Risk premia in general equilibrium

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Motivation

1 Motivation

“... the challenge now is to understand the economic forces that determine the stochastic discount factor, or put another way, the rewards that investors demand for bearing particular risks.”

Campbell (2000, J. Finance)
Motivation

Research question: Which economic forces determine the risk premium?

standard asset pricing model: risk tolerance, betas

risk-premium puzzle, rare-disaster hypothesis (Barro-Rietz)

non-linearities and non-normalities important for DSGE models
Motivation

This paper:

shows that non-linearities and non-normalities in DSGE models are important and useful to generate key features of the risk premium

sheds light on the fundamental determinants of the equity premium, that is the reward investors demand for bearing particular risks, and the premium investors are willing to pay to hedge systematic risk

proposes the continuous-time formulation of DGSE models as a workable paradigm in the macro-finance literature
Motivation

Findings:

closed-form expressions for the risk premium in general equilibrium as a function of fundamental uncertainty and structural parameters

non-linearities affect the risk premium through the curvature of the policy function, can generate time-varying risk premia with CRRA preferences (agents with lower risk aversion may be willing to pay higher risk premia)

non-normalities accounts for the observed risk premium puzzle (Barro-Rietz rare disaster hypothesis)

clarification on asset pricing implications, default risk, implicit risk premium
Plan of the talk

2 Barro-Rietz rare-disaster hypothesis

3 DSGE model with non-linearities

4 Conclusion
Barro-Rietz rare-disaster hypothesis

2 Barro-Rietz rare-disaster hypothesis

One-good pure exchange economy (Lucas 1978, Econometrica)

\begin{equation}
    dA_t = \bar{\mu} A_t \, dt + \bar{\sigma} A_t \, dB_t + \bar{J}_t \, A_t \, dN_t
\end{equation}

$A_t$ production (perishable fruits), $B_t$ a standard Brownian motion, $\bar{J}_t = \exp(\bar{\nu}) - 1$ jump-size distribution, $N_t$ is a Poisson process with arrival rate $\lambda$

Two-assets: (1) risky equity share, and (2) government bond with default risk

\begin{equation}
    dp_t = \mu p_t \, dt + \sigma p_t \, dB_t + p_t - J_t \, dN_t, \quad dp_0(t) = p_0(t) r \, dt + p_0(t_\cdot) D_t \, dN_t,
\end{equation}

\begin{equation}
    D_t = \begin{cases} 
        0 & \text{with } 1 - q \\
        \exp(\kappa) - 1 & \text{with } q
    \end{cases}
\end{equation}

For illustration $q = 1$. 
Barro-Rietz rare-disaster hypothesis

Representative consumer,

\[ U_0 \equiv E \int_0^\infty e^{-\rho t} u(C_t) dt, \quad u' > 0, \ u'' < 0 \]

\[ \text{s.t.} \quad dW_t = (\mu_M W_t - C_t) dt + \sigma_M dB_t - \zeta_M W_t dN_t, \]

Closed economy, \( A_t = C_t \)

Simple Ramsey problem of optimal consumption given an uncertain yield of a composite asset (Merton 1971, J. Econ. Theory), defining

\[ \mu_M \equiv (\mu - r)w_t + r, \quad \sigma_M \equiv w_t \sigma, \quad \zeta_M \equiv 1 - e^\kappa - (e^\nu - e^\kappa)w_t. \]
Barro-Rietz rare-disaster hypothesis

Value function

\[ V(W_0) \equiv \max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} E_0 e^{-\rho t} u(C_t)dt \]

First-order condition

\[ u'(C_t) = V(W_t) \quad \Rightarrow \quad u''(C_t)C_W = V(W_W(W_t)) \]

implies \( C_t = C(W_t) \) and from the Euler equation we obtain

\[ \mu_M - E \left[ -\frac{u''(C_t)}{u'(C_t)}C_W W_t \sigma_M^2 + \frac{u'(C((1-\zeta_M)W_t))}{u'(C(W_t))} \zeta_M \lambda \right] = \rho - \frac{1}{dt} E \left[ \frac{du'(C_t)}{u'(C_t)} \right] \]
Barro-Rietz rare-disaster hypothesis

