The Importance of Mixed Incentive Contracts in Partnerships

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Abstract

The paper illustrates the impact of mixed contracts on teamwork and welfare in a partnership when production depends on the efforts of agents at their own tasks as well as their efforts at helping other teammates. We show that a mixed contract that combines compensation based on team output with that of a tournament based on individual output can lead to higher welfare than a contract based purely on team output despite the possibility of sabotage and the possible adverse impacts that the contract may have on agent efforts. We also show that a mixed contract with a tournament based on alternative performance measures can lead to a higher level of welfare than a mixed contract with the tournament based on individual output. The alternative contract can also induce agents into choosing first-best levels of effort.

JEL Classification: D2; J3; L2; M5
I. Introduction

Partnerships have been widely used to organize production in many working places. Examples include, but are not limited to, a team of architects designing a new museum or a group of scientists working on a research project. Partnerships are also prevalent in most professional service industries such as law, accounting, consulting, medicine, investment banking, etc. According to Farrell and Scotchmer (1988), two chief features of partnerships are joint production and equal sharing among participants. In such organizations where team production is predominant and requires for example, agents to engage in two types of effort, their own and helping fellow team members, the problem of providing a proper balance between team and individual incentives can be a difficult one to achieve. An incentive scheme that emphasizes collective rewards over individual rewards, for instance, may promote stronger helping incentives but possibly at the expense of reduced individual effort. In addition, the effects of free riding may remain since the level of cooperation may still fall short of what is efficient, despite the increased levels of helping effort. On the other hand, a more competitive incentive scheme provides stronger incentives for individual effort but may also run the risk of diminishing incentives to help others and may, in certain cases, induce agents to sabotage the work of other team members (Lazear, 1989).

Part of this difficulty can be attributed to the interaction between team technology and agent incentives and how closely the two are aligned so as to motivate efforts that increase the value of the partnership. A tournament based on relative individual output for example, may move individual incentives away from the value of the partnership if teamwork is a significant

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1 Greenwood and Empson (2003) report the percentage of partnerships for the top 100 firms per industry: law 100%, accounting 56%, architecture 18%, management consulting 17%. 
contributor to partnership value. Moreover, this gap can widen if the difference between winning and losing the tournament increases for each individual agent and becomes even more pronounced. Alternatively, moving incentives in the other direction so that they are more in line with that of the partnership can run into other obstacles as well. Rewarding agents on the basis of their marginal social contributions to total output, for example, may eventually violate budget balance – the point at which the value of output is exceeded by the amount distributed as compensation (Holmström, 1999).

Given the fact that such tradeoffs are often necessary in order to reconcile individual and group incentives in team production, it is not sufficient to only determine and illustrate these tradeoffs under various proposed incentive schemes. As a practical matter as well as theoretical one, it is just as important to gauge and compare the impact of the various incentive contracts on partnership output and profitability. The additional information that we derive from the extension of the analysis to these two areas of concern helps complete the picture and enables us to judge whether or not a certain incentive scheme is preferable to another. By way of an illustration, it is not immediately obvious whether a pure-team contract in which agents equally split team output is preferable to a mixed contract that offers a combination of individual and group rewards. Even if the former contract results in free riding, combining it with a competitive tournament scheme may not be desirable if it gives rise to strategic behavior, as employees may then seek to promote their own interests at the expense of others. Although this possibility has been recognized in previous literature, the analysis remains incomplete without knowledge of the relative ranking of the two contracts.

Thus part of our goal is to extend the analysis and obtain a better picture of the impact of various proposed incentive schemes on contract choice. However in order to do so, the analysis
must remain tractable enough to permit welfare comparisons between alternative incentive contracts. Accordingly, we first introduce a specific model of team production that allows output to depend on both an agent’s own efforts and their helping efforts. Our specification is also sufficiently flexible enough so that it remains possible for agents to work at cross purposes to one another depending on the incentives that the agents face. Once this groundwork has been set, it is not difficult to show that under certain conditions, a combined contract of competitive and cooperative elements can lead to higher partnership profitability than a pure-team contract, even if the mixed contract does not lead to increased levels of helping effort. More specifically, we find that the team synergy plays a key role in determining the relative efficiency of contracts, where team synergy is defined as the sum of the marginal products of each agent’s helping efforts. Thus when team synergy is low, a mixed contract yields higher welfare than a pure-team contract. However, when team synergy is high, the former contract is less efficient than the latter.

In our analysis of mixed contracts, we also follow Baker (2002) and illustrate our results geometrically and give a trigonometric interpretation of the distortion introduced by a tournament scheme based on relative individual output. Not only does this technique help highlight the discrepancy between incentives and the first-best value of the partnership, it also helps us understand why partnerships may elect to choose other performance measures, other than individual output or productivity, to base the tournament upon. This can occur even if the measures are somewhat arbitrary and subject to member influence and politicking. Although such measures are obviously dysfunctional, it allows us to capture the very real possibility that a performance appraisal system can be subject to the influence activities of those who are being evaluated.
The contributions of the paper are twofold. First, our findings provide a possible explanation for the widespread of equal-sharing rule in the human-capital intensive partnerships. For example, the equal-sharing rule is used by more than 55% of medical groups (Encinosa, Gaynor, and Rebitzer, 2007) and by a majority of small law firms (Farrell and Scotchmer, 1988). Since members in these types of partnerships are usually professional specialists that often work closely with each other, we would expect team synergy to be quite high in these organizations. Consequently equal-sharing contracts will tend to yield higher levels of profitability and welfare than a mixed contract. Second, our findings have important policy implications for contract design. Unlike previous literature on partnerships, we emphasize the importance of technology and team synergy in determining optimal contract selection.

The rest of the paper is structured as follows. Section II briefly reviews the related literature. Section III introduces our basic model of team production and presents the results under the pure-team contract with equal sharing. Section IV investigates and illustrates the results of a contract that offers agents a mix of individual and team incentives. In Section V, agents face a similar mixed contract but one that is based on alternative performance measures. Section VI concludes the paper with a brief overview of our results.

II. Related Literature

In the area of contract theory, the most relevant for our purposes is the strand of study that examines the problem of moral hazard in teams and compares the efficiencies of various incentive contracts in teams. Alchian and Demsetz (1972) and Holmström (1982) have emphasized the role of free riding in an environment where agent efforts are unverifiable and output is determined by the effort of all team members. Thus, if agents are rewarded based on
team performance, they will tend to shirk whenever the benefits are a fraction of the costs of their effort. Farrell and Scotchmer (1988) also emphasize the inefficiency of the team-output contract with equal sharing from the perspective of exploiting economies of scale in partnerships. They argue that heterogeneous teams are more efficient given the possibility that there could be gains in combining different ability levels. However, equal-sharing tends to limit these gains and cause partnerships to be too homogeneous since the more talented agents are reluctant to admit those who are less talented given that the former ultimately expect that they will have to subsidize the latter.

The widespread of use equal-sharing therefore seems puzzling given the free-riding problem stressed in the literature. Some studies attempt to provide a rationale for this phenomenon by comparing the team-output contract with other contracts. In a theoretical paper, Itoh (1991) derives conditions under which contracts based on team output can yield higher welfare than contracts based on individual output. For example, a team-output contract will be optimal if each agent exerts more of their own effort in response to an increase in the helping efforts of other agents. Team-output contracts can also be optimal if tasks tend to be boring or tiresome enough so that agents look to relieve the tedium of their own tasks by allocating some of their own effort towards helping other agents.

