Calculating the Expectation in a Labor Division Model

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Demonstrate the process of heterogeneous agents emerge from initially homogeneous agents through an agent specialization process. Agent individuals specialize and globally we perceive division of labor.
Bridge bt. Macro-Micro Levels

Previous Phys. or Econ. did not cover the gap

Emergence:

Irreversible non-equilibrium macro level phenomena

Bottom-up Simulation

Simple reversible micro level interactions

Evolving Agents
Self-Reinforcement Learning

Previous Experiences (Memory)

Pos. & Neg. Feedback Balance

Adaptation
More Theory vs. More Reality

Games in Physics

• Simple Games to study cooperation & competition patterns. Strong in theory/weak in application

• B. Arthur’s El Farol bar problem 1994
• D. Challet and Y.-C. Zhang: Minority Game 1997
• Stanley, 2001, Similarities and Diffs. Between Phys. and Econ.

Agent-based Computational Economics (ACE)

Good at modeling adaptation

L. Tesfatsion, 2001-2002, *IEEE TEC, PNAS, Artificial Life*

Izumi 2001, foreign exchange market; Tassier 2001, labor market; Stock market (Stanley 2001 calibrated to S&P data, Le Baron 2001); M.W. Macy, Trust, cooperation, & market formation


Macy, Trust, cooperation, and market formation in the U.S. and Japan *PNAS* | May 14, 2002 | vol. 99 | Suppl. 3 | 7214-7220
Agents cooperation and specialization is an important issue

- John Holland, 2006, Agents study will not make significant progress if autonomous specialization is not achieved
• Bonabeau, E., Theraulaz, G. & Deneubourg, J.-L., a model of division of labor in social insects.

• Perspective of seeing the systems as the emergence of macroscopic patterns out of processes and interactions defined at the microscopic level.

• Can be extended to social insects to show that complex collective behavior may emerge from interactions among individuals that exhibit simple behaviors.


• Bonabeau et al.,
• Information sharing: pheromone (hormone).
• Threshold: The more individual performs a task, the lower is its response threshold with respect to stimuli associated with this task, and vice-versa.
• Learn to update the threshold.
• Some follow up works using pheromone in task allocation
Stick pulling experiment

No information shared

- A team of robots search a circular arena and pull sticks out of holes in the ground.
- The length of a stick has been chosen so that a single robot is incapable of pulling a stick out of the ground completely on its own, but collaboration between two robots is sufficient for solving this task.
- Agents learn the maximal length of time that a robot waits for the help of another robot while holding a stick (gripping time parameter, GTP).
• Researchers have all agreed that Specialization and division of labor should not be *a priori*, should develop in the process of system development.
• There are hardly any research on specialization and labor division in socio-economic systems (resemble human transactions).
Specialization of Agents with Expectations

- Specialization: agents tend to focus on one task, so on the macro level we observe division of labor

How can we model specialization and labor division?
Simple → Complex
Environment: N self-interested agents, resources, scattered on a $L \times L$ lattice. The initial quantity of each pack of resources is a constant $R$. Resources can be replenished.

Two Tasks: 1. Searching for resources. 2. Exploiting (mining) resources found.

Will agents specialize on one task?

Trade information of resources on a market.

No central planning.
No global info. sharing.
Periodic Boundary Conditions

• An agent steps out from one side, it will step in from the opposite side
  – If not, an agent may step out and disappear, we have to add new agents to the system which leads to wasting learning time
  – If not, an agent bounces back at the boundary, a small environment size has a large impact on the system, while a very large environment size consumes much computing power

• No more than one agent can occupy the same site on the lattice at the same time
• Trade takes place in a market
• A dealer determines the trading price based on willingness prices of sellers and buyers
• transaction cost

Previous Experiences
State Transition Graph

A: Mining → Mine → Exhausted? → Yes: Purchasing; No: Mining

B: Searching → Search → Found? → Yes: Possessing; No: Purchasing

C: Possessing → Offer sell willingness price to broker → Sold? → Yes: Searching; No: Mining

D: Purchasing → Offer buy willingness price to broker → Bought? → Yes: Mining; No: Searching
The Skills

- Searching range $r$
- $E$ equals the production per step
- Exploiting capability:
  \[ C_e = \frac{E}{R} \]
- Searching capability $C_s$ (Moore Neighborhood)
  \[ \frac{(2r + 1)^2}{(2r_{\text{max}} + 1)^2} \]
- density of resources $\rho$
- the probability of finding resources in one searching process is
  \[ 1 - (1 - \rho)^{(2r+1)^2} \]
Expectation | Willingness Prices

- If autarky, revenue is $R$
- If sell, get payment $p$, and save time $R/E$
- Expected productivity in this time interval: $\hat{R}$
- Agents would like to sell if $p + \hat{R} \times R/E \geq R$
- The relative price $p/R \geq 1 - \hat{R}/E$
Expectation | Willingness Prices

