Timing of Protectionism

Aurora Gómez-Galvarriato,* and César L. Guerrero-Luchtenberg†,+, ‡

*Centro de Investigación y Docencia Económicas (CIDE), Mexico  
†University of Castilla la Mancha, Spain.

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Abstract

Recent history gives us evidence of the different timing and results of the opening up of several economies. We present a model to explain this divergence. Accordingly with this evidence, we show that, provided the government prefers more competition than less competition irrespective of the firms’ nationality, essentially three concepts explain everything: The agents’ degree of impatience, the gap between the domestic and the foreign technologies and the costs due to the political environment. In sharp contrast to the existing literature, we show that a temporal protectionism can be time consistent, and domestic firms adopt new technologies under it.

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*Corresponding author. Division of Economics CIDE. Carretera México-Toluca 3655. Lomas de Santa Fe 12010, Mexico D.F., Mexico. Tel: +52 55-5727-9800. E-mail: aurora.gomez@cide.edu

†University of Castilla la Mancha, Spain, and University of Alicante, Spain. E-mail: cesarl.guerrero@uclm.es, and c.guerrero@merlin.fae.ua.es

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1 Introduction

Why and when policy-makers decide to open up the economy and whether firms choose to invest in new technology or not as a result of such decision? Can a temporal protectionist policy be time consistent? Why do countries follow different paths to openness, and what are the economic consequences of that? Why do some countries under protectionism have been successful in inducing firms to adopt new technologies and others have not? These are some of the questions our model can answer. These questions constitute the core of the issue of why different countries display different timing of protectionism.

During the second half of the 20th century, different countries have taken different paths away from the protectionist policies that became dominant during the Great Depression. Some, such as Spain or Korea, started opening-up early, others, as most countries in Latin America, began to open up as late as the mid-1980s. The pace of the opening up was also different: For some it was a slow and gradual process, for others it was an abrupt event carried out in only a few years. The results, in terms of the survival of their industries, and their ability to compete internationally have also been very different. See Williamson (2006) for more details on this timing.

A startling example is the contrast between the ways Mexico and Spain opened-up and the results they obtained. While Spain started opening-up in 1947 and did so in a gradual process that took over twenty years, Mexico pursued its protectionist policies until 1985 when in the course of a few years it achieved the same degree of openness that Spain had achieved by then. The figure 1 that follows gives an insight into the issue.¹

Although the figure 1 is persuasive, it is only given in order to suggest how deep can be the differences in the timing of openness. For more arguments on these differences, see, again, Williamson (2006).

The results for the industry, and for the economic developments of the two countries were also very different. Whereas Spain had around the same GDP per capita as Mexico by 1950 by 2000 Mexico’s GDP per capita was only half of that of Spain. This great divergence, probably, cannot be attributed exclusively to the timing of the opening of the economies but it may depended as well upon other factors. Yet the lack of productivity in Mexico in contrast to that in Spain, together with the better performance of the Spanish industry, suggest that some of the divergence must have been the result of the different opening-up paths. In particular, and very suggestive, in Spain the temporal protectionism induced the domestic firms to adopt new technologies, but in Mexico did not (see Carreras and Tafunnel, 2004; Gómez-Galvarriato and Silva-Castañeda, 2007). Although we obtain very general results, in order to highlight the strengh of our results, we explain in the propositions 1 and 2 below both this Spanish history and the Mexican history as subgame perfect equilibrium paths.
Some details about the history of Mexico are worth to mention, since they are, in essence, typical examples of the general situations that we describe in our model. Indeed, looking at the case of the Mexican textile industry we realized that behind the protectionist policies were political agreements between the government, the company owners, and the unions. High tariffs allowed a situation in which the companies were able to survive, the unions capable of maintain the jobs of their rank and file, and the government able to achieve social peace. Once this situation was reached in the late 1920s, it remained the same for almost fifty years. During those fifty years, firms were almost not able to adopt a new technology because of some costs due to the political environment, that is, costs that appeared as a consequence of those political agreements between the government, the company owners, and the unions. These costs were very high, as laying off costs and some other costs derived from the fact that firms were constrained to fix (self-forced, to some extent, since there were agreements) the maximum number of machines per worker and also constrained to specific wages per piece. Only for convenience we will call those type of costs as legal-political constraint costs. For it passed by so much time with no technological progress in the domestic industries, the gap between domestic and international technologies grew too large—the difference between the marginal costs, roughly—and it was only when the economy fell into a deep crisis that incentives to change arrived, and the economy started to be opened very quickly and suddenly, and therefore it was then too late to bridge the technological gap and most firms in the spinning and weaving of the textile industry went into bankruptcy. But, due to this gap, it appeared also many other costs, that we call, also only for convenience, extra-economic costs. The most clear example of that type of extra-economic costs are the costs derived from, if the firm requires a credit in order to buy the new technology—as usually is the case, the rate interest of the credit. Another typical example of that type of costs are those due to that a new technology usually entail the need of some human capital than may not be available in the
country, so that either must be formed or must be imported, both of which entail some additional
costs that tend to vanish as the time is passing by—this taxonomy is not crucial to our study; it
is clear that many times some costs may seem to be pure economic costs, but in the end are also
consequence of the political enviroment, as the credit’s costs, since the finantial system many times
is regulated by the government—. Historically speaking, this gap —a huge gap, indeed— between
the technologies, the extra-economic costs and the legal-political constraint costs played a crucial
role in the decisions taken by both government and firms in Mexico at that time, and in the final
outcome of the process or, to say it more appropriately, in the actual situation of the industry. See,
once again, Gómez-Galvarriato (2007), for details on this history.

In any case, the main points we are making now are the following: 1) The adoption of a new
technology entails time and is costly (not only because of it entails time), whose costs may include
some additional costs to the marginal cost that characterize the new technology; 2) The political
envioroment may make impossible the adoption of the new technology.

However, this Mexican history is similar to that of many other underdeveloped economies
(see Revenga, 1997), and this history jointly with the Spanish history allowed us to detect the
main factors that can explain the issue —items (1)-(3) above—, and to put them into a formal
model. Indeed, our model heavily relies on those facts, whose generalizations constitute some of its
fundamental assumptions.

2There are indeed many other reason for that a new inversion takes time to be effective. The very recent book
by Brynjolfsson and Saunders (2009) expands in detail this argument. Some of them are the following. It takes, at
least, the lapse of time during which the inversion is made: During this period of time, the benefits are lower than
can be, and will be, once the new inversion is totally paid. In other words: If it would not entail a period of time
in which the benefits are lower than can be once the inversion is totally paid, there would be no trade-off between
to install or not to install the new technology. It is a necessary and indeed a usual assumption in order to study the
issue of protectionism.
As we said, provided that a government prefers more competition irrespective of the nationality of the competing firms, only three factors matter: The degree of patience of the agents and the gap between the extant and the new technologies, and the political costs. Nonetheless, by no means the aim in this paper is only to explain the Mexican and Spanish histories and therefore, when we set the model, we describe those type of costs in general.

To the best of our knowledge, there are no formal models explaining neither the history of Mexico nor the history of Spain, and both the contrast between the two histories and the lack of a model explaining it, were some of the starting points of our study.

The detailed presentation of the model is a little complex —due to, mainly, the realism and great generality of our set-up, which allows us to explain very uneven situations as those of Mexico and Spain—. However, in the end, both the idea of the model and the reason of why it works are very simple, which runs as follows.

As we said, usually the adoption of new technologies takes time for many reasons, and given that at any moment of time, the government and firms take the gap between the new and the old technologies as given —the difference between the marginal costs of the respective technologies, roughly—, there are essentially two situations, one when this gap is very large, and the other when it is not very large. (For a given good, of course. Later on we display the idea for an economy with many goods).

In the first case, the government faces the following trade-off: If it opens the economy for a given good, the society will enjoy a better-foreign technology (lower prices) from this moment on, but at the cost of (possibly) widespread bankruptcies in the domestic industry and, on the other hand, if it keeps the economy closed until domestic firms adopt the new technology, the society in the future will enjoy a better technology that is also used by domestic firms, but at the cost of charging higher prices than those that can be charged by using the domestic old technology
meanwhile the new technology is totally adopted. Hence, if the firms are willing to adopt the new technology, depending on the government’s discount factor it opens the economy at the outset, or it keeps the economy closed until domestic firms adopt the new technology, and at that moment it opens the economy for that good. If the firms are not willing to adopt the new technology, there is no trade-off for the government, and it opens the economy at the outset.

In the second case, that is, if the gap between the technologies is not very large, the government never opens the economy, if firms do not adopt the new technology, provoking no bankruptcies at all. However, it keeps the economy closed until firms totally install the new technology, in the case that there exist firms willing to adopt the new technology.

Similarly, the domestic firms face the following trade-off: If a firm adopts the new technology, during a lapse of time —meanwhile the new technology is totally adopted—, that firm earns lower benefits than it can earn with the old one (no trade-off there is otherwise), but on the other hand, in the future —once the new technology is totally adopted— will earn higher benefits than with the old one or simply will be able to survive in the long run (with the old technology a firm never survive, either because a domestic firm adopts the new technology or because the economy is opened), so that depending on its discount factor it will adopt or not the new technology. Simply, the trade-off faced by firms is the following: To obtain high returns in the short run, at the cost of risking the possibility of surviving in the future, or to invest in the present, increasing the possibility of surviving in the long run, at the cost of low returns in the present. It is very important to notice that, in our set-up, the incentive of a patient firm to adopt the new technology is not that it will become a monopolist, since if it adopts the new technology, the economy will be opened and a foreign firm enters the market, so it will not become a monopolist: The incentive, indeed, is to survive in the long run, and nothing else.

In order to have a further justification of our model, see Brynjolfsson and Saunders (2009). In
this book, it is precisely argued how massive inversions in new technology can produce, at a cost of reduced benefits for a while, large benefits in the future, both for the firms and the society as a whole.

It turns out that all the equilibria we have found are sub-game perfect equilibrium, so that time-consistent.

The existing literature on the timing of opening-up the economy (see among others Staiger and Tabellini, 1987; Matsuyama, 1990; Tornell, 1991; and Wright, 1995), argues that temporal protectionist policies are always time inconsistent since the government cannot credibly commit to the promise of openness, and only in special cases temporal protectionism has been time consistent. In all these papers it is assumed that the adoption of the new technology takes time and it is costly. Otherwise, as said it before, there is no issue of temporary protection in order to induce firms to adopt new technologies.

Our model is in the line of the literature on the political economy of protectionism (see Hillman, 1989), since in it the trade policy is mostly endogenously determined by the interaction among some actors of the economy. In our case, the government, the industrialists, and the unions (who appear as exogenously given legal-political constraints; later in this introduction we comment this assumption). However, our model is neither a voting model, nor a pure lobbying model (i.e. Grossman and Helpman 1994, 1995a, 1995b and Hillman, 1982), although lobbies play a fundamental role.

Surprisingly enough —and essentially what it makes our paper to be in sharp contrast to others that proved time inconsistency of a promise of openness— we can make the following resume: Provided that the government prefers more competition than less competition irrespective of the firms’ nationality, independently of how large is the gap between the extant-domestic and the foreign technologies, both firms and government know that the economy will be opened, if there are patient firms, sooner or later. Soon, if the government has a very small discount factor. Late,
once the existing domestic firms have totally adopted the new technology, if the government has a large enough discount factor.

Furthermore, one of the most crucial points in our paper is the assumption that the government does not take into account the nationality of the competing firms in its welfare social function. In order to make the point clear, we have proven the results in detail using a standard utilitarian social welfare function. Obviously, if the government does take into account the nationality of the competing firms, our results may not hold true, not all of them, something that one can see at once after seen the proofs in detail.

Other models have considered the possible existence of dumping effects as a cause of protectionism (i.e. Blonigen and Park, 2004; Cheng, Wiu and Wong, 2001). There is no dumping in our model—foreign firms declare their true costs—, so antidumping policies cannot be a possible reason for protectionism.

Other results from the model show that, if the gap between the technologies is very large, whatever is the degree of patience of all the agents —therefore, even if all the agents are very patient—, if the agents have the expectation that things will go badly, things will indeed go wrong (see proposition 3), that is, no firms adopt the new technology and the government opens the economy at the outset, which is a sub-optimal outcome.

Lastly, we completely characterize the degree of patience of the agents. More precisely, we obtain that under general conditions there exists a critic value for the discount factor which satisfies that a firm adopts the new technology whenever its discount factor is no lower than that critic value, and an analogous statement applies for the government. The critic value satisfies the following intuitive property: The larger the gap between the domestic and the foreign technologies —in terms of costs—, the larger is the critic value. Analogously for the government: The larger the gap between the domestic and the foreign technologies, the larger is the critic value of the discount factor that
satisfies the condition that when the discount factor is larger than that critic value, the government will temporarily protect the industry. Also, there exists a minimum value of the discount factor such that if the discount factor of a firm is lower than that value, the firms prefers not to invest. Similarly, the larger is the initial investment necessary to buy the new technology, the more patient must the agent be to prefer higher future benefits than higher present benefits (see remark 1).

How do we explain the figure 1 then?

The idea is also very simple. Our model fix a sector of the economy, the car industry or the textile industry, whatever. Hence, given the gap between the new and the old technologies in that industry—which need not need be the same for all industries, as indeed are different in real life: In section II, when we set the assumptions of the model, we provide further justifications for that assumption—, and consequently the time needed to install the new technology is also given, our theorems apply. Now, imagine that in Spain the government and firms are patient enough—high discount factors—in order to display the equilibrium in which the economy is closed for that industry during the lapse of time needed to install the new technology. Therefore, as different industries need different periods of protection, the government is opening the economy gradually, sector by sector. On the other hand, imagine that in Mexico the government is too impatient (too low discount factor), then it opens the economy in all the sectors at once, and hence provoking widespread bankruptcies in those industries not prepared for competition.

To give more insights into one of the main points of our argument—the agents’ degree of patience—, we recall that the discount factor is usually related to the internal interest rate that a firm may earn by producing, by means of the formula $\beta = \frac{1}{1+r}$, so that the higher is the interest rate $r$, the lower is the discount factor $\beta$, and hence the more impatient is the firm, and vice versa. Consistently with this, one may argue that a reasonable proxy for an average degree of patience of firms is the expression $\frac{1}{1+r_g}$, where $r_g$ is the real interest rate offered by the government—it can
be argued that a firm for which its annual benefits are lower than \( r_g \) has incentives in order to not produce. In Mexico real interests rates at 1986 were about 6.7\%, whereas in Spain were about 1.56\%, and therefore the returns of a firm in Mexico was \( \frac{0.067}{0.0156} = 4.2949 \) times of the return of a firm in Spain in real terms, that is, more than four times. Therefore there are strong reasons to argue that in Spain government and firms were much more patient than government and firms in Mexico. (See, for these data, Aceña (2005), and Messmacher and Werner (2001)).

One may say that this numbers are for one year, and hence it is not reasonable to draw conclusions from this. But we do not do that. One of the main arguments of this paper is the following: The trade-off is always high present returns at the cost of the of risking the survival capacity of the firms in the long run, or low present returns but ensuring the survival capacity in the future. Therefore, if firms did not invest and the government opened suddenly the economy, it was because they discounted heavily future returns. It is in the end a sort of revealed preference argument. The given data above is only saying that at that year, the opportunity cost of investing was very high in Mexico —compared with those in Spain—, and therefore it is a possible reason of why people was impatient.

Both the fact that different goods need a different lapse of time in order to be prepared for competition and that this lapse of time is exogenously given are very important assumptions in this paper, and the reasons of why we make these hypotheses are also very important, and we will give these justifications in due time, when we present formally the model in the section II. However, in the end the reason is that there is no an only one reason, but there are many economic,

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3 Real interest rates are calculated using the Rational Expectations approach.

4 From Aceña (2005), and Messmacher and Werner (2001), it is possible to make a comparison of the real interest rates between Mexico and Spain for the period 1985-2000. As one may have expected, in Mexico there was a high variability with some years in which the real interest rate was negative (-41\% in 1987), and others in which it was about 25\%, in 1988.
political and cultural factors entailed in a very complex process. Our paper, from a more general perspective, can be thought as a first approach or a partial equilibrium approach in order to built a future endogenous model in which both cultural and pure economic factors interact in a game theoretic setting.

The rest of the paper is organized as follow. Section II lays down the model. Section III discusses its main results. Section IV presents a generalization of the model and conjectures about possible alternative equilibria to those we have found. In section V the conclusions are given. The full and complete results are given in the appendix I. Proofs are in the appendix II.