Autrey (2005, 2007) also studies the effect of contractual incentives on cooperation among teammates but in a dynamic setting. In addition, she adds a team synergy factor into the model in order to determine how the degree of synergy influences the relative efficiency of team-based and individual-based performance measures in the presence of career concerns. Her results follow accordingly: when a team-output contract is used, agents are willing to cooperate and exert more effort when team synergy is high. However, when synergy is low, it may be
optimal to switch to a contract that utilizes individual output as a measure of performance rather than team output.\(^2\)

On the other hand, there have been several studies that examine the possible efficiency of alternative types of contracts. The use of tournaments and the introduction of competition among employees to boost employee efforts, for example, is an incentive scheme that has long been a subject of study since Lazear and Rosen (1981). However, in a team setting or when positive externalities are present among agents, tournaments can remain susceptible to shirking since the existence of positive externalities can still work to reduce the possible benefits that can be derived from increased individual effort (Drago and Turnbull, 1988). Furthermore, as mentioned earlier, a relative compensation scheme can quickly pit agents against one another and can even encourage sabotage (Lazear, 1989, 1995). In a slightly different context, Drago and Turnbull (1991) look at two different types of promotion schemes – one competitive and one noncompetitive – in an environment where helping efforts are as important as own efforts. After allowing for a variety of possible conjectural variations that agents may have concerning the other’s response to their own and helping efforts, the authors show that the competitive tournament scheme often prevents the possibility of there being positive helping effort whereas

\(^2\) In other areas of the literature, other authors have used the presence of implicit incentives to explain the widespread use of the equal-sharing rule in partnerships. For example, Kandel and Lazear (1992) argue that peer pressure can effectively provide incentives that help attenuate free-riding even in the presence of the rule. Bartling and Von Siemens (2008), argue that the same may also hold when fairness is a major concern of an agent’s social preferences, where fairness depends on the difference between an agent’s monetary payoff and their cost of effort.
the same is not true of the noncompetitive scheme.\(^3\)

In a paper that is perhaps closest to ours in spirit and in terms of incentive design, Irlenbusch and Ruchala (2008) use a simple model to study how agents perform under a contract that rewards both team output and relative individual output. They show that the incentive compensation scheme can help raise individual effort when compared to a purely team-output scheme. However, the authors do not allow for situations where helping effort is an important component of team production. Consequently, the question of whether or not the impact of such mixed compensation schemes harms agents’ helping efforts, as well as whether or not such schemes are more efficient than pure-team contracts, remain unanswered. These are some of the issues that we begin to examine in the next section.

III. The Model

Setup and the first-best allocation

We consider the case where two homogenous risk-neutral agents work as a team. We assume that each agent \(i\) produces individual output \(y_i\) according to the following form:

\[
y_i = a_i + f_{own}h_i + f_{other}h_j + e_i, \quad i, j = 1, 2, \text{ and } i \neq j,
\]

where \(a_i\) represents agent \(i\)’s own effort and \(h_i\) and \(h_j\) respectively represent agent \(i\)’s and \(j\)’s helping efforts. We allow helping efforts to be negative in which case \(h_i\) and \(h_j\) can then be interpreted as describing each agent’s efforts at sabotage. The parameter \(f_{own}\) denotes the

\(^3\) The noncompetitive (quota) scheme is based on individual output and a fixed output standard that is independent of the other agent’s output. Thus the probability of winning a promotion is based on the agent’s own output exceeding that of the fixed standard.
marginal product of helping effort on own output and $f_{other}$ represents the marginal product of helping effort on the other teammate’s output. For short, we call $f_{own}$ the own marginal product and $f_{other}$ the other marginal product (or external effect) of helping effort. Note that because of our assumption of homogeneity, the own and other marginal products of helping effort are the same for both agents. Thus when agent $i$ helps agent $j$, helping effort, $h_i$, not only improves the output of agent $j$ through $f_{other}$, but also agent $i$’s own output through $f_{own}$. We assume both $f_{own}$ and $f_{other}$ are nonnegative and allow for $f_{other}$ to be less than or greater than one, which is the normalized marginal product of own effort.\textsuperscript{4} In addition, for future reference, we refer to the case, $f_{own} + f_{other} > 1$, as one where there is a high team return to helping effort whereas the opposite case, $f_{own} + f_{other} < 1$, can be referred to as one where there is a low team return to helping effort. It will also be helpful to refer to the situation where the own marginal product of helping effort is greater than its external effect, i.e., when $f_{own} > f_{other}$, as the case where there is high individual return to helping effort while we refer to the opposite case, $f_{own} < f_{other}$, as one where there is low individual return to helping effort. Note that in the case of sabotage, an agent’s own individual output will also be impacted and in the special case where $f_{own} > f_{other}$,

\textsuperscript{4} In practice, an agent may perform a variety of multiple tasks where some of these tasks entail intensive interaction among team members while others may not. Although we abstract from the more realistic cases by assuming only two types of effort, we do allow for the possibility of helping effort having a greater impact than own effort as a proxy for those cases where interaction among coworkers is more important from the point of view of the firm or partnership than effort spent on own tasks.
the effect will reduce the agent’s own output even more than the other agent’s output. Lastly, \( \varepsilon_i \) represents the random shock term for individual output, which for simplicity, we assume to be to be \( i.i.d. \) for all agents and uniformly distributed over the interval: \([-\bar{\varepsilon}, +\bar{\varepsilon}]\).

An agent’s production of output is assumed to be costly in terms of effort. Agent \( i \)'s costs are therefore assumed to be an increasing and convex function of the agent’s own and helping (or sabotage) efforts. This function takes the following form:

\[
C(a_i, h_i) = \frac{a_i^2 + h_i^2}{2} \quad i = 1, 2. \tag{2}
\]

Before analyzing the contracts, we first derive the first-best solution in order to obtain benchmark levels of the agents’ efforts with which to compare our subsequent results to. In order to keep the exposition as simple as possible, suppose that agents jointly agree to maximize the partnership value (or equivalently the expected total utility of both agents), i.e., the total value of their output (where the output price is normalized to one) minus the costs of their efforts. Given that the choice variables are the agents’ own and helping efforts, the problem then can be specified as:

\[
\max_{a_i, h_i, a_j, h_j} EV = E(y_i + y_j) - C(a_i, h_i) - C(a_j, h_j) \quad i, j = 1, 2, \text{ and } i \neq j. \tag{3}
\]

where \( EV \) represents the partnership value, or equivalently, the expected total utility of both agents, \( EV = EU_i + EU_j \). Deriving the first-order conditions and solving, yields the Pareto optimal (PO) solution:

\[
a_i = a_j = a_{PO} = 1, \tag{4a}
\]

\[
h_i = h_j = h_{PO} = f_{own} + f_{other}, \tag{4b}
\]

\[
EU_i = EU_j = EU_{PO} = \frac{1 + (f_{own} + f_{other})^2}{2}. \tag{4c}
\]
where $EU^{PO}$ is the Pareto optimal level of expected utility for each of the two homogeneous agents. Although the first-order conditions are not shown, it is easy enough to see that the first-best solution indicates that an agent’s marginal cost of effort (left-hand side of (4a)-(4b)) should be set equal to the marginal product of that agent’s effort on the total team output (right-hand side of (4a)-(4b)).

*The pure-team contract*

In the pure team-output contracting scenario, assume that the two homogeneous agents equally split total team output. Given our assumptions concerning output and agent costs, we can characterize the solution of each agent’s expected utility maximization problem by solving the following problem for each agent:

$$\max_{a_i, h_i} EU_i^{pure\_team} = \frac{1}{2} E(y_i + y_j) - C(a_i, h_i)$$  \hspace{1cm} (5)

Solving (5) yields

$$a_i^{pure\_team} = \frac{1}{2},$$  \hspace{1cm} (6a)

$$h_i^{pure\_team} = \frac{f_{own} + f_{other}}{2}.$$  \hspace{1cm} (6b)

$$EU_i^{pure\_team} = \frac{3\left[1 + \left(\frac{f_{own} + f_{other}}{2}\right)^2\right]}{8}.$$

(6c)

From the above results we observe that, due to the free-riding problem, agents provide only a half of the efforts that are required under the first-best allocation. From (6b) we can also see that if there is a high team return to helping effort, $(f_{own} + f_{other} > 1)$, then agents will devote more
effort to helping their teammate than on their own task. Also observe that an agent’s expected utility under the pure-team contract is lower than it is under first best.

