Assessing the Importance of Transaction Costs in Option Pricing: Evidence from the Australian Index Option Market

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Outline

• Introduction
• Review of option pricing models with transaction costs
• Option pricing models used
• Data
• Method
• Findings
• Future research
Introduction

• Aim
  - to assess the importance of transaction costs in option pricing models based on Australian index option data
    • effectiveness of model – measured by the mispricing errors for systematic tendencies

• Motivation of study
  - no empirical studies on option pricing model with transaction costs based on S&P/ASX 200 index option
  - development of Leland (1985) model
  - the unresolved questions of whether Leland’s method can be used to price options with realistic trading costs and rebalancing intervals
  - findings may offer option sellers and traders to appropriately apply option pricing models in pricing out-of-money, at-the-money and in-the-money options
## Review of option pricing models with transaction costs

<table>
<thead>
<tr>
<th>Author</th>
<th>Method/Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leland (1985)</td>
<td><em>Perfect replication</em>, modify BSM model via adjusted volatility, single options</td>
</tr>
<tr>
<td>Merton (1989)</td>
<td><em>Perfect replication</em>, two period binomial model</td>
</tr>
<tr>
<td>Boyle &amp; Vorst (1992)</td>
<td><em>Perfect replication</em>, several periods binomial model</td>
</tr>
<tr>
<td>Bensaid et al. (1992) &amp; Edirisinghe, Naik &amp; Uppal (1993)</td>
<td><em>Super-replication</em> that dominates the option payoff at lower initial cost</td>
</tr>
<tr>
<td>Hoggard, Whalley &amp; Wilmott (1994)</td>
<td>Work with same assumptions as Leland, valid not only on single options but portfolio of options</td>
</tr>
<tr>
<td></td>
<td>Incorporate initial trading costs of trading with the assumptions of initial portfolio consists of all cash and all stock positions</td>
</tr>
</tbody>
</table>
Review of option pricing models with transaction costs (contd.)

<table>
<thead>
<tr>
<th>Author</th>
<th>Method/Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hodges &amp; Neuberger (H&amp;N) (1989)</td>
<td><strong>Utility maximisation</strong> – Stochastic optimal control problem</td>
</tr>
<tr>
<td>Davis, Panas &amp; Zariphopoulou (1993)</td>
<td>Modify H&amp;N to include proportional costs to amount of stocks traded</td>
</tr>
<tr>
<td>Clewlow &amp; Hodges (1997)</td>
<td>Modify H&amp;N to include fixed and proportional costs</td>
</tr>
<tr>
<td>Whalley &amp; Wilmott (1997)</td>
<td>Addressed the computational problem of H&amp;N by providing asymptotic analysis</td>
</tr>
<tr>
<td>Barles &amp; Soner (1998)</td>
<td>Extend H&amp;N – provide alternative analysis</td>
</tr>
<tr>
<td>Zakamouline (2006)</td>
<td>Extend Davis et al. – provide alternative to asymptotic analysis – approximation strategy</td>
</tr>
</tbody>
</table>
Option pricing models used

a) **Black-Scholes-Merton (BSM) model**

\[ c = S_0 N(d_1) - Ke^{-rT} N(d_2) \]  \hspace{1cm} (1)

and

\[ p = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \]  \hspace{1cm} (2)

where

\[
d_1 = \frac{\ln \left( \frac{S_0}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}}
\]

\[
d_2 = \frac{\ln \left( \frac{S_0}{K} \right) + \left( r - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T}
\]

\(N(x)\) is the cumulative probability distribution function for a standardised normal distribution;

c and p are the European call and European put price respectively;

\(S_0\) is the price of the underlying asset;

\(K\) is the strike price;

\(r\) is the continuously compounded risk-free rate;

\(\sigma\) is the underlying asset price volatility;

\(T\) is the time to maturity of the option.
Option pricing models used (cont.)

