output is associated with mainly 2 kinds of shocks in the model:
  - Pos. demand shock (govt. spending shock)
    - causes both output and potential output to ↑
  - Pos. supply shock (productivity shock)
    - causes both output and potential output to ↑

- The magnitude of the inefficient shocks corresponding to a positive surprise in output growth is too low to have any significant impact on the output gap in the model
## Results

<table>
<thead>
<tr>
<th>Shocks\FE</th>
<th>Δy</th>
<th>Δc</th>
<th>Δi</th>
<th>hours</th>
<th>Δw</th>
<th>π</th>
<th>int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>productivity</td>
<td>0.61</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.88</td>
<td>-0.09</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>risk premium</td>
<td>0.00</td>
<td>-0.33</td>
<td>0.00</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.38</td>
</tr>
<tr>
<td>govt. spend.</td>
<td>0.67</td>
<td>-0.63</td>
<td>-0.17</td>
<td>0.39</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>mon. policy</td>
<td>0.05</td>
<td>-0.14</td>
<td>0.00</td>
<td>-0.16</td>
<td>-0.04</td>
<td>-0.37</td>
<td>1.02</td>
</tr>
<tr>
<td>price mark-up</td>
<td>0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.08</td>
<td>-0.13</td>
<td>0.40</td>
<td>0.04</td>
</tr>
<tr>
<td>investment</td>
<td>-0.05</td>
<td>-0.21</td>
<td>0.24</td>
<td>0.09</td>
<td>-0.04</td>
<td>0.09</td>
<td>0.22</td>
</tr>
<tr>
<td>wage mark-up</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-0.01</td>
<td>-0.03</td>
<td>0.42</td>
<td>0.43</td>
<td>0.08</td>
</tr>
</tbody>
</table>

**Table:** Kalman Gains- Regression coefficients from running structural shocks on the one step ahead forecast errors in observed variables.
Figure: Prior and Posterior graphs for Kalman gains on structural shocks. Green line is prior, blue line is posterior, and red line is population posterior mode.
Conclusion

■ Show how to think about policy in terms of forecast errors
  - Important to consider the entire $\Omega$ instead of just the main diagonal
  - Evaluate the model based on the structural interpretation the model puts on News.

■ The arbitrary priors defined over the structural parameters do not necessarily correspond to our actual priors over the features of interest.

■ Whether the priors imposed on the model are informative or non-informative depends on the question being asked of the model. The same priors can be informative for some features and non-informative for others as shown here.

■ The posterior does a good job of matching the diagonal elements of $\hat{\Omega}$ but fails w.r.t. certain key off-diagonal one period ahead forecast error correlations.

■ The structural interpretation of the model suggests, contrary to our actual prior, that a forecast error on output growth has no impact on the output gap.