IV. The Team-based Contract with Bonus Reward Based on Relative Individual Output

The mixed contract

In this section, we consider rewarding agents based on their relative output levels. In this case the agent receives a positive bonus, $\Delta$, if the agent’s output exceeds that of the other team member.\(^5\) The bonus is financed by both team members in equal shares given that the partnership is owned by two homogenous agents. The probability of agent $i$ winning the tournament is given by $P_i(a_i, h_i, a_j, h_j)$.\(^6\) In addition to the possibility of earning a bonus, each agent also receives a similar equal-sharing contract as before: one half of the total team output. Each agent’s problem can then be written as:

$$\max_{a_i, h_i} EU_{i, \text{bonus\_team}} = \left(1 - P_i(a_i, h_i, a_j, h_j)\right)\left(\frac{1}{2} E\left(y_i + y_j\right)\right)$$

$$+ P_i(a_i, h_i, a_j, h_j)\left(\frac{1}{2} E\left(y_i + y_j\right) + \Delta\right) - C(a_i, h_i) - \frac{\Delta}{2},$$

or more simply as

\(^5\) We preclude the negative bonus case since it is hardly observed in practice. In addition, this possibility can also be viewed as a byproduct of a perverse incentive system given that as we proceed, we will see that this possible outcome can be eliminated by the use of an alternative incentive scheme that induces less distortion between team and individual incentives.

\(^6\) See Appendix A for details about the derivation of the arguments for this function.
\[
\max_{a_i, h_i} E U_{i \text{bonus,team}}^{i} = \frac{1}{2} E \left( y_i + y_j + P_i \left( a_i, h_i, a_j, h_j \right) \Delta - C \left( a_i, h_i \right) - \frac{\Delta}{2} \right). \tag{7}
\]

Solving (7) yields the two first-order conditions:

\[
\frac{1}{2} \frac{\partial P}{\partial a_i} \Delta - a_i = 0, \tag{8a}
\]

\[
\frac{f_{\text{own}} + f_{\text{other}}}{2} + \frac{\partial P}{\partial h_i} \Delta - h_i = 0. \tag{8b}
\]

Note that with the implementation of a tournament scheme, each agent now faces an additional term in their calculus as represented by the marginal increase in the probability of winning times the bonus. This holds for both types of efforts. In equilibrium, the marginal probabilities are given by:

\[
\frac{\partial P_i}{\partial a_i} = \frac{1}{2 \varepsilon}, \quad \frac{\partial P_i}{\partial h_i} = \frac{f_{\text{own}} - f_{\text{other}}}{2 \varepsilon}. \tag{7}
\]

Using these expressions we can then solve for the optimal levels of the agent’s own and helping efforts and expected utility under the mixed contract for a given prize level:

\[
a_{\text{bonus,team}} = \frac{1}{2} + \frac{\Delta}{2 \varepsilon}, \tag{9a}
\]

\[
h_{\text{bonus,team}} = \frac{f_{\text{own}} + f_{\text{other}}}{2} + \frac{\Delta \left( f_{\text{own}} - f_{\text{other}} \right)}{2 \varepsilon}. \tag{9b}
\]

\[
E U_{\text{bonus,team}} = \left[ 1 + \left( f_{\text{own}} - f_{\text{other}} \right)^2 \right] \frac{\Delta^2}{8 \varepsilon^2} + \frac{\left( 1 + f_{\text{own}}^2 - f_{\text{other}}^2 \right) \Delta}{4 \varepsilon} + \frac{3 \left[ 1 + \left( f_{\text{own}} + f_{\text{other}} \right)^2 \right]}{8}. \tag{9c}
\]

In comparing these results with respect to (6a) and (6b), we can refer to the additional

\[\text{See Appendix A for the derivation of both marginal probabilities of winning.}\]
terms in (9a) and (9b) as the bonus incentives for own and helping efforts respectively. From (9a) we can easily see that the bonus incentive for own effort, $\Delta / 2\bar{\epsilon}$, is always positive. In contrast, it is only when there is high individual return to helping effort, i.e., when $f_{\text{own}} > f_{\text{other}}$, that the bonus incentive in (9b) is positive for helping effort. This result seems intuitive given the fact that when this condition holds, the agent’s chance of winning the tournament is increasing in $h_i$. On the other hand, when exerting helping effort increases the other teammate’s output more than one’s own, and therefore lowers an agent’s chances of winning the tournament, i.e., when $f_{\text{own}} < f_{\text{other}}$, then using an additional bonus reward scheme reduces helping effort. In this latter case, a mixed incentive contract has a similar effect on helping effort as a pure tournament has. This last result is also similar to the one found in Drago and Garvey (1998), who investigate the impact of a combined exogenous incentive scheme on both own and helping efforts. We can also observe in this case, that if the bonus is large enough relative to the uncertainty that the agent faces (as represented by $2\bar{\epsilon}$, the length of the support for random shock in individual production), it is possible that an agent will engage in negative helping efforts (i.e., sabotage) in order to increase their chances of winning.

Rewriting (9c) yields

$$E U_{\text{bonus, team}} = -\frac{1 + (f_{\text{own}} - f_{\text{other}})^2}{8\bar{\epsilon}^2} \left[ \Delta - \frac{\bar{\epsilon} \left(1 + f_{\text{own}}^2 - f_{\text{other}}^2\right)}{1 + (f_{\text{own}} - f_{\text{other}})^2} \right]^2$$

---

8 The result for this case differs with previous theoretical findings in the literature that indicate that a larger prize (or prize spread) leads to less cooperation. See Coupé et al. (2003) for empirical evidence that a wage gap may not lead to less cooperation.
\[
\frac{1 + \left(f_{\text{own}}^2\right)^2 + \left(1 - 2f_{\text{own}}^2\right)f_{\text{other}}^2 + f_{\text{other}}^4}{2\left[1 + (f_{\text{own}} - f_{\text{other}})^2\right]}, \quad (9c')
\]

from which we can see, after some algebraic manipulation, that when \(\Delta = 0\), \(EU_{\text{bonus}_{\text{team}}} = EU_{\text{pure}_{\text{team}}}\). Using the above expression, it can also be shown that \(EU_{\text{bonus}_{\text{team}}}\) is a concave function of \(\Delta\). Furthermore, maximizing \((9c')\) with respect to \(\Delta\) and then solving, we can obtain the expression for the optimal bonus:

\[
\Delta_{\text{bonus}_{\text{team}}} = \frac{\bar{e}(1 + f_{\text{own}}^2 - f_{\text{other}}^2)}{1 + (f_{\text{own}} - f_{\text{other}})^2}, \quad (10)
\]

For future reference, we refer to the case where the sum of the squares of the marginal products of an agent’s own and helping effort on own output is greater than the square of the marginal product of helping effort on the other member’s output, \(1 + f_{\text{own}}^2 > f_{\text{other}}^2\), as the low synergy setting whereas the opposite case, \(1 + f_{\text{own}}^2 < f_{\text{other}}^2\), is referred to as the high synergy case (where there is low individual return to helping effort). Thus from (10) we can see that in the low synergy setting, the optimal bonus is positive and increases with the amount of uncertainty faced by agents in terms of their individual output. In addition, since \(EU_{\text{bonus}_{\text{team}}}\) is a concave function of \(\Delta\), \(EU_{\text{bonus}_{\text{team}}} > EU_{\text{pure}_{\text{team}}}\) whenever \(\Delta\) is an element between

\[
\left[0, \frac{2\bar{e}(1 + f_{\text{own}}^2 - f_{\text{other}}^2)}{1 + (f_{\text{own}} - f_{\text{other}})^2}\right].
\]

However note that expected utility reaches its peak when the bonus value lies in the middle of this interval, as indicated by the expression for the optimal level of the bonus given in (10). In the high synergy setting, however, the optimal bonus is negative. Since the use of a negative bonus is not an option by assumption, it is better for the partnership to use
the pure-team contract when team synergy is high given that the pure team contract would yield a higher level of welfare than the mixed contract with a positive bonus. This result is of some interest since it provides an alternative explanation for the widespread use of equal-sharing schemes in human-capital intensive partnerships. Since members in these types of partnerships would tend to work closely with one another in tasks that are interrelated, we would expect team synergy to be high enough to warrant the profitable use of these kinds of contracts.