b) Leland models

• Leland (1985) model

Leland formula for a call and a put:

\[ c = S_0 N(d_1^*) - Ke^{-rT} N(d_2^*) \]  \hspace{1cm} (3)

\[ p = Ke^{-rT} N(-d_2^*) - S_0 N(-d_1^*) \]  \hspace{1cm} (4)

Similar to BSM formula except that \( d_1^* \) and \( d_2^* \) are based on adjusted volatility for trading costs:

\[ \sigma^* = \sigma \left( 1 + \frac{k \sqrt{2 \pi}}{\sigma \sqrt{\Delta t}} \right)^{1/2} \]

\( \sigma \) is the underlying risky asset standard deviation

\( \Delta t \) is the rebalancing interval

\( k \) is the transaction cost rate
Option pricing models used (cont.)

- **Leland (2007) models**
  
i) Assuming initial trade costs are all cash positions:
  
  \[
c = \left(1 + \frac{k}{2}\right)S_0N(d_1^*) - Ke^{-rT}N(d_2^*)
  \]  
  
  (5)

  ii) Assuming initial trade costs are all stock positions:
  
  \[
c = \left(\frac{k}{2}\right)S_0 + \left(1 - \frac{k}{2}\right)S_0N(d_1^*) - Ke^{-rT}N(d_2^*)
  \]  
  
  (6)

  Formula (1), (3), (5) and (6) are used.
Data

- Uses data on S&P/ASX index call option (XJO index call option), S&P/ASX 200 index levels and Australian 90-day Bank Accepted Bill interest rate
- Daily index option data: trading date, expiration date, closing price, strike price and trading volume for each trading option
- Daily closing index levels
- Prior to 2\textsuperscript{nd} April 2001, excessive movements due to changes of underlying asset of S&P/ASX 200 index option
- Sample period: 2\textsuperscript{nd} April 2001 to 27\textsuperscript{th} July 2005
**Sampling procedure**

- Apply some filter rules to remove offending daily option prices
  - Remove observations that do not satisfy minimum value arbitrage constraints (Bakshi, Cao & Chen 1997; Sharp & Li 2008)
    
    $$C(\tau) \geq \max[0, S_0 - KB(\tau)]$$
    
    $C(\tau)$ is the price of call maturing in $\tau$ periods (years)
    
    $K$ is the exercise price of the option
    
    $S_0$ is the initial index level
    
    $r$ is the risk-free rate of return
    
    $B(\tau)$ is the current price of a $1$ zero coupon bond with the same maturity as the option
  - Remove observations that have less than 6 days to maturity (Bakshi, Cao & Chen 1997)
  - Remove observations with exercise price of zero - LEPOs
### Table 1. Sample Properties of S&P/ASX 200 Index Options

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Time to maturity in days (T)</th>
<th>S/K</th>
<th>T &lt; 30</th>
<th>30 ≤ T &lt; 90</th>
<th>T ≥ 90</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>OTM</td>
<td>m &lt; 0.97</td>
<td></td>
<td>5.79 pts</td>
<td>19.46 pts</td>
<td>44.41 pts</td>
<td>29.39 pts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>291</td>
<td>2014</td>
<td>1789</td>
<td></td>
<td>4094</td>
</tr>
<tr>
<td>ATM</td>
<td>0.97≤m&lt;1.03</td>
<td></td>
<td>33.75 pts</td>
<td>64.96 pts</td>
<td>103.11 pts</td>
<td>64.40 pts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1599</td>
<td>3714</td>
<td>1213</td>
<td></td>
<td>6526</td>
</tr>
<tr>
<td>ITM</td>
<td>m ≥ 1.03</td>
<td></td>
<td>258.22 pts</td>
<td>194.26 pts</td>
<td>274.82 pts</td>
<td>228.04 pts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>158</td>
<td>209</td>
<td>49</td>
<td></td>
<td>416</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>47.09 pts</td>
<td>54.08 pts</td>
<td>71.45 pts</td>
<td>57.58 pts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2048</td>
<td>5937</td>
<td>3051</td>
<td></td>
<td>11036</td>
</tr>
</tbody>
</table>

