The Impact of International Trade on Institutions and Infrastructure

Gal Hochman∗, Chrysostomos Tabakis†, and David Zilberman‡

December 2009

Abstract

We develop a theoretical model that explores the impact of international trade on both institutions and infrastructure, while explicitly addressing the correlation between institutional quality and infrastructure investment. We show that trade leads to higher infrastructure investment so that domestic firms become more productive and thus can better compete internationally. However, infrastructure investment also has a detrimental effect on firms as it is partly financed through firm taxation. As a result, when some firms have stronger political ties than others, trade leads to weaker institutions as the government attempts to lower the tax burden on the politically connected firms. Moreover, we show that trade with a partner characterized by high aggregate firm productivity induces a country to invest more heavily in both its infrastructure and its institutional framework.

JEL classification: F12; H54; P48

Keywords: International trade; Market institutions; Infrastructure; Connected entrepreneurs; Corruption

∗UC Berkeley, Berkeley CA, 94702; galh@berkeley.edu
†Universidade Nova de Lisboa, Lisbon Portugal, 1099-032; ctabakis@fe.unl.pt
‡UC Berkeley, Berkeley CA, 94702; zilber11@berkeley.edu
1 Introduction

Institutional quality, such as the quality of property rights or rule of law, and public infrastructure are key determinants of long-run economic prosperity and can help explain the underperformance of many developing countries.\(^1\) It is therefore important to understand what factors give rise to differences in institutions or infrastructure across countries. In this paper, we adopt the view that trade openness is a significant such factor, while recognizing that the level of infrastructure in a country is not independent of the quality of its institutions. For instance, in the presence of weak institutions and thus a high degree of corruption, many firms might decide to operate in the underground economy, reducing tax revenues and hence the ability of the government to fund infrastructure investment.\(^2\) We formalize here these ideas by developing a theoretical model that explores the impact of international trade on both institutions and infrastructure, while explicitly addressing the correlation between institutional quality and infrastructure investment.

Specifically, we consider a government that is about to undertake a two-part reform. On the one hand, it wants to increase domestic firms’ productivity by improving infrastructure (e.g., by building a more reliable electricity grid in order to reduce production costs). On the other hand, it wishes to reform the existing institutional framework (e.g., by increasing the transparency of the tax mechanisms or reducing red tape). The latter part of the reform will have broad consequences for the workings of the economy as the new institutions put in place will determine the extent to which the government will be able to exploit some firms, while favoring others by allowing them to evade taxes or avoid bureaucratic costs. In other words, the quality of the chosen institutions will determine the level of corruption of the economy and thus, its ability to fund infrastructure investment. We then investigate how opening this economy to international trade affects the overall outcome of the aforementioned reform.

\(^1\)For the impact of institutions on economic performance, see Hall and Jones (1999), Easterly and Levine (2003), Rodrik et al. (2004), and Acemoglu et al. (2005), among others. For the effect of infrastructure on economic growth, see, for example, Aschauer (1989), Easterly and Rebelo (1993), Canning and Pedroni (2004), and Egert et al. (2009).

\(^2\)For empirical support of these ideas, see, for example, Loayza (1997), Friedman et al. (2000), and Johnson et al. (2000).
More formally, we assume the government first decides whether to open the economy to international trade, subsequently picks the quality of domestic institutions, and finally selects the level of infrastructure investment, while taking into account both labor real income and real firm profits. Afterwards, production takes place and firms profits are realized. Institutional quality is represented by a parameter that lies between 0 and 1, the market-institutions coefficient. A coefficient value of zero indicates that the economy is in total anarchy (e.g., tax mechanisms are very intransparent, there is extreme corruption, and political ties matter a lot), whereas a coefficient value of 1 indicates that the economy is a perfect market economy (e.g., tax mechanisms are very transparent, there is no corruption, and political ties do not matter at all). Moreover, infrastructure investment is assumed to affect firms in two very different ways. On the one hand, it improves their productivity and thus their ability to compete internationally and export their products. On the other hand, infrastructure investment raises firms’ fixed cost of operation and exporting, since the government finances its public expenditures partly through firm taxation.

We maintain the assumption that some (but not all) firms have political ties to the current government, which allows them for a given market-institutions coefficient to face lower fixed costs, due perhaps to their ability to evade taxes or avoid the bureaucratic channels in order to carry out their everyday business. This introduces firm heterogeneity with respect to their cost structure. However, as the quality of domestic institutions increases (i.e., as the market-institutions coefficient rises), political ties matter less and firms’ fixed cost of operation converges to the same level.\textsuperscript{3} Note that the timing of our game reflects the fact that before the government decides on how much to invest in infrastructure, it needs to know how much in taxes it can collect from the domestic firms, which in our setting is assumed to be a function of institutional quality.

We find that in equilibrium only the firms that are relatively politically connected find it optimal to engage in production. Furthermore, only the most connected of them export. We then compare the closed- and open-economy equilibria, assuming that the trading partner of

\textsuperscript{3}This paper, therefore, extends previous work on connected firms (e.g., Roberts, 1990; Kroszner and Stratmann, 1998; Ang and Boyer, 2000; Morck et al., 2000; Fisman, 2001; Johnson and Mitton, 2003; and Faccio, 2006, among others) by explicitly modeling the interaction between political ties and institutional quality.
the domestic economy has a market-institutions coefficient equal to 1, i.e., it is a perfect market economy. We show that although the quality of domestic institutions deteriorates under trade, infrastructure investment and national welfare increase. In an open economy, the government has a stronger incentive to invest in infrastructure so that firms' productivity and thus their international competitiveness increase. However, greater infrastructure investment implies a higher firm tax burden. Therefore, the government caring about the politically connected firms, “compensates” them by distorting institutions in their favor. We also find that the domestic economy’s benefits from international trade are a function of its trading partner. The higher the firm productivity in the foreign country, the higher the infrastructure investment and the institutional quality in equilibrium in the home country. Institutional quality rises in this case because the government needs a larger tax base in order to finance the higher spending on domestic infrastructure. Finally, we demonstrate that as the government favors firms over consumers, the domestic economy ends up with higher infrastructure investment, more corruption, and a higher volume of exports.

Our modelling of the production side of the economy relies heavily on Dixit and Stiglitz’s (1977) model of monopolistic competition, and is clearly inspired by the Melitz (2003) model of trade with heterogeneous firms and monopolistic competition, as well as by Helpman et al. (2004). Nevertheless, there is one major difference. As we discussed above, in our model, firms within a country differ with respect to their fixed cost of operation and exporting but not with regard to their productivity. Our model allows only for firm productivity differences across countries.