**Graphical analysis**

In order to interpret our findings along similar lines as Baker (2002), we introduce some new notation and parameters. The measures that are of primary concern to us are the partnership value and the incentives that agents face, which are ultimately based on the performance measures used to reward agent efforts. Starting with the former, let \( w \) be the vector of marginal products of an agent’s efforts on partnership value. We will also refer to this vector as the first-best vector. Also let \( W \) represent the vector’s length. We then have: \( w = (1, f_{own} + f_{other}) \) and \( W = \|w\| = \sqrt{1 + (f_{own} + f_{other})^2} \). With respect to the team performance measure, denote \( m \) as the

\[^9\] We have also compared the expected utility of the pure-team contract with that of the pure-individual contract in which agents receive their individual output as compensation. Expected utility under the latter is given by \( \frac{1 + f_{own}^2 + 2f_{own}f_{other}}{2} \). However, we find that expected utility is greater under the team contract than the individual output contract in the high synergy setting. Moreover, in the low synergy setting, the expected utility of the mixed contract with an optimal bonus is greater than expected utility of the individual output contract.
vector of the marginal products of an agent’s efforts on this measure, and let its length be given by $M$. Since this (absolute) measure is also based on total team output, which is also the value of the partnership, we have: $m = w$ and $M = W$. For the (relative) performance measure which the bonus is based on, let $r$ be the vector of marginal products of an agent’s efforts on this measure and let $R$ be its length. Consequently, we have: $r = \left( \frac{\partial P_i}{\partial a_j}, \frac{\partial P_i}{\partial h_k} \right) = \left( \frac{1}{2\tilde{E}}, \frac{f_{own} - f_{other}}{2\tilde{E}} \right)$ and $R = \sqrt{1 + \left( \frac{f_{own} - f_{other}}{2\tilde{E}} \right)^2}$. We can now rewrite the expression in (10) as

$$\Delta^{\text{bonus\_team}} = \frac{w \cdot r}{R^2} - \frac{m \cdot r}{2R^2} = \frac{WR \cos \sigma}{R^2} - \frac{MR \cos \sigma}{2R^2} = \frac{W \cos \sigma}{R} - \frac{M \cos \sigma}{2R}$$

(10')

where $\sigma$ represents the angle between vectors $w$ and $r$ and the angle between that of $m$ and $r$, since $m = w$.\(^\text{10}\) Note that the more closely aligned the vectors are, the higher $\cos \sigma$ will be, and when the two vectors are orthogonal, i.e. when $w \cdot r = 0$, then $\cos \sigma = 0$.

The first term on the right-hand side of (10') represents the amount needed to motivate agents in order to get them to choose the best possible pattern of efforts – from the perspective of partnership value – as a function of the parameters that impact partnership value and the performance measures that the tournament is based upon (such as $f_{own}$, $f_{other}$, and $2\tilde{E}$). From this term, we can easily see that the less aligned the vectors are, the smaller this term will be. All else constant, this will tend to lower the bonus given that the bonus will likely generate more distortion than motivation. However, the larger $W$ is relative to $R$ (or in other words, the higher

\(^{10}\) Here we apply the relationship between the dot product, vector lengths, and their angles. For two vectors, $a$ and $b$, intersecting at the origin, this relationship can be written as: $a \cdot b = ||a|| \cdot ||b|| \cos \theta$, where $\theta$ is the angle between the two vectors.
the effect that a unit of effort has on the partnership value relative to the performance measure), the larger the tournament prize will be. The second term of (10′) reflects the impact that team performance has on the bonus. An increase in \( \cos \sigma \) will tend to lower the bonus, all else held constant, since an increase in this parameter will raise the unit value of the tournament bonus as a complementary incentive device and therefore a smaller amount of the bonus is needed in order to motivate agent effort.

Combined, these two expressions demonstrate how (10′) captures the interplay between partnership value and the mix of incentives that the agents face. To illustrate this further, note that when the vectors are orthogonal, the bonus should be zero since the tournament cannot provide any incentives in the direction of partnership value under these conditions. A slightly more subtle example is the case where \( f_{\text{other}} = 0 \). In this scenario, where an agent’s individual return to helping effort is the same as the team return (i.e., there is no synergy in the partnership), the vectors \( \mathbf{w} \) and \( \mathbf{r} \) are exactly aligned so that \( \cos \sigma = 1 \). The bonus should thus be set just equal to the measure of uncertainty that the agents face in a tournament under the mixed contract, \( \Delta^{\text{bonus,team}} = \bar{\varepsilon} \). In addition, we can see from the right-hand side of (10′) that when \( \cos \sigma < 0 \) (i.e., when \( \sigma > 90^\circ \)), the optimal bonus should be negative. However this can only occur in the high synergy case.

We can now translate the above results and their implications graphically. These are summarized in Figure 1 below for the low synergy case.\(^{11}\) The first-best vector, \( \mathbf{w} \), with length \( W \), is depicted in the figure by the vector \( \mathbf{OPO} \) where point \( \mathbf{PO} \) represents the effort levels

\(^{11}\) Before we translate (10′) into the graph, we rearrange the expression slightly to get:

\[
\Delta R + \frac{M \cos \sigma}{2} = W \cos \sigma \quad (\text{where the superscript on } \Delta \text{ has been removed for clarity}).
\]
associated with the first-best allocation and $O$ represents the origin. Incentives associated with the team contract are given by vector $\frac{m}{2}$ and is represented by vector $OB$ in the diagram. Since $m = w$, the vectors $OB$ and $OPO$ are precisely aligned in the same direction. We also observe that the length of vector $OB$ is given by $\frac{M}{2}$, which is one half of that of vector $OPO$. With respect to the incentives associated with the bonus, the vector $\Delta r$ is represented by $OA$ in the graph and its length is given by $\Delta R$. Observe that the line of this vector must cross point $E = (1, f_{own} - f_{other})$ in the diagram.

Of key interest is the relation between the first-best partnership value and the incentives generated by the tournament. This is represented by the angle $\sigma$ between the vectors $OA$ and $OPO$ in the diagram. Also of interest is the combined effect of the incentives faced by agents as a result of the mixed contract. This is equal to $OA + OB$ and is represented by the vector $OC$. This vector can also be written as:

$$OC = \left[ \frac{1}{2} + \frac{\Delta}{2\epsilon}, \frac{f_{own} + f_{other}}{2} + \frac{\Delta(f_{own} - f_{other})}{2\epsilon} \right].$$

As one can readily determine, these elements are just the optimal levels of efforts for a given prize level as given by the expressions (9a)-(9b) under the mixed contract.

Figure 1 and equation (10′) also help illustrate how to find the optimal effort level represented by point $C$ in the diagram. Finding this point is equivalent to finding the optimal length of vector $OA$ for the given vector $OB$. Inasmuch as $OA$ depends on the exogenous parameters $f_{own}$ and $f_{other}$, the direction of $OA$ is itself exogenous and consequently, the position of point $C$ on line $BGCF$ depends on the length of $OA$. Moreover, from (10′) we see that the
optimal length of the bonus incentive vector $OA$ should be equal to $\|OD\| - \|AD\|$, (i.e., (the length of $OPO$)$\cdot \cos \sigma$ – (the length of $OB$)$\cdot \cos \sigma$). This means that point $C$ must lie exactly at the intersection point of the two lines $BGCF$ and $POCD$ (where the latter vertically intersects the line $BGCF$). Utilizing Proposition 1 below, we can then show why this point must represent an optimum while other points, like $F$ and $G$, and their associated levels of effort, cannot.

**Proposition 1:** The partnership value generated by the effort pair decreases with the distance between it and the effort pair under the first-best allocation.

