Money and Reality*

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Abstract

The use of cash for purchasing consumption goods in the U.S. has constantly declined since the end of the 80’s. Correspondingly, we find that the celebrated result of a causal effect of money on GNP (Sims, 1972) does not hold anymore for the U.S after 1988. We interpret these facts as suggesting the hypothesis that the primary role of money has changed from facilitating transactions towards serving as an unit of account as in Woodford (2003). To test this hypothesis and to understand its consequences, we develop and estimate a DSGE model with real resource costs of transactions, and compare it to a cashless economy. We find empirical support for our hypothesis, and explain that and how the loss in predictive power of money contributed to the well established decline in the volatilities of output and inflation.

JEL classification: E32, C51, E52.

Keywords: Bayesian model estimation, cashless economy, transaction role of money

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1 Introduction

The use of cash for purchasing consumption goods in the U.S. has constantly declined since the end of the 80’s (Humphrey, 2004; Schreft and Smith, 2000). Correspondingly we document that the celebrated result of a causal effect of money on GNP (Sims, 1972) does not hold anymore for the U.S. in a sample that starts after 1988.\(^1\) We interpret these facts as suggesting the hypothesis that the primary role of money has changed from facilitating transactions towards serving as an unit of account as in Woodford (2003). To test this hypothesis and understand its consequences, we develop and estimate a DSGE model with a transaction role for money, and compare it to a cashless model. We find empirical support for this hypothesis, and explain that and how the loss in predictive power of money contributed to the well established decline in the volatilities of output and inflation (Galí and Gambetti, 2009; Stock and Watson, 2002; Kim and Nelson, 1999).

The role of money as a unit of account is emphasized in the textbook cashless New Keynesian model (Woodford, 2003), in which current money balances are provided residually exactly in the amount requested by households’ equilibrium current consumption choices and the equilibrium interbank nominal interest rate as the operating target of the monetary authority. To capture the transaction role of money, we extend the DSGE model with a transaction friction: beginning-of-period nominal money balances are assumed to reduce transaction costs of purchasing consumption goods (Feenstra, 1986; Schmitt-Grohé and Uribe, 2004a). Here, the amount of money balances restricts and predicts consumption expenditures and thus output. The timing assumption (Svensson, 1985) and the existence of real balance effects allow predetermined real money balances to serve as an endogenous state variable for output and inflation, and thus constitutes an endogenous propagation mechanism that may amplify the effects of exogenous disturbances. To fix ideas, consider a technology shock. As a first consequence, output increases and inflation decreases on impact. The latter triggers an increase in real money balances, increases households’ consumption possibilities, and fuels the boom in future periods.

To test whether the main role of money has changed, we estimate the two competing models using Bayesian estimation techniques. As observable variables we treat output, inflation and real wages of the U.S. economy. We divide the data set ranging from 1964 to 2009 into two sub-samples. The first sample captures the pre-Volcker era from 1964 to the end of 1978. To identify the loss in predictability of money, we subsequently vary the starting date of the second sample from 1983 – after the disinflation years – to 1991. For each of the sub-samples we compute posterior odds for the two competing models. To find

\(^1\)The results of this exercise can be found in Appendix A.1.
supporting evidence for our hypothesis, first posterior odds has to favor the model with a
transaction friction before 1979. To the best of our knowledge, we are first in developing a
monetary model that outperforms a cashless model in explaining non-monetary observable
variables. After the disinflation years the cashless economy has to continuously gain ground
and the posterior odds should be in favor of that model after 1988. The latter excludes many
alternative explanations that introduce an endogenous state variable like capital, indexation
or habit formation. We are not aware of any evidence that claims a gradual or discrete change
in these features. Moreover, these features even nowadays constitute useful extensions of the
cashless economy (Del Negro, Schorfheide, Smets, and Wouters, 2007).

We find that the primary role of money indeed has changed. First, the cash economy
outperforms the cashless economy for the pre-Volcker era. In this time period, the ampli-
fying effect of real money balances is helpful to capture the substantial fluctuations in our
observable variables. In particular, we find that 40 percent of all fluctuations in output,
and almost 90 percent of all fluctuations in inflation are explained by the endogenous state
variable. Second, from 1983 onwards the economy with the transaction friction continuously
loses explanatory power relative to the cashless economy. From the beginning of 1989 on-
wards and consistent with evidence on the use of cash in transactions, the cashless economy
provides a better explanation. Comparing the pre-Volcker era with the time period after
1988 reveals a substantial loss in predictive power of money: in the second sample only 14
percent of all variations in output are due to variations in real money balances, and also the
fraction of inflation fluctuations explained by the endogenous state variable drops.

The remainder of the paper is organized as follows. In the next section we describe the
two competing models. The analysis of the two models is conducted in section 3. We present
our econometric strategy in Section 4. In Section 5 we deliver and explain our main result.
The last section concludes.