Our paper relates to a broad literature on international trade, infrastructure, and institutions. One strand of this literature addresses the effect of various aspects of institutional quality on trade flows. Specifically, Levchenko (2007) and Ranjan and Lee (2007) look at contract enforcement, Anderson and Marcouiller (2002) employ a broad measure of institutional quality in their analysis, whereas Dutt and Traca (2008) share our focus on corruption. A second strand of this literature investigates the impact of infrastructure on international trade, including Bougheas et al. (1999), and Limao and Venables (2001). Moreover, a few papers, such as
Francois and Manchin (2007), look at both infrastructure and institutions and study empirically their joint impact on trade flows.

Another strand of this literature, to which this paper relates the most, explores how trade liberalization affects the quality of domestic institutions. In particular, Cheptea (2007) focuses on two factors promoting trade integration between Central and Eastern Europe and the European Union – trade liberalization and institutional reforms – and finds a positive correlation between the two. Ades and Di Tella (1997) restrict their attention to corruption and demonstrate that trade openness is associated with less corruption when looking at a cross-section of countries. On the other hand, using theoretical frameworks, Segura-Cayuela (2006) argues that given a set of weak political institutions, trade might increase the inefficiency of domestic economic policies, whereas Do and Levchenko (2009) argue that trade openness may be detrimental to institutional quality when firms differ with regard to their productivity, which in their model directly affects their political power. All of these papers, however, do not look at either infrastructure investment or the interaction between firms’ political ties and institutional quality. Both though are key for economic performance as the former affects countries’ export performance, whereas the latter determines firms’ political power, and thus influences the efficiency of government policies.

The setup of our model is provided in Section 2. The autarky equilibrium is derived in Section 3, whereas the trade equilibrium is obtained in Section 4. The equilibrium level of infrastructure investment and institutional quality is numerically derived and characterized in Section 5. Section 6 discusses some possible avenues for future research and concludes. All proofs are relegated to the Appendix.

2 The Model

We now formally model the economy of a developing country that is about to undergo institutional reform and infrastructure development, while considering whether to open up to trade. To this end, we begin by describing the consumption and production decisions faced by the economic agents. We then define the country’s market-institutions coefficient and its govern-
ment’s objective function. We conclude this section by specifying the timing of the different actions that will be undertaken in this economy.

2.1 Consumption

Consider an economy consisting of two sectors. The first sector produces a homogeneous good $z$, while the second one produces a continuum of differentiated goods indexed by $v$. The preferences of a representative consumer are captured by the following utility function:

$$ U = \left[ \int_{v \in V} x(v)^{\alpha} \, dv \right]^{\frac{\beta}{\alpha}} z^{1-\beta}, $$

where $V$ is the set of available varieties of the differentiated good, $x(v)$ is the quantity consumed of variety $v$, and $\alpha, \beta \in (0, 1)$, with $\beta$ measuring the share of total expenditure spent on the differentiated good. Standard utility maximization results in the following demand functions:

$$ x(v) = Ap(v)^{-\varepsilon} \quad \text{and} \quad z = (1 - \beta) \frac{E}{p_z}, \quad (2) $$

where $A \equiv \frac{\beta E}{\int_{v \in V} p(v)^{1-\varepsilon} \, dv}$ is the shift parameter of the demand for any variety of the differentiated good that is exogenous from the point of view of an individual firm, $E$ is total expenditure, $p(v)$ and $p_z$ are the goods prices, and $\varepsilon = \frac{1}{1-\alpha} > 1$ measures the elasticity of substitution between different varieties of the differentiated good. The economy’s aggregate price level is then given by:

$$ P = \left( \int_{v \in V} p(v)^{1-\varepsilon} \, dv \right)^{\frac{\beta}{1-\varepsilon}}. \quad (3) $$

2.2 Production

Labor is the only factor of production and is inelastically supplied at its aggregate level $L$. The homogeneous good $z$ is produced with a linear technology of the form $l_z = z$. We normalize $p_z$ to 1, implying that the wage in the economy also equals to 1 in equilibrium.
In the differentiated sector, there is a continuum of firms, each of which can only produce a single variety. The mass of the potentially produced varieties is fixed and equals to $N$. Variety $v$ can be supplied according to the following production technology:

$$l_v = a(v) f(F) + \frac{x(v)}{F},$$

(4)

where $F$ is the level of infrastructure in the economy and $a \geq 0$ is a parameter representing the discriminatory treatment the government applies across firms. Firms share the same $f(F) \geq f > 0$, with $\partial f / \partial F > 0$, but have a different $a$ and thus, a heterogeneous fixed cost of operation, $a f(F)$. Of course, each firm is free not to operate and therefore, not to incur this fixed cost. The technology specified in (4) reflects two key assumptions. First, in order to operate, firms have to incur a fixed cost that consists of (i) the costs levied by bureaucracy, such as regulatory compliance costs; and (ii) a lump-sum tax imposed by the government that is used to partly finance infrastructure spending, meaning that this tax is increasing in $F$. Since firms receive differential treatment (in accordance with their $a$), some of them avoid part of these costs and hence incur a lower fixed cost of operation, while others bear a disproportionately heavier financial burden. Second, active firms also face a variable cost of operation including production as well as distribution costs. This cost is homogeneous across firms. In addition, it is decreasing in the infrastructure level since, for example, a more developed transportation system reduces distribution costs, while a more reliable electricity grid has a dampening effect on the firms’ production costs.

We can then index firms in the differentiated sector by $a$. Any firm deciding to produce faces a residual demand curve given by (1). Profit maximization yields the following pricing strategy for all firms:

$$p(a) = \frac{1}{F^{\alpha}}.$$ 

[4] This assumption considerably simplifies the analysis. It has also been used by Chaney (2008) and Arkolakis (2008), among others.

[5] Note that all firms in the differentiated sector within our economy are characterized by the same productivity (although as will become evident below, firm productivity does differ across countries). We maintain this assumption because we choose to remain agnostic on the relation between firm productivity and political connectedness.
Thus, the equilibrium price is decreasing in $F$ as well as in the elasticity of substitution $\varepsilon$ (since $\varepsilon = \frac{1}{1-\alpha}$). Firm profit is then:

$$\pi(a) = p(a) x(a) - l(a) = A \left( \frac{1}{F\alpha} \right)^{1-\varepsilon} (1 - \alpha) - af(F),$$

implying that firms facing a lower discriminatory-treatment parameter $a$ enjoy higher profitability.

