We consider one-to-one matching markets in which agents can either be matched as pairs or remain single. These markets are known as roommate markets and they include as special cases the well-known marriage markets (Gale and Shapley, 1962).

We are interested in analyzing consistency properties for roommate markets. In two papers (Sasaki and Toda, 1992; Toda, 2006) consistency is analyzed for marriage markets and various characterizations of the core using consistency together with other properties are derived. Here, we focus on Toda’s (2006) results concerning consistency and research in how far they can be extended to roommate markets. Toda (2006) characterizes the core by weak unanimity, population monotonicity, and consistency. Klaus (2007) obtains two characterizations of the core of solvable roommate markets by weakening population monotonicity to either resource or competition sensitivity (population monotonicity will cause an incompatibility for the more general class of roommate markets). This leads to the following conjecture.

Conjecture 1. On the class of solvable roommate markets,

(a) a solution satisfies unanimity, consistency, and competition sensitivity if and only if it equals the core;

(b) a solution satisfies unanimity, consistency, and resource sensitivity if and only if it equals the core.

A first step to prove the conjecture is to follow Toda’s (2006) proof steps. But here, we encounter first difficulties when trying to extend the following auxiliary result.

Lemma 1 (Toda, 2006, Lemma 3.4). For marriage markets, if a solution satisfies individual rationality, mutually best, and consistency, then it is a subsolution of the core.

Clearly, this statement cannot be extended for roommate markets with an empty core. However, even for solvable roommate markets Toda’s (2006) Lemma 3.4 does not hold: we construct a supersolution $\varphi$ of the core that is individually rational, mutually best, and consistent and for some economies assigns matchings that are not in the core. With this result, we can also demonstrate that Conjecture 1 does not hold on the class of solvable roommate markets.
Chung (2000) gives a sufficient condition (the well-known “no odd cycles” condition) on the preference domain to guarantee that a roommate market is solvable and lists various very appealing economic domains that satisfy his condition. Many of these economic domains in fact satisfy a further domain restriction we would like to refer to as “generalized no cycles” condition. Loosely speaking, we will consider preferences that do not permit certain types of 3- and 4-cycles.\(^1\)

Then for any preference domain \(\mathcal{D}\) that guarantees solvability and satisfies the generalized no cycles condition, we obtain the following results:

**Lemma 2.** If a solution satisfies mutually best, and consistency, then it is a subsolution of the core.

**Lemma 3.** The core does not have a consistent strict subsolution.

**Lemma 4.**

(a) Unanimity and competition sensitivity imply mutually best.

(b) Unanimity and resource sensitivity imply mutually best.

**Theorem 1.** A solution satisfies mutually best and consistency if and only if it equals the core.

**Theorem 2.**

(a) A solution satisfies unanimity, consistency, and competition sensitivity if and only if it equals the core.

(b) A solution satisfies unanimity, consistency, and resource sensitivity if and only if it equals the core.

**References**


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\(^1\)In this extended abstract we will only refer to (characterization) results pertaining to preference domains that satisfy the generalized no cycles condition. Alternatively, we could impose a milder domain restriction by only excluding certain 3-cycles and adding Pareto optimality to our list of properties.