Dynamic Consumption and Portfolio Decisions with Estimated Low Frequency Movements of Asset Returns

Willi Semmler (Dept. Economics, New School for Social Research, New York)
Chih-Ying Hsiao, School of Finance and Economics, UTS)
1. Introduction

- Since Merton (1973, 1974) the literature has moved away from the static portfolio theory (the Markovitz mean-variance principle) and studied the allocation issues when investment opportunities change over time.
- In this view what is needed is a joint modelling of dynamic 1) consumption and 2) portfolio decisions as the returns may vary over time.
- Campbell and Viceira (1999, 2002) have made seminal contributions to such a dynamic decision problem, the latest in their book (2002).
- Based on CV (1999) they use some linearization techniques and constant expected risk premia as well as approximately constant consumption wealth ratio.
- They explore 1) the role of risk aversion and 2) time horizon for dynamic asset allocation and consumption decisions.
1. Introduction

- In our paper we model time varying returns through Fast Fourier Transform (FFT) and use a harmonic fit to monthly U.S. asset market time series data, 1983.6 to 2007.6
- The estimated low frequency movements of the asset returns are imported into a dynamic programming algorithm
- The effects of low frequency movement in returns on consumption decisions, asset allocation, consumption-wealth ratio and path of wealth are studied
- We do this, as CV for: 1) varying risk aversion across investors and 2) varying time horizon across investors.
2. Literature: Campbell and Viceira (2002, ch.3)

\[ \alpha = \frac{1}{\gamma} \left( \frac{E_t r_{t+1} - r_{f,t+1}}{\sigma_t^2} + \frac{\sigma_t^2}{2} \right) + \left( 1 - \frac{1}{\gamma} \right) \left[ r_{t+1} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \delta^j r_{f,t+1+j} \right] \]
2. Literature: Campbell and Viceira (2002, ch. 6), Role of Discount Rates

- If the discount rate goes to infinity a myopic decision problem arises (see CV, 2002, ch. 2 and ch. 6), and investors behave like single period investors.
- With the discount rate large (discount factor small), the investors place a relative high weight on the near future.
- With discount rate small (discount factor large) the investors place less weight on the near future and more on the distant future.
3. Time Varying Asset Returns and Dynamic Programming Solution

- Estimates of harmonic fit (FFT) for monthly U.S. time series data on asset returns: 1983.6 to 2007.6 (risk free rate and equity return)

\[
R_{f,t} = -0.0021(t) + 0.0521 + \sum_{i=1}^{2} \left( a_i \sin \left( \frac{2\pi}{\tau_i} (t) \right) + b_i \cos \left( \frac{2\pi}{\tau_i} (t) \right) \right)
\]

\[
R_{e,t} = -0.0046(t) + 0.1259 + \sum_{i=1}^{2} \left( a_i \sin \left( \frac{2\pi}{\tau_i} (t) \right) + b_i \cos \left( \frac{2\pi}{\tau_i} (t) \right) \right)
\]
3. Time Varying Asset Returns and Dynamic Programming Solution

- Dynamic Programming Method (grid refinement): HJB Equation

\[
V(x) = \max_u \int_0^\infty e^{-r} f(x, u) dt \\
\text{s.t. } \dot{x} = g(x, u)
\]

\[
V_h(x) = \max_j \left\{ h f(x, u_0) + (1 + \theta h) V_h(x_h(1)) \right\}
\]

\[
\eta_i := \max_{k \in c_i} | T_h(V_h^\Gamma)(k) - V_h^\Gamma(k) |
\]
3. Time Varying Asset Returns and Dynamic Programming Solution

- Dynamic Programming, using estimated time series data on returns

\[
\max_{\{C, \alpha\}} \int_0^\infty e^{-\delta t} U(C_t) dt
\]

subject to
\[
\dot{W}_t = \alpha_t R_{e,t} W_t + (1 - \alpha_t) R_{f,t} W_t - C_t
\]

\[
\dot{x}_t = 1.
\]

\[
U(C_t) = \frac{C^{1-\gamma}}{1-\gamma}
\]
4. Varying Risk Aversion Across Investors

- Role of risk aversion, gamma=0.5

Figure 1: Long swings in asset build up for $\gamma = 0.5$

Figure 2: Value function for $\gamma = 0.5$
4. Varying Risk Aversion Across Investors

- Role of risk aversion, gamma=0.8

![Figure 3: Long swings in asset build-up for \( \gamma = 0.8 \)](image1)

![Figure 4: Value function for \( \gamma = 0.8 \)](image2)
4. Varying Risk Aversion Across Investors (see Platen and Semmler, 2009)

- Role of risk aversion, gamma=5,
4. Varying Risk Aversion Across Investors: cyclical variation of C, alpha, and C/W, for gamma=0.75
5. Varying Time Horizon Across Investors (Blanchard 1985)

- Role of the time horizon, discount rate delta

\[
1 = E_t \left\{ \pi^e \rho \left( \frac{C^e_{t+1}}{C^e_t} \right)^{-\gamma} + (1 - \pi^e) (1 - \pi^d) \rho \left( \frac{C^r_{t+1}}{C^e_t} \right)^{-\gamma} \right\} (1 + R_{p,t+1}) \right\} 
\]
5. Varying Time Horizon Across Investors, wealth for delta=0.05

Figure 6: Long swings in asset build up for $\gamma = 0.8$ and $\delta = 0.05$
5. Varying Time Horizon Across Investors, value function for $\delta=0.05$

Figure 7: Value function for $\gamma = 0.8$ and $\delta = 0.05$
5. Variation of Time Horizon Across Investors, wealth for delta=0.5

Figure 8: Long swings in asset build-up for $\gamma = 0.8$ and $\delta = 0.5$
5. Variation of Time Horizon Across Investors, value function for delta=0.5

Figure 9: Value function for $\gamma = 0.8$ and $\delta = 0.5$
6. Conclusions

• We estimate low frequency components in asset returns from a harmonic fit to the financial data. The consumption and asset allocation is solved through dynamic programming (we can also extend this to include labor income).

• Our examples show that when returns follow a low frequency movements, so also does the fraction of assets allocated to risky assets and the consumption wealth ratio, when properly computed.

• As shown the consumption wealth ratio is not constant as in Campbell and Viceira (1999, 2002) and the path of wealth can globally be computed (which is only locally computed in Campbell and Viceira).
6. Conclusions

- Our procedure can be used for a higher dimensional problem and thus can have multiple assets and returns: 1) estimate the low frequency movements (harmonic fit, filters) and 2) read in the low frequency components into a dynamic programming algorithm to compute online the solution of consumption and asset allocation.

- The above model works with theory of “risk aversion” (see parameter of risk aversion, gamma, in CRRA) and preferences are concave. Recent financial market theory uses theory of “loss aversion”, preferences are convex-concave, see Gruene and Semmler, Journal of Economic Dynamics and Control (2008), and Computational Economics (2008).