### 2.3 The Market-Institutions Coefficient

In this economy, some firm owners in the differentiated sector have stronger political ties than others, allowing them to receive preferential treatment from the government. This preferential treatment could translate into lighter taxation, relaxed regulatory oversight by the authorities, or stiffer regulatory supervision of their competitors. By contrast, firms with weak political connections get exploited. The political ties of a given firm are represented by $a_0$, with a lower $a_0$ signifying that the firm is more politically connected. We assume that $a_0$ is uniformly distributed on $[0, 2]$. However, the ability of the government to discriminate among firms hinges on the checks and balances of the economy, summarized in our model by a market-institutions coefficient $\theta \in (0, 1)$. A higher $\theta$ signifies higher institutional quality, which mitigates the impact of a firm’s political ties on its profits.

More specifically, the effect of institutional quality on the significance of political ties is captured by the discriminatory-treatment parameter $a$:

$$a = a_0 (1 - \theta) + \theta,$$

with a cumulative distribution function $G(a) = \frac{a - \theta}{2(1-\theta)}$. Therefore, for given market institutions, a firm with closer ties to the government (i.e., a firm with a lower $a_0$) has to incur a lower fixed cost in order to supply the differentiated good. Intuitively, such a firm enjoys lower regulatory compliance costs and/or lighter taxation. As institutional quality though improves (i.e., as $\theta$ rises), political ties to the current government matter less and firms’ fixed cost of operation converges to $f(F)$. It is apparent that firms with $a_0 < 1$ prefer poor institutions as in a corrupt economy they can take advantage of their political connections, whereas firms with $a_0 > 1$ prefer institutions of high quality in order to avoid government exploitation.
2.4 Government Objective Function

The government maximizes a linear objective function \( \Gamma \) comprising four terms: (i) labor real income; (ii) real firm profits weighed by the discriminatory-treatment parameter \( a \), such that firms with closer ties to the government receive a larger weight; (iii) the cost in real terms of the institutional reform; and (iv) the real cost of partly funding infrastructure investment through nontax sources. Formally:

\[
\Gamma = \frac{L}{P} + \frac{N}{P} \int_{a \in \Delta} \lambda (a) \pi (a) dG (a) - \frac{C (\theta)}{P} - \frac{\varphi F}{P},
\]

where \( \varphi \in [0, 1], C (\theta) \geq 0 \) is the (nominal) cost of establishing market institutions of quality \( \theta \), \( \Delta \) denotes the set of firms operating in equilibrium, and \( \lambda (a) \) is the weight the government places on firm \( a \), with \( \frac{\partial \lambda}{\partial a} < 0 \).

2.5 The Multistage Game

We assume the country is initially in autarky and is characterized by very weak institutions and very limited infrastructure. The government is about to undertake a two-part reform and is considering the benefits from opening up the economy to trade. The reform in question consists of choosing new market institutions (i.e., \( \theta \)) and the new level of infrastructure (i.e., \( F \)). Our aim in this paper is to compare the equilibrium outcome of this reform in autarky and under trade. For either regime, we consider the following multistage game:

- **Stage 1:** The government picks the market-institutions coefficient \( \theta \) so as to maximize its objective function \( \Gamma \) defined by (7).

- **Stage 2:** The government selects the level of infrastructure \( F \) in order to maximize \( \Gamma \) subject to the constraint that infrastructure spending does not exceed the tax revenues collected from firms plus the nontax funds allocated to infrastructure investment.

\(^6\text{The assumption that infrastructure investment is not exclusively funded by firm taxation is made so that our model is more consistent with real-world experiences. For instance, the government could privatize some state-owned enterprises or borrow money from abroad in order to partly fund infrastructure investment (and not overtax the economy). Furthermore, relaxing this assumption would not affect the qualitative nature of our findings.}\)
• Stage 3: Firms choose whether to actually engage in production or not. Production then takes place, and profits are realized.

This outlines the basic structure of our model. We solve our game recursively. Specifically, we first derive the closed- and open-economy equilibria for a given $\theta$ and $F$. Subsequently, we turn to Stage 2, and solve numerically for the optimal $F$ in autarky and under trade: $F^*_A$ and $F^*_T$, respectively. Last, we look at Stage 1, and obtain numerically the optimal $\theta$ in autarky and under trade: $\theta^*_A$ and $\theta^*_T$, correspondingly.

3 Production Equilibrium in a Closed Economy

In this section, we assume the economy is closed, and that both $\theta_A$ and $F_A$ are given, where subscript $A$ refers to the autarky values. To characterize the autarky production equilibrium, we need to derive the cutoff level of $a$, $a_A$, such that all firms above this $a$ (i.e., firms facing a relatively high fixed cost of operation) decide not to supply the differentiated good.

In equilibrium, since only firms with $a \leq a_A$ find it optimal to operate, we have that:

$$\int_{v \in V} p_A(v)^{1-\varepsilon} \, dv = \frac{N}{(F_A \alpha)^{1-\varepsilon}} G(a_A) \Rightarrow A_A = \frac{(F_A \alpha)^{1-\varepsilon} \beta E_A}{NG(a_A)}. \quad (8)$$

Moreover, the cutoff firm $a_A$ makes by definition zero profit, i.e., $\pi_A(a_A) = 0$, a condition that can be rewritten using (8) as:

$$\frac{\beta E_A}{NG(a_A)} (1-\alpha) = a_A f_A, \quad (9)$$

where $f_A \equiv f(F_A)$. We can now use equation (9) to determine $a_A$, but first the equilibrium value of total expenditure $E_A$ is required. To obtain the latter, we impose the goods market-clearing condition that total expenditure must equal national income, which is the sum of labor income plus the profits accruing to all active firms:

$$E_A = L + N \int_{\theta_A}^{a_A} \pi_A(a) \, dG(a).$$

Using equation (9), it is straightforward to show that:

$$E_A = L + N f_A \frac{(a_A - \theta_A)^2}{4(1-\theta_A)}. \quad (10)$$
Plugging finally (10) into (9), we derive $a_A$.

**Lemma 1** The cutoff value $a_A$ in the autarky production equilibrium equals:

$$
a_A = \frac{NF_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{NF_A [4L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + NF_A \theta_A^2]}}{NF_A [2 - \beta (1 - \alpha)]},
$$

such that $a_A > \theta_A$.