2 Two competing models: cashless and monetary

In this section we describe two models that differ with respect to the primary role of money.
While in the cashless economy ($M_{CE}$) money is supplied after the allocation is deter-
mined, predetermined real money balances serve as an endogenous state variable and thus
directly influence the allocation in the monetary economy ($M_{ME}$).

The economies are populated by a continuum of infinitely-lived households indexed with
$j \in [0, 1]$ that have identical initial asset endowments and identical preferences. Household
$j$ acts as a monopolistic supplier of labor services $l_j$. Lower (upper) case letters denote real
(nominal) variables. At the beginning of period $t$, households’ financial wealth comprises a
portfolio of state contingent claims on other households yielding a (random) payment \( Z_{jt} \), and one-period nominally non-state contingent government bonds \( B_{jt-1} \) carried over from the previous period. Assume that financial markets are complete, and let \( q_{t,t+1} \) denote the period \( t \) price of one unit of currency in a particular state of period \( t+1 \) normalized by the probability of occurrence of that state, conditional on the information available in period \( t \). Then, the price of a random payoff \( Z_{t+1} \) in period \( t+1 \) is given by \( E_t[q_{t,t+1}Z_{jt+1}] \).

The only difference between the two economies is the budget constraint of household \( j \). In the cashless economy it reads

\[
B_{jt} + E_t[q_{t,t+1}Z_{jt+1}] + P_t c_{jt} \leq R_{t-1} B_{jt-1} + Z_{jt} + P_t w_{jt} l_{jt} + \int_0^1 D_{jit} di - P_t T_t,
\]

where \( c_t \) denotes a Dixit-Stiglitz aggregate of consumption with elasticity of substitution \( \zeta \), \( P_t \) the aggregate price level, \( w_{jt} \) the real wage rate for labor services \( l_{jt} \) of type \( j \), \( T_t \) a lump-sum tax, \( R_t \) the gross nominal interest rate on government bonds, and \( D_{it} \) dividends from monopolistically competitive firms.

As competing specification, we consider a monetary economy with an explicit transaction friction. As in Schmitt-Grohé and Uribe (2004a) or Feenstra (1986), purchasing consumption goods is costly, and holding and using real money balances in transactions reduces theses costs. This is captured by a modified budget constraint in the monetary economy

\[
M_{jt} + B_{jt} + E_t[q_{t,t+1}Z_{jt+1}] + P_t c_{jt} + P_t \phi(c_{jt}, M_{jt-1}/P_t) \leq M_{jt-1} + R_{t-1} B_{jt-1} + Z_{jt} + P_t w_{jt} l_{jt} + \int_0^1 D_{jit} di - P_t T_t,
\]

with \( \phi(c, h) > 0 \) as real resource costs of transactions, and with \( M_{jt-1} \) and \( h_{jt} = M_{jt} - 1/P_t \) as beginning-of-period nominal and real money balances. We assume transaction costs that increase in consumption (\( \phi_c > 0 \)) and decrease in real money balances (\( \phi_h < 0 \)). Marginal transactions costs of consumption are assumed to be decreasing in real money balances (\( \phi_{ch} < 0 \)) and the marginal resource gains of holding money are decreasing (\( \phi_{hh} > 0 \)). To simplify the analysis, we assume linear marginal transaction costs of consumption purchases \( \phi_{cc} = 0 \). Since beginning-of-period money balances \( M_{t-1} \) provide transaction services, we follow Svensson (1985)’s timing of markets assumption within one period where the goods market is closed before the asset market is opened (see also Woodford, 1990; McCallum and Nelson, 1999; Persson, Persson, and Svensson, 2006).
The objective of household $j$ in both models is

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \{u(c_{jt}) - v(l_{jt})\} \quad \beta \in (0,1),$$

where $\beta$ denotes the subjective discount factor. The instantaneous utility function is assumed to be non-decreasing in consumption, decreasing in labor time, strictly concave, twice differentiable, and to fulfill the Inada conditions. Households are wage-setters supposed to be non-decreasing in consumption, decreasing in labor time, strictly concave, where $\beta$ denotes the subjective discount factor. The instantaneous utility function is as-