2 The model

The Histories, Strategies and Payoff Functions

We propose a dynamic discrete time model with infinite horizon, with infinitely lived agents. More precisely, we will define an Extensive Game with Perfect Information and Simultaneous Moves, following the presentation in Osborne and Rubinstein (1994). The model is at the same time very simple and complex. Simple, since it mimics the real timing of a society year by year—and therefore with infinite horizon—but it turns complex because both we set as an objective to find sub-game perfect equilibria, and the costs structure of the economy. The model runs as follows.

From the very beginning we want to make the following remark.

Remark 1 In what follows we will restrict the formal analysis to Cournot Competition, a linear inverse demand function and some specific preferences for the government, only for the sake of the exposition. As we said, in the section IV we show the extent of the generality of our model.

The set of players is \{G, I, P\}, where G stands for the government, I stands for the impatient
firm, and $P$ for the patient firm. Roughly, the timing of the game is such that each of the players at each period of time move simultaneously, the firms deciding which technology is going to use the current year, the government deciding if to open—or to keep opened—or not to open the economy for this year—or to keep closed the economy—. Periods in this model are counted, as we said, in years. In the remark 1 below we justify to take $\{G, I, P\}$ as the set of players.

Now we formalize these ideas. Let the set $\{N, T\}$ be the set of firms’ actions. That is, if a firm $i \in \{I, P\}$ at $t - 1$ is facing costs according to some technology (the foreign or the domestic one), if that firm at $t$ decides $N$ —not to change—, it means that it has decided—for this period $t$—to continue with the technology that it was using at $t - 1$ either is the new one or the old one and, logically, the action $T$ means exactly the opposite, that is, to change the technology that it was using at $t - 1$.

The government at each $t \geq 0$ decides to open or to close the economy for this period, that is, for this year. Concretely, at a given $t$, to choose “to open” (which is denoted by $O$) means, simply, that the economy is opened during this period $t$, and to chose “to close” means the economy is closed during the period $t$ (which is denoted by $C$). If the economy is opened at $t$, then a foreign firm enters the market and will compete with domestic firms à la Cournot at $t$, in the way that we will show in detail later in this paper. (Below, in the figure 2, we present a possible equilibrium path of the game. It might be useful to see it right now as well, although it must be kept in mind that in that figure 2 we only picture a possible equilibrium paths of the game: The game has an infinite number of finite histories and an infinite number infinite histories.) This last agent is not included as a player because its behaviour is trivial, since it possesses the best technology already installed, and hence will not consider to change it for the old technology —one could, perhaps, argue that the foreign firm may have some costs in order to operate in this new market; even in this situation its behaviour is quite clear: If it has a large enough discount factor, enters the market, otherwise not;
on the other hand, our study only has sense if it enters the market—. It is, in principle, possible to consider more domestic and foreign firms to enter the market. However, this will complicate resolution of the model too much, with no gains or new insights over the argument.\footnote{As can be guessed from the preceeding paragraph, in contrast to most of the literature on the issue of time consistency, we will allow the government to chose more complex strategies than simply to open or not to open the economy at future times, as it is the case in real life economies.}

\textbf{Remark 2} Two comments are in order. First, in relation of why we have chosen to set two domestic firms and no more, no less. No more, just by simplicity. After the proofs are read, one can observe that to consider more firms only will difficult the notation and proofs, but no new issues may arise. No less, in order to observe which might be the consequences of that assumption, and one interesting issue appeared: As have said, the degree of patience of the firms turned to be one crucial factor in the general issue, since a patient firm survive, an impatient firm does not. Further, this fact rule out the possibility of collusion in that situation —if both firms have the same degree of patience, they may collude, but this does not modify any of our results—. Second, in relation of why we allow for only one foreign firm to enter in the market when the economy is opened. Once again, this is only for simplicity. However, from the proofs, one can see that if more foreign firms can enter when the economy is opened, our arguments are even stronger.

Now we define the set of histories.

The Set of Histories

The general set of histories $H$ then is given as follows. First we define $A^F = \{N,T\} \times \{N,T\}$, $A^G = \{C,O\}$ then, if we denote by

$$Z = \left\{ (a_t^G, a_t^F)_{t=0}^{t=\infty} \mid (a_t^G, a_t^F) \in A^G \times A^F, t \geq 0 \right\},$$

the set of terminal histories, that is, no finite history is terminal.
Therefore, and we set

\[ H = \{\emptyset\} \cup (H \setminus Z) \cup Z. \]

Now we specify the timing of the game.

**Timing and Interpretation.**

Given \( h = (a_t)_{t=0}^{\infty} \in H \), we use the interpretation that, for any \( a_t = (a_t^G, a_t^I, a_t^P) \), the first coordinate of the triple \((a_t^G, a_t^I, a_t^P)\) is the action chosen by the government during the period \( t \), the second is the action of the firm \( I \) during the period \( t \) and, finally, the third coordinate is the action chosen by the patient firm during \( t \). For instance, if \( a_t^G = O \), it means that the economy is opened during the period \( t \), and if \( a_t^I = N \), means that the firm \( I \) is using during the period \( t \) the same technology than it was using at \( t - 1 \), and so on and so forth.

The player function \( \bar{P} : H \to \{G, I, P\} \) is given by

\[ \bar{P}(h) \in \{G, I, P\} \text{ for all } h \in H. \]

**The strategies**

Therefore, the set of strategies for the player \( i \in \{I, P\} \) is given by

\[ S^i = S^F = \{\{s_h\}_{h \in H} | s_h \in \{N, T\}, \text{ for all } h \in H \} \]

and,

\[ S^G = \{\{s_h\}_{h \in H} | s_h \in \{C, O\}, \text{ for all } h \in H \}. \]

Therefore, if \( s^G \in S^G \), \( h_t \in (H \setminus Z) \), \( s^G(h_t) = O \), it means that the government has decided to keep the economy opened during \( t + 1 \), and so on and so forth.

**Remark 3** Observe that the set of government’s strategies is not simply to open or not to open the economy at a given future time. Neither the set of firms’ strategies is not simply to adopt or not the new technology. However, in equilibrium, the strategies will be ‘as if the government has decided to
open the economy at a future time, and the firms have decided to adopt the new technology,’ along the equilibrium paths.

As in any game, given a profile of strategies \( s = (s^G, s^I, s^P) \in S^G \times S^I \times S^P \), this triple determines a path that is actually played, according to those strategies, of the form \( (a_t)_{t=0}^\infty = ((a^G_t, a^I_t, a^P_t))_{t=0}^\infty \), with \( a^I_t \in \{N, T\} \) for all \( t \geq 0 \) (\( i \in \{I, P\} \)) and \( a^G_t \in \{C, O\} \) for all \( t \geq 0 \), which in turn, logically, determines a sequence of costs and periods of openness of the form

\[
((C^I_t, C^P_t))_{t=0}^\infty, Op((a_t)_{t=0}^\infty),
\]

where \( C_i^t \) is the cost faced by the firm \( i \in \{I, P\} \) at time \( t \), and

\[
Op((a_t)_{t=0}^\infty) = \{ t \geq 0 | a^G_t = O \}
\]

is the set of periods in which the economy is opened according to \( s \). Below we will specify the costs.

We assume that the firms at each period compete à la Cournot, with and inverse function given by \( p : \mathbb{R} \to \mathbb{R} \), given by \( p(Q) = \max\{0, a - Q\} \), where \( 0 < a \leq 1 \), just for simplicity.

Remark 4 In this paper, the currency is not an issue. Without loss of generality, we can assume that all costs and benefits are given in real terms. Furthermore, it is even more aproprate to set the model in those terms.

With this preliminaries in place, we define the preferences.

The Preferences

Denote by \( 0 < \beta^i < 1, i \in \{G, I, P\} \), the discount factors of the players, which will be assumed as constant over time, just for simplicity. Now, if the profile of strategies \( (s^G, s^I, s^P) \in S^G \times S^I \times S^P \) is such that the corresponding associated sequence of costs and periods of openness is given by
\[ (((C_i^I, C_i^P))_{t=0}^\infty, Op((a_i)_{t=0}^\infty), \text{then the payoff function of the firm } i \text{ is given by} \]

\[
\Pi^i(s^i, s^j, s^G) = \begin{cases} 
\sum_{t \in Op((a_i)_{t=0}^\infty)} (\beta^i)^t \pi^i(C_i^I, C_i^j) + \\
\sum_{t \in Op((a_i)_{t=0}^\infty)} (\beta^i)^t \pi^i(C_i^I, C_i^j, C_F) 
\end{cases}
\]

with \( i, j = I, P \), where \( \pi^i(C_i^I, C_i^j) \) is the Cournot profit of firm \( i \in \{I, P\} \) at time \( t \), if the respective costs for that period are \( C_i^I \) and \( C_i^j \) and with the economy closed and, similarly, \( \pi^i(C_i^I, C_i^j, C_F) \) is the Cournot profit of firm \( i \) at time \( t \), if the national firms face \( C_i^I \) and \( C_i^j \) and the foreign firm faces \( C_F \), at periods with the economy opened. The costs’ structure, that is the costs that domestic and foreign firms face in each situation, is presented in detail later in this paper.

For the government’s payoff we explicitly present three possible scenarios, one that we call a consumer oriented government, and the others that we can encompass in a one class that we call a utilitarian government.

The objective of presenting many scenarios is to give more robustness to our results since, unlike the case of the firms, there is no a widely accepted candidate for the preferences that governments may have. The consumer oriented government payoff may be thought as representing a government that puts all the welfare of the society in the consumers weight, since the lower is the price of the good, the better is for the society. The utilitarian government is representing the standard set-up in welfare economics, in which the welfare is the measured as the sum of the consumer’s surplus, plus the firms’ surplus, roughly.

**A Consumer Oriented Government**

Take a profile of strategies \((s^G, s^I, s^P) \in S^G \times S \times S\) such that the corresponding associated sequence of costs and periods of openness are given by \(((C_i^I, C_i^P))_{t=0}^\infty, Op((a_i)_{t=0}^\infty)\), then the payoff
function of the consumer oriented government is given by

$$\Pi^G((s^G, s^I, s^P)) = \begin{cases} 
\sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left( \frac{1}{2} (Q_T(C^i_t, C^p_t))^2 \right) + \\
\sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left( \frac{1}{2} (Q_T(C^i_t, C^p_t, C^F))^2 \right) 
\end{cases}$$

where $Q_T(\cdot)$ is the total quantity of the good in the market, and hence $\frac{1}{2} (Q_T(\cdot))^2$ is the consumer’s surplus at the corresponding period.

**A Utilitarian Government**

Take a profile of strategies $(s^G, s^I, s^P) \in S^G \times S \times S$ and take the corresponding associated sequence of costs and periods of openness given by $((C^i_t, C^p_t))_{t=0}^{\infty}, Op((a_t)_{t=0}^{\infty}))$, then the payoff function of the utilitarian government, we present two possibilities. First, consider:

$$\Pi^G((s^G, s^I, s^P)) = \begin{cases} 
\sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \frac{1}{2} (Q_T(C^i_t, C^p_t))^2 \right] + \\
\sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \sum_{i \in \{I, P\}; j \neq i} \pi^i(C^i_t, C^j_t) \right] + \\
\sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \sum_{i \in \{I, P\}; j \neq i} \pi^i(C^i_t, C^j_t, C^F) \right] \right] + \\
\sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \sum_{i \in \{I, P\}; j \neq i} \pi^i(C^i_t, C^j_t) \right] + \\
\sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \sum_{i \in \{I, P\}; j \neq i} \pi^i(C^i_t, C^j_t, C^F) \right] + [q^F(C^i_t, C^p_t, C^F)]^2, 
\end{cases}$$

and second:

$$\Pi^G((s^G, s^I, s^P)) = \sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \frac{1}{2} (Q_T(C^p_t, C^F))^2 \right] + \sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \frac{1}{2} (Q_T(C^p_t, C^F))^2 \right] + \sum_{t \in Op((a_t)_{t=0}^{\infty})} (\beta^G)^t \left[ \sum_{i \in \{I, P\}; j \neq i} \pi^i(C^i_t, C^j_t, C^F) + \tau q^F(C^i_t, C^p_t, C^F) \right],$$

where $q^F(C^i_t, C^p_t, C^F)$ is the Cournot quantity offered by the foreign firm, $\beta^G \in (0, 1)$, and $\tau \in (0, 1)$ may be interpreted as tariffs in some contexts.

In (3), at each $t$ the term in the sum is simply the standard felicity function of the society at that time used in welfare economics, that is, the consumer’s surplus plus the profits of the firms —both, domestic and foreign firms operating in the domestic marked—, and it has the property that if it is maximized, the result is Pareto Optimal, for this reason is that we consider this case one of the most interesting.
The game is then given by

\[ \Gamma = \left\{ \{I, P, G\}, H, \hat{P}, (\Pi^i)_{i \in \{I, P, G\}} \right\} \]  \hspace{1cm} (5)

where \( \Pi^i \) is given by (1) for \( i = I, P \) and \( \Pi^G \) is given either by (2), (3) or (4).

Definition of equilibrium.

**Definition 1** Given the game (5), a profile \((s^G, s^I, s^P) \in S^G \times S \times S\) is an equilibrium if it is Subgame Perfect Equilibrium.

For the precise statement of subgame perfect equilibrium, see Osborne and Rubinstein (1994).

The idea is that a profile \((s^G, s^I, s^P)\) is subgame perfect equilibrium if for any player \( i \in \{I, P, G\} \), and for any history \( h_t \) in which a player \( i \) is going to play, the strategy is a Nash Equilibrium in the game that follows from \( h_t \).

**Remark 5** Observe that the game we have defined is neither a repeated game nor it has an recursive structure. On the other hand, in the way it is settled, it allows us to face a great variety of situations, as it is shown in the sections where the results are presented (next section and the appendix). For this reason, in the proof we used in the proofs the one deviation property (see Osborne and Rubinstein (1994), and Fudenberg and Tirole (2004)). In particular, our theorems in which firms adopts the new technology are not folk theorems.

Another important remark must be done.

**Remark 6** Due to the objective of our paper, the infinite horizon set-up of the game is unavoidable, since the issue of openness and timing of openness is intrinsically a dynamic situation, which would not be properly faced in a finite horizon set-up.\(^6\) Furthermore, both the infinite horizon set-up and the possibility given to the government to open and close the economy at any time are mostly

\[^6\text{It is well known that an infinite horizon Overlapping Generation Model does not possesses the same properties}\]
responsible of the complexity of our game, however both are crucial in order to put our results in the realm of complex and real life economies. For example, we can describe situations in which the economy is opened and closed cyclically by allowing not only the costs to change over time, but also the degree of patience of governments and firms—as it may be the case in real economies—. It will be easy to prove that, if firms expect that governments will never give enough time to install new technologies, in spite of some periods of protection, they would never adopt the new technology; or, further, if they expect that at some moment in the future there will be a government that will give them time to install the new technology, they adopt the new technology only at the moment the government that enters in office is patient enough.

Now we will show an example in order to fix ideas.

An Example of a Terminal History

Figure 2 below gives a flavour of the game and its timing, by isolating a single (infinite) history of the game, which is both one of the possible equilibrium paths of our game and one of the infinite possible histories of the game.

In the figure the government keeps the economy closed for the first two periods, and then opens the economy for ever. The patient firm adopts the new technology and the impatient firm does not adopts the new technology.

However, we insist on noticing that the history represented in the previous figure is obtained as a result of one of the equilibria of the model, but it is not an a priori exogenously given history of the game. There is no such a history.

It may not be superfluous to say that in real life economies, this example above does not seem of the finite horizon OLG. In particular, in the infinite horizon model, a competitive equilibrium need not be Pareto Optimal. Therefore, to infer from a finite horizon set-up properties and apply them to infinite the infinite horizon set-up is, to the less, a blind inference.
to represent a typical outcome, and it is not the unique history in our game either.

