An Estimated Small Open Economy Model Under Learning

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• RE: private agents and policy-makers know the ‘true model of the economy’, except for unforecastable random shocks.
• Learning: relax the assumption that agents know the structure of the economy – put the agents and the econometrician on the same footing (Sargent, 1993; Evans and Honkapohja, 2001).
• Agents have a *perception* of the equilibrium law of motion, and they use available data generated by the economy to update their perceived law of motion using recursive algorithms, such as recursive least squares.
• The self-referential system: agents beliefs about the future affect the current state of the economy, which feeds back into the learning rule affecting future beliefs.
Learning can be a source of persistence

- With constant gain learning, policymakers should lean more strongly on inflation stabilisation (Orphanides and Williams 2007, 2008; Gaspar, Smets and Vestin 2006).
- Simple policy rules that are robust to other types of model uncertainty are robust to uncertainty about how expectations are formed.
- Shocks can lead to ‘inflation scares’ - expectations deviate persistently from RE.

But these results are based on calibrated models.
Introduction - estimated models with learning

- DSGE models with RE often fail to match the persistence of aggregate data.
  - Add potential sources of persistence - habit formation in consumption, indexation to lagged inflation in price setting, persistent structural shocks etc.

- Can learning provide some or all of the persistence needed to fit the data?
  - Structural inertia (habits and indexation) is systematically reduced when learning is introduced (Milani, 2007).

- Milani’s model is very stylised - Slobodyan and Wouters (2008) evaluate the role of learning in a medium-scale DSGE model.
  - Habit formation is stronger under learning; indexation more or less unchanged; the adjustment cost of investment lower (prices and wages less sticky).
  - The persistence of the shocks stays high

- These are closed economy models
• A small open economy setting
• Sources of persistence: Habits and ‘rule-of-thumb’, but also the dynamics in the foreign block may provide persistence
  • A priori not as clear-cut as Milani
• The uncovered interest rate parity (UIP) condition
  • Usually amended with a risk premium shock. The estimated risk premium shock very persistent (for Australia Justiniano and Preston, 2008; Nimark, 2008; Jaaskela and Nimark, 2008).
• SVAR models suggests that the impulse response function for the real exchange rate after a monetary policy shock is hump-shaped with a peak effect after about one year (Eichenbaum and Evans, 1995; Faust and Rogers, 2003; Liu, 2008).
  • The standard UIP condition implies a peak effect within the quarter followed by a relatively quick mean reversion.
The SOE Model – ALM

- Consumers: external habit formation
- Firms: Calvo pricing - rule-of-thumb price setters and optimal price setters
- Cost to being a net foreign borrower (risk premium)
- Perfectly competitive labour market
- Central bank: Taylor-type interest rate rule \((t - 1\) timing)
- RoW: Exogenous, VAR(1)
Expectations formation – PLM

• MSV learning

• Agents forecast the lead variables with a linear function of states and exogenous variables

\[ x_t = a_{t-1} + b_{t-1}x_{t-1} + c_{t-1}v_t \]
\[ v_t = \rho v_{t-1} + \varepsilon_t \]

\[ \mathbb{E} x_{t+1|t} = (I + b_{t-1})a_{t-1} + b_{t-1}b_{t-1}x_{t-1} + (b_{t-1}c_{t-1} + c_{t-1}\rho)v_t \]

\[ \mathbb{E} x_{t+1|t}^f = S\mathbb{E} x_{t+1|t} \]

• In our model \( \mathbb{E} x_{t+1|t}^f = [\mathbb{E} c_{t+1}, \mathbb{E} \pi_t^m, \mathbb{E} \pi_t^d, \mathbb{E} \Delta s_{t+1}]' \), i.e. agents form fwd-looking expectations on consumption, inflation and the exchange rate
Agent update their beliefs ($\Phi_t$ and $R_t$) with the constant gain least squares algorithm

$$\Phi_t = \Phi_{t-1} + \bar{g} R_{t-1}^{-1} Z_{t-1} (x'_t - Z'_{t-1} \Phi_{t-1})$$

$$R_t = R_{t-1} + \bar{g} (Z_{t-1} Z'_{t-1} - R_{t-1})$$

- $\bar{g}$ is the gain parameter; $Z_t = [1, x'_{t-1}, v'_t]'$ denotes ‘data’ and $\Phi_t = [a_t, b_t, c_t]'$
Estimation very briefly

- Substitute $\mathbb{E}X_{t+1}^f | t$ into the model to get the implied “Actual Law of Motion” for the learning model:

$$\xi_t = A_t + F_t \xi_{t-1} + G_t \varepsilon_t$$

where $\xi_t = [v_t, x_t]$

- Map the ALM to the data using the measurement equation

$$Y_{data}^t = H\xi_t + e_t$$

- The observable time series:

$$Y_{data}^t = [y^*_t, \pi^*_t, r^*_t, y_t, \pi_t, r_t, c_t, \Delta q_t]$$

- $e_t$ is measurement error.

