Unplanned Inventories and the Decline in GDP Volatility

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17 July 2009
Great Moderation

- Real GDP growth became less volatile in the mid-1980s.
- Possible explanations: Monetary Policy, Good luck, Inventory Management
- Why consider inventories?
First feature of the data

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Aggregate (I)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s.d.($\Delta y_t$)</td>
<td>1.082</td>
<td>0.510</td>
</tr>
<tr>
<td>s.d.($\Delta s_t$)</td>
<td>0.837</td>
<td>0.500</td>
</tr>
<tr>
<td>s.d.($\Delta^2 i_t$)</td>
<td>0.693</td>
<td>0.412</td>
</tr>
<tr>
<td>corr($\Delta s_t$, $\Delta^2 i_t$)</td>
<td>$-0.008$</td>
<td>$-0.392$</td>
</tr>
</tbody>
</table>
Pre-1984

- output much more variable than sales—undermined production smoothing role of inventories.

Post-1984

- output and sales equally variable, but both experienced a decline in variability
Second feature of the data

Figure 1: Output, sales and change in inventories
Forecasting role of inventories

- Simple VECM model

\[
\Delta y_t = c_{y,0} + \alpha_y (y_{t-1} - s_{t-1}) + \sum_{j=1}^{2} \gamma_{yy,j}\Delta y_{t-j} + \sum_{j=1}^{2} \gamma_{ys,j}\Delta s_{t-j} + e_{y,t}
\]

\[
\Delta s_t = c_{s,0} + \alpha_s (y_{t-1} - s_{t-1}) + \sum_{j=1}^{2} \gamma_{ss,j}\Delta s_{t-j} + \sum_{j=1}^{2} \gamma_{sy,j}\Delta y_{t-j} + e_{s,t}
\]
### Table 2. Error-correction coefficient estimates

<table>
<thead>
<tr>
<th></th>
<th>Pre-moderation sample</th>
<th>Post-moderation sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate (I)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>$-0.713 (0.184)$</td>
<td>$-0.320 (0.148)$</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>$-0.143 (0.165)$</td>
<td>$0.452 (0.150)$</td>
</tr>
</tbody>
</table>
Conclude

- forecasting role of inventory changes has altered
- inventory changes are less “unplanned” and more deliberate.

Why fewer unplanned inventories?

- change in the process of sales and/or inventories
- deliberate change in firm’s behavior towards more production smoothing resulting in more intentional inventory movements.
What we do

- Estimate an unobserved components model of output, sales, and change in inventories.
- Decompose inventory changes into planned and unplanned components.
- Study the role of inventories in moderating output using counterfactuals.
- Reconcile the estimates of the UC model with the VECM results.
- Comment on the production smoothing role of inventories.
Related literature and main findings

- Better inventory management as an explanation of the Great Moderation
  - sales versus inventories:
    - our finding: inventories can explain about half of the overall decline in output volatility.
  - within inventories, is it shocks or propagation?
    - our finding: shocks (inventory mistakes)
- Altered forecasting role of inventories also explained by the diminished role of inventory mistakes.
Basic equations

• Process of output, sales and inventories

\[ y_t = \tau_t + (y_t - \tau_t), \]
\[ s_t = \tau_t + (s_t - \tau_t), \]
\[ i_t = i^*_t + (i_t - i^*_t). \]

where \( i^*_t \) is target inventory, \( \tau_t \) is the common unobserved stochastic trend with a deterministic drift \( \mu \).
Permanent and transitory components

- Permanent components

\[ i_t^* = \gamma_{i\tau} \tau_t + \kappa_t \quad \text{where} \quad 0 \leq \gamma_{i\tau} \leq 1, \]
\[ \tau_t = \mu + \tau_{t-1} + \eta_t, \quad \eta \sim i.i.d. N(0, \sigma_\eta), \]
\[ \kappa_t = \kappa_{t-1} + \nu_t, \quad \nu \sim i.i.d. N(0, \sigma_\nu). \]

- Transitory components

\[ \Psi_y(L)^{-1}(y_t - \tau_t) = \lambda_{y\eta} \eta_t + \lambda_{y\nu} \nu_t + \lambda_{y\varepsilon} \varepsilon_t + u_t, \]
\[ \Psi_s(L)^{-1}(s_t - \tau_t) = \lambda_{s\eta} \eta_t + \varepsilon_t, \]
\[ \Psi_i(L)^{-1}(i_t - i_t^*) = \lambda_{i\eta} \eta_t + \lambda_{i\nu} \nu_t + \lambda_{i\varepsilon} \varepsilon_t + u_t. \]
Interpretation of shocks