The final consumption good $Y_t$ is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in [0,1]$ and defined as $y_i^\xi = \int_0^1 y_{it}^{\xi} di$, with $\zeta > 1$. Let $P_{it}$ and $P_i$ denote the price of good $i$ set by firm $i$ and the price index for the final good. The demand for each differentiated good is $y_i^d = (P_{it}/P_i)^{-\zeta} y_i$, with $P_i^{1-\zeta} = \int_0^1 P_{it}^{1-\zeta} di$. A firm $i$ produces good $y_i$ using a technology that is linear in the labor bundle $l_i = \int_0^1 \int_{\epsilon_i^{(t-1)}}^{\epsilon_i^t} \int_{\epsilon_i^{(t-1)}}^{\epsilon_i^t} dj_{(i)} e_{(i)} l_{it} d\epsilon_i$ and $\alpha_i$ is a productivity shock with mean 1. Labor demand satisfies: $mc_{it} = w_i/a_i$, where $mc_{it} = mc_t$ denotes real marginal costs independent of the quantity that is produced by the firm. We allow for a nominal rigidity in form of a staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with probability $1 - \alpha$ independently of the time elapsed since the last price setting. A fraction $\alpha \in [0,1)$ of firms are assumed to keep their previous period’s prices, $P_{it} = P_{it-1}$. In each period a measure $1 - \alpha$ of randomly selected firms set new prices $\tilde{P}_{it}$ as the solution to

$$\max_{P_{it}} \sum_{T=t}^{\infty} \alpha^{T-t} q_{it,T}(\tilde{P}_{it}y_{iT} - P_T mc_{iT} y_{iT}), \quad s.t. \ y_{iT} = (\tilde{P}_{it})^{-\zeta} P_T^\zeta y_T,$$

where we assume that firms have access to contingent claims.

The central bank as the monetary authority is assumed to control the short-term interest rate $R_t$ with a simple feedback rule contingent on inflation:

$$R_t = f(\pi_t) \quad with \quad f'(()) > 0.$$

The consolidated government budget constraint reads $R_{t-1} B_{t-1} + P_t G_t = B_t + P_t T_t$ in the cashless economy, and $M_{t-1} + R_{t-1} B_{t-1} + P_t G_t = M_t + B_t + P_t T_t$ in the monetary economy.
The exogenous government expenditures $g_t$ evolve around a mean $\bar{g}$, which is restricted to be a constant fraction of output, $\bar{g} = \bar{y}(1 - sc)$. We assume that tax policy $T_t$ guarantees government solvency.

The aggregate resource constraint is given by

$$y_t = a_t l_t / \Delta_t,$$

where $\Delta_t = \int_0^1 (P_{it}/P_t)^{-\zeta} di \geq 1$ and thus $\Delta_t = (1 - \alpha)(\bar{P}_t/P_t)^{-\zeta} + \alpha \pi_t \Delta_{t-1}$. The dispersion measure $\Delta_t$ captures the welfare decreasing effects of staggered price setting. Goods’ market clearing requires

$$c_t + g_t = y_t.$$

We collect the exogenous disturbances in the vector $\xi_t = [a_t, g_t, \mu_t]$, where $\mu_t = \frac{\epsilon_t}{\epsilon_{t-1}}$ is a wage mark-up shock. It is assumed that the percentage deviations of the first two elements of the vector from their means evolve according to autonomous AR(1)-processes with autocorrelation coefficients $\rho_a, \rho_g \in [0, 1)$. The process for $\log(\mu_t/\bar{\mu})$ and all innovations, $z_t = [\epsilon^a_t, \tilde{\epsilon}_g^t, \epsilon^\mu_t]$, are assumed to be i.i.d..

The recursive equilibrium is defined as follows:

**Definition 1** Given initial values $P_{t_0-1} > 0$, $\Delta_{t_0-1} \geq 1$, $M_{t_0-1}$, $R_{t_0-1}$, a monetary policy, and a Ricardian fiscal policy $T_t \forall t \geq t_0$, a rational expectations equilibrium (REE) for $R_t \geq 1$, is a set of sequences $\{y_t, c_t, l_t, m_{ct}, w_t, \Delta_t, P_t, \bar{P}_t, R_t\}_{t=t_0}^{\infty}$ for the cashless economy and additionally $\{m_t\}_{t=t_0}^{\infty}$ for the monetary economy, for $\{\xi_t\}_{t=t_0}^{\infty}$

(i) that solve the firms’ problem (4) with $\bar{P}_{it} = \bar{P}_t$,

(ii) that maximize households’ utility (3) s.t. their budget constraints (1) in the cashless economy, or s.t. to (2) in the monetary economy

(iii) that clear the goods market (7),

(iv) and that satisfy the aggregate resource constraint (6) and the transversality conditions

$$\lim_{i \to \infty} E_t \beta^i \lambda_{t+i}(B_{t+i} + Z_{t+i+1})/P_{t+i} = 0 \text{ for the cashless economy, and } \lim_{i \to \infty} E_t \beta^i \lambda_{t+i}(M_{t+i} + B_{t+i} + Z_{t+i+1})/P_{t+i} = 0 \text{ for the monetary economy.}$$

3 Model Analysis

In this section, we focus on the models’ behavior in the neighborhood of their deterministic steady states, and apply a log-linear approximation to the true non-linear equilibrium equa-
tions. We therefore assume that the support of the natural logarithm of shocks, log $\xi_t$, is sufficiently small such that they remain in the neighborhood of their steady state value.