Lemma 1 shows that a strictly positive mass of firms operate in equilibrium.\(^7\) This mass decreases with the level of public spending on infrastructure, since a higher $F_A$ results in an increased tax burden across firms.

**Lemma 2** The cutoff value $a_A$ decreases with $F_A$, i.e., $\frac{\partial a_A}{\partial F_A} < 0$.

Using equation (3) and Lemma 2, we obtain Proposition 1.

**Proposition 1** Higher infrastructure investment results in a lower aggregate price index provided that \((\varepsilon - 1)(\theta_A - a_A) F_A - \frac{\partial a_A}{\partial F_A} < 0\). This condition is satisfied if $\frac{\partial f_A}{\partial F_A}$ is sufficiently small.

Intuitively, two offsetting forces are at work here. On the one hand, a higher $F_A$ lowers the firms’ marginal cost of operation (i.e., $\frac{1}{F_A}$) and therefore prices, decreasing $P_A$. On the other hand, it raises the firms’ fixed cost of operation (i.e., $af_A$) and hence reduces the number of active firms (as demonstrated by Lemma 2), which acts to increase $P_A$. In other words, infrastructure investment has two rival effects on firms: a beneficial productivity effect and a detrimental fixed-cost effect. If the latter is sufficiently weak (i.e., if $\frac{\partial f_A}{\partial F_A}$ is sufficiently small), then $\left| \frac{\partial a_A}{\partial F_A} \right|$ is sufficiently small and the inequality stated in the proposition is satisfied (recall that $\varepsilon > 1$ and $\theta_A < a_A$).

\(^7\)As we show in the Appendix, a second equilibrium exists in which no firm produces. We choose though to focus on the more interesting (and standard) case where the firm with the lowest fixed cost of operation does produce.
4 Production Equilibrium in an Open Economy

We next examine the production equilibrium that would arise in our economy under trade with a country characterized by strong market institutions. To this end, we model international trade between the home economy ($H$) described above and a foreign one ($F$) that only differs in two respects: (i) its market-institutions coefficient $\theta$ equals 1, i.e., the foreign country is a perfect market economy; and (ii) there is a large (unbounded) mass of potential entrants into its differentiated sector. It is important to emphasize that the two countries are assumed to have the same endowment of labor $L$, i.e., they are of equal size, which simplifies the comparison between the two economies.

Moreover, we assume that $z$ can be traded costlessly, implying that as long as both countries produce some $z$, wages in the two countries equal to 1. Nevertheless, firms in the differentiated sector have to incur two additional costs in order to export: a fixed per-period cost $af_i^X > 0$, where $i \in \{H,F\}$ and $f_i^X \equiv f_X (F_i)$, and a per-unit transportation cost $\tau$. The former consists of the bureaucratic costs imposed on exporting firms (e.g., the cost of obtaining an export permit) as well as of a government lump-sum tax levied on them in order to partly finance infrastructure investment, is a function of the discriminatory-treatment parameter $a$, and is measured in terms of units of labor. The latter is of the iceberg type, is common across firms, and is denoted by $\tau > 1$. $\tau$ represents the number of units of a particular variety that need to be shipped so that 1 unit arrives at destination. Without loss of generality, we assume that home and foreign firms in the differentiated sector face the same transportation costs.

Note that regardless of whether an exporting firm sells domestically or not, it still incurs the fixed cost $af^i_i$. A straightforward implication of this assumption is that for all exporting firms, it is optimal to also produce for their domestic market. Furthermore, observe that $\theta = 1$ in the foreign country means that all firms in its differentiated sector have the same fixed cost of operation $f^F_X$ and the same fixed cost of exporting $f^F_X$ (i.e., $a = 1$ across country-$F$ firms).

---

8It could be argued that an exporting firm also faces a fixed per-period cost associated with the importing-country bureaucracy. However, introducing such a cost would not affect the qualitative nature of our findings, and therefore, it is omitted for expositional simplicity.
Let us begin our analysis from the home country. To characterize its open-economy equilibrium, we need to determine the cutoff values of \( a \) for production and exporting: \( a_D \) and \( a_X \), respectively. Each firm’s domestic pricing strategy is still \( p^D_H (a) = \frac{1}{F^H_\alpha} \), resulting in the following (domestic) profit:

\[
\pi^H_D (a) = A^H \left( \frac{1}{F^H_\alpha} \right)^{1-\varepsilon} (1 - \alpha) - af^H. \tag{11}
\]

For the exporting firms, the prices in the foreign market are higher due to the transportation costs: \( p^H_X (a) = \frac{\tau}{F^H_\alpha} \). Their export profits are then equal to:

\[
\pi^H_X (a) = A^F \left( \frac{\tau}{F^H_\alpha} \right)^{1-\varepsilon} (1 - \alpha) - af^H. \tag{12}
\]

By definition, we have that for the cutoff firms \( a_D \) and \( a_X \): \( \pi^H_D (a_D) = 0 \) and \( \pi^H_X (a_X) = 0 \), correspondingly. We also have that in equilibrium, total expenditure must equal national income:

\[
E^H = L + N^H \left( \int_{a^D}^{a_X} \pi^H_D (a) dG (a) + \int_{a^D}^{a_X} \pi^H_X (a) dG (a) \right) = L + N^H \left( f^H (a_D - \theta)^2 \frac{2}{4(1 - \theta)} + f^H (a_X - \theta)^2 \frac{2}{4(1 - \theta)} \right), \tag{13}
\]

where the second equality uses the zero-cutoff-profit conditions.

