Asset Markets and Monetary Policy

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Introduction

- financial market and economic activity strongly interact

- appropriate monetary policy?

- Taylor rule Taylor (1993)
  respond to inflation and output gap

Svensson (1997, 1999)
Woodford (2003)
• **current crises**
  triggered by subprime crisis
  global financial market meltdown
  banking crisis
  downturn in real economic activity
  partial deflation

**monetary policy and asset markets:**
U.S.: zero interest rates
quantitative easing
- dynamic portfolio approach

reveals transmission mechanism and limitations of monetary policy

**static:** Tobin (1969, 1980)
Frankel (1995)

**dynamic:** Campbell & Viceira (2002)

constant variances
constant expected risk premia
constant consumption-wealth ratio
risk neutral assumptions
Benchmark Approach

Pl. & Heath (2006)

distinction between nominal and real assets
real world martingales
general stochastic processes

⇒

interest rate rule
inflation rate rule
• interest rate rule

\[ i_t = (a_t - \gamma \sigma_t^2 (1 - |1 - \alpha_t|))^+ \]

- \( a_t \) expected return
- \( \gamma \) risk aversion
- \( \alpha_t \) fraction of wealth invested in the equity market
- \( \sigma_t \) volatility of the equity index
• inflation rate rule

\[ \pi_t = a_t - c_t + \frac{\gamma \sigma_t^2}{2} \alpha_t (\alpha_t - 2) \]

\( c_t \) consumption rate
Asset Market Dynamics

Merton (1992)
Cochrane (2001)
Campbell & Viceira (2002)

• savings account

\[ \beta_t = \exp \left\{ \int_0^t i_s \, ds \right\} \]

\( i_t \) nominal interest rate

• risky asset

\[ dP_t = P_t \left( a_t \, dt + \sigma_t \, dz_t \right) \]

\( z_t \) standard Wiener process
\( \sigma_t > 0 \)
• consumer price index

$$I_t = \exp \left\{ \int_0^t \pi_s \, ds \right\}$$

$$\pi_t$$ inflation rate

• consumption rate

$$c_t = c_0 \exp \left\{ \int_0^t e_s \, ds \right\}$$

$$c_0 > 0$$

$$e_t$$ growth rate of consumption rate
Budget equation

total wealth

\[ dW_t = W_t ((1 - \alpha_t) i_t \, dt - c_t \, dt + \alpha_t (a_t \, dt + \sigma_t \, dz_t)) \]

\(\alpha_t\) fraction in the risky asset index \(P_t\)

\(W_0 > 0\).
Maximization of aggregate consumed real wealth per unit of time:

\[
\frac{c_s W_s}{I_s}
\]

- objective

\[
\max_\mathcal{W} \mathbb{E}_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right)
\]

for all \(0 \leq t < s < \infty\)

real world expectation

\[
U'(\cdot) > 0 \quad U''(\cdot) < 0
\]
Example:

- power utility

\[ U(x) = \frac{x^{(1-\gamma)}}{1 - \gamma} \]

\(\gamma > 0, \gamma \neq 1,\)

\(\gamma \rightarrow 1\) logarithmic utility

no time horizon

consumption rate \(c_t\) given
Market Dynamics

- **martingale approach**
  
  assumes risk neutral martingales
  
  Cox & Huang (1989)  
  Campbell & Viceira (2002)  
  trends in real world dynamics are ignored

- **benchmark approach**
  
  assumes real world martingales
  
  Pl. & Heath (2006)  
  no equivalent risk neutral probability measure needed  
  trends in real world dynamics are taken into account
Optimal Wealth Dynamics

- benchmark as best performing strictly positive portfolio

numeraire portfolio $S^*_t$

$$dS^*_t = S^*_t (i_t \, dt + \theta_t (\theta_t \, dt + dz_t))$$

market price of risk

$$\theta_t = \frac{a_t - i_t}{\sigma_t}$$


growth optimal portfolio
Kelly (1956), Merton (1973)
• accumulated total wealth $W_t G_t$

$$G_t = \exp \left\{ \int_0^t c_s \, ds \right\}$$

benchmarked accumulated total wealth

$$\tilde{W}_t = \frac{W_t G_t}{S_t^*}$$

$$d\tilde{W}_t = \tilde{W}_t (\alpha_t \sigma_t - \theta_t) \, dz_t$$

local martingale, supermartingale

Pl. & Heath (2006)
• Law of the minimal price

Pl. (2008)

⇒

real world pricing formula

\[ V_t = S_t^* E_t \left( \frac{H_s}{S^*_s} \right) \]
• constraint optimization problem

\[
\nu_t = \max_{\mathcal{W}} E_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right) - \ell_t E_t \left( \tilde{W}_s - \tilde{W}_t \right)
\]

\[
= \max_{\mathcal{W}} E_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right) - \ell_t \left( \frac{W_s G_s}{S_s^*} - \frac{W_t G_t}{S_t^*} \right)
\]

for \( 0 \leq t < s < \infty \)