The dynamic evolution of output and inflation in the cashless economy is described by the following familiar structural equations (Woodford, 2003), the Euler equation and the aggregate supply curve:

$$\sigma(E_t \hat{y}_{t+1} - E_t \hat{g}_{t+1}) = \sigma(\hat{y}_t - \hat{g}_t) + \hat{R}_t - E_t \hat{\pi}_{t+1}$$

(8)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa ((\omega + \sigma)\hat{y}_t - (1 + \omega)\hat{a}_t - \sigma \hat{g}_t + \hat{\mu}_t),$$

(9)

where $\sigma = -u_{cc}/(u_{cc} + u_{sc})$, $\omega = v_{lt}/v_l$ and $\kappa = (1 - \alpha)(1 - \alpha \beta)/\alpha$. Furthermore, $\hat{k}_t$ denotes the percentage deviation of a generic variable $k_t$ from its steady-state value $k$ except in the case of government expenditures where $\hat{g}_t = (G_t - y)/y$ denote deviations from steady-state output. The model is closed by a simple Taylor-rule as an approximation to (5):

$$\hat{R}_t = \rho_\pi \hat{\pi}_t,$$

(10)

with $\rho_\pi \equiv f' (\pi) \pi / R$. It is well known that for local stability and uniqueness of equilibrium sequences in the cashless economy it is necessary and sufficient to follow the Taylor-principle and to react actively to changes in inflation with changes in the nominal interest rate, i.e. $\rho_\pi > 1$ (Woodford, 2003). While in the cashless economy, there is no motive for holding money balances, predictions of this model for output and inflation are not affected by a money demand function of the following form:

$$\hat{m}_t = -\eta_R \hat{R}_t + (\hat{y}_t - \hat{g}_t),$$

(11)

where $\eta_R > 0$ denotes the interest-rate elasticity of money demand. A money demand equation of this kind can be derived under various specifications without influencing output and inflation in equilibrium, for example when real money balances are assumed to contribute separately to households utility (Woodford, 2003; Gali, 2008). In this case however, money plays the role of a unit of account, and the amount of real money balances follows residually after output, inflation and interest rate have been determined.

In the monetary economy, maximizing (3) with respect to (2) leads to the following set of equations describing households optimal choices for consumption, labor and real money balances:

$$v_l(l_{jt}) \mu_t(1 + \phi_c(m_{jt-1}/\pi_t)) = w_t u_c(c_{jt})$$

(12)

$$\frac{u_c(c_{jt})}{1 + \phi_c(m_{jt-1}/\pi_t)} = \beta R_t E_t \frac{u_c(c_{jt+1})}{1 + \phi_c(m_{jt}/\pi_{t+1})/\pi_{t+1}}$$

(13)
Assume that \( \max(1 - \sigma, \bar{\sigma}) > 0 \) that is sizeable enough and a passive monetary policy \( \rho_t\equiv Y > 0 \) that is empirically relevant. The proof can be found in Appendix A.2. The main message of this proposition is: local stability and uniqueness if and only if

\[ \rho_{\sigma} < \rho_{\sigma} < 1 \text{ and } \eta_{ch} < \frac{\sigma + \omega}{1 + 2\omega/\sigma_h} \text{ leading to non-oscillatory equilibrium sequences} \]

1. \[ \max(1 - \sigma, \bar{\sigma}) < \rho_{\sigma} < 1 \text{ for } \eta_{ch} < \frac{\sigma + \omega}{1 + 2\omega/\sigma_h} \text{ and } \max(1 - \sigma, \bar{\sigma}) < \rho_{\sigma} < \frac{\sigma + \omega}{1 + 2\omega/\sigma_h} \text{ leading to oscillatory equilibrium sequences}, \]

2. \[ \rho_{\sigma} \equiv \frac{(2(1 + \beta) + \kappa)\gamma + 2\kappa}{\kappa(\eta_{ch} + 1 + 2\omega/\sigma_h) - (\sigma - \omega)} \text{ and } z = R/(R - 1). \]

The proof can be found in Appendix A.2. The main message of this proposition is: local stability and uniqueness of non-oscillatory equilibrium sequences as the empirically relevant case, requires a transaction friction \( \eta_{ch} \) that is sizeable enough and a passive monetary policy \( \rho_{\sigma} < 1 \). In other words, the Taylor-principle does not constitute a stabilizing advice in

\[ (R_t - 1)E_t \frac{u_c(c_{jt+1})}{1 + \phi_c(m_{jt}/\pi_{t+1})}/\pi_{t+1} = -E_t \phi_a(c_{jt+1}, m_{jt}/\pi_{t+1}) u_c(c_{jt+1})/\pi_{t+1}, \]
the monetary economy, and even results in explosive equilibrium sequences which marks a
sharp difference compared to the cashless economy. To see this, suppose that a negative
technology shock occurs and that equilibrium sequences are non-oscillatory. As a reaction
to the increase in real marginal costs, firms raise their prices and inflation exceeds its steady
state value (16). Since the inflation elasticity is positive, $\rho_\pi > 0$, the central bank increases
the nominal interest rate, which ceteris paribus causes households to reduce their end-of-
period real money holdings $\hat{m}_t$, by (17). According to (15), the expected real interest rate
is now negatively related to the growth rate of real balances. Thus, an active interest rate
setting, $\rho_\pi > 1$, leads to a decline in the level and the growth rate of real balances, such
that the sequences of real balances and, thus, of output and inflation do not converge to the
steady state.

