Heterogeneous Expectations in Monetary DSGE Models

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Overview

- Benchmark for monetary policy analysis: New Keynesian model with a representative rational agent (RE)

- Learning as alternative to RE

- Evidence for heterogeneous expectations (HE)
  - Survey data: Carrol (2003), Branch (2004), Pfajfar and Santoro (2006)
Contribution of the paper

- Derivation of a general New Keynesian framework consistent with heterogeneous expectations
  - Explicit solution of the microfoundations underpinning the model
  - RE benchmark as a special case
- Estimation of the “degree of rationality” in the economy
  - Relevant for the conduct of a sound monetary policy
Households

- Intertemporal optimization problem

\[
\max \quad \tilde{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{c_{s}^{1-\sigma}}{1-\sigma} - \chi \frac{h_{s}^{1+\gamma}}{1+\gamma} \right)
\]

s.t. \quad c_{s} + b_{s} \leq w_{s} h_{s} + R_{s-1} \pi_{s}^{-1} b_{s-1} + d_{s}

- First order conditions in log-linear terms

\[
\tilde{E}_{i,t} \hat{c}_{i,s} = \tilde{E}_{i,t} \left[ \hat{c}_{i,s+1} - \sigma^{-1} \left( \hat{R}_{s} - \hat{\pi}_{s+1} \right) \right]
\]

\[
\tilde{E}_{i,t} \hat{h}_{i,s} = \frac{1}{\gamma} \tilde{E}_{i,t} \left( \hat{w}_{s} - \sigma \hat{c}_{i,s} \right)
\]
Individually consumption rule

- Iterate the flow budget constraint and impose the No Ponzi constraint to get the perceived lifetime budget constraint

- Iterating forward the Euler equation and substituting it in the intertemporal budget constraint together with the labor supply equation we finally get the individual consumption rule for agent $i$

\[
\hat{c}_{i,t} = \zeta_b \hat{b}_{i,t-1} + \hat{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} \left[ (\zeta_w \hat{w}_s + \zeta_d \hat{d}_s) - \frac{\beta}{\sigma} (\hat{R}_s - \hat{\pi}_{s+1}) \right]
\]
Firms

- Profit maximization

\[
\max \tilde{E}_{j,t} \sum_{s=t}^{\infty} \omega^{s-t} Q_s \left( \frac{P_{j,t}}{P_s} - \frac{W_s}{P_s} \right) \left( \frac{P_{j,t}}{P_s} \right)^{-\eta} c_s
\]

- First order condition

\[
\tilde{E}_{j,t} \sum_{s=t}^{\infty} \omega^{s-t} Q_s \left[ (1 - \eta) \frac{1}{P_s} + \eta \frac{1}{P_{j,t}} w_s \right] \left( \frac{P_{j,t}}{P_s} \right)^{-\eta} c_s = 0
\]
Individual pricing rule

- Log-linearizing firms’ first order condition we get the individual pricing rule

\[ \hat{p}_{j,t} = (1 - \omega \beta) \hat{E}_{j,t} \sum_{s=t}^{\infty} (\omega \beta)^{s-t} \hat{w}_s + \omega \beta \hat{E}_{j,t} \sum_{s=t}^{\infty} (\omega \beta)^{s-t} \hat{\pi}_{s+1} \]
Aggregate equations

- Aggregating individual decision rules and using equilibrium conditions we get the aggregate relations

**heterogeneous expectations-IS**

\[
\hat{y}_t = (1 - \beta) \tilde{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \hat{y}_s - \frac{\beta}{\sigma} \tilde{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \hat{R}_s - \hat{\pi}_{s+1} \right)
\]

**heterogeneous expectations-NKPC**

\[
\hat{\pi}_t = k \tilde{E}_t \sum_{s=t}^{\infty} (\omega/\beta)^{s-t} \hat{y}_s + (1 - \omega) \beta \tilde{E}_t \sum_{s=t}^{\infty} (\omega/\beta)^{s-t} \hat{\pi}_{s+1}
\]

where \( \tilde{E}_t = \int_0^1 \tilde{E}_{i,t} f(i) \, di = \sum_{h=1}^{H} n_{h,t} \tilde{E}_{h,t} \)
Introduction The HE model Dynamic feedback system Bayesian estimation

Specification of expectations

- Agents face **cognitive problems** in understanding and processing information (e.g., Kahneman and Thaler (2006) and Della Vigna (2007)). Therefore they eventually make **mistakes in forecasting** macroeconomic variables.

- Some agents, by paying some **information gathering and processing costs** $C \geq 0$ per period, have **rational expectations**.
Perfectly rational and boundedly rational agents

- Aggregate equations of the economy populated by perfectly rational and boundedly rational agents:

\[
\hat{y}_t = n_{1,t} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( (1 - \beta) \hat{y}_s - \frac{\beta}{\sigma} (\hat{R}_s - \hat{\pi}_{s+1}) \right) \\
+ \int_{n_{1,t}}^{1} \tilde{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t} \left( (1 - \beta) \hat{y}_s - \frac{\beta}{\sigma} (\hat{R}_s - \hat{\pi}_{s+1}) \right) f(i) di
\]

\[
\hat{\pi}_t = n_{1,t} E_t \sum_{s=t}^{\infty} (\omega \beta)^{s-t} \left( k \hat{y}_s + (1 - \omega) \beta \hat{\pi}_{s+1} \right) \\
+ \int_{n_{1,t}}^{1} \tilde{E}_{i,t} \sum_{s=t}^{\infty} (\omega \beta)^{s-t} \left( k \hat{y}_s + (1 - \omega) \beta \hat{\pi}_{s+1} \right) f(i) di
\]
Dynamic feedback system (I)