• The minimum price that the agent would like to accept to sell is called the willingness price for selling

\[ V_s = 1 - \frac{\hat{R}}{E} \]

• willingness price for selling

\[ V_s = 1 - \frac{\hat{R}}{E} \]
Expectation | Willingness Prices

- If an agent purchases resources, it can get revenue $R-p$ in the time interval $R/E$
- If it sticks to autarky, its expected revenue during the same time interval will be $\bar{R} \cdot R/E$
- The agent will buy the resources if $R - p \geq \bar{R} \cdot R/E$
- willingness price for purchasing $V_p = 1 - \hat{R} / E$
Expected Productivity in autarky

• Now we calculate the agent’s expected productivity $\hat{R}$ in the state of autarky.
• Assume $t_s$ is the expected time used to find one pack of resources. The time for an agent to find and finish exploiting a pack of resources is $t_s + R/E$. Then the productivity can be written as follows.

$$\hat{R} = \frac{R}{t_s + R/E}$$
The Market
- Double Auction

• The dealer list two series for the purchasing willingness price (in descending order) and selling willingness price (in ascending order)

• The dealer then select a price at which the number of sellers and the number of buyers are equal

\[ p = \frac{p_{sk} + p_{pk}}{2} \]
uniform-price or discriminatory-price auction

- *uniform*-price auction (set equal across all matched buyer-seller pairs)
- *discriminatory*-price auction (set individually for each matched buyer-seller pair)
In order to carry out a market transaction it is necessary to discover who it is that one wishes to deal with, to conduct negotiations leading up to a bargain, to draw up the contract, to undertake the inspection needed to make sure that the terms of the contract are being observed, and so on.


- The broker collects a portion of $p$ from the seller $\alpha \cdot p, 0 \leq \alpha \leq 1$
Transaction cost

\[ p(1 - \alpha) + \hat{R} \cdot R/E \geq R \]

\[ V_s = \frac{(1 - \hat{R} / E)}{(1 - \alpha)} \]
To what extent is information shared

• The information about resources locations to be shared at the market is confined to only individuals that trade with each other.
• No information about the willingness price of any agent could be accessed by any other agent(s).
Learning

Reinforcement learning: tendency to implement an action should be

strengthened if it produces favorable results
weakened if it produces unfavorable results

Cambridge, MA: Cambridge Univ. Press, 1998
A.E. Roth and I. Erev, Learning in extensive form games: Experimental data and
J. Nicolaisen, V. Petrov, Leigh Tesfatsion, Market Power and Efficiency in a
Computational Electricity Market With Discriminatory Double-Auction Pricing,
Roth–Erev Algorithm

- Track the observed intermediate-term behavior of human subjects over a wide variety of multi-agent repeated games with unique equilibria achievable using stage-game strategies.
Our Learning Mechanism

• RE algorithm updates the probability of selecting the possible prices next round
• We need a mechanism that the price can adapt instead of the probability to select that price
Update parameters at the end of each $\tau$ round

- A time interval $\tau$ is given
- Agents try to learn and adjust their parameters at each $\tau$ rounds (steps)
- Advantages:
  - Not possible to calculate profits each round
  - The system fluctuates less
Learning - Logistic skill adaptation

• Assume \( \Delta R_e \) and \( \Delta R_s \) are revenues achieved through exploiting and searching respectively in the time interval \( \tau \)

\[
\Delta C_s = \Delta R_s \cdot k \cdot C_s \cdot (1 - C_s)
\]

\[
\Delta C_e = \Delta R_e \cdot k \cdot C_e \cdot (1 - C_e)
\]

• will result in super agents with both high searching and high exploiting capabilities
Learning
- Normalized skill adaptation

• Searching capability+mining capability=1

\[
\Delta C_e = k \cdot \frac{\Delta R_e}{\Delta R} - C_e / (1 + k)
\]

\[
\Delta C_s = k \cdot \frac{\Delta R_s}{\Delta R} - C_s / (1 + k)
\]

• No growth for the total capabilities. No one is smarter than the other.
Learning
- Biased skill adaptation

• A recency factor: Agents may forget

• When an agent learns some new knowledge, it will forget a part of knowledge in another field
Learning
- Biased skill adaptation

• When \( \Delta R_e - \Delta R_s > 0 \), the agent gains more profit through mining instead of searching, so the agent’s mining ability will increase and searching ability will decrease a little. So, we have

\[
\Delta C_e = k_1 \cdot C_e \cdot (1 - C_e)
\]
\[
\Delta C_s = -k_2 \cdot C_s \cdot (1 - C_s)
\]

• where \( k_1 > k_2 > 0 \)
Learning - Biased skill adaptation

• When $\Delta R_e - \Delta R_s < 0$, the agent gains more profit through searching, so we have

$$\Delta C_e = -k_2 \cdot C_e \cdot (1 - C_e)$$

$$\Delta C_s = k_1 \cdot C_s \cdot (1 - C_s)$$
Learning
- Biased skill adaptation