We next turn to the foreign country. Here, in the differentiated sector, all firms have an identical cost structure. In addition, unlike the home country, there is an unbounded mass of potential entrants. Therefore, to characterize the trade production equilibrium of the foreign country, we need to obtain the equilibrium number of firms selling domestically and exporting: \( n^F_D \) and \( n^F_X \), correspondingly. The domestic and export prices are analogous to the home-country ones: \( p^F_D = \frac{1}{F^F_\alpha} \) and \( p^F_X = \frac{\tau}{F^F_\alpha} \), respectively. The corresponding profits from domestic sales and exports are then:

\[
\pi^F_D = A^F \left( \frac{1}{F^F_\alpha} \right)^{1-\varepsilon} (1 - \alpha) - f^F \quad \text{and} \quad \tag{14}
\]

\[
\pi^F_X = A^H \left( \frac{\tau}{F^F_\alpha} \right)^{1-\varepsilon} (1 - \alpha) - f^F. \tag{15}
\]

These profits are identical across firms because in a perfect market economy, all firms face the same fixed costs of operation and exporting. However, given there is free entry, equilibrium firm profits equal to zero, implying that the income-equals-expenditure condition is simply:

\[
E^F = L. \tag{16}
\]
Let us now look at the home and foreign aggregate price indices. In equilibrium, since only firms with \( a \leq a_D \) operate in country \( H \), its aggregate price index equals to:

\[
P^H = \left( N^H \left( \frac{1}{F^H\alpha} \right)^{1-\varepsilon} \frac{a_D - \theta}{2(1 - \theta)} + n^F \left( \frac{\tau}{F^F\alpha} \right)^{1-\varepsilon} \right)^{\frac{\beta}{1-\varepsilon}}. \tag{17}
\]

At the same time, since only country-\( H \) firms with \( a \leq a_X \) export, the equilibrium aggregate price level of country \( F \) is given by:

\[
P^F = \left( N^H \left( \frac{\tau}{F^H\alpha} \right)^{1-\varepsilon} \frac{a_X - \theta}{2(1 - \theta)} + n^F \left( \frac{1}{F^F\alpha} \right)^{1-\varepsilon} \right)^{\frac{\beta}{1-\varepsilon}}. \tag{18}
\]

Straightforward algebra then reveals that:

\[
a_D = \frac{f^F_X}{f^H_X} \left( \frac{F^H\tau}{F^F\tau} \right)^{\varepsilon-1}, \tag{19}
\]

\[
a_X = \frac{f^F}{f^H_X} \left( \frac{F^H\tau}{F^F\tau} \right)^{\varepsilon-1}, \tag{20}
\]

\[
n^F_D = \frac{(1 - \alpha) \beta E^F}{f^F_X} - N^H \left( \frac{F^H\tau}{F^F\tau} \right)^{\varepsilon-1} \frac{a_X - \theta}{2(1 - \theta)}, \quad \text{and} \quad \tag{21}
\]

\[
n^F_X = \frac{(1 - \alpha) \beta E^H}{f^F_X} - N^H \left( \frac{F^H\tau}{F^F\tau} \right)^{\varepsilon-1} \frac{a_D - \theta}{2(1 - \theta)}, \tag{22}
\]

where \( E^F \) and \( E^H \) are given by equations (16) and (13), respectively. A few remarks are here in order. First, infrastructure investment \( F^H \) has a direct positive impact on the number of country-\( H \) firms selling domestically (i.e., on \( a_D \)) as well as on the number of the ones exporting (i.e., on \( a_X \)). This is due to its productivity effect: As country-\( H \) firms become more efficient, they can better compete against the country-\( F \) firms both domestically and abroad. Nonetheless, as equations (19) and (20) demonstrate, investing in infrastructure also has a detrimental fixed-cost effect on country-\( H \) firms that is reflected in the rise of both \( f^H \) and \( f^H_X \), which acts to reduce the home-country cutoff values for production and exporting. In other words, the overall effect of an increase in \( F^H \) on the number of country-\( H \) firms being active in their domestic or export market is ambiguous. Of course, infrastructure investment in the foreign country \( F^F \) has the exact opposite ramifications for \( a_D \) and \( a_X \). Furthermore, note that \( a_D > a_X \iff f^F_X f^H \tau^{2(\varepsilon-1)} > f^F f^H \), which is a standard assumption in the literature on
trade with heterogeneous firms. This condition is clearly satisfied when both \( f^H \) and \( f^X \) are sufficiently large. Last, observe that the level of foreign (home) consumer expenditure on the differentiated good \( \beta E^F (\beta E^H) \) affects directly in a positive manner the number of country-\( F \) firms that are active in their domestic (export) market.

Let now \( \eta^H = \frac{\partial f^H}{\partial F^H} f^H \) and \( \eta^X = \frac{\partial f^X}{\partial F^H} f^X \). In the remainder of this section, we do comparative statics.

**Lemma 3** If the elasticity of substitution is sufficiently large, then the country-\( H \) cutoff values for production and exporting increase with infrastructure investment \( F^H \). In particular, if \((\varepsilon - 1) > \eta^H\), then \( \frac{\partial a_D}{\partial F^H} > 0 \); moreover, if \((\varepsilon - 1) > \eta^X\), then \( \frac{\partial a_X}{\partial F^H} > 0 \).

In the trade regime, unlike the autarky regime, the home-country cutoff value for production \( a_D \) might increase with infrastructure investment \( F^H \). Similarly, the cutoff value for exporting \( a_X \) might too rise with \( F^H \). Intuitively, as we discussed above, an increase in \( F^H \) has two offsetting effects on \( a_D \) and \( a_X \): a positive productivity effect and a negative fixed-cost effect. If the elasticity of substitution is sufficiently large, the former effect dominates because consumers are then highly responsive to price changes. The productivity effect outweighs the fixed-cost effect also when \( \eta^H \) and \( \eta^X \) are sufficiently low, since then raising infrastructure investment does not entail a heavy additional tax burden for country-\( H \) firms.

**Lemma 4** If the elasticity of substitution is sufficiently large, then the number of country-\( F \) firms selling domestically decreases with infrastructure investment \( F^H \). In particular, if \((\varepsilon - 1) > \eta^X\), then \( \frac{\partial n^F_D}{\partial F^H} < 0 \).

The intuition underlying Lemma 4 is straightforward. If the elasticity of substitution is large enough (or \( \eta^X \) is low enough), the productivity effect of infrastructure investment \( F^H \) on country-\( H \) firms outweighs its fixed-cost effect, resulting in fewer country-\( F \) firms being active in their domestic market in equilibrium (i.e., a lower \( n^F_D \)). However, it is important to note that in this case, the number of foreign firms exporting (i.e., \( n^F_X \)) might still rise. This is due

---

9See, for instance, Melitz (2003), and Helpman et al. (2004).
to an income effect. More specifically, if $\varepsilon$ is sufficiently large, income increases in the home country as $F^H$ rises due to larger aggregate firm profits, which might potentially lead to a higher demand for imports (despite the lower domestic-firm prices).