For the case of the given history of the figure 2 the preferences given by (1) for \( i = I, P \) and \( \Pi^G \) given by (3) are as follows. If the profile of strategies \( \bar{s} = (\bar{s}^G, \bar{s}^I, \bar{s}^P) \) are such that the path of the game is the one in that figure, then

\[
\Pi^i((\bar{s}^G, \bar{s}^I, \bar{s}^P)) = \begin{cases} 
\sum_{t=1}^{\infty} (\beta^i)^t \pi^i(C_i^G, C_i^I) + \\
\sum_{t=2}^{\infty} (\beta^i)^t \pi^i(C_i^G, C_i^I, C^F) 
\end{cases}
\]

is the firm \( i \)'s payoff, and

\[
\Pi^G((\bar{s}^G, \bar{s}^I, \bar{s}^P)) = \begin{cases} 
\sum_{t=0}^{\infty} (\beta^G)^t \left[ \frac{1}{2} (Q_T(C_i^I, C_i^P))^2 + \sum_{j \in \{I, P\}, j \neq i} \pi^i(C_i^G, C_i^I, C^F) \right] + \\
\sum_{t=2}^{\infty} (\beta^G)^t \left[ \frac{1}{2} (Q_T(C_i^I, C_i^P, C^F))^2 + \sum_{j \in \{I, P\}, j \neq i} \pi^i(C_i^G, C_i^I, C^F) \right] + \\
\sum_{t=2}^{\infty} (\beta^G)^t \left[ \sum_{j \in \{I, P\}, j \neq i} \pi^i(C_i^G, C_i^I, C^F) + [q^F(C_i^I, C_i^P, C^F)]^2 \right]
\end{cases}
\]

As usual, the payoff of a profile is determined by the value of its path.

We assume, just for simplicity, a linear inverse demand function \( p : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) given by \( p(Q) = \max\{a - Q, 0\} \), where \( Q \) denotes the total quantity of the good in the market. In the section IV we relax this assumption.

Now we proceed to describe the cost structure, in order to be able to evaluate the strategies and finish the set-up of the model.
The Costs

For the reason given in the introduction, the costs associated to the adoption of a new technology will be divided into two type of costs, the extra economic costs and the legal-political constraint costs. However, in order to formalize all the costs included in the process, a further separation is convenient: The proper cost associated to the technologies —to the production per se—, from the costs of adopting the new technology.

The proper or per se costs of the technologies are modeled in a standard way, as follows. The domestic firms may use the extant domestic technology, characterized by its constant marginal cost $C_N$ in each period, or they may adopt the foreign technology, characterized by $C_F$—of course, we will require $C_F < C_N$—, which is the cost that the foreign firm that owns it has to face. If the domestic firms want to adopt the new technology, they have to face not only $C_F$ but also some additional costs for a lapse of time—the time needed to adopt the new technology—, which, as we said, we call extra economic costs and legal-political constraints costs. Just for simplicity, we assume that $C_F$ and $C_N$ are constant over time. Given the generality of the game in (5), this assumption is not unavoidable.

Now we proceed to model the costs of adopting the new technology, which, like we said, will take into account the time needed to adopt it. First, observe that the time needed to adopt the new technology must be modeled—in our set-up—without making a more detailed description of the sequence of costs. Otherwise we would be constrained to a given good or industry in particular, and no general analysis would be possible.

In the first place, the extra economic costs.

- The extra economic costs

At a given moment of time, the gap between the new and the old technologies is in real life given, and hence the time needed to totally install a new technology is given as well—further
justifications to this assumptions are given below—. However, like we argued before, that
time depends on the negotiations between unions, industrialists and the government, and
therefore it may change in accordance to those negotiations, and thus it may endogenously
change in the future. For this reason, argument that we will expand below in detail, we take
that time —thus the costs— as exogenously given and, to begin with, constant (this can be
easily relaxed: See the remark below). Formally, we assume that the extra economic costs
are defined by a decreasing finite sequence

\[ C^e_0, C^e_1, \ldots, C^e_n \quad (C^e_t > C^e_{t+1} \text{ for all } 0 \leq t \leq n - 1, n \geq 1), \]

where \( C^e_n \) is a permanent extra cost that the domestic firm adopting the new technology may
have to pay to the owner of said technology. Think of that a domestic firm that adopts the
new technology pays first, \( CF + C^e_0 \), the next period it pays \( CF + C^e_1 \), the next \( CF + C^e_2 \), an
so on, until it pays \( CF + C^e_n \). \( C^e_n \) may represent the royalty paid to the owner of the foreign
technology. In any case, \( C^e_n \) may be zero as well, which certainly is the simplest case.\(^7\)

Hence, if at \( t = \tilde{t} \) the new technology is adopted, the economic costs faced by the firm from
that moment are \( C_{\tilde{t} + i} = CF + C^e_{\tilde{t}} \) for all \( 0 \leq \tilde{t} \leq n - 1 \), and the firm faces \( CF + C^e_n \) from
t = \( \tilde{t} + n \), that is, \( C_t = CF + C^e_n \) for all \( t \geq \tilde{t} + n \). In other words, if the foreign technology is
adopted from \( t = \tilde{t} \) and forever, the sequence of economic costs that the firm faces from \( \tilde{t} \) on
is given by \( (C_t)_{t=\tilde{t}}^{\infty} \), where \( C_t = CF + C^e_{t-\tilde{t}} \) for all \( \tilde{t} \leq t \leq \tilde{t} + n - 1 \), and \( C_t = CF + C^e_n \) for all \( t \geq \tilde{t} + n \).

The number \( n \) is therefore the time needed to install the new technology, so that \( n \geq 1 \) (with
\(^7\)An alternative interpretation for the permanent cost \( C^e_n \) can be given: The owner of the technology is the sole
person who produces it. Therefore, \( C^e_n \) may represent his profits, if we understand that he is not selling the new
technology but only the strategic elements to use it. These elements cannot be produced by anyone but the owner;
thus, the buyer cannot develop that new technology.
this notation, \( n = 0 \) would be equivalent to say that there is no trade-off for the firm, since domestic firms that adopted the new technology would be facing \( C^F + C^e_0 < C^N \) for ever; see below the technical assumption A3).

In order to encompass the wide range of situations that may arise, one for each good in particular, we have expressed these costs with no further specifications.

There are, therefore, many interpretations for the assumptions we make about these costs. The most direct one is that in which we capture the idea that to adopt a new technology only in exceptional cases entails an at once cost, so that normally takes time, in a way that at the beginning the economic costs of installing a new technology are high but decrease over time until stabilizing at the level \( C^F + C^e_n \). The other is that there is a fix cost of installing the foreign technology that may not be affordable in one period, so that the domestic firm needs a credit in order to buy the new technology. As we said, later we provide further justifications to this assumption.

Mostly in this paper the sequence \( (C^e_t)_{t=0}^n \) is constant over time just for simplicity (as well as \( C^F \) and \( C^N \)), but it can be relaxed, given the generality of the game defined in (5). In the proposition 2 below we allow for those costs, \( C^N \) and \( C^F \) to change over time.

Finally, notice that \( (C^e_t)_{t=0}^n \) will be different for different goods, as we argued in the introduction, and hence we do not go further in the specification of those costs. In any case, to provide a precise specification of those costs for a given good is not the issue in this paper.

**Remark 7** It is important to notice that when at a given time a firm adopts the new technology, both the gap between the technology that it is using at that time and the new technology and the time needed to install the new technology are lower than in the previous period, precisely because \( (C^e_t)_{t=0}^n \) is decreasing. So in a sense the firm is making endogenous both the gap and the time needed to install the new technology. Observe that, if we allow that \( (C^e_t)_{t=0}^n \) be depending on time,
that is, to write $(C^p_l(t))_{l=0}^{l=n(t)}$, we can make the following two assumptions over them. One is to impose that both $C^p_l(t)$ (for $l = 0, \ldots, n(t)$) and $n(t)$ are increasing on $t$, and all the theorems of this paper can be proven taking $n(t) = n(0)$ for all $t \geq 0$, if we have in mind a situation as the one described for Mexico. On the other hand, the second can be, exactly because of we have said above in this remark, to assume that both $C^p_l(t)$ (for $l = 0, \ldots, n(t)$) and $n(t)$ are decreasing on $t$, if we are modeling a situation as the one described for Spain.

- **The legal-political constraint costs**

Like we said, to introduce in our model the possibility that a protectionist policy fails to induce domestic firms to adopt new technology not only because of the degree of patience of the agents, but for some other non more pure economic reasons, we assume the existence of some costs that may be legally imposed over a firm if it decides to adopt a foreign technology, whose imposition may be a consequence of agreements between government, unions and firms.

We are thinking of the Mexican textile experience and others — so that this is not an *ad hoc* hypothesis —, which were commented in the introduction. We call those costs *legal-political constraints costs*, which are exogenously given and defined by a possibly infinite sequence $(C^p_l)_{l=0}^{l=\infty}$. Each $C^p_l$ represents the extra cost that the firm has to pay if it adopts the new technology at time $t$, but once and for all — for simplicity: We can assume that the firm may have to face those costs for a while —, due to, for instance, the fact that the firm may have to dismiss some workers who are not useful anymore. Then, if the new technology is adopted from $t = \bar{t}$ on, the sequence of total costs that the firm faces from $\bar{t}$ on is given by $(C_t)_{t=\bar{t}}^{\infty}$, where $C_{\bar{t}} = C^F + C^e_{\bar{t}} + C^p_{\bar{t}}$, $C_t = C^F + C_{t-1}^e$ for all $\bar{t} + 1 \leq t \leq \bar{t} + n - 1$, and $C_{\bar{t} + n} = C^F + C^e_n$ for all $t \geq \bar{t} + n$, if $n > 1$. For instance, if $n = 1$, $(C_t)_{t=\bar{t}}^{\infty}$ is given by $C_{\bar{t}} = C^F + C^e_{\bar{t}} + C^p_{\bar{t}}$ and $C_t = C^F + C^e_t$ for all $t \geq \bar{t} + 1$. 

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If we think of the costs that are a consequence of negotiations between government and unions, it is reasonable to assume that the more powerful the trade unions are, the larger these costs will be. It would be also reasonable to assume that those costs increase through time since, as the gap between the domestic and the foreign technology widens it is likely that more workers will be redundant when the foreign technology is adopted, as indeed was the case in Mexico. Nonetheless, without this assumption, the model can be used to assess situations under which those costs can become constant or even decreasing —at least temporarily— as was the case of some countries in Europe during the last years when labor laws were reformed in order to provide more flexibility to the labor market and laying off workers became cheaper —Spain, for instance—.

Before we further justify all our assumptions about costs, it is necessary to make the following observation. In relation to the time needed to completely install the new technology, there are implicitly two concepts. First, that at the outset it is given, and second, that this lapse of time does not change in the future, or it may be larger in the future, if the new technology is not adopted. The first assumption need not too much justification —although we do it—, since it is determined by the previous history; it is the initial value of a variable in a dynamic system, which is always a parameter of the system. However, the lapse of time may change in the future, and in relation to this issue, we assume that this time in the future is exogenously given and, for simplicity, the same as at the outset of the game, that is, the moment from which the dynamical system is studied.

Now we proceed to the justification for these assumptions and for the important assumption that different goods need different spans of time in order to be prepared for competition, which will be part of the justification of the former ones.

First of all, that different goods need different spans of time in order to be prepared for com-
petition is a fact. The economic history of Mexico and its textile industry in particular are vivid examples of this, and also is an example of why at a given moment of time, this lapse of time is a data that must be taken as given (see, again, Carreras and Tafunnel, 2004; Gómez-Galvarriato and Silva-Castañeda, 2007).

However, there are many other arguments that support these assumptions. The NAFTA (1994) is another vivid example.\footnote{Our model can be easily interpreted to take into account the possibility of announced dates of opening the economy in advance. But the virtue of the model is that it does not need a credible written promise of opening, since the equilibrium is credible because it is a sub-game perfect equilibrium. Further, the NAFTA in our set-up may be used to select one the equilibria that the model display when agents are patient agents.} That treat took into account this timing in the different sectors of the economies, and took that time as given at the moment of the treat. An extrem example is that some goods have just been opened to foreign competition in Mexico (in 2009 for second hand cars from USA); some other agricultural goods suffered the lack of competition because Mexico lacks the infrastructure necessary for competition, such as efficient railroads and highways, and therefore those goods needed more time than others to become competitive (fifteen years, against ten years in other goods). All these data can be confirmed in any of the official web-pages of Canada, Mexico or USA, as for instance (www.international.gc.ca).

Another factor for which different goods need different periods of modernization is how efficient is the financial system: High investments may need more time to repay the capital borrowed than small investments, or simply it may not be possible to borrow from inside or even outside the country —today in Spain banks are almost not lending, then almost nobody buy houses, and most firms cannot make new investments—. Also, related to the financial system, it is the velocity of the financial transactions (indeed, this is part of the efficiency of a financial system: Its velocity in making available the needed capital), which at the same time depends on the rule of low and bureaucracy, which are usually different for different goods, since some goods may be related with
some lobbies that may influence the financial institutions more than others. Another factor for which that time may be different for different goods is that the new technology may need some specialized human capital, and that human capital may not be available in the country —mainly in emergent economies, as Mexico and others in Latin America—, and then the new technology will not be in use at least until foreign specialists are available or that human capital is formed in the country, and this takes time and is costly, and need not be the same for different goods.

We take that time as exogenously given just for simplicity, for that otherwise it is a very complex issue. Indeed, many of the factors named above are pure economic factors to some extent. However, the rule of low is clearly a factor that depends on the political swinging in the country, and there may be other type of factors that may influence the time needed for an industry to become competitive, as lobbies, many of which may depend upon the political swinging. Strong lobbies —unions in particular— against competition may force the negotiations between the government and firms even to the point of making almost not affordable the adoption of new technology —if there are no efficient financial markets—, as it was the case during sixty years in the textile Mexican industry (see, once again, Gómez-Galvarriato, 2007). Furthermore, both bureaucracy and the legal-political costs not only need not be the same for different goods, but it may also change over time as some other cultural determinants of the society change —as indeed changed in the case of the textile industry in Mexico, in which case what was changing it was the ideology of the unions—, as ideology, religion, other institutions in general, as justice (which is related to the rule of low), or the formal education system, social capital and democracy, all of which have been convincingly argued to be crucial to economic development, exactly because of the time that in general can be saved, depending on the characteristics of those determinants (see Besley, 2006; Fukuyama, 1996; Guido et. al. (2009), North 1994-2004; Weber, 2002). Of among all those studies, we want to put emphasis in the very recent paper by Guido et. al. (2009), since it deals exactly with —and
directly to—the issue of how cultural factors—perceptions of the degree of social capital in this case—bias international economic trade, and therefore development.

From the examples just commented here (Mexico, Spain, and those in Revenga, 1997) and the literature above cited, one can guess how deep and complex the dynamic interrelations among cultural, political and economic factors can be in real life societies. A moving example of this complexity, is the actual 2008’s grave financial crisis in Europe and USA. In this crisis, banks have lost credibility—which can be interpreted as a cultural trait: See, specially, Cavalli-Sforza and Feldman (1983), and also Guiso et. al. (2009)—among themselves and inside themselves, which at the same time has lead to a heavy cut in lending, and hence to a deep contraction in the economy.

Nobody predicted this crisis, but, to be honest to the end, it should be questioned if this financial crisis must be thought—and modeled—as an endogenous crisis bursted by pure economic causes. However, How one could know the extent to which this kind of events are endogenously predictable?

With all this in mind, we think the two following sentences are justified. First, that different goods need different spans of time in order to totally install a new technology. Second, that a full endogenous analysis of the issue—endogenous legal-political costs and the future time needed to install the new technology—would need a model describing how a society changes its political preferences and cultural determinants over time jointly with its economic consequences. Therefore, we take as exogenously given both the time needed to install the new technology in the future—or, equivalently, the sequence of costs needed to install the new technology will be exogenous and constant (for simplicity)—, and the legal-political costs.

Probably, our set-up could be a starting point in order to face that ambitious objective of setting a full endogenous model of crisis.

Now we present for further technical assumptions over the costs above described.

- Technical assumptions
Some fundamentals of the economy satisfy the following general conditions:\footnote{In order to see the formal expression from which we drew the assumptions’ interpretations, we refer to the standard results in Cournot Competition under constant marginal costs.}

**A1** \( C^N < a \).

This is the minimal hypothesis to assume in order to make sensible the maximization problem of the firms: It simply implies that it is possible to produce positive quantities of the good.

**A2** \( a - C^N \leq \frac{a - C^F}{2} \).

This means that the foreign technology not only is more efficient than the domestic one but also that the domestic one is not competitive, in the sense that it can only produce zero quantities of the good if it competes face to face with the foreign technology. Notice that A2 implies that \( C^F < C^N \).

**A3**

\hspace{1em} A3.1) \( C^N < C^n_t + C^F < a \) for all \( 0 \leq t \leq n - 1 \);  
\hspace{1em} A3.2) \( \frac{a + C^N}{2} > C^e_0 + C^F \);  
\hspace{1em} A3.3) \( \frac{C^F + a}{2} > C^n_n + C^F \).