- The system estimated with Bayesian methods

- $R_0$ and $\Phi_0$ initialiased at the estimated RE values
## Results I: parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habit Formation</td>
<td>$\eta$</td>
<td>0.87</td>
</tr>
<tr>
<td>Rule of thumb</td>
<td>$\omega$</td>
<td>0.27</td>
</tr>
<tr>
<td>Domestic Calvo</td>
<td>$\theta^d$</td>
<td>0.83</td>
</tr>
<tr>
<td>Import Calvo</td>
<td>$\theta^m$</td>
<td>0.61</td>
</tr>
<tr>
<td>Feedback output</td>
<td>$\phi_y$</td>
<td>0.20</td>
</tr>
<tr>
<td>Feedback Inflation</td>
<td>$\phi_\pi$</td>
<td>1.56</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>$\phi_i$</td>
<td>0.74</td>
</tr>
<tr>
<td>Gain parameter</td>
<td>$\bar{g}$</td>
<td>-</td>
</tr>
</tbody>
</table>

- Shock persistence: mixed
- Shocks have a lower standard deviation under learning
Results II: IRFs - Monetary Policy shock

- **Cash Rate**
- **CPI Inflation**
- **GDP**
- **Real Exchange Rate**
- **Consumption**

Graphs showing the impact of a monetary policy shock on various economic indicators over time.
Results II: IRFs - Monetary Policy shock

- Cash Rate
- CPI Inflation
- GDP
- Real Exchange Rate
- Consumption

Rational expectations with learning parameterisation
- Learning
Results II: IRFs - Productivity shock
Results II: IRFs - Productivity shock
Results III: the role of priors

- Re-estimate the model with looser priors

- Smaller differences between the learning parameter estimates than between the rational expectations estimates. The increase in the parameter uncertainty less pronounced under learning.

- Differences between the impulse responses always smaller under learning. Also the change in the width of the confidence intervals is smaller under learning.

- Parameter estimates under learning less sensitive to the choice of priors.
Results IV: Shift in the gain parameter

- Re-estimate the learning model from 1984 (the floating of the AUD) onwards. IT introduced in 1993.

- Let the gain parameter to evolve as follows:

\[
\bar{g} = \begin{cases} 
\bar{g}_1, & t < 1993 : Q2 \\
\bar{g}_2, & t \geq 1993 : Q2 
\end{cases}
\]

- All the other model parameters sample-invariant
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior</th>
<th>Mode</th>
<th>Std</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{g}_{LS}$</td>
<td>$U[0,1)$</td>
<td>0.0024</td>
<td>0.0003</td>
<td>0.0020</td>
<td>0.0028</td>
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<tr>
<td>$\bar{g}_{1,pre-IT}$</td>
<td>$U[0,1)$</td>
<td>0.0044</td>
<td>0.0005</td>
<td>0.0037</td>
<td>0.0052</td>
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<tr>
<td>$\bar{g}_{2,IT}$</td>
<td>$U[0,1)$</td>
<td>0.0009</td>
<td>0.0006</td>
<td>0.0001</td>
<td>0.0022</td>
</tr>
<tr>
<td>Long sample, no break</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>$U[0,1)$</td>
<td>0.0070</td>
<td>0.0006</td>
<td>0.0059</td>
<td>0.0082</td>
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<tr>
<td>IT sample, no break</td>
<td></td>
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<tr>
<td>$\bar{g}$</td>
<td>$U[0,1)$</td>
<td>0.0007</td>
<td>0.0003</td>
<td>0.0004</td>
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</tbody>
</table>
Conclusions

- The importance of mechanical forms of structural inertia is not reduced when learning is introduced.
- Learning generates hump-shaped impulse response functions consistent with more data-driven (SVAR) models.
- Learning improves identification. The parameters of the model can be estimated with less informative prior distributions without compromising the models ability to generate plausible dynamics.
- Evidence of a downward shift in the estimated gain parameter in the wake of the introduction of the inflation-targeting framework, suggesting improved credibility and macroeconomic stability.