• When \( \Delta R_e - \Delta R_s = 0 \)

\[
\Delta C_e = \frac{1}{2} (k_1 - k_2) \cdot C_e \cdot (1 - C_e)
\]
\[
\Delta C_s = \frac{1}{2} (k_1 - k_2) \cdot C_s \cdot (1 - C_s)
\]

\[
X = (k_1 - k_2)/2, Y = (k_1 + k_2)/2
\]
\[
\Delta C_e = (X + Y \cdot \text{sign}(\Delta R_e - \Delta R_s)) \cdot C_e \cdot (1 - C_e)
\]
\[
\Delta C_s = (X + Y \cdot \text{sign}(\Delta R_s - \Delta R_e)) \cdot C_s \cdot (1 - C_s)
\]
Measure of labor division

Order---Entropy

$P_i$ is the probability that agents spend $i, i = 0,1,2,\ldots,\tau$ time steps searching in the time interval $\tau$

Entropy: $S = -\sum_i P_i \log P_i$

Degree of Labor Div.: 0, when no labor division
1, with complete labor division

\[ m_1 = \frac{1}{N} \sum_{i=1}^{N} \left| 2 \cdot \frac{R_{s,i}}{R_i} - 1 \right| \]

\[ m_2 = \frac{1}{N} \sum_{i=1}^{N} \left| 2 \cdot \frac{T_{s,i}}{\tau} - 1 \right| \]

$m_1$: measure the portion of revenue due to search
$m_2$: measure the portion of time spend in searching
Simulation & Results

• Simulation Platforms: Swarm, Repast, our platform with C++

• Initial state:
  • agents randomly scattered,
  • resources randomly distributed with density $\rho$

• Parameter settings:

$L = \sqrt{100000} \approx 316$  $N = 100$  $R = 20$  $\rho = 0.001$  $\tau = 100$
Figure. 1. Degree of labor division versus transaction cost $\alpha$ under Logistic learning mechanism. $m_1$ is measured by revenue.
Which state has the largest entropy value?

- Total labor division?
- Autarky?
Figure. 2. Diagram for Entropy evolution under biased learning mechanism with $k_1=0.1$, $k_2=0$. 
**Figure. 3.** Entropy versus transaction cost $\alpha$ under biased learning mechanism. When $\alpha$ is between 0.4 and 0.7, entropy is high.
Figure 4. Entropy versus transaction cost $\alpha$ under logistic learning mechanism. Logistic learning mechanism cannot sustain complete labor division with even a small value of transaction cost, e.g., 0.1.
Figure. 5. Entropy versus transaction cost $\alpha$ under normalized learning mechanism. Under biased and logistic mechanism, the search efficiency will approach maximum value 1 in autarky state. Under normalized mechanism, the search efficiency will only be 0.5 when in autarky state, half the maximum value.
Why the high tail in the entropy vs. transaction cost curve for the normalized model (previous 3 slides)
Figure 5. The distribution of the probability that agents spend $tsi$ time steps searching in the time interval.

Normalized model corresponds to more micro-level states.
Figure 4. Diagram for productivity evolution under biased learning mechanism with $k_1=0.1$, $k_2=0$. 
Figure 5. Diagram for productivity evolution under capability normalization learning mechanism with $k=0.1$. 
The Sudden Jump in Productivity Curves

• We find that the jumps are due to the discrete mechanism in this model which prohibits agents to do multiple tasks in one time step. When an agent finishes mining the resources, it has to wait until the next time step to do another task.

• So, when a large portion of agents shift from spending three time steps (3-step-agents) to mine one pack of resources to spending two time steps (2-step-agents), or from two steps to one steps (1-step-agents), the productivity curve jumps.
Corresponding to Econ. Literature

• Management services and transaction cost are fundamentally interrelated in the determination of the level of division of labor and specialization, and will identify the key mechanisms of their co-evolution.

• X. Yang, and Y.-K. Ng, Specialization and Economic Organization, Elsevier, 1993.
Corresponding to Econ. Literature

- Each individual’s labor productivity increases as she narrows down her range of production activities.
- As shown by Yang (2001, Chapter 2), the aggregate production schedule for three individuals discontinuously jumps from a low profile to a high profile as each person jumps from producing three goods to a production pattern in which at least one person produces only one good (specialization).
Conclusion

• Homogeneous agents → heterogeneous agents through specialization (labor division) in a decentralized society
• The degree of labor division depends on transaction costs
• Entropy decreases in the evolution process. It’s the state of partial labor division that has the highest entropy.
• Productivity increases with specialization.
• Necessary condition: Initially homogenous agents turn to be heterogeneous agents must involve adaptation (learning).
• The symmetry breaking: path dependent
Thank You!

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