**Value function**

\[
V(W_0) \equiv \max_{\{C_t\}_{t=0}^{\infty}} E_0 \int_0^{\infty} e^{-\rho t} u(C_t) dt
\]

**First-order condition**

\[
u'(C_t) = V_W(W_t) \quad \Rightarrow \quad u''(C_t)C_W = V_{WW}(W_t)
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\]

\[
\underbrace{\text{certainty equivalent return on saving}}_{\text{cost of forgone consumption}}
\]
Barro-Rietz rare-disaster hypothesis

Value function

\[ V(W_0) \equiv \max_{\{C_t\}_{t=0}^\infty} E_0 \int_0^\infty e^{-\rho t} u(C_t) dt \]

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\]

- risk premium
- cost of forgone consumption
Barro-Rietz rare-disaster hypothesis

Implicit risk premium,

\[ RP \equiv -\frac{u''(C_t)}{u'(C_t)} C_W W_t \sigma_M^2 + \frac{u'(C((1 - \zeta_M) W_t))}{u'(C(W_t))} \zeta_M \lambda \]

Optimal consumption and portfolio choice

\[ V_W = u'(C_t), \quad w_t = -\frac{V_W(W_t) \mu - r}{V_{WW}(W_t) W_t} \frac{\sigma^2}{\sigma^2} - \frac{V_W((1 - \zeta_M) W_t) e^\nu - e^K}{V_{WW}(W_t) W_t} \frac{\sigma^2}{\sigma^2} \lambda \]

\[ \Rightarrow \quad \mu_M - r = RP - \frac{u'(C((1 - \zeta_M) W_t))}{u'(C(W_t))} (1 - e^K) \lambda \]

which defines the risk premium of the market portfolio (conditioned on no disaster)
Barro-Rietz rare-disaster hypothesis

Closed-form solution

CRRA preferences with risk aversion \( \theta \), general equilibrium conditions pin down asset prices \( \sigma_M = \bar{\sigma} \) and \( \zeta_M = 1 - e^\bar{V} \), conditioned on no disasters, imply

\[
\mu_M - r = RP - \left(1 - \zeta_M\right)^{-\theta}(1 - e^\kappa)\lambda
\]

\[
= \theta\bar{\sigma}^2 + e^{-\bar{V}\theta}(1 - e^\bar{V})\lambda - e^{-\bar{V}\theta}(1 - e^\kappa)\lambda
\]

The unconditional market risk premium, i.e., the expected rate of return on the market portfolio net of the expected rate of return on bills,

\[
\mu_M - r - (e^\kappa - e^\bar{V})\lambda
\]
<table>
<thead>
<tr>
<th>Parameters</th>
<th>(1)</th>
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<tbody>
<tr>
<td>No disasters</td>
<td>Baseline</td>
<td>Low $\theta$</td>
<td>High $\lambda$</td>
<td>Low $q$</td>
<td>Low $\bar{\mu}$</td>
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<td>$1 - e^\nu$ (size of disaster)</td>
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### Barro-Rietz rare-disaster hypothesis

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<td>0.067</td>
<td>0.028</td>
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<tr>
<td>Expected market rate</td>
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<td>0.06</td>
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<td>0.038</td>
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<td>Expected market rate, conditional</td>
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<td>Sharpe ratio, conditional</td>
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DSGE model with non-linearities

3 DSGE model with non-linearities

One-good production economy with constant returns to scale technology (Merton 1975, Rev. Econ. Stud.)

\[ Y_t = A_t F(K_t, L) \]

where

\[
\begin{align*}
    dA_t &= \bar{\mu} A_t dt + \bar{\sigma} A_t dB_t \\
    dK_t &= (I_t - \delta K_t) dt + \sigma K_t dZ_t + (e^\nu - 1) K_t dN_t
\end{align*}
\]

\( A_t \) total factor productivity, \( B_t \) and \( Z_t \) standard Brownian motions (uncorrelated), \( N_t \) is a standard Poisson process with arrival rate \( \lambda \)

One asset: physical capital can be accumulated
DSGE model with non-linearities

Representative consumer,

\[ U_0 \equiv E \int_0^\infty e^{-\rho t} u(C_t) dt, \quad u' > 0, \quad u'' < 0 \]

s.t. \[ dW_t = ((r_t - \delta)W_t + w_t^L - C_t)dt + \sigma W_t dZ_t + (e^\nu - 1)W_t dN_t \]