4 Econometric strategy

When the transaction frictions vanishes ($\eta_{ch} = 0$), the monetary economy collapses to the
cashless economy. A straightforward way thus would be to estimate the monetary economy
only and describe how the distribution of $\eta_{ch}$ evolved over time. If the role of money has
changed over time, $\eta_{ch}$ should decrease and its posterior distribution should be close to zero
at the end of the 80’s, i.e. the cashless economy should emerge. However, as Proposition 1
states, the joint posterior distribution of $\eta_{ch}$ and $\rho_\pi$ will display discontinuities due to the
existence of explosiveness regions. This sections sheds light on how we deal with this issue.

4.1 Model estimation and comparison

The models set out in Section 2 are rich enough to be thought of as the data generating
process for three observable variables. Therefore, the models can be estimated using nowa-
days common Bayesian estimation techniques. This amounts in approximating the mode
of the posterior distribution and to describe the posterior distribution by sampling around
that mode using a random walk Metropolis-Hastings algorithm. The algorithm works well
in characterizing the posterior distribution in the case the surface of the posterior distribu-
tion does not display large discontinuities and has one maxima only. The reason for this is,
that multivariate posterior distribution of the parameters is assumed to be approximately
multivariate normal around the posterior mode. Convergence to the true joint posterior
distribution fails, when it is necessary to discriminate between two local maxima, which are
separated from each other by a large region of discontinuity.

As Proposition 1 implies, we are confronted with exactly this issue. There are two regions
of determinate, non-oscillatory equilibria: one with a sizeable enough transaction friction \( \epsilon_{ch} > 0 \) and passive monetary policy \( \rho_\pi < 1 \), i.e. the monetary economy, and one with \( \epsilon_{ch} = 0 \) and \( \rho_\pi > 1 \), i.e. the cashless economy. These two regions form two different local maxima with a region of explosive equilibrium sequences, i.e. a discontinuity, in between.

We cope with this issue by first estimating each determinacy region discretely, i.e. we estimate the posterior distribution of the monetary economy and cashless economy. In a second step we compute the marginal data density for each model and form the joint posterior distribution by weighting each model by its corresponding posterior probability.

### 4.2 Prior choice and calibrated parameters

We calibrate the discount factor to \( \beta = 0.99 \), the steady-state fraction of private consumption relative to GDP \( c/y = 0.8 \) and the elasticity of substitution between differentiated goods to \( \zeta = 6 \) (see Woodford, 2003). In order to clearly identify the key parameters, i.e. the Taylor coefficient on inflation \( \rho_\pi \) and the transaction cost technology parameter \( \eta_{ch} \), we employ log preferences (\( \sigma_c = 1 \)) and a Frisch elasticity of labor supply of one quarter (\( \omega = 1/4 \)). \( \sigma_h = \eta_{hc} = 2.5 \) are set, to yield a unit output elasticity of money demand and a semi-interest elasticity of -4.47 at an average post-war nominal interest rate of 1.083 percent per year (Woodford, 2003; Lucas, 2000).

Whenever possible, we choose the prior distribution for the remaining parameters similar to Smets and Wouters (2007) as the benchmark estimation for the US economy: the prior distribution for the Calvo parameter defined as a Beta distribution with mean 0.5 and standard deviation 0.1. For the parameters related to the shock processes we choose a standard prior: for the autoregressive coefficients we employ a Beta distribution, centered around 0.5 with a wide standard deviation, for the standard deviations we select an Inverted-gamma distribution.

For the coefficient on inflation in the Taylor rule we use a Gamma distribution with mean 1.5 and a high standard deviation of 1. That way, we can employ the same prior distribution for both determinacy regions. On a first glimpse the prior set might favor the cashless economy. But, since the Taylor rule is only specified in terms of inflation and does not feature a smoothing coefficient on past interest rates nor a coefficient on the output gap, we expect the coefficient on inflation to be higher than 2. Since prior information about the elasticity \( \eta_{ch} \) is scarce, we formulate a not very informative prior distribution with mean 0.075 and a high standard deviation of 0.1. A complete overview of the prior specification can be found in Table 4 in the Appendix.
4.3 Data

We treat the variables real wages, output per capita and inflation captured by the GDP deflator as observable\(^3\). The quarterly observations range from the first quarter 1964 to the third quarter 2009. For our purpose and as it is commonly done (Lubik and Schorfheide, 2004; Clarida, Galí, and Gertler, 2000), we split the sample into two parts: the first sample \(S_1\) runs from the first quarter of 1964 to 1979. We exclude the disinflation years and subsequently vary the starting date of the second sample \(S_2, t = 1983, 1984, \ldots 1991\).