\( \ell_t \) Lagrange multiplier

\( \tilde{W}_t \) martingale

only real world probability used

\[\text{c} \cdot \text{⃝ Copyright E. Platen 09 Asset Markets} \]
• candidate for optimal wealth process

\[
\begin{align*}
&\left[ U \left( \frac{c_s W_s}{I_s} \right) - \ell_t \left( \frac{W_s G_s}{S^*_s} - \frac{W_t G_t}{S^*_t} \right) \right] \rightarrow \text{max} \\
&U' \left( \frac{c_s W_s}{I_s} \right) \frac{c_s}{I_s} - \ell_t \frac{G_s}{S^*_s} = 0 \\
&\frac{c_s W_s}{I_s} = U'^{-1} \left( U' \left( \frac{c_s W_s}{I_s} \right) \right) = U'^{-1} \left( \ell_t \frac{G_s}{S^*_s} \frac{I_s}{c_s} \right)
\end{align*}
\]
\[
W_s = \frac{I_s}{c_s} U'^{-1}(\ell_t \phi_s)
\]

with
\[
\phi_s = \frac{G_s I_s}{S^*_s c_s}
\]

Lagrange multiplier \( \ell_t = \ell_0 = c_0 U'(c_0 W_0) \)

\[0 \leq t < s < \infty\]

generally \( W \) maximizes
\[
E_t \left( U \left( \frac{c_s W_s}{I_s} \right) \right)
\]
• benchmarked accumulated total wealth

\[ \frac{W_t}{S_t^*} G_t = F(\phi_t) \]

with

\[ F(\phi) = \phi U'^{-1}(\ell_0 \phi) \]

\[ d\phi_t = \phi_t (\left[ c_t + \pi_t - e_t - i_t \right] dt - \theta_t d\omega_t) \]
Itô formula

\[ d\tilde{W}_t = \frac{\partial}{\partial \phi} F(\phi_t) \phi_t [c_t + \pi_t - e_t - i_t] \, dt - \theta_t \, dz_t \]

\[ + \frac{1}{2} \frac{\partial^2}{\partial \phi^2} F(\phi_t) \phi_t^2 \theta_t^2 \, dt \]

Comparison of the drift coefficients

\[ 0 = c_t + \pi_t - e_t - i_t - \frac{\theta_t^2}{2 \gamma} \]
comparison of diffusion coefficients \[\Rightarrow\]

\[\tilde{W}_t (\alpha_t \sigma_t - \theta_t) = -\frac{\partial}{\partial \phi} F(\phi_t) \phi_t \theta_t\]

\[\Rightarrow\]

\[\alpha_t = \frac{\theta_t}{\gamma_t \sigma_t}\]
Optimal Interest Rate

assume power utility

⇒ optimal interest rate:

\[ \tilde{i}_t = a_t - \gamma \alpha_t \sigma_t^2 \]

cannot become negative  $\implies$  

adjusted interest rate  

$$i_t = (\hat{i}_t)^+$$  

$\hat{i}_t$ - theoretical interest rate  

- **negative optimal interest rates**  

  Japanese stagnation  
  great depression  
  current financial crisis  
  “quantitative easing”
• change in consumption rate

\[
\frac{dc_t}{dt} = c_t e_t
\]

\[
= c_t \left( c_t + \pi_t - i_t - \frac{(a_t - i_t)^2}{2\gamma \sigma_t^2} \right)
\]

\[
= c_t \left( \pi_t - \tilde{\pi}_t - \frac{(i_t - \phi_t)^2}{2\gamma \sigma_t^2} \right)
\]

with critical inflation rate

\[
\tilde{\pi}_t = a_t - c_t - \frac{\gamma \sigma_t^2}{2}
\]

and intrinsic interest rate

\[
\phi_t = a_t - \gamma \sigma_t^2
\]
Figure 1: Inflation versus interest.
1. **Subcritical Inflation**

\[ \pi_t < \tilde{\pi}_t \]

\[ \implies c_t \text{ decreases} \]

Interest rate does not matter!

Nothing can stop downward trend!

\[ a_t \text{ may decrease} \]

\[ \implies \text{recession} \]

**Subcritical inflation dangerous!**

\[ \implies \text{Low inflation rate targeting questionable!} \]
2. **Supercritical Inflation**

set interest rate $i_t$ slightly above or below

$$\bar{i}_t = \phi_t + \sqrt{2\gamma \sigma_t^2 (\pi_t - \tilde{\pi}_t)}$$

- $i_t > \bar{i}_t$

$$\Rightarrow$$

$$(i_t - \phi_t)^2 \geq 2\gamma \sigma_t^2 (\pi_t - \tilde{\pi}_t)$$

$$\Rightarrow \ c_t \text{ decreasing}$$

- $\phi_t < i_t < \bar{i}_t$

$$\Rightarrow \ c_t \text{ increasing}$$

convenient mechanism
for $\alpha_t < 1$ and

$$\bar{i}_t = \tilde{i}_t \text{ optimal interest rate}$$

$$\implies \pi_t - \tilde{\pi}_t = \frac{\gamma \sigma^2_t}{2} (\alpha_t - 1)^2$$

For $\alpha_t > 1$

$\bar{i}_t$ not close to optimal interest rate $\tilde{i}_t$

Keep the inflation rather small!