- The **beliefs parameters** $\theta_{i,t}$ are random variables distributed according to $\psi_t(\theta)$.
- The distribution of beliefs evolves over time as a function of past performance according to the *continuous choice model* (Diks and van der Weide (2005)):

$$\psi_t(\theta) = \frac{e^{\delta U_{t-1}(\theta)}}{Z_{t-1}}$$

where $Z_{t-1} = \int_{\Theta} e^{\delta U_{t-1}(\vartheta)} d\vartheta$ is a normalization factor.
Dynamic feedback system (II)

Assuming that the performance measure is given by past squared forecast errors

\[ U_{t-1}(\theta) = - (\theta - x_{t-1})^2 \]

it can be shown that \( \psi_t(\theta) \) follows a normal distribution, whose evolution is characterized by

\[ \mu_t = x_{t-1} \]
\[ \sigma_t^2 = \frac{1}{2\delta} \]
To be or not to be (rational)?

- **Discrete choice model** (Brock and Hommes (1997))

\[ n_{h,t} = \frac{e^{\delta U_{h,t-1}}}{\sum_{h=1}^{H} e^{\delta U_{h,t-1}}} \]

- \( n_{h,t} \) fraction of agents using predictor \( h \)

- Agents compare cost \( C \) for rationality with the heuristics’ average past forecast error

- Fraction of rational agents constant over time:

\[ n_1 = \frac{e^{-\delta d C}}{e^{-\delta d C} + e^{-\delta d \frac{3}{2\delta c}}} \]
Monetary policy rule

- Close the model by specifying a forward-looking interest rate rule of the form
  \[
  \hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)(\phi_\pi E_t \hat{\pi}_{t+1} + \phi_y E_t \hat{y}_{t+1}) + \varepsilon_{mp}^t
  \]

- Assuming a demand and a supply shock we can write the HE model in the canonical form
  \[
  \Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi \varepsilon_t + \Pi \eta_t
  \]
Basic mechanics of Bayesian estimation

- Let $\xi$ be the vector that collects the **structural parameters** of the model and denote with $Y_T$ **historical data** on output, inflation and interest rate (sample 1982:04-2008:03)

- All the information about the parameters is summarized by the **posterior distribution**

$$p(\xi|Y_T) = \frac{p(Y_T|\xi)p(\xi)}{p(Y_T)}$$

- Use the **Metropolis-Hastings algorithm** to generate draws from the posterior and the **Kalman filter** to recursively evaluate the likelihood
### Posterior moments

<table>
<thead>
<tr>
<th>Name</th>
<th>Mean</th>
<th>Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1$</td>
<td>0.7794</td>
<td>[0.7515, 0.8075]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.2275</td>
<td>[0.2239, 0.2321]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.8460</td>
<td>[0.4858, 1.1911]</td>
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<td>$\Omega$</td>
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<td>[0.1316, 0.3842]</td>
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<td>$\phi_\pi$</td>
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<td>[1.8776, 2.1914]</td>
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<tr>
<td>$\phi_y$</td>
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<td>[0.1714, 0.2850]</td>
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<tr>
<td>$\pi^*$</td>
<td>1.9690</td>
<td>[0.4650, 3.4233]</td>
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<tr>
<td>$\rho_g$</td>
<td>0.5934</td>
<td>[0.5487, 0.6403]</td>
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<tr>
<td>$\rho_u$</td>
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<td>[0.8733, 0.9577]</td>
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<tr>
<td>$\rho_R$</td>
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<td>[0.0062, 0.0887]</td>
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<tr>
<td>$\sigma_g$</td>
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<tr>
<td>$\sigma_u$</td>
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<td>[0.4977, 1.8731]</td>
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<tr>
<td>$\sigma_R$</td>
<td>0.2459</td>
<td>[0.2261, 0.2669]</td>
</tr>
</tbody>
</table>
Robustness check and model comparison (I)

- Estimation under a more diffuse prior for \( n_1 \), namely a prior uniform distribution in the interval \([0, 1]\). The posterior mean slightly increases from 0.7794 to 0.7934 with other parameter estimates being almost unchanged.

- Model comparison via posterior odds ratio

\[
\frac{p(HE|Y_T)}{p(RE|Y_T)} = \frac{p(HE) p(Y_T|HE)}{p(RE) p(Y_T|RE)}
\]
Robustness check and model comparison (II)

- Marginal data densities $p(Y_T|\mathcal{M})$ where $\mathcal{M} = HE, RE$ computed using a modified harmonic mean approximation (Geweke (1999))

| Model | $\ln p(Y_T|\mathcal{M})$ | Bayes factor versus HE |
|-------|--------------------------|------------------------|
| HE    | -945.3                   | 1                      |
| RE    | -1264.2                  | $\exp(318.9)$          |

- According to Jeffreys (1961), the value of the Bayes factor $\exp(318.9)$ represents decisive evidence of the HE model versus the RE benchmark

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Findings and future lines of research

- Decisive empirical evidence in support of the HE model versus the representative agent RE benchmark

- Findings comparable with the results of Gali and Gertler (1999) who found a proportion of forward looking firms between 0.6 and 0.8 by estimating a reduced form NKPC with GMM

- Investigation of the implications of heterogeneous boundedly rational expectations in the context of optimal monetary policy design