We de-trend the data before estimating the models. We employ an one-sided backward looking HP filter for output and real wages\(^4\). Inflation is de-trended using a linear quadratic time trend. As can be seen from Table 3 in Appendix C, the unconditional standard deviations of the filtered time series of output, inflation and real wages feature the well-known decline over time (Galí and Gambetti, 2009; Stock and Watson, 2002; Kim and Nelson, 1999).

5 Results

In this section we deliver our main result that the primary role of money has changed from facilitating transactions towards just serving as a unit of account in a cashless economy.

In the time period before Volcker, cash was the main means of transactions and fluctuations in the observable variables were higher than after the disinflation years. Our findings are consistent with this. As displayed in the first column of Table 6 in the appendix, the monetary economy with an explicit transactions role for cash balances exhibits a substantially higher log data density \(\ln p(Y|\mathcal{M}_{ME})\) than the cashless economy \(\ln p(Y|\mathcal{M}_{CE})\). In this time period, the amplifying effect of real money balances as a state variable is useful to explain the fluctuations in our observable variables.

This changed after the disinflation years. The economy with the transaction friction continuously loses explanatory power relative to the cashless economy. From the beginning of 1989 onwards and consistent with evidence on the use of cash in transactions, the cashless economy provides a better explanation. In Figure 1, we plot the evolution in explanatory power of the cashless economy relative to the monetary economy, \(100 \times \frac{\ln p(Y|\mathcal{M}_{CE}) - \ln p(Y|\mathcal{M}_{ME})}{\ln p(Y|\mathcal{M}_{CE})}\) over time. The loss in money’s predictive power is modest in a second sample that starts in the mid-eighties and then accelerates from 1987 onwards until the beginning of 1989 when the cashless economy for the first time has a higher marginal data density than the monetary economy.\(^5\) From 1989 onwards, we find that the cashless

\(^3\)A complete description of the data set and its source can be found in Appendix B

\(^4\)The one sided HP circumvents the end point problem and is initialized with 12 quarters.

\(^5\)Marginal data densities and posterior probabilities for the estimated models \(S_{2,88} - S_{2,90}\) are reported in
economy provides a better explanation of the data than the monetary economy. To deliver a
clearer understanding of our result, we compare the estimation results before 1979 with the
ones obtained in the sample that starts 1989. The key parameters in this comparison are
the Taylor coefficient on inflation $\rho_\pi$ and the money elasticity of marginal transaction costs
of consumption $\eta_{ch}$. Furthermore, we conduct a variance decomposition in endogenous and
exogenous states, to assess the loss in predictive power of money as an endogenous state.

In line with the conventional view, we estimate for both models a smaller coefficient $\rho_\pi$ in
the sample period before 1979 than afterwards. Furthermore, the data favor a combination
of $\rho_\pi$ and $\eta_{ch}$ that results in non-oscillatory equilibrium sequences. This requires a Taylor
coefficient smaller than 1 and a transaction friction that is sufficiently large. A detailed
overview of all estimated parameters is given in Table 5. Plots of the posterior distribution
vs. the prior distribution are displayed in Figures 2 -5.

The decreasing importance of predetermined real money to predict the observable vari-
ables is reflected in the estimates of the recursive law of motion. Let this be given by
\[ \tilde{x}_t = \delta_{xm}\hat{m}_{t-1} + \delta_{x\xi}\hat{\xi}_t \] and \[ \hat{m}_t = \delta_{mm}\hat{m}_{t-1} + \delta_{m\xi}\hat{\xi}_t, \]
where $x_t = y_t, \pi_t, w_t$. To assess the loss in predictability of money, we decompose the unconditional variances of output, inflation

Table 6.
and wages into two parts: fluctuations that result directly from shocks and fluctuations that are moderated by real money balances as the endogenous state variable. The fraction of all fluctuations in $\hat{x}_t$ that are induced by variations in real money balances $var(\hat{m}_{t-1})$ in percentage terms is given by:

$$\varrho_{xm} \equiv 100 \times \frac{\delta^2_{xm} var(\hat{m})}{var(\hat{x})}. \quad (18)$$

We compute decompositions for fluctuations in observable variables induced by all shocks and compare the pre-Volcker and with the sample that starts in 1989. The results in Table 1 clearly support our hypothesis that the primary role of money changed. In particular, we find that while predetermined money was capable to explain 40 percent of all fluctuations in output in the first sample, this fraction falls to 14 percent for the second sample. This finding is consistent with the finding that the causality result found by Sims (1972) does not hold anymore after 1988. Remarkably, money accounted for 87 percent of all fluctuations in inflation in the first sample, and loses explanatory power in the second sample.