- Sales shocks ($\eta_t$ and $\epsilon_t$): technology and/or demand shocks.
- Inventory shocks
  - $v_t$ – shocks to inventory management practices
  - $u_t$ – errors made in correctly measuring permanent and temporary shocks to sales within the period.
- Key distinction between sales shocks and $u_t$
  - both reflect surprises, but by adjusting current output
    - ...firms can respond to sales shocks
    - and firms cannot respond to $u_t$ shocks and therefore cause unintentional inventory movements.
Inventories: planned and unplanned

Unplanned inventories changes defined as

$$\Delta i_{t}^{\text{unplanned}} = \Delta i_t - E_{t-1}(\Delta i_t)$$
$$= (\lambda_{y\eta} - \lambda_{s\eta})\eta_t + (\lambda_{ye} - 1)\epsilon_t + \lambda_{yv}\nu_t + u_t$$

Two components: (i) unintentional inventory mistakes and (ii) intentional changes in inventories in response to $\eta_t$, $\epsilon_t$, and $\nu_t$ shocks.

Planned inventory changes are

$$\Delta i_{t}^{\text{planned}} = \Delta i_t - \Delta i_{t}^{\text{unplanned}} = E_{t-1}(\Delta i_t).$$
Data and methodology

Data
- real GDP and sales data are from the BEA
- change in inventories is a residual given logarithms of output and sales data
- each series is multiplied by 100.

Methodology
- conduct maximum likelihood estimation
- explicitly model sales and inventory changes (output is modeled implicitly)
- assume an AR(2) process for transitory sales and an AR(1) process for transitory inventories.
### Table 3. Estimates of UC Model Parameters.


<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Estimates for the sales process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>1.407</td>
<td>0.780</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>0.214</td>
<td>0.256</td>
</tr>
<tr>
<td>$\phi_s^*$</td>
<td>0.612</td>
<td>0.750</td>
</tr>
<tr>
<td>$\lambda_{s\eta}$</td>
<td>$-0.476$</td>
<td>$-1.000$</td>
</tr>
<tr>
<td><strong>Estimates for the inventory process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>1.420</td>
<td>0.632</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.227</td>
<td>0.000</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>0.731</td>
<td>0.657</td>
</tr>
<tr>
<td>$\gamma_{it}$</td>
<td>0.387</td>
<td>0.386</td>
</tr>
<tr>
<td>$\lambda_{y\eta}$</td>
<td>$-0.476$</td>
<td>$-0.944$</td>
</tr>
<tr>
<td>$\lambda_{y\epsilon}$</td>
<td>0.000</td>
<td>0.192</td>
</tr>
<tr>
<td>$\lambda_{iv}$</td>
<td>$-1.000$</td>
<td>$-0.808$</td>
</tr>
</tbody>
</table>
Table 3: main findings

- Parameters of the process of sales and inventories have altered significantly across the two samples.
- Variability of shocks has declined after 1984, supporting the “good luck” hypothesis.
- Propagation parameters have changed after 1984.
Table 4

**Table 4. Volatility of key variables.**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$\Delta y_t$</td>
<td>1.029</td>
<td>0.491</td>
</tr>
<tr>
<td>$\Delta s_t$</td>
<td>0.838</td>
<td>0.485</td>
</tr>
<tr>
<td>$\Delta i_t$</td>
<td>0.687</td>
<td>0.369</td>
</tr>
<tr>
<td>$\Delta i_t^{unplanned}$</td>
<td>0.312</td>
<td>0.243</td>
</tr>
<tr>
<td>$\Delta i_t^{planned}$</td>
<td>0.612</td>
<td>0.277</td>
</tr>
</tbody>
</table>
Counterfactual experiments

- Objective
  - disentangle the role of inventories from that of sales in explaining the overall volatility decline.
  - within inventory process, is it shocks or changes in the propagation parameters?

- Example: Isolate the impact of reduced variability of inventory shocks
  - in the post-moderation period hold all the estimates, except $\sigma_v$, $\sigma_u$, at their pre-moderation levels
  - compute the implied volatility of output and sales growth.
### Table 5. Counterfactuals.

<table>
<thead>
<tr>
<th>Aggregate (I)</th>
<th>s.d.($\Delta y_t$)</th>
<th>s.d.($\Delta s_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-moderation</td>
<td>1.029</td>
<td>0.838</td>
</tr>
<tr>
<td>Post-moderation</td>
<td>0.491</td>
<td>0.485</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Implied post-moderation volatility</th>
<th>s.d.($\Delta y_t$)</th>
<th>s.d.($\Delta s_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory process alone</td>
<td>1.296</td>
<td>0.838</td>
</tr>
<tr>
<td>Inventory shocks</td>
<td>0.887</td>
<td>0.838</td>
</tr>
<tr>
<td>$u_t$ shocks alone</td>
<td>0.960</td>
<td>0.838</td>
</tr>
<tr>
<td>Propagation</td>
<td>1.387</td>
<td>0.838</td>
</tr>
<tr>
<td>Sales process alone</td>
<td>0.738</td>
<td>0.485</td>
</tr>
</tbody>
</table>
Inventory changes constitute 0.5 percent of total output.