**Proposition 2** One set of conditions guaranteeing that the aggregate price index in country $H$ decreases as $F^H$ rises is that (i) the elasticity of substitution is sufficiently large; and (ii) infrastructure investment in country $F$ is relatively large as compared with the country-$H$ one. In particular, if:

\[
(\varepsilon - 1) > \max \{\eta^H, \eta^H_X\} \quad \text{and} \quad 1 > \frac{F^H \frac{\partial a_D}{\partial F^H}}{F^H \frac{\partial a_D}{\partial F^H} + (a_D - \theta)(\varepsilon - 1)} > \left(\frac{F^{H \tau}}{F^H}\right)^{\varepsilon - 1},
\]

then $\partial P^H / \partial F^H < 0$.

To gain some insight for Proposition 2, recall that as $F^H$ increases, the domestic prices of the home firms decrease. At the same time, the conditions stated in Proposition 2 ensure that product variety in the home country rises. As a result, the country-$H$ aggregate price index declines and real income increases.

**Proposition 3** If the elasticity of substitution is sufficiently large and infrastructure investment $F^H$ increases (decreases) with the market-institutions coefficient $\theta$, then the country-$H$ cutoff values for production and exporting increase (decrease) with $\theta$. In particular, if $(\varepsilon - 1) > \eta^H$ and $\partial F^H / \partial \theta > (\)0, then $\frac{\partial a_D}{\partial \theta} > (\)0. Moreover, if $(\varepsilon - 1) > \eta^H_X$ and $\partial F^H / \partial \theta > (\)0, then $\frac{\partial a_X}{\partial \theta} > (\)0.

A high elasticity of substitution implies that both $a_D$ and $a_X$ increase with $F^H$. Proposition 3 then follows trivially when the second condition is also met.

### 5 Infrastructure and Institutions Equilibrium

We next turn to Stages 1 and 2 of our game. To complete our analysis, we need to derive the optimal market-institutions coefficient $\theta$ and infrastructure investment $F^H$ in autarky and
under trade, utilizing the production equilibria characterized above. To this end, we resort to numerical analysis. Let us now define the following three relationships. First, we assume firms’ fixed costs are a linear function of infrastructure investment:

\[ af_j^i = a f_j^i + b_j^i \cdot F^i, \]  

(23)

where \( f_j^i, b_j^i \geq 0, i \in \{H, F\}, \) and \( j = X \) when the fixed cost of exporting is considered. Second, recall that firm owners in the differentiated sector differ with respect to their political ties to the government as captured by their parameter \( a_0 \). This affects firms in two ways. On the one hand, they share unequally the infrastructure burden as equation (23) demonstrates. On the other hand, the government weights them differently when choosing infrastructure investment and market institutions. In particular, the government places weight \( \lambda(a) \) on firm \( a \)'s profits, where \( \lambda(a) \) is decreasing in the discriminatory-treatment parameter \( a \). We make here the assumption that:

\[ \lambda(a) = (2 - a) \omega, \]  

(24)

with \( \omega \geq 0 \). Last, we have to specify the monetary cost \( C(\theta) \) of establishing market institutions of quality \( \theta \). A quadratic function is assumed:

\[ C(\theta) = \sigma \theta^2. \]  

(25)

The parameters we initially use in order to obtain the numerical solution for the benchmark case are listed in Table 1. A number of assumptions underlie these parameters. First, \( \omega = 1 \), meaning the home government places the same weight on labor income and the profit of the average potential firm (i.e., the one with \( a_0 = 1 \)). Second, consumers divide their income equally between the homogeneous and the differentiated good, i.e., \( \beta = 0.5 \). Third, we assume that only a small part of infrastructure investment is financed through nontax sources and set \( \varphi = 0.1 \). Finally, the elasticity of substitution equals 5, which is within the range suggested by the literature (see Anderson and van Wincoop, 2004).

5.1 The Benchmark Solution

We begin by deriving the autarky solution. Specifically, we obtain numerically the optimal level of infrastructure investment \( F^*_A \) and the optimal market-institutions coefficient \( \theta^*_A \), utilizing
The benchmark parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^H$</td>
<td>150</td>
<td>$f^H$</td>
<td>7</td>
</tr>
<tr>
<td>$L$</td>
<td>10,000</td>
<td>$b^H$</td>
<td>1/150</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1</td>
<td>$f_x^H$</td>
<td>5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
<td>$b_x^H$</td>
<td>1/150</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8</td>
<td>$f^F$</td>
<td>7</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.1</td>
<td>$b^F$</td>
<td>1/150</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>100</td>
<td>$f_x^F$</td>
<td>1</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.2</td>
<td>$b_x^F$</td>
<td>1/225</td>
</tr>
<tr>
<td>$F^F$</td>
<td>500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: The benchmark model

the relationships and the parameters specified above. We find that the former equals 61.11 and the latter 0.99. Our results indicate that although in autarky investment in infrastructure in the home country is low as compared with the foreign country ($F^F = 500$), the market-institutions coefficient is set close to 1, i.e., near the country-$F$ level. Moreover, the autarky equilibrium is characterized by large product variety (90% of potential firms do produce in the autarky equilibrium), which benefits the consumers. Nevertheless, the lack of infrastructure investment entails a large marginal cost of operation for the firms and thus, elevated prices, resulting in a small per-variety amount produced.

In contrast to autarky, under trade, infrastructure investment in country $H$ exceeds the one in country $F$, but the market-institutions coefficient is set at a lower level. In particular, infrastructure investment $F^*_T = 571.61$ and the market-institutions coefficient $\theta^*_T = 0.72$. Intuitively, the home government chooses a higher $F^H$ under trade so that domestic firms can better compete against their foreign counterparts both in the home market and abroad. However, although a higher $F^H$ raises firms’ productivity, it also implies, for a given $\theta$, higher fixed costs of operation and exporting. The home government therefore chooses now a lower $\theta$ in order to
benefit the politically connected firms by reducing their lump-sum-tax burden. Interestingly, infrastructure investment and market institutions emerge as substitutes in our setting.

Note that as compared with autarky, under trade, infrastructure investment in country $H$ is more than 9 times larger but the market-institutions coefficient is 28% lower. This results in less variety being produced but also in a significantly higher output per variety. While 59.8% of potential firms now produce for their domestic market versus 90% in autarky, the output per active firm is more than 14 times higher. Moreover, 44% of potential firms do export. Finally, a welfare comparison between autarky and trade reveals that aggregate welfare in the home country is more than three times higher under trade. This is mainly due to the fact that when its economy is open, its aggregate price index is 69% lower.