**Proof:** See Appendix B.

This proposition can best be understood by using Figure 1. Note that the circle centered on point $PO$ and tangent to point $C$ represents a given level of expected utility. Given the direction of vector $OA$, any movement away from point $C$ along line $BGCF$ (or away from $PO$) must result in a decrease in expected utility. Given that $BCPO$ is a right triangle and because point $C$ is closer to $PO$ than point $B$, the mixed contract will generate a higher partnership value than the pure-team contract when team synergy is low.\(^\text{12}\)

**Further results**

For the remainder of this section, we continue and complete our analysis of agent behavior under the low synergy setting. Using (10), (9a), (9b) and (9c'), we can now write, as a function of the basic parameters of the problem, the optimal levels of own and helping efforts

\(^{12}\) We also derive this result by comparing the mathematical expressions of an agent’s expected utility under the two incentive schemes in the following subsection.
and the expected utility of each agent. These expressions are stated as:

\[
\alpha_{\text{bonus\_team}} = \frac{1}{2} + \frac{1}{2} \frac{(1 + f_{\text{own}}^2 - f_{\text{other}}^2)}{1 + (f_{\text{own}} - f_{\text{other}})^2}
\]

\[(11a)\]

\[
\Phi_{\text{bonus\_team}} = \frac{f_{\text{own}} + f_{\text{other}}}{2} + \frac{(f_{\text{own}} - f_{\text{other}})(1 + f_{\text{own}}^2 - f_{\text{other}}^2)}{2(1 + (f_{\text{own}} - f_{\text{other}})^2)}
\]

\[= f_{\text{own}} + f_{\text{other}} - \frac{f_{\text{other}}}{1 + (f_{\text{own}} - f_{\text{other}})^2},\] \[(11b)\]

\[
EU_{\text{bonus\_team}} = \frac{(1 + f_{\text{own}}^2)^2 + (1 - 2f_{\text{own}}^2)f_{\text{other}}^2 + f_{\text{other}}^4}{2(1 + (f_{\text{own}} - f_{\text{other}})^2)}.
\]

\[EU_{\text{bonus\_team}}.\] \[(11c)\]

Comparing the expressions for own effort across the various contracts, we see that optimal own effort will be higher under the mixed contract than under the pure-team contract (comparing \(11a\) with \(6a\)). It can also be shown that whenever there is a high individual return to helping effort, \(f_{\text{own}} > f_{\text{other}}\), the mixed contract will induce agents to exert even higher levels of optimal own effort than they would if they had agreed and fully cooperated to implement the first-best allocation.\(^{13}\) On the other hand, optimal helping effort expressed in \(11b\) is strictly less than its Pareto optimal level, with the exception being the case where the two are equal, which only occurs when \(f_{\text{other}} = 0\). We also find at the optimum that there will be no sabotage and therefore helping effort will not be negative. Nonetheless when synergy is low, helping effort can be lower than it is under the pure-team contract (see Proposition 2 below for the relevant conditions).

We can also compare \(11c\) with \(6c\) and show that the mixed contract yields higher

\[^{13}\text{The reverse is also true, when the individual return on helping effort is low, optimal own effort under the mixed contract is strictly less than the socially efficient level of own effort.}\]
expected utility than the pure-team contract. However, the two are equal in the special case when \(1 + f_{own}^2 = f_{other}^2\). Under this condition, the distortion in incentives reaches its peak at \(\sigma = 90^\circ\) and \(\cos \sigma = 0\). In this case, the partnership might as well implement the team contract and set the bonus equal to zero and earn the same amount as the pure-team contract.

Before concluding this section, we summarize a few of the main results from the above discussion in the following proposition.

**Proposition 2:** In our comparisons of the various allocations under the alternative contracts, we are able to establish the following:

i. **When team synergy is low or not high, i.e., when** \(1 + f_{own}^2 \geq f_{other}^2\):

   i.a) \(a_{\text{bonus\_team}} \geq a_{\text{pure\_team}}\) (**equal when** \(1 + f_{own}^2 = f_{other}^2\) \(\text{and } a_{\text{bonus\_team}} \geq a_{\text{PO}}\) whenever \(f_{own} \geq f_{other}\);

   i.b) \(h_{\text{bonus\_team}} \leq h_{\text{pure\_team}}\) as \(\bar{X} < f_{own} \leq f_{other}\), where \(\bar{X} = \sqrt{f_{other}^2 - 1}\) if \(f_{other} > 1\), and \(\bar{X} = 0\) otherwise. Also \(h_{\text{bonus\_team}} \leq h_{\text{PO}}\) (**equal when** \(f_{other} = 0\)).

   i.c) \(EU_{\text{bonus\_team}} \geq EU_{\text{pure\_team}}\) (**equal when** \(1 + f_{own}^2 = f_{other}^2\)).

ii. **When team synergy is high, i.e., when** \(1 + f_{own}^2 < f_{other}^2\), and given the restriction on negative bonuses, the mixed contract reduces to the pure-team contract.

**Proof:** The proof follows along the arguments given above.

From our analysis in this section, we have shown that basing the bonus award on individual output induces a degree of distortion in the incentives agents face. That this distortion can move incentives away from the first-best partnership value can be seen from the fact that the
bonus vector has to pass through the point \((1, f_{own} - f_{other})\). Note that whenever \(f_{own} < f_{other}\), an increase in this difference between the marginal products of helping efforts increases the degree with which the tournament moves an agent’s efforts further away from partnership value. Thus in the high synergy case, when \(1 + f_{own}^2 < f_{other}^2\), implementing a tournament with any positive bonus in a team-output contract will only decrease partnership value. One way a partnership can avoid this distortion, or at least try to lower it, is to try and base the reward on an alternative performance measure, one that is more aligned with the first-best partnership value.

V. Team-based Contract with Bonus Reward based on Other Performance Measures

An alternative appraisal system

Although we have shown that a combined bonus and team contract can increase expected utility of agents in a partnership, we also know it can still fall short of what can be achieved under the first-best allocation (unless we have \(f_{other} = 0\)). This result is not surprising given that the relative performance measure based on individual output is not necessarily aligned with the first-best partnership value. It is therefore natural to consider other possible relative performance measures a partnership might use in order to motivate its members.

In this section we thus assume that the partnership utilizes an alternative “relative judgment” scheme designed to reward agent performance.\(^{14}\) Since we continue to assume that

\(^{14}\) A relative judgment scheme is a general scheme where agents are ranked in comparison with others performing similar tasks (Gomez-Mejia, Balkin, and Cardy, 1998). For our purposes in this section, we assume that agents are ranked along a variety of dimensions and categories using criteria other than individual output.
own and helping efforts remain important contributors to partnership value, we assume that the measures used by the partnership attempt to capture these efforts without replicating the contradiction that might be inherent in using individual output as a relative performance measure. For concreteness, we assume that the partnership implements a scheme based on paired comparisons or one based on a forced ranking with which to base incentive pay or promotions upon. For example, in order to reward helping effort we assume that the appraisal scheme in place is able to exogenously score and weight an agent’s efforts according to designated performance areas such as how well an agent works with others or whether an agent actively participates in meetings and group projects. The results are then tabulated and summarized for this particular category and then used along with other categories such as own effort in order to form an overall measure of the agent’s performance or effectiveness. This measure is then compared to those of other agents.

Accordingly, assume that the outcome of the partnership’s performance appraisal system for each individual agent can be represented by the following equation:

\[ z_i = ga_i + kh_i + lt_i + \theta_i. \]  

Note that in addition to own and helping efforts, we include another variable, \( t_i \), which represents an agent’s influence activities or the efforts by the agent to influence and manipulate the appraisal system in order to achieve a better outcome. Although the activity can improve

\[ \text{In paired comparison schemes, pair-wise comparisons are made among all agents of a group based on particular job characteristics. The results are then added up and arranged by rank. In a forced ranking (or forced distribution) scheme, a set number or proportion of agents are assigned to each of several possible performance categories. For example, only 10% of agents can be assigned the highest ranking. For further details, see Rudman (2003).} \]

15
one’s overall rating, we assume that the activity comes at a cost to an individual and does not contribute to individual and team output.\textsuperscript{16} The parameters \(g, k, l\) represent the weights that the performance scheme places on the three efforts (though we shall still refer to these parameters as the marginal products of an agent’s efforts). All are assumed to be nonnegative.\textsuperscript{17} Finally, we assume that the random component of the performance appraisal system, \(\theta_i\), is uniformly distributed over the interval \([-\overline{\theta}, +\overline{\theta}]\) and \(i.i.d.\) for all agents and may be correlated with \(\varepsilon_i\), the shock term for individual output, in which case, the correlation between the two is given by \(\zeta\).