...but they explain about 50 percent of the decline in output growth volatility.

Inventory shocks, $v_t$ and $u_t$, are critical

- explain about a quarter of the total decline in aggregate output growth
- out of this, unplanned inventory mistakes explain 50 percent.

A surprise: inventory propagation parameters have changed in way that would have made output growth about 20 – 30 percent more volatile.
Can unplanned inventories explain VECM results?

- VECM results indicate that inventory changes are less unintentional in the recent sample.
- UC model estimates suggest that there are no unplanned inventory mistakes—a key component of unplanned inventories—in the post-1984 sample.
- Can a change in the composition of unplanned inventories explain our VECM results?
UC model and VECM results

- Determine partial effects: $\frac{\partial \Delta y_{t+1}}{\partial \Delta i_{t}^{\text{unplanned}}}$ and $\frac{\partial \Delta s_{t+1}}{\partial \Delta i_{t}^{\text{unplanned}}}$

  - compute the impact of each shock on (i) unplanned inventories, and (ii) on future output and sales growth.
  - for instance for $\eta_t$, compute $\frac{\partial \Delta y_{t+1}}{\partial \eta_t}$ and $\frac{\partial \Delta i_{t}^{\text{unplanned}}}{\partial \eta_t}$

- Determine the weight of each shock

  - weights have changed as the variability of shocks has not declined proportionately.
  - weight is given by the ratio of the standard deviation of the shock relative to the standard deviation of unplanned changes in inventories.
Table 6. Marginal effects of shocks.

Permanent shocks

\[
\frac{\partial \Delta y_{t+1}}{\partial \Delta i_{t,\text{unplanned}}} = \frac{\lambda_s \eta (\phi_{s,1}-1) + \lambda_i \eta (\phi_i-2) - \gamma_i \tau}{\gamma_i \tau + \lambda_i \eta}
\]

\[
\frac{\partial \Delta s_{t+1}}{\partial \Delta i_{t,\text{unplanned}}} = \frac{\lambda_s \eta (\phi_{s,1}-1)}{\gamma_i \tau + \lambda_i \eta}
\]

Transitory shocks

\[
\frac{\partial \Delta y_{t+1}}{\partial \Delta i_{t,\text{unplanned}}} = \frac{\lambda_i \eta (\phi_i-2) - 1}{1 + \lambda_i \eta}
\]

\[
\frac{\partial \Delta s_{t+1}}{\partial \Delta i_{t,\text{unplanned}}} = 0
\]

\[
\frac{(\phi_{s,1}-1) + \lambda_i \epsilon (\phi_i-2)}{\lambda_i \epsilon}
\]
Table 7

<table>
<thead>
<tr>
<th></th>
<th>Permanent shocks</th>
<th>Transitory shocks</th>
<th>Total impact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta_t$</td>
<td>$\nu_t$</td>
<td>$\epsilon_t$</td>
</tr>
<tr>
<td>Aggregate (I)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-moderation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weights</td>
<td>0.00</td>
<td>0.00</td>
<td>0.47</td>
</tr>
<tr>
<td>$\frac{\partial \Delta y_{t+1}}{\partial \Delta i_{t, \text{unplanned}}}$</td>
<td>NA</td>
<td>NA</td>
<td>$-0.98$</td>
</tr>
<tr>
<td>$\frac{\partial \Delta s_{t+1}}{\partial \Delta i_{t, \text{unplanned}}}$</td>
<td>NA</td>
<td>NA</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-moderation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weights</td>
<td>0.03</td>
<td>0.25</td>
<td>0.72</td>
</tr>
<tr>
<td>$\frac{\partial \Delta y_{t+1}}{\partial \Delta i_{t, \text{unplanned}}}$</td>
<td>9.52</td>
<td>0.44</td>
<td>$-0.75$</td>
</tr>
<tr>
<td>$\frac{\partial \Delta s_{t+1}}{\partial \Delta i_{t, \text{unplanned}}}$</td>
<td>8.48</td>
<td>0</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 7. UC model and forecasting.
Change in weights

- Pre-1984: weight of $u_t$ greater than 50 percent
- Post-1984: role of $u_t$ completely diminished and weight of $\epsilon_t$ is almost 75 percent.

The weighted marginal impact of unplanned inventory changes on output and sales matches VECM results in Table 2.

Therefore, accumulation of unplanned inventories, weighted appropriately, predicts a pattern of behavior which is consistent with the pattern generated by overall inventories.
Is there production smoothing?