Table 1: Percentage of fluctuations in observable variables due to fluctuations in money

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_{2,89}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varrho_{ym}$</td>
<td>39.53</td>
<td>14.35</td>
</tr>
<tr>
<td>$\varrho_{\pi m}$</td>
<td>86.86</td>
<td>74.28</td>
</tr>
<tr>
<td>$\varrho_{wm}$</td>
<td>0.39</td>
<td>0.14</td>
</tr>
</tbody>
</table>

6 Conclusion

In this paper we have investigated the hypothesis that the primary role of money has changed from facilitating transactions towards serving as a unit of account. To test this hypothesis, we have developed and estimated a DSGE model with a transaction role for money, and have compared it to a cashless model. We find empirical support for this hypothesis.

The cash economy outperforms the cashless economy for the pre-Volcker era. In this time period, the amplifying effect of real money balances is helpful to capture the substantial fluctuations in our observable variables. From 1983 onwards the economy with the transaction friction continuously loses explanatory power relative to the cashless economy. From the beginning of 1989 onwards and consistent with evidence on the use of cash in transactions, the cashless economy provides a better explanation.
References


A Appendix

A.1 Money, income and causality

Sims (1972) proposed a procedure to test whether there is a causal relationship of money (monetary base $MB$) to GNP (see Theorems 1 and 2). The procedure in a nutshell is to run a regression of current income on current, past and future money. Then, if causality is unidirectional from money to income, the coefficients on future values of money should have coefficients that are as a group insignificantly different from zero. We replicate this exercise for two sub-samples: the first one starts in the first quarter 1947 and ends with the last quarter 1978, and the second one covers the first quarter 1989 until the third quarter 2008. In both cases, we measure both variables in natural logs and apply the filter suggested by Sims to run the following regressions of $GNP$ on $MB$ and vice versa:

\[ GNP_t = c(1)MB_t + c(2)MB_{t-1} + c(3)MB_{t-2} + c(4)MB_{t-3} + c(5)MB_{t-4} + c(6)MB_{t-5} + c(7)MB_{t-6} + c(8)MB_{t-7} + c(9)MB_{t-8} + c(10)MB_{t+1} + c(11)MB_{t+2} + c(12)MB_{t+3} + c(13)MB_{t+4} \]

\[ MB_t = c(14)GNP_t + c(15)GNP_{t-1} + c(16)GNP_{t-2} + c(17)GNP_{t-3} + c(18)GNP_{t-4} + c(19)GNP_{t-5} + c(20)GNP_{t-6} + c(21)GNP_{t-7} + c(22)GNP_{t-8} + c(23)GNP_{t+1} + c(24)GNP_{t+2} + c(25)GNP_{t+3} + c(26)GNP_{t+4}. \]

**Table 2: Test on four future coefficients**

<table>
<thead>
<tr>
<th>Regression Equation</th>
<th>$F_{47-78}$</th>
<th>$F_{89-08}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GNP$ on $MB$</td>
<td>0.13</td>
<td>4.52**</td>
</tr>
<tr>
<td>$MB$ on $GNP$</td>
<td>4.39**</td>
<td>1.72</td>
</tr>
</tbody>
</table>

** Significant at 0.05 level. All tests that apply to regressions from the first sample are assumed distributed $F(4,101)$, while the tests for the second sample are assumed distributed $F(4,52)$.**

A.2 Proof of Proposition 16

Combining $\hat{R}_t = \rho_{\pi t}$ with (15)-(17) leads to the following characteristic equation

\[ F(x) = x^3 - x^2 \left( \frac{\kappa + \beta + 1}{2} \right) \overline{Y} + \omega \kappa + x \frac{\rho_{\pi} \kappa \left( \sigma + \omega - \eta_{ch}(1 + z \omega / \sigma_a) \right)}{\beta \overline{Y}} + \frac{\omega \kappa \eta_{ch} z \rho_{\pi}}{\beta \overline{Y} \sigma_a} = 0 \]
Since $F(0) > 0$, non-oscillatory equilibrium sequences (exactly one positive stable root) necessarily require
\[ F(1) = (\rho - 1)\kappa \frac{\Psi + \omega}{\beta \Psi} < 0 \Rightarrow \rho < 1. \]

To rule out any further stable root in this case, we examine the conditions for $F(-1) > 0$
\[ \rho \kappa \left[ \eta ch(1 + 2z\omega/\sigma_h) - \sigma - \omega \right] > (2(1 + \beta) + \kappa)\Psi + \omega \kappa, \Rightarrow \rho > \bar{\rho} \text{ for } \eta ch > \frac{\sigma + \omega}{1 + 2z\omega/\sigma_h}, \]
for $\eta ch < \frac{\sigma + \omega}{1 + 2z\omega/\sigma_h}$ no stable non-oscillatory equilibrium sequences exist.