This assumption captures the following idea: The new technology is more expensive— but affordable— than the domestic one at the beginning (A3.1). It can be installed (A3.2) but, at some moment, once it is completely installed, it becomes not only more efficient than the domestic one but also, if it is used by the two domestic firms, it is capable of producing positive quantities even when the economy is already opened (A3.3). Notice that A2 and A3.1 imply that \( a - (C^n_t + C^F) < \frac{a - C^F}{2} \) for all \( 0 \leq t \leq n - 1 \) and, hence the only way for a firm to survive, after the economy is opened, is to have the new technology completely installed. Also, observe that A2 and A3.3 imply that \( C^N > C^n_n + C^F \).

Once the model is settled, we present the results.
3 The Main Results

First, the equilibrium results.

3.1 Equilibrium Results

For the sake of the exposition, in this section we only present the shape of the equilibrium strategies along the equilibrium path of the results considered. The full definition of the strategies and the complete formal results are presented in the theorems 1, 2 and 3 in the appendix I.

Recall that given a profile of strategies \( s = (s^G, s^I, s^P) \in S^G \times S^I \times S^P \), this triple determines a path that is actually played, according to those strategies, of the form \((a_t)_{t=0}^{\infty} = ((a^G_t, a^I_t, a^P_t))_{t=0}^{\infty}\), with \( a^i_t \in \{N, T\} \) for all \( t \geq 0 \) (\( i \in \{I, P\} \)), and \( a^G_t \in \{C, O\} \) for all \( t \geq 0 \).

In order to highlight the applications of our model, we first will present the two propositions that describes the situations that were commented in the introduction a part of the motivation of this paper, the Spanish case, and the Mexican case. Later in this section, we will provide an overview of all the equilibrium results of this paper.

The first proposition gives an explanation to the evidence that in Spain, some protectionist policies have been time consistent, and successful in order to induce domestic firms to adopt new technology (see Carreras and Tafunnel, 2004). This proposition is contained in the item (3.1) of the theorem 3. As we said, this result is in sharp contrast to the existing literature on the issue.

In the sequel when we say that a firm is willing to adopt the new technology, it means that it is patient enough or, equivalently, that it has a sufficiently large discount factor.

**Proposition 1** (Time consistency of a protectionist policy) If the government and the patient firm are patient enough, the impatient firm is too impatient, and the gap between the technologies is very large \((C^N - C^F\) very large, roughly), then the following history...
\((C, N, T), (C, N, N), (C, N, N), \ldots, (C, N, N), \ldots, (O, N, N), \ldots\), in which the economy is closed only the first \(n\) periods of the history (from \(t = 0\) to \(t = n - 1\)), the impatient firm does not adopt the new technology (hence, shuts down at \(t = n\)), and the patient firm adopts the new technology at the outset of the game, is a subgame perfect equilibrium path of the game.

The rationale of this equilibrium is very simple. Provided that there is a firm that is willing to adopt the new technology and that the gap between the technologies is very large, a patient government keeps the economy closed until the domestic firm has adopted the new technology, moment at which it opens the economy, since it prefers more competition in spite of that the competing firm is a foreign firm, than less competition with all domestic firms in the market. Once this state is reached, there are no incentives to close the economy, so the strategy must be time consistent. The patient firm, as its discount factor is large enough, is willing to sacrifice present high benefits, in order to ensure its surviving capacity in the future.

The following proposition describes the situation in which a government opens the economy even provoking widespread bankruptcies in the domestic industry after a long period of time in which the economy was closed. This gives an explanation to the evidence showed in Gómez-Galvarriato (2007), in which from 1920 to 1985 the economy was closed, none of the domestic firms adopted the new technology, and then the economy was abruptly opened at 1985, moment at which the gap between the technologies was very large. This result is contained in the item (1.3) of the theorem 1.

**Proposition 2** Assume the government is too impatient, the legal-political costs are very high for \(L\) periods (\(C_l^P\) for \(l = 0, \ldots, L\) are very large) and the gap between the domestic and foreign technologies is not very large (\(C^N - C^F\) small enough, roughly), but from \(t = L + 1\) those political costs are not too large (become affordable) and the gap between the foreign and the domestic technologies become very large, then the following history \(((C, N, N), \ldots, (C, N, N), (O, N, N), \ldots)\) in which the economy is
closed for $L+1$ periods (from $t = 0$ to $t = L$) but open from $t = L+1$, and the firms never adopt the new technology is an equilibrium path of the game—both domestic firms leave the market at $L+1$. 

The intuition of this proposition is the following. From $t = 0$ to $t = L$, the firms do not adopt the new technology because it is not affordable and, since the gap between the technologies is not very large, the government, even being impatient, does not open the economy. However, from $t = L + 1$, the trade-off faced by the government is to keep in the economy two domestic firms with an old and inefficient technology, or to open the economy at the outset and to allow a foreign firm to operate as a monopolist for ever, but with a very good technology. Given that the government is very impatient, it is not willing to wait for future possible high welfare for the society, so that it opens the economy at $t = L + 1$. We recall that only for simplicity we assumed that if the economy is opened only one foreign firm enters the market, so that if more firms can enter the market, our argument is even stronger. As long as the parameters of the agents do not change, it is clear that along the equilibrium path none of them have incentives to modify their strategies: Both the government and the firms are facing essentially the same situation all the periods. Out of equilibrium paths the situation is more subtle, and it is shown in detail in the appendix II, where the proofs are presented.

As for the proposition 1, there are other situations in which the economy is opened at the outset of the history, which are described in the appendix I.

Now we present the third result in this section, which tells that bad expectations may provoke bad economic outcomes (a monopolist operating in the market), even in presence of conditions in which better outcomes are possible (more than one firm operating with the new technology). This proposition is contained in the item (1.1.6) of the theorem 1, and it is a sort of self fulfilling prophesies.
Proposition 3 Assume the gap between the technologies is very large. Then, the history 

\(((O, N, N), (O, N, N), \ldots)\), in which the economy is opened at the outset of the game, the domestic firms shut down, and there are only foreign firms operating in the market, is an equilibrium of the game. Along the history, none of the agents have incentives to modify their behavior.

Observe that in this proposition the history \(((O, N, N), (O, N, N), \ldots)\) is a possible equilibrium path independently of the degree of patience of the agents and the legal-political costs, and this is exactly the point here. The conditions that ensure the existence of the equilibrium in the proposition 1 are therefore not ensuring a unique equilibrium. An interpretation or intuition of this fact runs as follows. If the government, independently of the degree of patience of the firm, perceives that they will not adopt new technology, and at the same time the firms think that the government is thinking that and hence will opens the economy at the outset, then the firms decide not to adopt the new technology, and so on and so forth. It is a sort of pessimism that provokes the outcome.

Notice, as we said in the introduction, that the NAFTA may function as a device that select the equilibrium in which firms adopt the new technology, the one given in the proposition 1.

Finally, we present an overview of all the equilibrium results of this paper (except the previous proposition, a very special case). There are results of two types, one in which the firms that are willing to adopt the new technology do that, and the other in which no firm adopt the new technology.

First, the conditions under which no firm adopts the new technology. Recall that the sentence ‘a firm is willing to adopt the new technology,’ means that it has a sufficiently large discount factor, and vice versa, ‘a firm is not willing to adopt the new technology,’ means that it has a very small discount factor.

Proposition 4 Given the game in (5). Then, no firm adopts the new technology, if either one of the following conditions holds:
1) Assume that the gap between the technologies is very large, and:

1.1) The government is too impatient; (the government opens the economy at the outset);

1.2) None of the firms are willing to adopt the new technology; (the government opens the economy at the outset);

2) Assume that the gap between the technologies is not too large, and:

2.1) The Legal-Political Costs are never affordable; (the government never opens the economy).

Second, the conditions under which the firms that are willing to adopt the new technology do that, and the government give enough time to do it. This proposition is essentially the theorem 1 in the appendix I.

**Proposition 5** Given the game in (5). Therefore, if either one of the conditions given below holds, we have that the government gives time to adopt the new technology and the firm(s) that is (are) willing to adopt the new technology do that:

1) Assume that the gap between the technologies is very large, the government is patient enough, and at least one firm is willing to adopt the new technology;

2) Assume that the gap between the technologies is not too large, and at least one firm is willing to adopt the new technology.

In the appendix I, theorem 3, we prove this proposition. There, the statement is more detailed than here. The presentation given right now is more concise, because isolates the crucial conditions that ensure the result, and for this reason we have written it in this way.

Observe that in the item (2) of this last proposition, we make no assumptions over the degree of patience of the government.

Next we characterize the when an agent is patient enough, and when it is too impatient.
3.2 Characterization of the Agents’ Degree of Patience

It is important to notice that it is possible to give a criteria that allows us to classify when an agent in this model is patient enough (she/he prefers to be better in the long run, than to have high benefits in the present), and when is too impatient. More precisely, we will prove:

1. There exists a value denoted by \( \beta_p(C^N) \)—all other parameters being constant—, so that for any discount factor larger than that, the agent will prefer to invest in new technology , so prefers to survive in the long run, than high present profits;

2. There exists a value denoted by \( \beta_I(C^N) \), so that for any discount factor lower than \( \beta_I(C^N) \), the agent will prefer not to invest in new technology, so prefers not to survive in the long run, than to survive. \( \beta_p(C^N) \) and \( \beta_I(C^N) \) satisfy that are increasing in \( C^N \), therefore the larger is the gap between the technologies —the larger is \( C^N \), the larger is the gap—, the more patient must be an agent in order to prefer future benefits. Similar statements apply if we consider the initial investment. For simplicity we will model it as the initial cost \( C_e^0 \) —notice that, if \( n = 1 \), \( C_e^0 \) is exactly standard the fix cost of installing the new technology—, that is, \( \beta_I(C^N) \) is also function of \( C_e^0 \), and it is increasing in \( C_e^0 \).

And analogous statements apply for the government.

Consequently, a given value of the discount factor may be large enough for some industries, but may be at the same time too small for other industries, and a government may be too impatient in some country due to the gap between the technologies, but may be patient enough in another country in which the gap is not that large.

The proof runs as follows. Define the function

\[
g(\beta) = h(\beta)(1 - \beta),
\]
with the function $h(\beta)$ given by

$$h(\beta) = \left( \sum_{t=0}^{t=n-1} (\beta)^t \pi_t(T) + \frac{\beta^n}{1 - \beta} \pi_n(T) \right) - \sum_{t=0}^{t=n-1} (\beta)^t \pi_t(NT),$$

where $\pi_t(NT)$ for $t = 0.., n - 1$ are the Cournot profits if it never adopts the new technology, and $\pi_t(T)$ for $t = 0.., n$ are the Cournot profits if it adopts the new technology forever. See the lemma 1 in the appendix to obtain the expressions of $\pi_t(NT)$ for $t = 0.., n - 1$ and $\pi_t(T)$ for $t = 0.., n$.

Let $\beta_p(C^N)$ be the maximum real root of the polynomial function $g$ in the interval $(0, 1)$. That value exists because $g(0) = \pi_0(T) - \pi_0(NT) < 0$, and $g(1) := \lim_{\beta \to 1} g(\beta) = \pi_n(T) > 0$. Notice that $\beta_p(C^N)$ equates the value of never adopting the new technology —not surviving—, to the value of adopting the new technology at $t = 0$ forever—surviving—, assuming that the economy is opened at $t = n$.

Also $\beta_p(C^F)$ satisfies that if $\beta \geq \beta_p(C^N)$, then $h(\beta) > 0$, due to that $h(1) = \pi_n(T) > 0$, and thus the firm prefers to invest. This proves the first part of our assertion.

We will show now that $\frac{\partial \beta_p(C^N)}{\partial C^N} > 0$, which implies the second part of our assertion. Observe that $g(\beta_p(C^N)) = 0$ if and only if $h(\beta_p(C^N)) = 0$. Applying the Implicit Function Theorem in the equation $h(\beta_p(C^N)) = 0$, we have

$$\frac{\partial \beta_p(C^N)}{\partial C^N} \frac{\partial h(\beta)}{\partial \beta} (\beta_p(C^N)) \pi_n(T) + \frac{\partial \pi_n(T)}{\partial C^N} \left[ \frac{(\beta_p(C^N))^n}{(1 - \beta_p(C^N))^2} \right] =$$

$$\sum_{t=0}^{t=n-1} (\beta_p(C^N))^t \left[ \frac{\partial \pi_t(NT)}{\partial C^N} - \frac{\partial \pi_t(T)}{\partial C^N} \right] + \sum_{t=0}^{t=n-1} t(\beta_p(C^N))^{t-1} \frac{\partial \beta_p(C^N)}{\partial C^N}(\pi_t(NT) - \pi_t(T)),$$

thus

$$\frac{\partial \beta_p(C^N)}{\partial C^N} \left[ n[\beta_p(C^N)]^{n-1}(1 - \beta_p(C^N)) + [\beta_p(C^N)]^n \pi_n(T) - \sum_{t=0}^{t=n-1} t(\beta_p(C^N))^{t-1}(\pi_t(NT) - \pi_t(T)) \right] =$$

$$\sum_{t=0}^{t=n-1} (\beta_p(C^N))^t \left[ \frac{\partial \pi_t(NT)}{\partial C^N} - \frac{\partial \pi_t(T)}{\partial C^N} \right],$$
since $\frac{\partial \pi_n(T)}{\partial C_N} = 0$ ($\pi_n(T)$ does not depend on $C_N$).

Also, note that

$$\left[ n[\beta_p(C_N)]^{n-1}(1-\beta_p(C_N)) + \frac{[\beta(N)]^n}{(1-\beta_p(C_N))^2} - \sum_{t=0}^{t=n-1} t(\beta_p(C_N))^{t-1}(\pi_t(NT) - \pi_t(T)) \right] = \frac{\partial h(\beta)}{\partial \beta} (\beta_p(C_N)) > 0$$

due to that $h$ is increasing close to $\beta_p(C_N)$ —we exclude the rare cases in which $h$ is increasing but that derivative is zero—.

Since $\frac{\partial \pi_t(NT)}{\partial C_N} < 0$, $\frac{\partial \pi_t(T)}{\partial C_N} > 0$ for all $t \leq n - 1$, the result follows. Notice that with $n = 1$ or $n = 2$ it is possible to obtain $\beta_p(C_F)$ explicitly. Notice also that if $n \geq 3$, then $g(\beta)$ may have more than one real root in the interval $(0,1)$, so that it may happen that for some $\beta < \beta(C_N)$, the firm also prefers to invest. Now taking $\beta_I(C_N)$ as the minimal positive root of the equation $h(\beta) = 0$ in the interval $(0,1)$, the same reasoning applies to prove that if $\beta < \beta_I(C_N)$, the firm prefers not to invest, and that $\beta_I(C_N)$ is increasing in $C_N$. Thus the point (b) is also proven. The proofs for the case in which we consider $C^e_0$ and the government are quite similar and hence omitted. Observe that in the cases in which $g(\beta)$ have only one root in $(0,1)$, $\beta_I(C_N) = \beta_p(C_N)$, and we obtain a complete characterization of the agents in patient or impatient.

Next, we isolates the fundamental assumptions that allows us to prove the theorems given in the appendix I, and we comment the existence of other equilibria.

### 4 Generalizations, Other Equilibria and Uniqueness

At first glance it may seem that our set-up is very restrictive since we assume a linear inverse demand function, and that firms compete à la Cournot, a given form for the preferences of the government, etc. However, we introduced these restrictions just for the sake of a clearer exposition, but they can be easily relaxed, and the results hold in much more general situations. The only three necessary requirements in order for our results to hold are the following: 1) The firms get
lower profits if more firms enter the market; 2) New investments reduce present profits but increase future profits for the firm that makes them; 3) If a firm enters the market with a better technology than the other firm is using, if it does not carry out new investments, the latter firm will have to shut down.

At the same time, we may generalize further about the instantaneous utility function of the government. The two key requirements for that utility function are the following: 1) The larger the number of firms competing in the market, the larger is the society’s instantaneous utility; 2) The better the technology used in the industry, the larger the society’s instantaneous utility.

Once we assume those requirements above described, we will obtain the same equilibria as those obtained with our explicit assumptions. That is, depending upon the agents’ degree of patience, the gap between the new technology and the domestic one, and the legal-political costs, the economy is opened or not, the government protects or not, and the firms adopt the new technology or not.

It is clear that other equilibria may appear in our model, if we allow for other conditions over the legal-political costs. Also, propositions 1 and 3 show that there is no uniqueness of equilibria in general.

The study of these two issues is left for future research.