5.2 Sensitivity Analysis

Last, we perform sensitivity analysis with regard to the elasticity of substitution $\varepsilon$, the political-economy parameter $\omega$, and the foreign infrastructure investment $F^F$. We begin with the parameter $\varepsilon$.

**The Elasticity of Substitution $\varepsilon$:** Reducing $\varepsilon$ by 5% results in an autarky equilibrium characterized by higher infrastructure investment $F^*_A$, a lower market-institutions coefficient $\theta^*_A$, more product variety, and higher aggregate welfare as compared with the benchmark case. A similar outcome is obtained in the open-economy case. Intuitively, a lower $\varepsilon$ raises firms’ market power, leading to an upward pressure on prices. The home government therefore increases infrastructure investment to countervail the pressure on the aggregate price index. To dampen the negative effect of a higher $F^H$ on the politically-connected firms’ fixed costs and therefore profits, a lower $\theta$ is now chosen, highlighting the fact that $F^H$ and $\theta$ are substitutes.

**The Political-Economy Parameter $\omega$:** We now assume the home government places a higher weight on the profit of the average potential firm (i.e., the one with $a_0 = 1$) than on labor income. In particular, we set $\omega = 2$. In the new autarky equilibrium, infrastructure
investment $F_A^*$ and the market-institutions coefficient $\theta_A^*$ are both lower, product variety is higher, and aggregate welfare is lower in comparison with the benchmark case. The intuition is straightforward. A larger $\omega$ implies a higher weight on firms’ profits in the government’s objective function. However, this effect is not symmetric across firms. Rather, it is biased in favor of the politically connected firms. The result then follows trivially once it is recalled that in autarky, there are no competitiveness considerations.

On the other hand, in the open-economy case, a higher $\omega$ results in a lower $\theta_T^*$ but a higher $F_T^*$ in equilibrium. The home government now increases $F^H$ in contrast to the closed-economy case in order to improve the domestic firms’ international competitiveness and thus their profitability in both the home and the foreign market. In other words, as the home government weighs firms more heavily than consumers, the home country ends up with weaker institutions and thus more corruption, higher infrastructure investment, and a higher export volume.

**The Foreign Infrastructure Investment $F^F$:** Finally, we investigate how a reduction in country $F$’s aggregate productivity affects the open-economy equilibrium. To this end, we reduce $F^F$ by 5% to 475. This results in a reduction in both $F_T^*$ and $\theta_T^*$, but an increase in product variety and aggregate welfare. As $F^F$ decreases and foreign firms become less competitive, the government of country $H$ finds it optimal to invest less in infrastructure. Moreover, a lower $F^F$ implies, ceteris paribus, a greater number of active firms in equilibrium in country $H$ (given a sufficiently high $\varepsilon$). Remember that the home government finances infrastructure investment mostly by taxing active firms. Therefore, as the financing of $F^H$ is now easier, the country-$H$ government chooses a lower $\theta$ benefitting the politically connected firms.

### 6 Conclusion

This paper has investigated the impact of international trade on both economic institutions and infrastructure investment. Our analysis has rested on the assumptions that (i) some firm owners have stronger political ties than others, allowing them to receive preferential treatment from the
government; (ii) a high institutional quality mitigates the significance of political connections; and (iii) infrastructure investment has two rival effects on firms: a detrimental fixed-cost effect since it is partly financed through lump-sum firm taxation and a beneficial productivity effect. We have demonstrated that trade leads to higher infrastructure investment so that domestic firms become more efficient and can therefore better compete internationally. However, trade results also in weaker institutions as the government attempts to lower the heightened tax burden on the politically connected firms. In other words, infrastructure investment and institutional quality have emerged as substitutes within our setting. Moreover, we have shown that trade with a partner characterized by high firm productivity induces a country to invest more in both its infrastructure and its institutional framework, even though the latter is against the interests of the firms with the stronger political ties.

In future work, we plan to extend our analysis in a number of dimensions. In particular, we intend to develop a dynamic multicountry trade model in order to study whether trade could have a domino effect on institutional quality and infrastructure investment across countries. For instance, we wish to address the following question: If a developing country engaged in trade with a developed market economy and as a result invested heavily in its infrastructure as our model predicts, would this be a catalyst for infrastructure development and institutional reform for the rest of its trading partners? The answer to such a question would have important policy implications for regions such as Africa or South America. Another avenue we plan to pursue is to empirically test our findings. Of course, the definition of market institutions in our paper has been quite broad. This was deliberate as we chose to abstract from institutional details for expositional simplicity. Nevertheless, a number of predictions our model generates could be readily tested. In particular, does trade liberalization lead to higher infrastructure investment? Does it simultaneously though lead to increased inequality or corruption? Such an exercise would provide us with significant insights regarding the overall gains and losses stemming from an extensive trade reform.
References


7 Appendix

7.1 Production Equilibrium in a Closed Economy

7.1.1 Proof of Lemma 1

Plugging (10) into (9), we obtain the following solutions for $a_A$:

$$a_{A,1,2} = \begin{cases} \frac{N f_A \theta_A [1 - \beta (1 - \alpha)] - \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} \\ \frac{N f_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} \end{cases}$$

Let us look first at $a_{A_1}$:

$$a_{A_1} = \frac{\theta_A [1 - \beta (1 - \alpha)]}{2 - \beta (1 - \alpha)} - \frac{\sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} < \frac{\theta_A [1 - \beta (1 - \alpha)]}{2 - \beta (1 - \alpha)} < \theta_A.$$

We now turn to $a_{A_2}$:

$$a_{A_2} = \frac{N f_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} > \theta_A \Leftrightarrow$$

$$\Leftrightarrow 4 N f_A L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) > 0,$$

which clearly holds. Therefore, we have a unique closed-economy equilibrium in which at least some of the potential producers in the differentiated sector do produce:

$$a_A = \frac{N f_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} > \theta_A.$$

This concludes the proof of Lemma 1.