At least one negative stable root and thus oscillatory sequences arise for $F(-1) < 0$ and $F(1) > 0$, i.e. for $\rho > 1$. To rule out two more stable roots (one additional stable can not occur), for $\eta ch < \frac{\sigma + \omega}{1 + 2z\omega/\sigma_h}$ it is sufficient to ensure $\rho > \frac{\beta \sigma}{\omega \eta ch \kappa z}$ such that $F(0) = -x_1 x_2 x_3 > 1$.
If $\eta ch > \frac{\sigma + \omega}{1 + 2z\omega/\sigma_h}$, $\rho < \bar{\rho}$ constitutes an additional condition for the existence of one negative stable root ($F(-1) < 0$).

### B Data appendix

The frequency of all data used is quarterly.

**Real GDP:** This series is *BEA NIPA table 1.1.6 line 1*.

**Nominal GDP:** This series is: *BEA NIPA table 1.1.5 line 1*.

**Implicit GDP Deflator:** The implicit GDP deflator is calculated as the ratio of Nominal GDP to Real GDP.

**Civilian noninstitutional population:** This series is taken from: 

**Nominal hourly wages total private industry:** This series is *AHETPI* obtained from Fred.
### C Tables and Figures

Table 3: Standard deviations of de-trended observable variables in $S_1$ and $S_2$

<table>
<thead>
<tr>
<th>std(y)</th>
<th>$S_1$</th>
<th>$S_{2,89}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>std(π)</td>
<td>0.0223</td>
<td>0.0147</td>
</tr>
<tr>
<td>std(w)</td>
<td>0.0121</td>
<td>0.0050</td>
</tr>
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</table>

Table 4: Prior distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>distribution</th>
<th>mean</th>
<th>std</th>
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</thead>
<tbody>
<tr>
<td>$\rho_{\pi}$</td>
<td>gamma</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>beta</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$\eta_{ch}$</td>
<td>normal</td>
<td>0.075</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>invgamma</td>
<td>0.04</td>
<td>0.026</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>invgamma</td>
<td>0.04</td>
<td>0.026</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>invgamma</td>
<td>0.04</td>
<td>0.026</td>
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</table>
Table 5: Posterior estimates of the structural parameters in each model

<table>
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<th>Parameter</th>
<th>$S_1$</th>
<th></th>
<th></th>
<th></th>
<th>$S_{2,89}$</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mathcal{M}_E$</td>
<td>$\mathcal{M}_E$</td>
<td>$\mathcal{M}_E$</td>
<td>$\mathcal{M}_E$</td>
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<td>$\mathcal{M}_E$</td>
<td>$\mathcal{M}_E$</td>
<td>$\mathcal{M}_E$</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>4.10</td>
<td>0.69</td>
<td>0.93</td>
<td>0.05</td>
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</tr>
<tr>
<td>$\eta_{ch}$</td>
<td>–</td>
<td>–</td>
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<td>$\rho_a$</td>
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<td>0.05</td>
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<td>$\sigma_a$</td>
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<td>0.002</td>
<td>0.01</td>
<td>0.001</td>
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<td>$\sigma_g$</td>
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<td>0.005</td>
<td>0.04</td>
<td>0.004</td>
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<td></td>
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</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.02</td>
<td>0.004</td>
<td>0.03</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Model comparison

|             | $S_1$ | | | | $S_{2,88}$ | | | | $S_{2,89}$ | | | | $S_{2,90}$ | | | |
|--------------|-------|---|---|---|-------|---|---|---|-------|---|---|---|-------|---|---|
| $\ln p(Y|\mathcal{M}_i)$ | $\mathcal{M}_E$ | $\mathcal{M}_E$ | | | $\mathcal{M}_E$ | $\mathcal{M}_E$ | | | $\mathcal{M}_E$ | $\mathcal{M}_E$ | | | $\mathcal{M}_E$ | $\mathcal{M}_E$ | |
| Posterior probabilities | 437.98 | 461.1593 | | | 934.29 | 942.46 | | | 893.73 | 893.64 | | | 847.12 | 845.56 |
|                          | 0 | 1 | | | 0 | 1 | | | 0.52 | 0.48 | | | 0.83 | 0.17 |
Figure 2: Prior distribution (white) vs. Posterior distribution (black) of the cashless economy $\mathcal{M}_{ME}$ Sample $S_1$
Figure 3: Prior distribution (white) vs. Posterior distribution (black) of the monetary economy $M_{ME}$ Sample $S_2$
Figure 4: Prior distribution (white) vs. Posterior distribution (black) of the cashless economy $\mathcal{M}_{ME}$ after 1988
Figure 5: Prior distribution (white) vs. Posterior distribution (black) of the monetary economy $M_{ME}$ after 1988