As in previous sections, our analysis can proceed along similar lines. With the added possibility of an agent exerting (unproductive) influence, let the agent’s costs now be given by:

\[
C(a_i, h_i, t_i) = \frac{a_i^2 + h_i^2 + t_i^2}{2} \quad i = 1, 2. \tag{2'}
\]

With “a” denoting “alternative” in the superscript, the agent’s problem can be written as:

\textsuperscript{16} In practice, these activities may involve attempts by agents to ingratiate themselves with their partners (if the tournament result is also affected by evaluations from coworkers) or a third party and politicking in order to exploit a possible institutional bias in the partnership’s appraisal system.

\textsuperscript{17} The nonrandom component of an individual agent’s overall performance measure, \(s_i\), and the marginal products of an agent’s efforts on this measure can be viewed as the final combination of all \((n)\) measures that the partnership might use. For example, if each effort has three different performance areas or measures, then the appraisal system that puts different weights on these measures might generate the following result: \(s_i^1 = 2a_i + 5h_i + 3t_i\), \(s_i^2 = 4a_i + 1h_i + 2t_i\), and \(s_i^3 = 5a_i + 3h_i + 1t_i\) so that \(s_i = 2s_i^1 + s_i^2 + 4s_i^3 = (4 + 4 + 20)a_i + (10 + 1 + 12)h_i + (6 + 2 + 4)t_i\).
where agent $i$ can win bonus $\Delta$ with probability $\tilde{P}_i()$ if $z_i$ is higher than $z_j$. Solving (13) then yields an agent’s optimal effort levels and level of expected utility for a given prize:

\[
\begin{align*}
\alpha^a_{\text{bonus team}} &= \frac{1}{2} + \frac{\Delta g}{2\vartheta}, \\
h^a_{\text{bonus team}} &= \frac{f_{\text{own}} + f_{\text{other}} + \Delta k}{2} + \frac{\Delta l}{2\vartheta}, \\
l^a_{\text{bonus team}} &= \frac{\Delta l}{2\vartheta}, \\
EU^a_{\text{bonus team}} &= -\frac{8^2 + k^2 + l^2}{8\vartheta^2} \left[ \Delta - \frac{\vartheta \left[ g + k \left( f_{\text{own}} + f_{\text{other}} \right) \right]}{8^2 + k^2 + l^2} \right]^2 \\
&\quad + \frac{3 \left[ 1 + \left( f_{\text{own}} + f_{\text{other}} \right)^2 \right]}{8} + \frac{\vartheta \left[ g + k \left( f_{\text{own}} + f_{\text{other}} \right) \right]^2}{8 \left( 8^2 + k^2 + l^2 \right)}.
\end{align*}
\]

One immediate difference between this scheme and the previous one is that there is no possibility of the combined scheme imposing negative incentives on an agent’s helping efforts. This result will remain true as long as the bonus is nonnegative. Similar to the expression of $EU^{\text{bonus team}}$ in (9c‘), $EU^a_{\text{bonus team}}$ is also a concave function of $\Delta$. From (14d) we can then obtain the following expression for the optimal bonus:

\[
\Delta^a_{\text{bonus team}} = \frac{\vartheta \left[ g + k \left( f_{\text{own}} + f_{\text{other}} \right) \right]}{g^2 + k^2 + l^2}.
\]

Note that unlike equation (10) in the high synergy setting, the bonus under the alternative incentive scheme is nonnegative. Comparing (15) and (10), we can observe that the marginal products of own and helping efforts in terms of production and in terms of the performance measures under this incentive scheme are now separate measures and that the marginal products
of helping efforts in terms of production, no longer pull in different directions. Utilizing a similar graphical framework as in the previous section, we precede to further investigate the implications of these differences.

**Graphical analysis**

As before, we can define the vector of the first-best partnership value as \( \vec{w} \) and that of the marginal products of an agent’s efforts on the absolute team performance measure as \( \vec{m} \). Since the two are exactly aligned, we have \( \vec{w} = \vec{m} = (1, f_{\text{own}} + f_{\text{other}}, 0) \). Now that there are three efforts, the space has been enlarged to three dimensions, but given the fact that influence activities does not affect either measure, the \( r^{th} \)-coordinate of these vectors is zero. The length of the two vectors are given by \( W \) and \( M \). With respect to the other measure of interest, let \( \vec{r} \) be the vector of marginal products of efforts on the relative performance measure, with \( \vec{r} = \left( \frac{g}{2\theta}, \frac{k}{2\theta}, \frac{l}{2\theta} \right) \). and let \( R \) denote its length: \( R = \sqrt{\frac{g^2 + k^2 + l^2}{2\theta}} \). Thus (15) can be rewritten as:

\[
\Delta^{a, \text{bonus_team}} = \frac{\vec{w} \cdot \vec{r}}{R^2} - \frac{\vec{m} \cdot \vec{r}}{2R^2} = \frac{\tilde{W} \cos \tilde{\sigma}}{\tilde{R}} - \frac{\tilde{M} \cos \tilde{\sigma}}{2\tilde{R}}
\]

(15’)

where \( \tilde{\sigma} \) represents the angle between vectors \( \tilde{w} \) and \( \tilde{r} \) and between \( \tilde{m} \) and \( \tilde{r} \) (since \( \tilde{m} = \tilde{w} \)).

One similarity between (15’) and (10’), is the fact that the tournament scheme introduces a distortion between the first-best partnership value and the incentives the agents face with the use of the relative performance measure. Nonetheless, under the present incentive scheme, the first-best allocation remains feasible whereas under the previous incentive scheme, such an
outcome is not possible unless \( f_{\text{other}} = 0 \). Moreover, under the present scheme, this goal could be accomplished if the partnership were free to calibrate its performance appraisal system. A proper choice or redesign of the ranking and weighting of agent efforts will help reduce the distortion that agents face and enable the partnership to move closer to the first-best allocation of efforts, until ultimately, \( \cos \tilde{\sigma} = 1 \). We return to this possibility once we complete our analysis of equation (15').

Although our graph, Figure 2, is now three-dimensional, most of the analysis remains unchanged from the previous section. As before, \( OPO \) is the first-best partnership value vector and point PO represents the allocation associated with Pareto optimal levels of effort. The vector associated with the incentives induced by the equal-sharing contract is \( OB \) and as in Figure 1, this vector coincides with \( OPO \) until point \( B \). The bonus incentive vector, \( \Delta \tilde{\varepsilon} \), with length \( \Delta \tilde{\theta} \), is given by \( OA \) in the diagram. The angle between this vector and \( OPO \) is depicted as \( \tilde{\sigma} \) and since \( OA \) resides in three-dimensional effort space, where efforts are nonnegative, \( \tilde{\sigma} \) cannot be greater than 90°. The vector of combined incentives, equal to \( OA + OB \), and depicted as \( OC \) in the diagram, is also given by the following expression:

\[
OC = \left( \frac{1}{2} + \frac{\Delta g}{2 \tilde{\theta}} + \frac{f_{\text{own}} + f_{\text{other}}}{2} + \frac{\Delta k}{2 \tilde{\theta}} + \frac{\Delta l}{2 \tilde{\theta}} \right).
\]

Also note that point \( C \) represents the optimal efforts of an agent under this contract and that this point can be viewed as lying somewhere along a line projecting out from the \( a-h \) plane towards the \( a-t \) plane (or as shown in Figure 2, in the middle of the line \( POCD \)).

\[
<<< \text{Insert Figure 2 Here} >>>
\]

\textsuperscript{18} In this special case, the relative performance measure automatically lines up with the first-best partnership value.
The reasoning of (15') can also be illustrated by following the logic of Proposition 1 given that a similar proposition holds here. However in this section, the given (iso-) levels of expected utility (not shown) would be depicted by a series of homocentric spheres centered around the point \((1, f_{own} + f_{other}, 0)\). Thus for a given sized sphere, point \(C\) represents the optimal effort levels under the alternative-mixed contract since the choice of any other point must result in a larger-sized sphere and a decrease in expected utility. In short, point \(C\) is the closest point to PO on the line \(BC\) and the optimal bonus is determined by the overall length of \(OA\).