\( W_t \equiv K_t/L \) individual wealth, \( r_t \equiv Y_K \) capital rewards, \( w_t^L \equiv Y_L \) labor income

Closed economy, \( Y_t = C_t + I_t \)
DSGE model with non-linearities

Value function

\[ V(W_0, A_0) \equiv \max_{\{C_t\}_{t=0}^{\infty}} E_0 \int_0^\infty e^{-\rho t} u(C_t) dt \]

First-order condition

\[ u'(C_t) = V_W(W_t, A_t) \quad \Rightarrow \quad u''(C_t) C_W = V_{WW}(W_t, A_t) \]

implies \( C_t = C(W_t, A_t) \) and from the Euler equation we obtain

\[
E(r_t) - E \left[ -\frac{u''(C_t)}{u'(C_t)} C_W W_t \sigma^2 + \frac{u'(C(e^\nu W_t, A_t))}{u'(C(W_t, A_t))} (1 - e^\nu) \lambda \right] = \rho + \delta - \frac{1}{dt} E \left[ \frac{du'(C_t)}{u'(C_t)} \right]
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DSGE model with non-linearities

Value function

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\]

risk premium
DSGE model with non-linearities

Closed-form solution (linear-policy-function)

Cobb-Douglas production, CRRA preferences and a specific value for risk-aversion

\[ Y_t = A_t K_t^\alpha L^{1-\alpha}, \quad \frac{u''(C_t)C_t}{u'(C_t)} = \theta, \quad \theta < 1 \]

Optimal consumption is \( C_t = C(W_t) = aW_t \), i.e., proportional to wealth for

\[ \alpha = \theta \quad \Rightarrow \quad RP = \theta \sigma^2 + e^{-\theta \nu}(1 - e^{\nu})\lambda \]

equivalent to Lucas fruit-tree model !!
DSGE model with non-linearities

**Closed-form solution (constant-saving-function)**

Cobb-Douglas production, CRRA preferences and a specific value for risk-aversion

\[
Y_t = A_t K_t^{\alpha} L^{1-\alpha}, \quad \frac{u''(C_t)C_t}{u'(C_t)} = \theta, \quad \theta > 1
\]

Optimal consumption is \( C_t = C(W_t, A_t) = bA_t W_t^\alpha \), i.e., nonlinear in wealth for \( \rho = \bar{\rho} \)

\[
RP = \alpha \theta \sigma^2 + e^{-\alpha \theta \nu} (1 - e^\nu) \lambda
\]

risk-premium depends on the curvature of the policy function
DSGE model with non-linearities

**Numerical solutions**

Cobb-Douglas production, CRRA preferences, \( \sigma = \bar{\sigma} = \bar{\mu} = 0 \)

\[
Y_t = AK_t^\alpha L^{1-\alpha}, \quad \frac{u''(C_t)C_t}{u'(C_t)} = \theta
\]

Reduced form DSGE model

\[
\begin{align*}
\frac{dK_t}{dt} &= ((r_t - \delta)K_t + w_t^LL - LC_t)dt - (1 - e^\nu)K_t-dN_t \\
\frac{dC_t}{dt} &= \left(\frac{r_t - \delta - \rho - \lambda}{\theta} + \frac{(C(e^\nu W_t))^{-\theta} e^\nu \lambda}{(C(W_t))^{-\theta} \theta}\right)C_tdt + (C(e^\nu W_{t-}) - C(W_{t-}))dN_t
\end{align*}
\]

Calibration \((\rho, \alpha, \theta, \delta, \lambda, 1 - e^\nu) = (.03, .75, \cdot, .25, .017, .4)\)
DSGE model with non-linearities

Calibration $\theta = 0.75$ (dotted), $\theta = 4$ (dashed), and $\theta = 6$ (solid)
Appendix

4 Conclusion

This paper

- proposes the continuous-time formulation of DGSE models
  - closed-form expressions for the risk premium in general equilibrium
- non-linearities are important for the risk premium
  - curvature of the policy function matters
  - can generate time-varying risk premia for CRRA preferences
- non-normalities are important (Barro-Rietz rare disaster hypothesis)