5 Conclusions

The model developed in this paper allows us to explain that a temporal protectionist policy can be time consistent and hence it can induce firms to adopt a new technology. It also allows us to better understand the reasons behind the divergence observed in the paths to openness taken by different countries, and their economic consequences. The crucial factor in almost all of our results is the degree of patience of the agents, crucial in the sense that a low degree of patience is a sufficient condition for the existence of some equilibria, those in which the economy is opened at
the outset. On the other hand, a high degree of patience is necessary to obtain the equilibria in which a patient firm adopts the new technology. Nevertheless, the degree of patience is relative to the gap between the technologies, in the sense that the lower is the gap, the higher is the degree of patience necessary in order to ensure that a firm, for instance, will choose to survive in the long run, and vice versa. At the same time, the degree of patience is relative to the initial investment, in the sense that the larger is this investment, the more patient must be a firm to prefer to invest. Therefore, our paper departs from the more standard analysis in which pure economic causes are thought as the unique causes of the performance of countries and firms.

Our paper hence begs the question: Is it reasonable to assume that the discount factors are not endogenous? One could think that in developing countries, firms tend to be impatient because in these countries there is usually more political and economic instability, plus the high opportunity costs that they face some periods. Uncertainty about the future may increase the degree of impatience. In fact, we may interpret the discount factor as proportional to the probability that company-owners assign to the possibility that the firm will survive the next period. However, our model suggests that company-owners in an unstable country will not necessarily be very impatient. More than that, to be impatient, may be the worst strategy in the long run. If our model reasonably describes how the decisions of the government and the firms work together in order to open or not to open an economy, a company-owner must think that if the government promised to open the economy at some moment in the future, it will do it, because the gap between the domestic and foreign technologies will eventually grow large and force, somehow, the government to open. Therefore if the firm wants to survive in the long run, it must not discount heavily future outcomes in order to survive. Thus, according to the model even when the environment is unstable a firm that wants to survive in the long run adopts a strategy that minimizes losses, and it will invest in
the new technology.\footnote{We want to emphazise that the gap between the technologies is not what generates the results in which the protectionist policy is time consistent. To say it once again: This equilibrium is the result of the high degree of patience of some of the agents in the game.}

Which are the determinants of the discount factors of company-owners? How do they decide these values? Some insights on this issue can be drawn from our model. Our model suggests: A firm that wants to survive in the long run, must adopt a high discount factor. On the other hand, if a firm adopts a low discount factor, it does not care about surviving in the long run. Why do one firm prefers to survive in the long run and the other prefers not to survive? Is it possible that the firm that decided the low discount factor, after the economy is opened, will regret of that decision? Was it mistaken?

If, in a society, we observe these two types of behavior, we may expect some kind of cultural and economic process determining the endogenous discount factors in that society. We conjecture that this process would eventually leave only patient firms in the market, in the long run, if at some moment of the society’s history there is a sufficient proportion of patient firms in the market. This analysis is left for future research.

6 Appendix I

In this section we present the full statements of all our results.

Unless other specification, in all the results that follow, when the condition \( a - (C_0^e + C^F) < C_t^p \) for all \( t \geq 0 \) is not required, it is implicit that the condition prevailing is \( a - (C_0^e + C^F) > C_t^p \) for all \( t \geq 0 \).

In the theorem 1, we state all the situations such that no firm adopt the new technology, and in the theorem 3, all the situations such that firms willing to adopt the new technology do that,
and the government gives time to them for the adoption of the new technology.

**Theorem 1** Assume the government is utilitarian with $\tau$ large enough (close to one). Then:

1.1) If $C^F \leq \frac{-a}{11} + a$ (the new technology is very efficient) and $\frac{1}{8}(a-C^F)^2 + \frac{(a-C^F)}{2} > \frac{1}{9}(a-C^N)^2$

(the gap between the domestic technology and the new technology is very large), then:

1.1.1) If the government is very impatient and the two national firms are very patient, then there is a subgame perfect Nash equilibrium, given by $(s^P, s^I, s^G) = \{(s_h(N, 1))_{h \in H}, (s_h(N, 1))_{h \in H}, s^G(0)\}$, where, given

\[ h_t = (a^I_t, a^P_t, a^G_t) \]

$s_{h_t}(N, 1)$ prescribes to use the domestic technology, unless the new technology can be totally installed at $t + 1$ or, at $t + 2$ (that is, the firm has paid all the costs of the new technology but $C^e_n$, or all the costs are paid but $C^e_{n-1}$ and $C^e_n$ and the economy is closed at $t + 1$), and $s^G(0)$ is such that $s^G(0)(h_t) = O$ for all $h_t = (a^I_t, a^P_t, a^G_t)_{t=0}^{T} \in H$. Furthermore, $s^G(0)$ is a strictly dominant strategy.

1.1.2) If the government is very impatient and the two national firms are very impatient, then \( (s_h(N, 2))_{h \in H}, (s_h(N, 2))_{h \in H}, s^G(0) \) is a subgame perfect Nash equilibrium, where \( (s_h(N, 2))_{h \in H} \) prescribes to use the domestic technology, unless the new technology can be totally installed at $t + 1$.

Furthermore, if a firm is very impatient, then \( (s_h(N, 2))_{h \in H} \) is a strictly dominant strategy.

1.1.3) If the government is very impatient and only one firm is very impatient, then \( (s_h(N, 1))_{h \in H}, (s_h(N, 2))_{h \in H}, s^G(0) \) is a subgame perfect equilibrium. \( (s_h(N, 1))_{h \in H} \) is adopted by the patient firm and \( (s_h(N, 2))_{h \in H} \) is adopted by the impatient firm.

1.1.4) If the two domestic firms are very impatient, but the government is very patient, then \( (s_h(N, 2))_{h \in H}, (s_h(N, 2))_{h \in H}, s^G(0) \) is a subgame perfect Nash equilibrium.

1.1.5) If $a - (C^0 - C^F) < C^P_t$ for all $t \geq 0$, then whatever be the degree of patience of the agents, \( (s_h(N, 2))_{h \in H}, (s_h(N, 2))_{h \in H}, s^G(0) \) is a sub-game perfect equilibrium.

\[ \dagger \text{Notice that if } (C^N, C^F) \rightarrow (a, 0), \text{ the two conditions of this item are satisfied.} \]
(1.1.6) Whatever be the degree of impatience of the agents and the legal-political costs, \( \{s_h(N, 2)\}_{h \in H} \) is a best response of \( s^G(0) \) and, reciprocally, \( s^G(0) \) is a best response of \( \{s_h(N, 2)\}_{h \in H} \). Further \( \{\{s_h(N, 2)\}_{h \in H}, \{s_h(N, 2)\}_{h \in H}, s^G(0)\} \) is a subgame perfect equilibrium.

(1.2) If \( \frac{4}{5}(a - C^N)^2 > \frac{1}{5}(a - C^F)^2 + \left(\frac{a - C^F}{2}\right) \) (the gap between the domestic technology and the new technology is not too large), then:

(1.2.1) Whatever be the degree of patience of the government, if both of the domestic firms are very impatient, then there is a subgame perfect Nash equilibrium, given by

\[
(s^p, s^I, s^G) = (\{s_h(N, 2)\}_{h \in H}, \{s_h(N, 2)\}_{h \in H}, s^G(\infty)),
\]

where \( \{s_h(N, 2)\}_{h \in H} \) is such that \( s_h(N, 2) = N \), if \( h_t = ((a^f_t, a^I_t, a^G_t))_{t=0}^{1} \in H \) is such that the technology that was in use at \( t \) was the domestic one, unless the new technology can be totally installed at \( t+1 \), and \( s^G(\infty) \) is given by \( s^G(\infty)(h_t) = 0 \) for all \( h_t \in H \) such that at least one of the domestic firms can have the new technology totally installed at \( t+1 \), otherwise \( s^G(\infty)(h_t) = C \).

(1.2.2) If it happens that \( a - (C^0_0 + C^F) < C^p_t \) for all \( t \geq 0 \) (the legal-political costs are too high) then, whatever be the degree of impatience of the agents, \( \{\{s_h(N, 2)\}_{h \in H}, \{s_h(N, 2)\}_{h \in H}, s^G(\infty)\} \) is a sub-game perfect equilibrium.

(1.3) If \( a - (C^0_0 + C^F) < C^p_t \) and the gap between the technologies is not too large for all \( 0 \leq t \leq L \), but \( a - (C^0_0 + C^F) > C^p_t \) and the gap between the technologies is too large for all \( t \geq L + 1 \), and the government is very impatient, then there is a sub-game perfect Nash equilibrium given by

\[
(s^p, s^I, s^G) = (\{s_h(N, 2)\}_{h \in H}, \{s_h(N, 2)\}_{h \in H}, s^G(L)),
\]

where \( s^G(L)(h_t) = s^G(\infty)(h_t) \) for all \( t \leq L \), and \( s^G(L)(h_t) = s^G(0)(h_t) \) for all \( t \geq L + 1 \).

We obtain similar result if the government is consumer oriented.

**Theorem 2** Assume the government is consumer oriented. Then, if in theorem 1 we replace the condition \( \frac{1}{5}(a - C^F)^2 + \left(\frac{a - C^F}{2}\right) \) (the gap between the domestic technology and the new technology is very large) for \( \frac{1}{5}(a - C^F)^2 > \frac{4}{5}(a - C^N)^2 \) and the condition \( \frac{1}{5}(a - C^F)^2 + \left(\frac{a - C^F}{2}\right) \) <
\[ \frac{4}{9}(a - C^N)^2 \] (the gap between the domestic technology and the new technology is not very large), the same equilibria as in theorem 1 exist.

The last results of this paper are the equilibria found when the government is very patient, and at most only one firm is very impatient, when the legal-political costs are not too high.

As one may have expected, the firms’ strategies are not the same in all situations. That is, a patient firm has a different strategy if the other firm is an impatient one than if the other firm is a patient one. When both firms are patient, both decides on the same strategy, which is, roughly, as follows: A firm adopts the new technology at \( t + 1 \) only if it has paid at least the same number of costs of the new technology as the other firm, otherwise it adopts the old technology. In the second situation, when only one is a patient firm, that one always decides to adopt the new technology, and the impatient firm never adopts the new technology unless at \( t + 1 \) that firm can have totally installed the new technology, that is, it adopts \( \{s_h(N, 2)\}_{h \in H} \).

Formally. Given \( h = (a_l)_{l=0}^{t} \in H \), denoting by \( ((C^l_i, C^P_i))_{l=0}^{t} \) the corresponding costs paid by the firms, define the set

\[
\varphi(i, h) = \begin{cases} 
  l \in \{0, \ldots, n - 1\} & \exists k \leq t, \text{ such that, } a_k^i = C \\
  C_k^i = C_e^i + C_k^P + C^F & \text{if } l = 0 \\
  C_k^i = C_e^i + C^F & \text{if } l \neq 0 \text{ and } \pi_k^i(C_k^i, C_k^j) > 0.
\end{cases}
\]

\( C(i, h) = |\varphi(i, h)| \) its cardinality. Simply, \( C(i, h) \) is the number of costs of the new technology that the firm \( i \) has indeed paid along the history \( h \) (notice that if even the economy is closed, the other domestic firm may have had the new technology totally installed before \( t \)). We define \( \{s_h(T, 1)\}_{h \in H} \) as follows: If \( h \in H \) is such that \( C(i, h) = n \) or \( C(i, h) < n \) and \( C(i, h) \geq C(j, h) \), with \( j \neq i \), then \( s_h(T, 1) = T \) if the technology used at \( t \) was the old one, and \( s_h(T, 1) = N \) if the technology used at \( t \) was the new one; but, if \( C(i, h) < n \) and \( C(i, h) < C(j, h) \), then \( s_h(T, 1) = N \) if the technology
used at $t$ was the old one, and $s_h(T, 1) = T$ if the technology used at $t$ was the new one.

On the other hand, define $\{s_h(T, 2)\}_{h \in H}$ as follows: $s_h(T, 2) = T$ if the technology used at $t$ was the old one, otherwise $s_h(T, 2) = N$.

The theorem.

**Theorem 3** Assume that the government is utilitarian with $\tau$ close to one, or it is consumers oriented, and the legal-political costs are not too high. Then:

3.1) Assume that the gap between the technologies is very large, the government and both domestic firms are patient enough, then, $(s^P, s^I, s^G) = (\{s_h(T, 1)\}_{h \in H}, \{s_h(T, 1)\}_{h \in H}, s^G(n))$ is a subgame perfect equilibrium where, given $h = ((a^P_t, a^I_t, a^G_t))_{t=0}^T \in H$, $s^G(n)(h) = C$ if none of the firms can have the new technology totally installed at $t + 1$, otherwise $s^G(n)(h) = O$.

3.2) Assume that the gap between the technologies is very large, the government is patient enough, one firm is patient enough ($P$) and the other is very impatient ($I$), then

$$(s^P, s^I, s^G) = (\{s_h(T, 2)\}_{h \in H}, \{s_h(N, 2)\}_{h \in H}, s^G(n))$$

is a subgame perfect equilibrium.

3.3) Assume that

$$\frac{4}{9}(a - (C^F + C^o))^2 > \frac{1}{8}(a - C^F)^2 + \left(\frac{a-C^F}{2}\right)$$

if the government is utilitarian or

$$\frac{4}{9}(a - (C^F + C^o))^2 > \frac{1}{8}(a - C^F)^2$$

if the government is consumer oriented (a stronger version of “the gap between the technologies is not very large”), the government is either patient or impatient, and both domestic firms are patient enough, then $(s^P, s^I, s^G) = (\{s_h(T, 1)\}_{h \in H}, \{s_h(T, 1)\}_{h \in H}, s^G(n))$ is a subgame perfect equilibrium.

3.4) Assume that

$$\frac{4}{9}(a - (C^F + C^o))^2 > \frac{1}{8}(a - C^F)^2$$

if the government is utilitarian, or

$$\frac{4}{9}(a - (C^F + C^o))^2 > \frac{1}{8}(a - C^F)^2$$

if the government is consumer oriented (the strongest version of “the gap between the technologies is not very large”), the government is either patient or impatient, one firm is patient enough ($P$) and the other is very impatient ($I$), then

$$(s^P, s^I, s^G) = (\{s_h(T, 2)\}_{h \in H}, \{s_h(N, 2)\}_{h \in H}, s^G(n))$$

is a subgame perfect equilibrium.
7 Appendix II

In this appendix there are the proofs of the theorems 1-3 of this paper.

First, we recall some well-known results in relation to Cournot Competence.

**Lemma 1** Suppose that the inverse demand function is given by \( P(Q) = a - Q \). Then

a) If there are two firms facing constant marginal costs \( C^1 \) and \( C^2 \) that compete à la Cournot, and \( a - C^i > 0 \) for \( i = 1, 2 \), then if \((q^1, q^2)\) denotes the Nash equilibrium, we have

\[
(q^k)_{k \in \{1,2\}} \text{ is given by }
\]

\[
\begin{cases}
q^i = \frac{a - 2C^i + C^j}{3} & \text{if } a - C^i > \frac{a - C^j}{2} \text{ for } i, j \in \{1,2\}, i \neq j \\
q^i = \frac{a - C^i}{2}, q^j = 0 & \text{if } a - C^i \leq \frac{a - C^j}{2} \text{ for } i, j \in \{1,2\}, i \neq j,
\end{cases}
\]

and the Cournot profits of the firm \( i \in \{1,2,3\} \) are given by \( \pi^i(C^i, C^j) = (q^i)^2 \) for \( i = 1, 2 \); and

b) If there are three firms facing constant marginal costs \( C^i \) with \( i = 1, 2, 3 \) that compete à la Cournot, then if \((q^k)_{k \in \{1,2,3\}}\) denotes a Nash equilibrium, we have that

\[
(q^k)_{k \in \{1,2,3\}} \text{ is given by }
\]

\[
\begin{cases}
q^i = \frac{a - 3C^i + \sum_{j \neq i} C^j}{4} & \text{if } a - C^i > \sum_{j \neq i} \frac{a - C^j}{3}, \text{ for } i \in \{1,2,3\}, \text{ or}
\\
q^i = 0, \text{ and } q^j = \frac{a - 2C^j + C^k}{3} & \text{if } a - C^i \leq \sum_{j \neq i} \frac{a - C^j}{3} \text{ or}
\\
\text{and } a - C^j > \frac{a - C^k}{2} & \text{for } j, k \in \{1,2,3\} \setminus \{i\}, j \neq k,
\end{cases}
\]

the Cournot profits of the firm \( i \in \{1,2,3\} \) are given by \( \pi^i(C^i, C^{-i}) = (q^i)^2 \).

Proof: Routine and omitted.

For all the proofs we will use the one-stage deviation principle for discrete-infinite-horizon games (theorem 4.2, in Fudenberg and Tirole (2002)). As we commented at due time, our game is not a repeated game, nor it has a recursive structure.