- Doubts about production smoothing role of inventories in the pre-1984 sample
  - output is much more variable than sales
  - positive correlation between sales and inventory.
- Table 1 suggests that these aspects of the data have changed after 1984.
- Results from counterfactual experiments suggest otherwise.
Has the objective for holding inventories—to smooth production or to avoid stock-outs—changed?

Use a standard cost minimization problem

- determine the implied values of $\lambda$s that are consistent with these objective
- examine whether the estimates of these coefficients can directly shed some light on the underlying reasons for holding inventories.
Cost minimization problem

\[
\lim_{T \to \infty} \min_{\{i_t+j\}_{j=0}^T} E_t \sum_{j=0}^T b^j C_{t+j}
\]

(1)

where

\[
C_t = 0.5a_0(\Delta y_t)^2 + 0.5a_1(y_t - y_t^*)^2 + 0.5a_2(\Delta i_t)^2 + 0.5a_3(i_t - i_t^*)^2 + u_{ct}y_t.
\]

- cost of changing output is given by \(a_0(\Delta y_t)^2\) and \(a_1(y_t - y_t^*)^2\).
- stock-out avoidance motive is given by \(a_2(\Delta i_t)^2\) and \(a_2(i_t - i_t^*)^2\).
- assume that the process of sales is given by our UC model.
## Table 8. Implied Role of Inventories.

<table>
<thead>
<tr>
<th></th>
<th>PS</th>
<th>SA</th>
<th></th>
<th>PS</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long run response (II)</strong></td>
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<tr>
<td><strong>Permanent shock to sales (A)</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\lambda_{s\eta}$</td>
<td>0</td>
<td></td>
<td>$\lambda_{s\eta}$</td>
<td>−1</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{y\eta}$</td>
<td>0</td>
<td>$\gamma_{i\tau}$</td>
<td>0</td>
<td>$\gamma_{i\tau}−1$</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{i\eta}$</td>
<td>$−\gamma_{i\tau}$</td>
<td>0</td>
<td>1 − $\gamma_{i\tau}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>Temporary shock to sales (B)</strong></td>
<td></td>
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<tr>
<td>$\lambda_{s\epsilon}$</td>
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<tr>
<td>$\lambda_{y\epsilon}$</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{i\epsilon}$</td>
<td>−1</td>
<td>0</td>
<td></td>
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<tr>
<td><strong>Permanent shock to inventories (C)</strong></td>
<td></td>
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</tr>
<tr>
<td>$\lambda_{y\nu}$</td>
<td>0</td>
<td>1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{i\nu}$</td>
<td>$−1$</td>
<td>0</td>
<td></td>
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</tr>
</tbody>
</table>
Estimated response coefficients

- Comparing the implied $\lambda$s of Table 8 with the estimated coefficients in Table 3.
- Pre-1984
  - short-run: Scenario B is consistent with PS
  - long-run: Scenarios B and C are consistent with PS
  - why output was more volatile than sales?
- Post-1984
  - no overt change in behavior.
  - production smoothing role is slightly diminished.
  - example: long-run scenario A with $\lambda_{s\eta} = -1$—implied value under PS is $\lambda_{y\eta} = 0$ and estimated coefficient is $\lambda_{y\eta} = -0.944$. Estimated coefficient more consistent with SA.
Have costs changed?

- Can the cost function explain the following results?
  - PS was relevant in the pre-1984 sample but it seems less relevant in the recent sample
  - some evidence of a movement towards more of stock-out avoidance motive in the post-1984 sample.

- Study inventory dynamics in a partial equilibrium setting.
  - find optimal solution to the firm’s problem
Optimal level of inventories

- Optimal level of inventories are

\[ i_t = \pi_i i_{t-1} - \pi_i \sum_{j=0}^{\infty} b \pi_i E_t \left( \frac{1}{a_1} \{ u_{c,t+j} \} - \{ b \phi_s - 1 \} s_{t+j} \right), \]

where

\[ \pi_i = \frac{(1 + b + \frac{a_3}{a_1}) - \sqrt{-4b + (1 + b + \frac{a_3}{a_1})^2}}{2b} \]

- \( \pi_i \) is increasing in \( a_1 \) and decreasing in \( a_3 \).
- The optimal solution is

\[ i_t = \pi_i i_{t-1} + \Gamma_s \epsilon_{s,t}, \quad (2) \]

- \( \pi_i \) directly maps into the autoregressive coefficient of transitory inventories \( \phi_i \) of our UC model.
- \( \phi_i \simeq 0.70 \) in both periods. No increased role of production smoothing.
Conclusion

- Inventories played a key role in moderating the economy above and beyond a change in the behavior of sales.
- Disappearance of inventory mistakes, and not changes in inventory dynamics, explain
  - part of the observed output stabilization and
  - the altered forecasting role of inventories.
- We also show that PS role of inventories more evident in the pre-1984 period.