**Further Analysis**

Substituting the solution of the optimal bonus as given by (15) into equations (14a)-(14d) yields the optimal effort levels \(a, h, t,\) and the expected utility for the agent under the mixed contract with alternative relative performance measures:

\[
a^{a,\text{bonus, team}}^* = \frac{1}{2} + \frac{g + k\left(f_{own} + f_{other}\right)}{2\left(g^2 + k^2 + l^2\right)}, \tag{16a}
\]

\[
h^{a,\text{bonus, team}}^* = \frac{f_{own} + f_{other} + k\left[g + k\left(f_{own} + f_{other}\right)\right]}{2\left(g^2 + k^2 + l^2\right)}, \tag{16b}
\]

\[
t^{a,\text{bonus, team}}^* = \frac{l\left[g + k\left(f_{own} + f_{other}\right)\right]}{2\left(g^2 + k^2 + l^2\right)}, \tag{16c}
\]

\[
EU^{a,\text{bonus, team}}^* = \frac{3\left[1 + \left(f_{own} + f_{other}\right)^2\right]}{8} + \frac{\left[g + k\left(f_{own} + f_{other}\right)\right]^2}{8\left(g^2 + k^2 + l^2\right)}. \tag{16d}
\]

Rather than compare effort levels across contracts, observe that the term associated with the bonus in expressions (16a)-(16c) differ only by the marginal product associated with that effort
level. For example, the only difference in this term between the optimal solutions for own effort and helping effort are the marginal products $g$ and $k$. Although we have assumed that the partnership’s appraisal system is exogenous, it is easy enough to imagine that if there were a prior design stage, the agents in the partnership would seek to choose an appropriate mix of scoring and weighting of the performance areas such that ultimately $g$, $k$, and $l$ lead to minimal distortion on agent incentives. If chosen properly, agents would then be induced into exerting levels of effort that are first best since $\cos \bar{\sigma} = 1$.

From (16d) we can easily observe that expected utility is positive. Moreover, we can see how utility changes as the marginal products of an agent’s effort changes. An increase in $g$ leads to an increase in utility as long as $g (f_{own} + f_{other}) - k < \frac{l^2}{k}$ holds, and decreases when the inequality is reversed. With respect to the marginal product of helping efforts, the agent’s expected utility increases with an increase in $k$ whenever the following condition holds:

$$g (f_{own} + f_{other}) - k > -\frac{l^2 (f_{own} + f_{other})}{g},$$

and decreases when this condition fails. Not surprisingly, expected utility is always decreasing in $l$, as a result of an agent’s influence activities.

By comparing (16) with the efficient benchmark in (4), we find that when

$k < g (f_{own} + f_{other})$ and $l < \sqrt{gk (f_{own} + f_{other}) - k^2}$, an agent’s own effort is greater than the efficient level. However, when $k > g (f_{own} + f_{other})$ and $l < \sqrt{\frac{gk - g^2 (f_{own} + f_{other})}{f_{own} + f_{other}}}$, the optimal helping effort is greater than the first-best level while own effort under this contract is less than efficient. More importantly, when $k = g (f_{own} + f_{other})$ and $l = 0$, the expected total utility
coincides with that under first-best, in which case, the bonus incentive vector overlaps the first-best partnership value vector and point $C$ coincides with point $PO$. Thus, if the partnership can adequately develop its performance appraisal system so that it is exactly aligned with the first-best partnership value, then the efficient outcome can be achieved.\footnote{Although we do not specify how this process might be carried out or what it entails, ultimately the system would have to generate the marginal products of an agent’s efforts that are consistent with each effort’s actual contribution to partnership value such that both $k = g \left( f_{own} + f_{other} \right)$ and $l = 0$ hold. For example, these two conditions can be satisfied by setting: $g = 1, \ k = f_{own} + f_{other}$, and $l = 0$.} Also note that the above results are unrelated to $\zeta$, the correlation between $\theta_i$ and $\epsilon_i$. This result is consistent with Baker’s (2002) finding that the quality of the performance measure does not depend on whether it is correlated with firm value but on whether or not it is aligned with firm value.\footnote{Although in the present paper, our focus is of course, on partnership value.}

In comparing the expected utility under the present contract with that under the pure-team contract, we find that the former is always higher than the latter. However the two are equal when $g = k = 0$, in which case there is no bonus since we have $\sigma = 90^\circ$. Both of these possible outcomes can be shown using Figure 2. Observe that $CPO$ is always shorter than $BPO$. However $CPO$ can be equal to $BPO$ when points $C$ and $B$ coincide and $\sigma = 90^\circ$. From Proposition 1, we also know that point $C$ cannot yield lower utility than point $B$. We can therefore state the following proposition.

**Proposition 3:** The alternative-mixed contract always yields higher partnership value (or equal
partnership value when \( g = k = 0 \) than the pure-team contract.

**Proof:** The proof follows directly from the above analysis.

Our results are ambiguous when comparing the expected utility under the alternative mixed contract with that under the mixed contract of Section IV where the bonus is based on individual output. From Figures 1 and 2, we can see that as long as the length of CPO in Figure 2 is shorter than that in Figure 1, it is better to use the alternative contract according to Proposition 1. The following proposition gives the conditions under which this can occur.

**Proposition 4:** The alternative-mixed contract yields higher partnership value than the mixed contract with the bonus based on individual output when

\[
\begin{align*}
i) & \quad f_{\text{other}} < \overline{A}, \; g \left( f_{\text{own}} - f_{\text{other}} \right) < k < \overline{B}, \; \text{and} \; l < \overline{C}, \; \text{or} \\
ii) & \quad f_{\text{other}} \geq \overline{A}, \; g \left( f_{\text{own}} - f_{\text{other}} \right) < k, \; \text{and} \; l < \overline{C},
\end{align*}
\]

where

\[
\overline{A} = \sqrt{\frac{3 + 4f_{\text{own}}^2 - f_{\text{own}}^3}{3}},
\]

\[
\overline{B} = \frac{8 \left[ f_{\text{own}} \left( 1 + f_{\text{own}}^2 \right) + f_{\text{other}} \left( 3 + f_{\text{own}}^2 \right) - f_{\text{own}}^2 f_{\text{other}} - f_{\text{other}}^3 \right]}{1 + \left( f_{\text{own}} - 3f_{\text{other}} \right) \left( f_{\text{own}} + f_{\text{other}} \right)}, \; \text{and}
\]

\[
\overline{C} = \sqrt{\left( g f_{\text{own}} - g f_{\text{other}} - k \right) \left[ g \left( f_{\text{other}}^2 - f_{\text{own}}^2 \right) \left( f_{\text{own}} + f_{\text{other}} \right) - f_{\text{own}}^2 - 3f_{\text{other}}^2 \right] + k \left( f_{\text{own}} - 3f_{\text{other}} \right) \left( f_{\text{own}} + f_{\text{other}} \right) \left( f_{\text{own}}^2 - f_{\text{other}}^2 \right)}.
\]

The condition, \( l < \overline{C} \), common to both (i) and (ii), is the simple requirement that influence activities cannot exceed a certain level. With respect to the remaining conditions, we
can interpret these in terms of Figures 1 and 2. The other common condition among (i) and (ii),
\[ g(f_{own} - f_{other}) < k \] (which can also be expressed as \( \frac{f_{own} - f_{other}}{1} < \frac{k}{g} \)), requires the mapping of
vector OA onto the a-h plane in Figure 2 to be closer to the vector OPO relative to their positions
in Figure 1. However, whenever the impact of helping effort on the other team member is
sufficiently low, \( f_{other} < \bar{A} \), the distortion induced by the mixed contract with the tournament
based on individual output is limited so that in order for the alternative-mixed contract to deliver
higher levels of total utility, the performance measure on helping effort must exceed
\[ g(f_{own} + f_{other}) \] but must also be less than \( \bar{B} : k < \bar{B} \). On the other hand, when the impact of
helping effort is quite high in terms of the other agent’s output, \( f_{other} \geq \bar{A} \), the distortion
generated by the alternative-mixed contract will tend to be lower than that induced by the team
contract with the bonus based on individual output. In this case, no further restrictions are
required for the performance parameters under the alternative-mixed contract.