We will prove in detail all the results, especially those that we think are clear from the preceding
arguments. Also, in order to make the exposition as short as possible, all the items of theorems 1 and 2 are proven, when possible, in one shot. That is, as we are making the arguments, we will be pointing out when an argument applies to another item and then when the corresponding result is proven. Only the most obvious proofs are dropped. We do it in that manner because arguments are common. Only theorem 3 is a little bit different, mainly because at least one domestic firm, a patient one, adopts the new technology, and the government is patient enough to give time to a patient firm to adopt the new technology.

The proposition 1 is the item (3.1) of theorem 3, the proposition 2 is the item (1.3) of theorem 1, and the proposition 3 is the item (1.1.6) of theorem 1. Therefore, once we prove the theorems, the propositions are proven.

For the sake of the exposition, in this section, the remarks’s numeration is independent of the remarks’ numeration of the rest of the paper, that is, they are numeredated from one to nine, as you will see.

1 Proof of theorems 1 and 2

Following Osborne and Rubinstein (1994), we introduce the following notation. Given the extensive game form with perfect information

\[ \Gamma = \left\langle \{I, P, G\}, H, \tilde{P}, \left(\Pi^i\right)_{i \in \{G, I, P\}} \right\rangle, \]

if \( \tilde{h} = ((a^I_i, a^P_i, a^G_i))_{i=0}^t \in H \), then \( \Gamma(\tilde{h}) = \left\langle \{I, P, G\}, H|_{\tilde{h}}, \tilde{P}|_{\tilde{h}}, \left(\Pi^i|_{\tilde{h}}\right)_{i \in \{I, P, G\}} \right\rangle \) will denote the sub-game of \( \Gamma \) that follows the history \( \tilde{h} \), where \( H|_{\tilde{h}} \) is the set of sequences \( h' \) of actions for which \( (\tilde{h}, h') \in H, \) \( \tilde{P}|_{\tilde{h}} \) is defined by \( \tilde{P}|_{\tilde{h}}(h') = \tilde{P}(\tilde{h}, h') \) for each \( h' \in H|_{\tilde{h}} \) and \( \Pi^i|_{\tilde{h}} \) is defined by \( h' \) is at least as good as \( h'' \) if and only if \( (\tilde{h}, h') \) is as good as \( (\tilde{h}, h'') \). Similarly, given a strategy \( s, s|_{\tilde{h}} \) will denote the strategy that \( s \) induces in the sub-game \( \Gamma(\tilde{h}) \), that is, \( s|_{\tilde{h}}(h') = s(\tilde{h}, h') \) for each \( h' \in H|_{\tilde{h}} \).

With this notation in place, we proceed to present the proofs. From now on, we assume \( C^i_n = 0 \).

In what follows, in order to take into account the initial history \( h = \emptyset \), one may think that the
game started at \( t = -1 \), but at that time there are no alternative decisions: The economy is closed, the firms are using the old technology, and that situation is taken as given.

First, the firms.

1.1.F Suppose that the government is too impatient.

Essentially, the same proof applies for all the items ((1.1.1)-(1.1.6), (2.1.1)-(2.1.6)). To fix ideas, consider \((s^I, s^P, s^G) = (\{s_h(N,1)\}_{h \in H}, \{s_h(N,1)\}_{h \in H}, s^G(0))\). From now on, we use the notation \(\{s^i_h(N,1)\}_{h \in H} = \{s_h(N,1)\}_{h \in H}\) for \(i \in \{I, P\}\). The intuition is the following: In all the situations in the results previously cited, the domestic firms either do not have time to install the new technology or at most they have two chances to do it, and then they will never adopt it if they need more than two periods, or they do not have incentives to invest due to their degree of impatience. Formally, we will prove that \(\{s^i_h(N,1)\}_{h \in H}\) is such that, for any \(\hat{h} \in H\), \(\{s_h(N,1)\}_{h \in H}|_{\hat{h}}\) is a best response to \((s^j, s^G)|_{\hat{h}}\) for \(i \neq j \in \{I, P\}\).

Take \(\tilde{h} = ((a^I_t, a^P_t, a^G_t))_{t=0}^t\) such that, for the firm \(i\), the new technology can neither be totally installed at \(t + 1\) nor at \(t + 2\) and consider the payoffs

\[
\Pi^i((s^i, s^j, s^G)|_{\tilde{h}})|_{\tilde{h}} \quad \text{and} \\
\Pi^i\left(\{s^i_h\}_{h \in H}|_{\tilde{h}}, (s^j, s^G)|_{\hat{h}}\right)|_{\tilde{h}},
\]

where \(s^i = \{s^i_h\}_{h \in H}\) is such that \(s^i_h = s_h(N,1)\) for all \(h \neq \tilde{h}\) and \(s^i_\tilde{h} \neq s_h(N,1)\). Then, \(s^i\) prescribes, given \((s^P, s^G)|_{\tilde{h}}\), to adopt the old technology for all \(l \geq t + 1\) and \(\tilde{s}^i = \{s^i_h\}_{h \in H}\) prescribes, given \((s^P, s^G)|_{\hat{h}}\), to adopt the new technology at \(t + 1\), but to adopt the old one for all \(l \geq t + 2\), because, as the government opens the economy at all \(l \geq t + 1\) along both game paths, the one defined by \((s^I, s^P, s^G)\) and the one defined by \((\tilde{s}^I, s^P, s^G)\), the firm \(i\) has neither time to install the new technology with \((s^I, s^P, s^G)\) nor with \((\tilde{s}^I, s^P, s^G)\), and hence
we have, for \( i \in \{I, P\} \),

\[
\Pi^i((s^i, s^j, s^G)|_{\bar{h}})|_{\bar{h}} = \Pi^i \left( \left\{ \tilde{s}^i_{\bar{h}} \right\}_{h \in H} \left, (s^j, s^G) \right|_{\bar{h}} \right) = 0,
\]

since, by A1-A3, that firm \( i \) shuts down at \( t + 1 \). Notice that the same reasoning applies if we consider \( \{ s^i_h(N, 2) \}_{h \in H} \) instead of \( \{ s^i_h(N, 1) \}_{h \in H} \).

If \( \tilde{h} \) is such that the new technology can be totally installed at \( t + 1 \), the reasoning is simpler: \( \tilde{s}^i \) prescribes to use the old technology at \( t + 1 \), but the new technology for all \( l \geq t + 2 \).

Thus

\[
\Pi^i((s^i, s^j, s^G)|_{\bar{h}})|_{\bar{h}} - \Pi^i \left( \left\{ \tilde{s}^i_{\bar{h}} \right\}_{h \in H} \left, (s^j, s^G) \right|_{\bar{h}} \right) = \\
\left( \pi_{i+1}^i((s^i, s^j, s^G)|_{\bar{h}}) - \pi_{i+1}^i \left( \left\{ \tilde{s}^i_{\bar{h}} \right\}_{h \in H} \left, (s^j, s^G) \right|_{\bar{h}} \right) \right)
\]

where \( \pi_{i+1}^i((s^i, s^j, s^G)|_{\bar{h}}) \) is the Cournot profit of the firm \( i \) at \( t + 1 \) using the technology according to \( s^i \), that is, the old technology, and an analogous definition applies to \( \pi_{i+1}^i \left( \left\{ \tilde{s}^i_{\bar{h}} \right\}_{h \in H} \left, (s^j, s^G) \right|_{\bar{h}} \right) \), but using the new technology not totally installed (the firm \( j \) is using the same technology in both situations, either with \( \{ s^j_h(N, 1) \}_{h \in H} \) or with \( \{ s^j_h(N, 2) \}_{h \in H} \)),

thus

\[
\left( \Pi^i((s^i, s^j, s^G)|_{\bar{h}})|_{\bar{h}} - \Pi^i \left( \left\{ \tilde{s}^i_{\bar{h}} \right\}_{h \in H} \left, (s^j, s^G) \right|_{\bar{h}} \right) \right) > 0,
\]

since the new technology, once it is totally installed, is more efficient than the old one, due to A1-A3.

Finally, given \( s^G \), the case when the new technology can be totally installed at \( t + 2 \) but not at \( t + 1 \) is not a possible path of the profile considered, since the government never keeps the economy closed, and hence there is nothing to prove.

Notice that we have shown that \( \{ s^i_h(N, 1) \}_{h \in H} \) is such that, for any \( \bar{h} \in H \), \( \{ s^i_h(N, 1) \}_{h \in H} |_{\bar{h}} \) is a best response to \( (s^j, s^G)|_{\bar{h}} \) for \( i \neq j \in \{I, P\} \), independently of the conditions of the degree of patience of the firms, that is, the optimality of the firms’ strategies for the items (1.1.1)-(1.1.3) and (1.1.6) is proven. Nevertheless, notice that from what we have done, it
follows at once that if a domestic firm is very impatient or the legal-political costs are very
large, it has a dominant strategy, namely \{s_h(N, 1)\}_{h \in H}, and hence that optimality for the
items (1.1.4) and (2.1.5) is also proven. Therefore, due to these last comments and remarks
3-6, the proof is done.

1.1.G The government

As for the case of the firms, the same proof applies for all the items ((1.1.1.)-(1.1.6), and
(2.1.1)-(2.1.6)). To fix ideas, consider \((s^I, s^P, s^G) = ((s_h(N, 1))_{h \in H}, \{s_h(N, 1)\}_{h \in H}, s^G(0))\).

We will prove that \(\{s_h^G(0)\}_{h \in H}\), for any \(\tilde{h} \in H\), \(\{s_h^G(0)\}_{h \in H}\) is a best response to \((s^I, s^P)|_{\tilde{h}}\).

We have two situations: a) If \(\tilde{h} = ((a^I_l, a^P_l, a^g_l))_{l=0}^t\) is such that none of the firms can have
totally installed the new technology at \(t + 1\); b) If at least one can.

a) Take \(\tilde{h} = ((a^I_l, a^P_l, a^g_l))_{l=0}^t\) such that none of the firms can have totally installed the new
technology at \(t + 1\). Since \(s^G(0)(\tilde{h}) = O\), then \(s^G(\tilde{h}) = C\).

A priori, we have two cases, the first if for one or two domestic firms it happens that at \(t + 2\)
the new technology can be totally installed, the other if none of them can. Clearly, if we were
considering

\(\{(s_h(N, 2))_{h \in H}, \{s_h(N, 2)\}\}\), neither case would be a possible equilibrium path, nor a possible
alternative path, and therefore we have nothing to prove.

In the former case, when the two domestic firms can install the new technology (if only
one can, the inequality between the Cournot profits at \(t + 1\) is lower than in the previous
case, but anyhow is positive) at \(t + 2\), both domestic firms react to \(s^G(\tilde{h})\) adopting the new
technology for all \(l \geq t + 1\), independently of what the government does at that period —but
the government opens the economy at $t + 2$, and then we have that

$$\left( \Pi^G((s^G, s^I, s^P)|\bar{h})\bigg|_{\bar{h}} - \Pi^G\left( \{s^G_h\}_{h \in H}|\bar{h}, (s^I, s^P)|\bar{h}\right)\bigg|_{\bar{h}} \right)(\beta^G)^{-(t+1)} = \frac{1}{2} \left(\frac{(a-C^F)}{2}\right)^2 + \tau\left(\frac{(a-C^F)}{2}\right) - \frac{1}{2} \left(\frac{2}{3}(a - (C^F + C^e_{n-1}))\right)^2 + 2 \left[\frac{1}{3}(a - (C^e_{n-1} + C^F))\right]^2.$$ 

and then

$$\lim_{(\beta^P, r) \to (0, 1)} \left( \Pi^G((s^G, s^I, s^P)|\bar{h})\bigg|_{\bar{h}} - \Pi^G\left( \{s^G_h\}_{h \in H}|\bar{h}, (s^I, s^P)|\bar{h}\right)\bigg|_{\bar{h}} \right)(\beta^G)^{-(t+1)} = \frac{1}{2} \left(\frac{(a-C^F)}{2}\right)^2 + \frac{(a-C^F)}{2} - \frac{1}{2} \left(\frac{2}{3}(a - (C^F + C^e_{n-1}))\right)^2 + 2 \left[\frac{1}{3}(a - (C^F + C^e_{n-1}))\right]^2.$$ 

Hence, 

$$\lim_{\beta^G \to 0} \left( \Pi^G((s^G, s^I, s^P)|\bar{h})\bigg|_{\bar{h}} - \Pi^I\left( \{s^G_h\}_{h \in H}|\bar{h}, (s^I, s^P)|\bar{h}\right)\bigg|_{\bar{h}} \right)(\beta^G)^{-(t+1)} > 0 \text{ if } \frac{1}{8}(a - C^F)^2 + \frac{1}{4}(a - C^N)^2 > 0,$$ 

since $\frac{1}{8}(a - C^N)^2 > \left\{ \frac{1}{2} \left(\frac{2}{3}(a - (C^F + C^e_{n-1}))\right)^2 + 2 \left[\frac{1}{3}(a - (C^F + C^e_{n-1}))\right]^2 \right\},$ due to A1-A3.

Then,

$$\lim_{\beta^G \to 0} \left( \Pi^G((s^G, s^I, s^P)|\bar{h})\bigg|_{\bar{h}} - \Pi^I\left( \{s^G_h\}_{h \in H}|\bar{h}, (s^I, s^P)|\bar{h}\right)\bigg|_{\bar{h}} \right)(\beta^G)^{-(t+1)} > 0.$$

This case is proven.

**Remark 1** The case of the consumer-oriented government. Here we have

$$\lim_{\beta^P \to 0} \left( \Pi^G((s^G, s^I, s^P)|\bar{h})\bigg|_{\bar{h}} - \Pi^G\left( \{s^G_h\}_{h \in H}|\bar{h}, (s^I, s^P)|\bar{h}\right)\bigg|_{\bar{h}} \right)(\beta^G)^{-(t+1)} = \frac{1}{2} \left(\frac{(a-C^F)}{2}\right)^2 - \frac{1}{2} \left(\frac{2}{3}(a - (C^F + C^e_{n-1}))\right)^2 = \frac{1}{8}(a - C^F)^2 - \frac{2}{9}(a - C^e_{n-1})^2.$$ 

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Then, if \( \frac{1}{9}(a - CF)^2 - \frac{2}{9}(a - CN)^2 > 0 \), we have \( \frac{1}{9}(a - CF)^2 \)

\[-\frac{2}{9}(a - (CF + C_{n-1}^c))^2 > 0. \) This remark is in order to prove (2.1) in the theorem 2.

\textbf{Remark 2} Notice that the condition \( \beta^G \) being small enough in the two previous reasonings is unavoidable, so that if the government is very patient, either utilitarian or consumer-oriented it is optimal for it to close the economy at \( t + 1 \) if none of the firms can have the new technology totally installed until \( t + 1 \), but at least one can install it at \( t + 2 \). However, as we will see in what follows, here is the unique step at which the impatience of the government is necessary, as a response to \( \{s_h(N, 1)\}_{h \in H} \). As commented before, if we consider \( \{s_h(N, 2)\}_{h \in H}, \{s_h(N, 2)\} \)

instead of \( \{s_h(N, 1)\}_{h \in H}, \{s_h(N, 1)\} \) or \( \{s_h(N, 1)\}_{h \in H}, \{s_h(N, 2)\} \), we do not need to assume the impatience of the government at the previous step, since according to \( \{s_h(N, 2)\}_{h \in H} \) the firms only adopt the new technology if at the next period is totally installed. Thus, the items ((1.1.4)-(1.1.6)) and ((2.1.4)-(2.1.6)) will be done after the next steps.

In the other case, when none of the firms can totally install the new technology at \( t + 2 \), both firms react to \( s^G(h) \) using the old technology, the same reaction to \( s^G(0)(\tilde{h}) \).