less economic growth
Interest Rate Rule:

\[ i_t = \left( \bar{i}_t \right)^+ = \left( a_t - \gamma \sigma^2_t \left( 1 - |1 - \alpha_t| \right) \right)^+ \]

assuming optimal inflation
• Create an economic environment where

\[ a_t > \gamma \alpha_t \sigma_t^2 \implies \tilde{i}_t > 0 \]

\[ \implies \text{Avoid extended long boom with subsequent crash!} \]

No cheap credit!

otherwise economic trap with zero interest
target inflation rate level

\[ \pi_t = a_t - c_t + \frac{\gamma \sigma_t^2}{2} \alpha_t (\alpha_t - 2) \]

after crash may be deflationary period if \( \alpha_t \in (0, 1) \)
• when credit market clears

$$\alpha_t = 1$$

$$\implies$$ target inflation rate minimal

least likely to have economic trap
For $c_t = c$, minimal inflation, $\alpha_t = 1 \implies$

$$U \left( \frac{c W_t}{I_t} \right) = E_t \left( U \left( \frac{c W_s}{I_s} \right) \right)$$

fair monetary policy
<table>
<thead>
<tr>
<th>Country</th>
<th>$\alpha$</th>
<th>$i$</th>
<th>$\pi$</th>
<th>$\sigma$</th>
<th>$d$</th>
<th>$\theta$</th>
<th>$\gamma \alpha$</th>
<th>$c$</th>
<th>$\pi_{min}$</th>
<th>$\pi - \pi_{min}$</th>
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<tr>
<td>Australia</td>
<td>0.119</td>
<td>0.045</td>
<td>0.041</td>
<td>0.177</td>
<td>0.048</td>
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<td>0.0910</td>
<td>0.034</td>
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<td>0.052</td>
<td>0.055</td>
<td>0.228</td>
<td>0.028</td>
<td>0.132</td>
<td>0.577</td>
<td>0.0056</td>
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<td>0.016</td>
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<td>0.168</td>
<td>0.041</td>
<td>0.286</td>
<td>1.701</td>
<td>0.0590</td>
<td>0.032</td>
<td>-0.001</td>
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<td>Denmark</td>
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<td>0.201</td>
<td>0.044</td>
<td>0.095</td>
<td>0.470</td>
<td>0.0334</td>
<td>0.035</td>
<td>0.006</td>
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<td>France</td>
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<td>0.079</td>
<td>0.231</td>
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<td>0.158</td>
<td>0.489</td>
<td>0.0075</td>
<td>0.035</td>
<td>0.016</td>
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<tr>
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<td>0.045</td>
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<td>0.001</td>
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<tr>
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<td>0.0055</td>
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<tr>
<td>Netherlands</td>
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<td>0.252</td>
<td>1.202</td>
<td>0.0390</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>S. Africa</td>
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<td>0.057</td>
<td>0.048</td>
<td>0.228</td>
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<td>0.276</td>
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<td>-0.016</td>
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<tr>
<td>Spain</td>
<td>0.100</td>
<td>0.065</td>
<td>0.061</td>
<td>0.220</td>
<td>0.047</td>
<td>0.159</td>
<td>0.723</td>
<td>0.0300</td>
<td>0.036</td>
<td>0.025</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.116</td>
<td>0.058</td>
<td>0.037</td>
<td>0.228</td>
<td>0.035</td>
<td>0.254</td>
<td>1.116</td>
<td>0.0530</td>
<td>0.052</td>
<td>-0.015</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.076</td>
<td>0.033</td>
<td>0.022</td>
<td>0.204</td>
<td>0.029</td>
<td>0.211</td>
<td>1.033</td>
<td>0.0330</td>
<td>0.026</td>
<td>-0.004</td>
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<td>UK</td>
<td>0.101</td>
<td>0.051</td>
<td>0.041</td>
<td>0.200</td>
<td>0.038</td>
<td>0.250</td>
<td>1.250</td>
<td>0.0410</td>
<td>0.030</td>
<td>0.003</td>
</tr>
<tr>
<td>US</td>
<td>0.101</td>
<td>0.041</td>
<td>0.032</td>
<td>0.202</td>
<td>0.041</td>
<td>0.297</td>
<td>1.470</td>
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<td>Average</td>
<td>0.103</td>
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<td>0.049</td>
<td>0.227</td>
<td>0.039</td>
<td>0.236</td>
<td>1.090</td>
<td>0.0330</td>
<td>0.038</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 1: Estimates from sixteen markets
• market price of risk

\[ \theta \approx \frac{a - i}{\sigma} \approx 0.236 \]

• fraction times risk aversion

\[ \gamma \alpha \approx \frac{a - i}{\sigma^2} = \frac{\theta}{\sigma} \approx 1.09 \]
• consumption rate for $\alpha = 1$

\[ c \approx i - \pi + \frac{(a - i)^2}{2\gamma \sigma^2} \approx 0.033 \approx 0.039 = d \]

• critical inflation

\[ \tilde{\pi} \approx a - c - \frac{\gamma \sigma^2}{2} \approx 0.038 \leq 0.049 \approx \pi \]
References