#### Sample Average

<table>
<thead>
<tr>
<th>S/K</th>
<th>Maturity (days)</th>
<th>Volume</th>
<th>Open interest</th>
<th>Series traded per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98</td>
<td>70.63 days</td>
<td>75.93 contracts</td>
<td>813.58 contracts</td>
<td>10.07 series</td>
</tr>
</tbody>
</table>
Method

• Effectiveness of option pricing models with transaction costs
  - Examine mispricing errors for systematic tendencies related to option’s moneyness and time to maturity
    - compute signed and unsigned pricing errors both in percentage and index points terms
  - Compare and contrast results with BSM model

• Examine the effect of rebalancing intervals on the magnitude of the pricing errors of the models prices relative to market prices
  - Apply different rebalancing intervals: daily, weekly, monthly and quarterly rebalancing intervals on Leland models
Method (contd.)

Variables

- Time to maturity
  - $T$ is measured by the number of trading days between the day of trade and the day immediately prior to expiry days divided by the number of trading days per year. There are 252 trading days per year (Hull 2003).
  - Expiry date is not taken into account

- Realised volatility
  - Use realised volatility to determine the return standard deviation

Daily return of index:

\[
R_i = \ln \left( \frac{S_i}{S_{i-1}} \right)
\]

- $S_i$ is the index level
- $R_i$ is the log-return on the $i$th day during the remaining life of the option
- $\bar{R}_t$ is the mean of daily log-returns during the period $t$
Method (contd.)

- Therefore, the annualised realised volatility is
  \[ \sigma_{r,t} = \sqrt{\frac{252}{n-1} \sum_{i=1}^{n} (R_{i,t} - \overline{R}_t)^2} \]

- Risk-free interest rate
  - Use Australian 90-day Bank Accepted Bill rate as proxy for risk-free interest rate
    Convert interest rates to continuous compounding risk-free interest rates

- Transaction costs
  - Trading fee is 0.2 basis points (0.2%) prior 1 July 2006 (ASX media release 15 Dec 2005)
  - Use transaction cost rate, \( k = 0.002 \)

- Rebalancing intervals
  - Apply rebalancing intervals: daily, weekly, monthly and quarterly
# Findings

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Empirical results</th>
</tr>
</thead>
</table>
| **OTM**  | • all models overprice OTM calls  
          | • BSM model superior to Leland models  
          |   - lowest index pt error – short-term calls; lowest percentage error – long term calls  
          | • Leland models generate lower errors as rebalancing becomes less frequent  
          | • Among Leland models – Leland (1985) produce lowest errors |
| **ATM**  | • all models overprice ATM calls  
          | • BSM model superior to Leland models  
          |   - lowest index pt and percentage errors – short-term calls  
          | • Leland models generate lower errors as rebalancing becomes less frequent  
          | • Among Leland models – Leland (1985) produce lowest errors |
| **ITM**  | • all models underprice ITM calls  
          | • Leland models superior to BSM model  
          | • Leland models generate lower errors as rebalancing becomes more frequent  
          | • other rebalancing intervals: lowest error – Leland (2007) cash model |
### Findings (contd.)

#### Summary of results

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Which model perform better?</th>
</tr>
</thead>
</table>
| OTM       | • BSM model superior to Leland models for pricing OTM & ATM calls  
            • BSM model generate lowest pricing errors for ATM short-term calls  
            • BSM model generate largest errors for OTM calls regardless of maturities  
            • Leland models produce lower errors as rebalancing intervals become less frequent  
            • Among Leland models – original Leland (1985) performs the best for OTM & ATM calls across all maturities |
| ATM       | • Leland models superior to BSM model  
            • Leland models produce lower errors as rebalancing becomes more frequent  
            • Leland (2007) cash model performs the best – produce lowest errors for medium-term ITM calls |
| ITM       |                             |
Future research

• Extend empirical study to other various approaches of option pricing models with transaction costs
  - assess and compare overall performances based on S&P/ASX 200 index option data
• Enhance study using a high frequency data