Then,

\[
\lim_{\tau \to 1} \left( \Pi^G((s^G, s^I, s^P)|\tilde{h})|_{\tilde{h}} - \Pi^G \left( \{s^G_h\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|\tilde{h} \right) \right) (\beta^G)^{-(t+1)} =
\]

\[
\lim_{\tau \to 1} \left( \left\{ \frac{1}{2} \left( \frac{a - CF}{2} \right)^2 + \tau \left( \frac{a - CF}{2} \right)^2 \right\} - \left\{ \frac{1}{2} \left[ \frac{2}{3} (a - CN)^2 + 2 [\frac{1}{3} (a - CN)^2] \right] \right\} \right) =
\]

\[
\left( \frac{1}{9} (a - CF)^2 + \left( \frac{a - CF}{2} \right)^2 - \frac{4}{9} (a - CN)^2 \right) > 0
\]

for all \( \beta^G \in (0, 1] \). Then, \( \Pi^G((s^G, s^I, s^P)|\tilde{h})|_{\tilde{h}} - \Pi^G \left( \{s^G_h\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|\tilde{h} \right) > 0 \) for all \( \beta^G \in (0, 1] \).
Remark 3. The case of the consumers-oriented government. Now we have
\[
\left( \Pi^G(\{s^G, s^I, s^P\}|_{\cdot} |_{\cdot} |_{\cdot} |_{\cdot}) \right)_{h^*} - \Pi^G \left( \{s^G_{h^*}\}_{h^* \in H} |_{\cdot} |_{\cdot} |_{\cdot} |_{\cdot} \right) (\beta^G)^{-(t+1)} = \\
\left( \left( \frac{1}{2} \left( \frac{a-C^F}{2} \right)^2 \right) - \left( \frac{1}{2} \left( \frac{2}{3} (a-C^N)^2 \right) \right) \right) = \\
\frac{1}{8} (a-C^F)^2 - \frac{2}{3} (a-C^N)^2 > 0,
\]
for all $\beta^G \in (0, 1]$. This remark is in order to prove theorem 2.

Notice that in the last two reasonings the condition that $\beta^G$ be small enough is not necessary.

The case (a) is concluded.

b) Now, to end the proof, take $\tilde{h} = ((a_1^t, a_1^p, a_1^g), t = 0$ such that one or two of the national firms can have totally installed the new technology at $t + 1$. Once again, we have that $s^G(0)(\tilde{h}) = O$, then $s^G(\tilde{h}) = C$. We will show first the reasoning for the situation in which both domestic firms can have totally installed the new technology at $t + 1$. In this situation, none of the firms change their strategy, that is, they continue with the new technology forever, and then we have
\[
\lim_{\tau \to 1} \left( \Pi^G(\{s^G, s^I, s^P\}|_{\cdot} |_{\cdot} |_{\cdot} |_{\cdot}) \right)_{h^*} - \Pi^I \left( \{s^I_{h^*}\}_{h^* \in H} |_{\cdot} |_{\cdot} |_{\cdot} |_{\cdot} \right) (\beta^G)^{-(t+1)} = \\
\left[ \frac{1}{8} (a-C^F)^2 + 2 \left( \frac{a-C^F}{4} \right)^2 + \frac{a-C^F}{4} \right] - \left[ \frac{1}{2} \left( \frac{2}{3} (a-C^F)^2 \right) + 2 \left( \frac{1}{3} (a-C^F)^2 \right) \right] = \\
- \frac{11}{288} a^2 + \frac{11}{144} aC^F - \frac{11}{288} C^2F + \frac{1}{4} a - \frac{1}{4} C^F = (a-C^F) \left( \frac{1}{4} - \frac{11}{288} (a-C^F) \right).
\]

The roots of the polynomial $(a-C^F) \left( \frac{1}{4} - \frac{11}{288} (a-C^F) \right)$ in $C^F$ are $\{C^F = a\}$ and $\{C^F = -\frac{72}{11} + a\}$, therefore, we have that
\[
\lim_{\tau \to 1} \left( \Pi^G(\{s^G, s^I, s^P\}|_{\cdot} |_{\cdot} |_{\cdot} |_{\cdot}) \right)_{h^*} - \Pi^I \left( \{s^I_{h^*}\}_{h^* \in H} |_{\cdot} |_{\cdot} |_{\cdot} |_{\cdot} \right) (\beta^G)^{-(t+1)} \geq 0
\]
if and only if
\[
C^F \leq -\frac{72}{11} + a.
\]
The case is done.

One comment here is in order.

Notice that

\[
\lim_{\tau \to 0} \left( \Pi^G((s^G, s^I, s^P)|_{\bar{l}}) - \Pi^i \left( \{ s^G_h \}_{h \in H} |_{\bar{l}} , (s^I, s^P)|_{\bar{l}} \right) \right) (\beta^G)^{-(t+1)} = \\
\left( \frac{1}{2} \left( \frac{3}{4} \right)^2 + 2 \left( \frac{1}{4} \right)^2 \right) - \left( \frac{1}{2} \left[ \frac{3}{4} \right]^2 + 2 \left[ \frac{1}{4} \right]^2 \right) (a - C^F)^2 (\frac{13}{32} - \frac{8}{16}) < 0.
\]

Therefore, it is necessary to impose \( \tau \) close to one for the proposed strategy to be optimal.

Remark 4 Suppose that the government is consumer-oriented and take \( \bar{l} = (a_l)_{l=0} \) such that both national firms can have totally installed the new technology at \( t+1 \). As before, we have that \( s^G(0)(\bar{l}) = O \), then \( s^G(\bar{l}) = C \). Hence

\[
\left( \Pi^G((s^G, s^I, s^P)|_{\bar{l}}) - \Pi^i \left( \{ s^G_h \}_{h \in H} \right) \right) (\beta^G)^{-(t+1)} = \\
\left[ \frac{1}{2} \left( \frac{3}{4} (a - C^F) \right)^2 \right] - \left[ \frac{1}{2} \left( \frac{3}{4} (a - C^F) \right)^2 \right] = \frac{1}{2} (a - C^F)^2 (\frac{9}{16} - \frac{4}{9}) > 0.
\]

This remark is in order to prove theorem 2.

It remains to show when only one of the domestic firms can have totally installed the new technology at \( t+1 \). We have again that \( s^G(0)(\bar{l}) = O \), \( s^G(\bar{l}) = C \), and even if the other firm can have totally installed the new technology at \( t+2 \), none of the firms changes its strategy from \( l \geq t + 2 \) and one is having the new technology totally installed and the other does use the old technology—the firm that may have totally installed the new technology cannot do it, even having the economy closed, and the other domestic firm has the new technology totally installed at \( t+1 \)—, so

\[
\left( \Pi^G((s^G, s^I, s^P)|_{\bar{l}}) - \Pi^i \left( \{ s^G_h \}_{h \in H} \right) \right) (\beta^G)^{-(t+1)} = \\
\left( \frac{1}{2} (a - C^F) \right)^2 + \left( \frac{(a - C^F)}{3} \right)^2 + \tau (a - C^F) - \left( \frac{1}{2} (a - C^F)^2 \right) + (a - C^F)^2,
\]

since after \( t + 2 \) the two game paths coincide.
We have that
\[
\lim_{\tau \to 1} \left( \Pi^G((s^G, s^I, s^P)|_{\tilde{h}}) \right)_{\tilde{h}} - \Pi^i \left( \{ s^G_h \}_{h \in H} |_{\tilde{h}}, (s^I, s^P) |_{\tilde{h}} \right)_{\tilde{h}} (\beta^G)^{-(t+1)} = \\
\left( \frac{1}{2} (\frac{2}{3}(a - C^F))^2 + \left( \frac{a-C^F}{3} \right)^2 \right) - \left( \frac{1}{2} \left( \frac{a-C^F}{2} \right)^2 \right) = \\
- \frac{1}{24} a^2 + \frac{1}{12} a C^F - \frac{1}{24} C^2 F + \frac{1}{3} a - \frac{1}{3} C^F = \frac{1}{3} (a - C^F)(1 - \frac{a-C^F}{8}) > 0,
\]

since \((1 - \frac{a-C^F}{8}) > 0\)—recall that \(a \leq 1\)—because the firm that cannot have totally installed the new technology at \(t + 1\) shuts down due to the fact that the other firm is much more efficient (it has \(C^F\) as its marginal costs). Hence, we have
\[
\left( \Pi^G((s^G, s^I, s^P)|_{\tilde{h}}) \right)_{\tilde{h}} - \Pi^i \left( \{ s^G_h \}_{h \in H} |_{\tilde{h}}, (s^I, s^P) |_{\tilde{h}} \right)_{\tilde{h}} (\beta^G)^{-(t+1)} > 0
\]
for all \(\tau\) large enough. Thus, the case is proven. Notice how important it is, for the firm that can have the new technology totally installed at \(t + 1\), that the other firm can have the new technology totally installed at \(t + 1\).

**Remark 5** Suppose that the government is consumer-oriented and take \(\tilde{h} = (a)|_i=0\) such that only one of the domestic firms can have totally installed the new technology at \(t + 1\). As before, we have that \(s^G(0)(\tilde{h}) = 0\), then \(s^G(\tilde{h}) = C\). Hence
\[
\left( \Pi^G((s^G, s^I, s^P)|_{\tilde{h}}) \right)_{\tilde{h}} - \Pi^i \left( \{ s^G_h \}_{h \in H} |_{\tilde{h}}, (s^I, s^P) |_{\tilde{h}} \right)_{\tilde{h}} (\beta^G)^{-(t+1)} = \\
(\beta^G)^{-(t+1)} \left( \frac{1}{2} \left( \frac{2}{3}(a - C^F) \right)^2 \right) - \left( \frac{1}{2} \left( \frac{a-C^F}{2} \right)^2 \right) > 0,
\]

if \(C^e_n\) is small enough, as in the preceding reasoning. This remark is in order to prove theorem 2.

As commented in due time, all the remarks in this section but 4, are in order to prove item (2.1) of theorem 2. Also, as the impatience of either the firms or the government only was necessary
when considering
\[ \{s_h(N, 1)\}_{h \in H}, \{s_h(N, 1)\}, \] the proof of items (1.1) and (2.1) of theorems 1 and 2 is done.

1.2 The proofs of (1.2) and (2.2).

1.2.F First, the firms. Consider \((s^j, s^P, s^G) = (\{s_h(N, 2)\}_{h \in H}, \{s_h(N, 2)\}_{h \in H}, s^G(\infty))\). As we will see, the arguments here are similar to the ones in the case when the economy is opened at the outset. Nevertheless, in order to reinforce those intuitions and to clearly show how the firms’ impatience is a necessary condition, we present the following reasoning. We will prove that \(\{s^j_h(N, 2)\}_{h \in H}\) is such that, for any \(\tilde{h} \in H\), \(\{s^j_h(N, 2)\}_{h \in H} \mid_{\tilde{h}}\) is the best response to \((s^j, s^G) \mid_{\tilde{h}}\) for \(i \neq j \in \{I, P\}\).

Take \(\tilde{h} = ((a^i_t, a^P_t, a^G_t))_{t=0}^T\) such that, for the firm \(i\), the new technology cannot be totally installed at \(t + 1\). Consider the payoffs \(\Pi^i((s^i, s^j, s^G) \mid_{\tilde{h}})\) and \(\Pi^i(\{\tilde{s}^j_h \mid_{\tilde{h}}\}_{h \in H} \mid_{\tilde{h}}, (s^j, s^G) \mid_{\tilde{h}})\), where \(\tilde{s}^j = \{\tilde{s}^j_h \mid_{\tilde{h}}\}_{h \in H}\) is such that \(\tilde{s}^j_h = s_h(N, 2)\) for all \(h \neq \tilde{h}\) and \(\tilde{s}^j_h \neq s_h(N, 1)\). We have that \(s^i\) prescribes, given \((s^P, s^G) \mid_{\tilde{h}}\), to adopt the old technology for all \(l \geq t + 1\). \(\tilde{s}^j = \{\tilde{s}^j_h \mid_{\tilde{h}}\}_{h \in H}\) prescribes, given \((s^P, s^G) \mid_{\tilde{h}}\), to adopt the new technology at \(t + 1\), but the old technology for all \(l \geq t + 2\), if the new technology cannot be totally installed at \(t + 2\), and the new technology for all \(l \geq t + 2\), in the other case. Whatever be the situation, we have

\[
\left(\Pi^i((s^i, s^j, s^G) \mid_{\tilde{h}}) \mid_{\tilde{h}} - \Pi^i(\{\tilde{s}^j_h \mid_{\tilde{h}}\}_{h \in H} \mid_{\tilde{h}}, (s^j, s^G) \mid_{\tilde{h}}) \mid_{\tilde{h}}(\beta^i)^{-(t+1)} \right) \rightarrow \\
\left(\pi^i_{t+1}((s^i, s^j, s^G) \mid_{\tilde{h}}) - \pi^i_{t+1}(\{\tilde{s}^j_h \mid_{\tilde{h}}\}_{h \in H} \mid_{\tilde{h}}, (s^j, s^G) \mid_{\tilde{h}})\right) \text{ as } \beta^i \rightarrow 0,
\]

where \(\pi^i_{t+1}((s^i, s^j, s^G) \mid_{\tilde{h}})\) is the Cournot profit of the firm \(i\) at \(t + 1\) using the technology according to \(s^i\), that is, the old technology, and an analogous definition applies to \(\pi^i_{t+1}(\{\tilde{s}^j_h \mid_{\tilde{h}}\}_{h \in H} \mid_{\tilde{h}}, (s^j, s^G) \mid_{\tilde{h}})\) but using the new technology not totally installed. Now, if the firm \(j \neq i\) can have the new technology totally installed at \(t + 1\), the economy is open at \(t + 1\), then \(\pi^i_{t+1}((s^i, s^j, s^G) \mid_{\tilde{h}}) = \pi^i_{t+1}(\{\tilde{s}^j_h \mid_{\tilde{h}}\}_{h \in H} \mid_{\tilde{h}}, (s^j, s^G) \mid_{\tilde{h}}) = 0\). In the other case, the
economy is closed, because none of the firms can have the new technology totally installed at \( t + 1 \), then

\[ \pi^i_{t+1}((s^i, s^I, s^G)\mid \bar{h}) > \pi^i_{t+1} \left( \{ s^I_h \}_{h \in H} \mid \bar{h}, (s^I, s^G) \mid \bar{h} \right), \]

since the old technology is more efficient than the new one, if that new technology is not totally installed (due to A1-A3).

In any case

\[ (\Pi^i((s^i, s^I, s^G)\mid \bar{h})\mid \bar{h}) - \Pi^i \left( \{ s^I_h \}_{h \in H} \mid \bar{h}, (s^I, s^G) \mid \bar{h} \right) \geq 0. \]

Now, take \( \tilde{h} = ((a^I_t, a^P_t, a^G_t))_{t=0}^T \) such that, for the firm \( i \), the new technology can be totally installed at \( t + 1 \). This case, as in (1.1.F), is quite intuitive, since the new technology, once it is totally installed, it is more efficient than the old one, providing more Cournot benefits (the other firm does not change its strategy if it can have the new technology totally installed, nor if it cannot).

The optimality of the firms’ strategies is finished.

1.2.G The government

Take \( \tilde{h} = ((a^I_t, a^P_t, a^G_t))_{t=0}^T \) such that at least one of the firms can have the new technology totally installed at \( t + 1 \). Since \( s^G(\infty)(\tilde{h}) = O \), then \( s^G(\tilde{h}) = C \), but the government, as with \( s^G(\infty) \), will open the economy for all \( l \geq t + 2 \). Then, assumed that \( \tau \) is large enough or that the government is consumer-oriented, due to the same reasonings done before, the case is done (the society is better off when there is one more firm in the market).

Assume now that none of the domestic firms can have the new technology totally installed at \( t + 1 \). Since \( s^G(\infty)(\tilde{h}) = C \), then \( s^G(\tilde{h}) = O \), but the government, as with \( s^G(\infty) \), will close the economy for all \( l \geq t + 2 \). Then

\[
\lim_{\tau \to 1^-} (\beta^G)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)\mid \tilde{h})\mid \tilde{h}) - \Pi^G \left( \{ s^G_h \}_{h \in H} \mid \tilde{h}, (s^I, s^P) \mid \tilde{h} \right) =
\]

\[
\lim_{\tau \to 1^-} \left[ \left\{ \frac{1}{2} \left( \frac{a-C^N}{3} \right)^2 + 2 \left( \frac{a-C^F}{3} \right)^2 \right\} - \left\{ \frac{1}{2} \left( \frac{a-C^F}{3} \right)^2 + \tau \left( \frac{a-C^F}{2} \right) \right\} \right] =
\]

\[
\frac{4}{5}(a-C^N)^2 - \frac{1}{5}(a-C^F)^2 - \left( \frac{a-C^F}{2} \right) > 0,
\]

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by assumption.

Remark 6 If the government is consumer-oriented, we have

\[
(\beta^G)^{-t(t+1)}(\Pi((s^G, s^I, s^P)|_{\tilde{h}})|_{\tilde{h}} - \Pi^G \left( \left\{ s^G_h \right\}_{h \in H} |_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}} \right) = \\
\left\{ \frac{1}{2} \left( \frac{2(a - CN)}{3} \right)^2 + 2 \left( \frac{(a - CN)}{3} \right)^2 \right\} - \left\{ \frac{1}{2} \left( \frac{(a - CF)}{2} \right)^2 \right\} > 0,
\]

by assumption.

The proof of (1.3) follows at once from (1.1.5) and (1.2.2).