7.1.2 Proof of Lemma 2

Taking the first-order derivative of $a_A$ with respect to $F_A$, we obtain:

$$\frac{\partial a_A}{\partial F_A} = -\frac{1}{f_A \theta A} \frac{2 L \beta (1 - \alpha) (1 - \theta_A)}{\sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}} < 0,$$ (26)

since $\frac{\partial f_A}{\partial F_A} > 0$ by assumption. This concludes the proof of Lemma 2.
7.1.3 Proof of Proposition 1

Straightforward algebra reveals:

\[
\frac{\partial P_A}{\partial F_A} = \frac{\beta}{2(\epsilon - 1)(1 - \theta_A)} \left( \frac{a_A - \theta_A}{2(1 - \theta_A)} \right)^{\frac{\beta + \epsilon - 1}{1 - \epsilon}} N_{\theta_A}^{\theta_A} (F_A \theta)^{-\beta} \left[ \frac{(\epsilon - 1)(\theta_A - a_A)}{F_A} - \frac{\partial a_A}{\partial F_A} \right] < 0 \Leftrightarrow \\
\Leftrightarrow \frac{(\epsilon - 1)(\theta_A - a_A)}{F_A} - \frac{\partial a_A}{\partial F_A} < 0,
\]

where the “\(\Leftrightarrow\)” follows since the terms multiplying the square brackets are all strictly positive.

The first term in (27) is strictly negative since \(a_A > \theta_A\) by Lemma 1, whereas the second one is strictly positive since \(\frac{\partial a_A}{\partial F_A} < 0\) by Lemma 2. However, if \(\frac{\partial f_A}{\partial F_A}\) is sufficiently small, then \(\left| \frac{\partial a_A}{\partial F_A} \right|\) is sufficiently small by (26), and the inequality in (27) is satisfied. This concludes the proof of Proposition 1.

7.2 Production Equilibrium in an Open Economy

7.2.1 Proof of Lemma 3

Straightforward calculations yield:

\[
\frac{\partial a_D}{\partial F^H} = a_D \left( \frac{\epsilon - 1}{F^H} - \frac{1}{f^H} \frac{\partial f^H}{\partial F^H} \right) > 0 \Leftrightarrow \\
\Leftrightarrow \frac{\epsilon - 1}{F^H} - \frac{1}{f^H} \frac{\partial f^H}{\partial F^H} > 0 \Leftrightarrow \epsilon - 1 > \frac{F^H}{f^H} \frac{\partial f^H}{\partial F^H} \equiv \eta^H,
\]

where the first “\(\Leftrightarrow\)” follows since \(a_D > \theta > 0\). Moreover, we have:

\[
\frac{\partial a_X}{\partial F^H} = a_X \left( \frac{\epsilon - 1}{F^H} - \frac{1}{f_X^H} \frac{\partial f_X^H}{\partial F^H} \right) > 0 \Leftrightarrow \\
\Leftrightarrow \frac{\epsilon - 1}{F^H} - \frac{1}{f_X^H} \frac{\partial f_X^H}{\partial F^H} > 0 \Leftrightarrow \epsilon - 1 > \frac{F^H}{f_X^H} \frac{\partial f_X^H}{\partial F^H} \equiv \eta^H,
\]

where the first “\(\Leftrightarrow\)” follows due to \(a_X > \theta > 0\). This concludes the proof of Lemma 3.

7.2.2 Proof of Lemma 4

Simple algebra reveals:

\[
\frac{\partial n^D}{\partial F^H} = -\frac{N^H}{2(1 - \theta)} \left( \frac{F^H}{F^H f^H} \right)^{\epsilon - 1} \left[ \frac{(\epsilon - 1)(a_X - \theta)}{F^H} + \frac{\partial a_X}{\partial F^H} \right].
\]
If \( \varepsilon - 1 > \eta_X^H \), then \( \frac{\partial a_X}{\partial F^H} > 0 \) by Lemma 3, implying that the sum in the square brackets is strictly positive, and thus, \( \frac{\partial \pi}{\partial F^H} < 0 \). This concludes the proof of Lemma 4.

### 7.2.3 Proof of Proposition 2

We know from (17) that:

\[
p^H = \left( N^H \left( \frac{1}{F^H \alpha} \right)^{1-\varepsilon} \frac{a_D - \theta}{2(1 - \theta)} + n_X^F \left( \frac{\tau F^H}{F^F \alpha} \right)^{1-\varepsilon} \right)^{\frac{\beta}{1-\varepsilon}}.
\]

As \( F^H \) rises, the domestic prices of the home firms (i.e., \( p^H_D(a) = \frac{1}{F^H \alpha} \)) decrease, whereas the export prices of the foreign firms (i.e., \( p^F_X = \frac{\tau F^H}{F^F \alpha} \)) remain unchanged. At the same time, it is direct to show that total product variety in the home country equals:

\[
n_X^F + N^H \frac{a_D - \theta}{2(1 - \theta)} = \left( \frac{1 - \alpha}{f_X^F} \left( L + N^H \left( f^H (a_D - \theta)^2 \frac{4(1 - \theta)}{4(1 - \theta)} + f_X^H (a_X - \theta)^2 \right) \right) \right)^{\frac{\beta}{1-\varepsilon}}.
\]

It can be readily shown that \( \frac{\partial \pi}{\partial F^H} > 0 \) if \( \frac{\partial a_D}{\partial F^H} > 0 \) and \( \frac{\partial a_X}{\partial F^H} > 0 \). These conditions are true (i.e., \( \frac{\partial a_D}{\partial F^H} > 0 \) and \( \frac{\partial a_X}{\partial F^H} > 0 \)) as long as \( (\varepsilon - 1) > \max \{ \eta_X^H, \eta_X^H \} \) by Lemma 3. Moreover, we have that:

\[
\frac{\partial \Lambda}{\partial F^H} > 0 \Leftrightarrow \frac{\partial a_D}{\partial F^H} > \left( \frac{F^H \tau}{F^F} \right)^{\varepsilon-1} \left( \frac{\partial a_D}{\partial F^H} + \frac{(a_D - \theta)(\varepsilon - 1)}{F^H} \right).
\]

The proposition then follows.

### 7.2.4 Proof of Proposition 3

Using the chain rule, we obtain:

\[
\frac{da_D}{d\theta} = \frac{\partial a_D}{\partial \theta} + \frac{\partial a_D}{\partial F^H} \frac{\partial F^H}{\partial \theta} = \frac{\partial a_D}{\partial \theta} \frac{\partial F^H}{\partial \theta} \quad \text{and} \quad \frac{da_X}{d\theta} = \frac{\partial a_X}{\partial \theta} + \frac{\partial a_X}{\partial F^H} \frac{\partial F^H}{\partial \theta} = \frac{\partial a_X}{\partial \theta} \frac{\partial F^H}{\partial \theta}.
\]

The proposition then follows directly from Lemma 3.