Theorems 1 and 2 are proven.

3 Proof of theorem 3.

Without loss of generality, we assume that the legal-political costs are zero. (Recall that we are assuming that they are not very large.)

3.1 First consider \((s^P, s^I, s^G) = (\{s_h(T, 1)\}_{h \in H}, \{s_h(T, 1)\}_{h \in H}, s^G(n))\).

3.1.1 F The firms. We will prove that \(\{s_h(T, 1)\}_{h \in H}\) is such that, for any \(\tilde{h} \in H\), \(\{s_h(T, 1)\}_{h \in H}|_{\tilde{h}}\) is the best response to \((s^I, s^G)|_{\tilde{h}}\); for \(i \neq j \in \{I, P\}\). We have three possible situations: a) If \(\tilde{h} \in H\) is such that \(C(i, h) = n\); b) If \(\tilde{h} \in H\) is such that \(C(i, h) \geq C(j, h)\) and \(C(i, h) < n\); c) If \(\tilde{h} \in H\) is such that \(C(i, h) < C(j, h)\) and \(C(i, h) < n\).

a) Take \(\tilde{h} \in H\) such that \(C(i, h) = n\) for \(i \in \{I, P\}\). Suppose first that \(C(i, h) = n\). Then, \(s^I_h(T, 1)\) prescribes to use the new technology at \(t + 1\) and that technology is totally installed at \(t + 1\). As in other situations analyzed before, this decision gives to the firm \(i\) more benefits than any other decision. This case is done.

b) Now, assume \(C(i, h) \geq C(j, h)\) and \(C(i, h) < n\) for \(i \in \{I, P\}\). Then, none of the firms has the new technology totally installed at \(t + 1\) and \(s^I_h(T, 1)\), given \((s^I, s^G)|_{\tilde{h}}\), prescribes
to use the new technology at $t+1$, since the government closes the economy at $t+1$ and will keep the economy closed until a firm can have the new technology totally installed, the firm $i$ in this case. Since $C(i, h) \geq C(j, h)$ the firm $i$, according to $\{s_h(T, 1)\}_{h \in H}$, will continue using the new technology until it pays all the remaining costs and will totally install the new technology sooner or later—the other firm may be adopting the new technology or may be not doing it, depending upon if $C(i, h) > C(j, h)$, or if $C(i, h) = C(j, h)$—. However, if we consider $\bar{s}^i = \{\bar{s}^i_h\}_{h \in H}$ such that $\bar{s}^i_h = s_h(T, 1)$ for all $h \neq \tilde{h}$ and $\bar{s}^i_\tilde{h} \neq s_\tilde{h}(T, 1)$, we have that $\bar{s}^i_\tilde{h}$ prescribes to use the old technology at $t+1$. To continue the reasoning, consider the history $(\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h))$ determined by the reactions to $\bar{s}^i_\tilde{h}$ of the firm $j$ and the government, according to their proposed strategies, and consider $C(i, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h)))$. Necessarily, $C(i, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h))) < n$, since $\bar{s}^i_\tilde{h}$ prescribes to use the old technology at $t+1$.

We have two situations, one if $C(i, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h))) \geq C(j, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h)))$ and the other if $C(i, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h))) < C(j, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h)))$—notice that given $\bar{s}^i_\tilde{h}$, the firm $j$, according to $s^j$, may decide to adopt the new technology: Imagine the case when $C(i, h) = C(j, h)$—. If $C(i, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h))) \geq C(j, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h)))$, then $\bar{s}^i$ prescribes to adopt the new technology and to install it, the government keeps the economy closed until the firm $i$ finishes installing the new technology. Thus, if $C(i, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h))) > C(j, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h)))$—the case when $C(i, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h))) = C(j, (\bar{h}, (\bar{s}^i_h, s^j_h, s^G_h)))$ is quite similar and omitted—,

$$
(\Pi^i((s^j, s^j, s^G)|_{\bar{h}}|_h) - \Pi^i((s^j|_{\bar{h}}|_h, (s^j, s^G)|_{\bar{h}}|_h)))(\beta^i)^{-(t+1)} = 
$$

$$
\pi^i_{t+1}((C^e_{C(i, h)} + C^F, C^N) - \pi^i_{t+1}((C^N, C^N) + 

\sum_{l=1}^{l=n-(C(i, h)+1)} (\beta^i)^l \left[\pi^i_{l+1}((C^e_{C(i, h)+C(i, h)} + C^F, C^N) - \pi^i_{l+1}((C^e_{C(i, h)+t-1} + C^F, C^N) + 

\frac{(\beta^i)^{n-(C(i, h))}}{(1-\beta^i)} \pi^i_{l+1}((C^F, C^F), 

\right)

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Then

$$\lim_{\beta^t \to 1} (\Pi^i((s^i, s^j, s^G)|_{\tilde{h}})|_{\tilde{h}} - \Pi^i((s^i, s^j, s^G)|_{\tilde{h}})|_{\tilde{h}})(\beta^t)^{-1} = \infty,$$

(in spite of having $\pi^i_{t+1}((C_{C(i, h)}^e + C_F), C_N) - \pi^i_{t+1}((C_N, C_N) < 0$, and $\pi^i_{t+1}((C_{t+C(i, h)}^e + C_F, C_N) - \pi^i_{t+1}((C_{t+C(i, h)-1}^e + C_F, C_N) < 0$ for all $0 \leq l \leq n$ ($C_l^e$ is decreasing).

This case is done.\(^{12}\)

Notice how crucial is the assumption that the firm $i$ is patient enough.

Now, if $C(i, (\tilde{h}, (s_{i}^j, s_{i}^j, s_{i}^G))) < C(j, (\tilde{h}, (s_{j}^j, s_{j}^j, s_{j}^G)))$

—notice that $C(j, (\tilde{h}, (s_{j}^j, s_{j}^j, s_{j}^G))) = C(i, h) + 1$—, then $\tilde{s}^i$ prescribes to adopt the old technology for all $l \geq t + 1$, the firm $j$ adopts the new technology at for all $l \geq t + 1$, and the government keeps the economy closed until the firm $j$ finishes installing the new technology.

Then

$$\left(\Pi^i((s^i, s^j, s^G)|_{\tilde{h}})|_{\tilde{h}} - \Pi^i((s^i, s^j, s^G)|_{\tilde{h}})|_{\tilde{h}})(\beta^t)^{-1} = \pi^i_{t+1}((C_{C(i, h)}^e + C_F), C_N) - \pi^i_{t+1}((C_N, C_{C(i, h)}^e + C_F) + \sum_{l=1}^{n-C(i, h)+1} (\beta^t)^{-1} \pi^i_{l+1}((C_{C(i, h)}^e + C_F), C_N) - \pi^i_{l+1}((C_N, C_{C(i, h)}^e + C_F) + \frac{(\beta^t)^{n-C(i, h)+1} \pi^i_{t+1}((C_F, C)}{1-\beta^t})$$

since, according to $\tilde{s}^i$, the firm $i$ leaves the market at $t + 1 + n - C(i, h)$. Therefore,

$$\lim_{\beta^t \to 1} (\Pi^i((s^i, s^j, s^G)|_{\tilde{h}})|_{\tilde{h}} - \Pi^i((s^i, s^j, s^G)|_{\tilde{h}})|_{\tilde{h}})(\beta^t)^{-1} = \infty,$$

as before.

c) Take $\tilde{h} \in H$ such that $C(i, h) < C(j, h)$ and $C(i, h) < n$ for $i \in \{I, P\}$. This case is the simplest one: The firm $i$ never can finish installing the new technology before the firm $j$, then it is better

\(^{12}\)If $P(i, \tilde{h}) + 1 = n$,

the term $\sum_{l=1}^{n-C(i, h)+1} (\beta^t)^{-1} \pi^i_{l+1}((C_{C(i, h)}^e + C_F, C_N) - \pi^i_{l+1}((C_{C(i, h)+1}^e + C_F, C_N) + C_{C(i, h)+1}^e + C_F))$ disappears, and the argument is the same. (Recall that we have assumed $C_n^e = 0.$)
for \( i \) not to adopt the new technology at \( t + 1 \). The proof is finished. Notice that the proof applies also for the item (3.2).

Remark 7  \textit{In order to prove the optimality of} \( \{s_h(T, 2)\}_{h \in H} \), \textit{it suffices to observe that this case is quite analogous to the cases (a) and (b) above, and then it is omitted. Further, the proof applies also for items (3.3) and (3.4), since the proofs only use that the government plays} \( \{s^G_h(n)\}_{h \in H} \), \textit{not an explicit assumption over the gap between the technologies.}

\section*{3.1.G The government.} For simplicity, we show the argument in the case of the consumer-oriented utility function. The argument for the utilitarian utility function is analogous and thus omitted. Consider \( (s^I, s^P, s^G) = (\{s^I_h(T, 1)\}_{h \in H}, \{s^P_h(T, 1)\}_{h \in H}, s^G(n)) \). We will prove that \( \{s^G_h(n)\}_{h \in H} \), for any \( \tilde{h} \in H \), \( \{s^G_h(n)\}_{h \in H} \bigg|_{\tilde{h}} \) is the best response to \( (s^I, s^P)\big|_{\tilde{h}} \). Take a history \( \tilde{h} = (a_i)_{i=0}^{l-1} \) such that none of the firms can have the new technology totally installed at \( t + 1 \).

We have that \( s^G_h(n) = C \), hence \( \tilde{s}^G_{\tilde{h}} = O \). We have two possibilities, one if \( C(i, h) = C(j, h) \), the other if \( C(i, h) \neq C(j, h) \). Consider first the case when \( C(i, h) \neq C(j, h) \); without loss of generality we assume that \( C(j, h) > C(i, h) \). As the firm \( i \) does not adopt the new technology, we have

\[
\left(\beta^G\right)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)|_{\tilde{h}})|_{\tilde{h}} - \Pi^G\left(\{s^G_h\}_{h \in H} \bigg|_{\tilde{h}} , (s^I, s^P)|_{\tilde{h}} \right) = \right.
\]

\[
\left. \left(\frac{1}{2}\left(\frac{3}{2}(a - 2(C_{C(j,h)+1}^e + C^F) - C^N) + \frac{3}{2}(a - 2C^N +
\right) + \right.
\]

\[
\sum_{l=1}^{n-(C(j,h)+1)} \frac{1}{4} \left( \frac{3}{2}(a - 2(C_{C(j,h)+1+l}^e + C^F) + C^N) + \right.
\]

\[
\left. \frac{1}{3}(a - 2C^N + C_{C(j,h)+1+l}^e + C^F )^2 - \right.
\]

\[
\left. \frac{1}{2}(a - 2C^N + C_{C(j,h)+1+l}^e + C^F )^2 + \right.
\]

\[
\left. \frac{1}{2}(a - 2C^N + C_{C(j,h)+1+l}^e + C^F )^2 + \right.
\]

\[
\left. \left(\beta^i\right)^{n-C(j,h)} \frac{1}{1-\beta^i} \left( \frac{1}{2}\left(2\frac{a-C^F}{3} \right)^2 \right) \right)
\]

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because the firm \(i\) leaves the market at the moment the economy is opened, that is, at the moment the firm \(j\) has totally installed the new technology, if \(C(j, h) < n\).

Remark 8 \(Observe\ that\ if\ we\ assume\ the\ stronger\ version\ of\ “the\ gap\ between\ the\ technologies\ is\ not very\ large,”\ then\)

\[
\left(\frac{1}{2}\left(\frac{1}{3}(a - 2(C_{C(j,h)+1}^c + C^F) - C^N) + \frac{1}{3}(a - 2C^N + (C_{C(j,h)+1}^c + C^F))^2\right) - \frac{1}{2}\left(\frac{a - C^F}{2}\right)^2 > 0.\right.
\]

\[\text{Also, notice that}\]

\[
\sum_{l \geq 1} (\beta^G)^l \left(\frac{1}{2}\left(\frac{1}{3}(a - 2(C_{C(j,h)+1+l}^c + C^F) + C^N) + \frac{1}{3}(a - 2C^N + C_{C(j,h)+1+l}^c + C^F))^2 - \right)\]

\[
\left(\frac{1}{2}\left(\frac{1}{3}(a - 2(C_{C(j,h)+1+l-1}^c + C^F) + C^N) + \frac{1}{3}(a - 2C^N + C_{C(j,h)+1+l-1}^c + C^F))^2\right) > 0
\]

\[\text{for all } l \geq 1, \text{ since the economic costs of installation are decreasing. Therefore, we have}\]

\[
(\beta^G)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)|_{\tilde{h}})|_{\tilde{h}} - \Pi^G\left(\{s^G\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}}\right)|_{\tilde{h}} > 0
\]

\[\text{for all } \beta^G \in [0, 1].\]

If \(C(j, h) = n\), we have

\[
(\beta^G)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)|_{\tilde{h}})|_{\tilde{h}} - \Pi^G\left(\{s^G\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}}\right)|_{\tilde{h}} = \]

\[
(\beta^G)^{n-C(j,h)} \frac{1}{1-\beta^G} \left(\frac{1}{2}\left(\frac{a - C^F}{2}\right)^2\right).
\]

Therefore, in either case,

\[
\lim_{\beta^G \to 1} (\beta^G)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)|_{\tilde{h}})|_{\tilde{h}} - \Pi^G\left(\{s^G\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}}\right)|_{\tilde{h}} = \infty.
\]

Now, if \(C(j, h) = C(i, h)\), similarly, we have
\[
\begin{align*}
(\beta^G)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)|_h)|_{\tilde{h}} - \Pi^G \left( \{s^G_h\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}} \right) =
& \left( (\frac{1}{2}(\frac{2}{3}(a - C^e_{C(i,\tilde{h})+1}) - C^F))^2 - (\frac{1}{2}(\frac{a-C^F}{2})^2) \right) + \\
& \sum_{l \geq 1} \left( (\beta^G)^l \left( (\frac{1}{2}(\frac{2}{3}(a - C^e_{C(i,\tilde{h})+1+l}) - C^F))^2 - \right) \right) \\
& \left( \frac{1}{2}(\frac{2}{3}(a - C^e_{C(i,\tilde{h})+1+l-1}) - C^F))^2 \right) \\
& (\beta^{i-C(i,\tilde{h})}) \frac{1}{1-\beta^{\sigma}} \left( (\frac{1}{2}(\frac{3(a-C^F}{4})^2) \right),
\end{align*}
\]
therefore

\[
\lim_{\beta^G \to 1} (\beta^G)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)|_h)|_{\tilde{h}} - \Pi^G \left( \{s^G_h\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}} \right) = \infty
\]
as well —if \( C(i, h) = n \), we apply the same reasoning as before—. Now, if we take a history
\( \tilde{h} = (a_l)_{l=0}^t \) such that at least one of the firms can have the new technology totally installed at \( t+1 \),
the reasoning is simpler, since the firm/s that can do it will do it, and therefore the two payoffs
\( \Pi^G((s^G, s^I, s^P)|_h)|_{\tilde{h}} \) and \( \Pi^G \left( \{s^G_h\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}} \right) \) differ only at time \( t+1 \), which difference
is positive, provided that letting a foreign firm enter the market gives more instantaneous utility
to the government than not allowing it. Therefore, items (3.1) and (3.2) are proven.

Remark 9 Observe, once again, that if we assume the stronger version of “the gap between the technolo-
gies is not very large,” then \( (\frac{1}{2}(\frac{2}{3}(a - C^e_{C(i,\tilde{h})+1}) - C^F))^2 - (\frac{1}{2}(\frac{a-C^F}{2})^2) > 0 \) and, as in the
previous remark,

\[
\sum_{l \geq 1} \left( (\beta^G)^l \left( (\frac{1}{2}(\frac{2}{3}(a - C^e_{C(i,\tilde{h})+1+l}) - C^F))^2 - \right) \right) \\
\left( \frac{1}{2}(\frac{2}{3}(a - C^e_{C(i,\tilde{h})+1+l-1}) - C^F))^2 \right) > \\
0 \text{ for all } l \geq 1, \text{ because of the same reason. Therefore, we have}
\]

\[
(\beta^G)^{-(t+1)}(\Pi^G((s^G, s^I, s^P)|_h)|_{\tilde{h}} - \Pi^G \left( \{s^G_h\}_{h \in H}|_{\tilde{h}}, (s^I, s^P)|_{\tilde{h}} \right) > 0
\]
for all \( \beta^G \in [0, 1] \).
Due to the last three remarks, items (3.3) and (3.4) are proven.

The proof of theorem 3 is concluded.
8 References


Tornell, A., 1991. Time Inconsistency of Protectionist Programs. Quarterly Journal of Eco-
nomics 106, 963–974.