VI. Conclusion

Our goals were to examine under what conditions the incorporation of a relative bonus
reward with an equal-sharing team-output contract can improve efficiency in a partnership. We
have shown that the typical tournament scheme based on individual output can improve
efficiency when compared to the pure-team contract in the low-team-synergy setting. However,
the fact that the mixed contract can still distort the incentives that agents face seems to suggest
that partnerships might do better if they can base their tournaments on alternative performance
measures. To this end, we have shown that a partnership can indeed improve upon the mixed
team contract with the tournament based on individual output. The conditions with which this can occur are stated in the form of our Proposition 4.

Our findings also have important implications where team synergy is high. In human-capital intensive partnerships, such as law firms, incorporating a relative bonus reward based on individual output along with equal sharing may only serve to reduce efficiency. Our results show that if agents can base a tournament on an appraisal system that is more aligned with partnership value, they can increase total utility. However, in a partnership without a third party to supervise the tournament, or because of excessive politicking that might be involved when the results of the tournament depend on the mutual evaluation among team members, the alternative mixed contract may be difficult to implement in practice. In such a case, an equal-sharing scheme would tend to be the prevailing incentive scheme. In contrast, in a partnership run by an independent office manager and where the different aspects of a member’s performance (including how she assists teammates) can be measured and evaluated with minimal political interference, a partnership may be better off with an alternative mixed incentive scheme despite the presence of high team synergies.

Finally, our analysis of the paper also seems to suggest that as partnerships face changes in technology and production relationships, they will have incentives to revisit their performance appraisal systems so as to ensure that these productivity and efficiency gains are more fully realized in the workplace.
Appendix

Appendix A

In this part of the Appendix, we closely follow Harbring and Irlenbusch (2005, 2008) and derive the marginal probabilities of winning for own effort and helping effort in equilibrium under the mixed contract of Section IV.\textsuperscript{21}

First, recall that the random shock terms of individual outputs, $\varepsilon_i$ and $\varepsilon_j$, are $i.i.d.$ and uniformly distributed over the interval $[-\overline{\varepsilon},+\overline{\varepsilon}]$. Denote the probability density function of $\varepsilon_i$ (or $\varepsilon_j$) as $f(\varepsilon)$, and the cumulative distribution function as $F(\varepsilon)$. We can then write each agent’s density function as $f(\varepsilon) = \frac{1}{2\overline{\varepsilon}}$.

In addition, in order for agent $i$ to win the tournament, agent $i$’s individual output must be higher than agent $j$’s. Given that output is a function of agent efforts, let the probability of winning for agent $i$ be defined as $P((a_i, h_i, a_j, h_j) = P(y_i > y_j)$ or equivalently,

$$P(a_i + f_{own} h_i + f_{other} h^* + \varepsilon_i > a^* + f_{own} h^* + f_{other} h_i + \varepsilon_j)$$

$$= P(a_i + f_{own} h_i + f_{other} h^* - a^* - f_{own} h^* - f_{other} h_i > \varepsilon_j - \varepsilon_i)$$

$$= F_X(a_i + f_{own} h_i + f_{other} h^* - a^* - f_{own} h^* - f_{other} h_i)$$

(A1)

where $X \equiv \varepsilon_j - \varepsilon_i$, $F_X(x)$ is the cumulative distribution function of $X$, $f_X(x)$ is the probability density of $X$, and $x \equiv a_i + f_{own} h_i + f_{other} h^* - a^* - f_{own} h^* - f_{other} h_i$. Also note, that $a^*$ and $h^*$ represent agent $j$’s equilibrium level of efforts.

\textsuperscript{21} Derivation of the equilibrium marginal probabilities under the alternative mixed contract of Section V follows along similar lines.
Since $X = \varepsilon_j - \varepsilon_i$ and $\varepsilon_i$ and $\varepsilon_j$ are i.i.d., the following holds:

$$f_X(x) = \frac{\partial F_X(x)}{\partial x} = \int_{x=-\varepsilon_i}^x f(\varepsilon_i) f(\varepsilon_i + x) d\varepsilon_i = \int_{x=-\varepsilon_i}^x \frac{1}{(2\varepsilon_i)^2} d\varepsilon_i. \quad (A2)$$

As in Harbring and Irlenbusch (2005, 2008), we can also write: $-\varepsilon \leq \varepsilon_i \leq \varepsilon$ and $-\varepsilon \leq \varepsilon_j \leq \varepsilon_i + x \leq \varepsilon \Leftrightarrow -x - \varepsilon \leq \varepsilon_i \leq \varepsilon - x$. Moreover, since the random variable $X$ is distributed over the interval $[-2\varepsilon, 2\varepsilon]$, it can be divided into two subintervals: $[-2\varepsilon, 0]$ and $[0, 2\varepsilon]$, so that when $x \leq 0$, $-x - \varepsilon \leq \varepsilon_i \leq \varepsilon$ and when $x > 0$, $-\varepsilon \leq \varepsilon_i \leq \varepsilon - x$. Therefore $f_X(x)$ can be broken down as:

$$f_X(x) = \int_{-x-\varepsilon}^{\varepsilon} \frac{1}{(2\varepsilon_i)^2} d\varepsilon_i \quad \text{if} \quad -2\varepsilon \leq x \leq 0,$$

and

$$f_X(x) = \int_{-\varepsilon}^{-x} \frac{1}{(2\varepsilon_i)^2} d\varepsilon_i \quad \text{if} \quad 0 < x \leq 2\varepsilon.$$

Given that both agents exert the same level of efforts in a symmetric equilibrium, the following result obtains since $x = 0$:

$$f_X(0) = \int_{-\varepsilon}^{\varepsilon} \frac{1}{(2\varepsilon_i)^2} d\varepsilon_i = \frac{1}{(2\varepsilon_i)^2} \int_{-\varepsilon}^{\varepsilon} d\varepsilon_i = \frac{\varepsilon - (-\varepsilon)}{(2\varepsilon)^2} = \frac{1}{2\varepsilon}. \quad (A3)$$

Finally, the marginal probabilities of winning for both own and helping effort in equilibrium can be derived as follows. The marginal probability for own effort is given by:

$$\frac{\partial P}{\partial a_i} = \frac{\partial F_X(x)}{\partial a_i} = \frac{\partial F_X(x)}{\partial x} \frac{\partial x}{\partial a_i} = f_X(x) \frac{\partial x}{\partial a_i}.$$

Since $\frac{\partial x}{\partial a_i} = 1$ and equilibrium entails $x = 0$, we have, according to (A3), $\frac{\partial P}{\partial a_i} = \frac{1}{2\varepsilon}$. Similarly, the marginal probability of winning for helping effort is given by:
\[ \frac{\partial P_i}{\partial h_i} = \frac{\partial F_x(x)}{\partial h_i} = \frac{\partial F_x(x)}{\partial x} \frac{\partial x}{\partial h_i} = f_{xx}(x) \frac{\partial x}{\partial h_i}. \]

Since \[ \frac{\partial x}{\partial h_i} = f_{own} - f_{other} \] and in equilibrium \( x = 0 \), we obtain: \[ \frac{\partial P_i}{\partial h_i} = \frac{f_{own} - f_{other}}{2\bar{\epsilon}}. \]

Appendix B

In this section, we prove Proposition 1 in the text, which is stated below for convenience.

**Proposition 1:** The partnership value generated by the effort pair decreases with the distance between it and the effort pair under the first-best allocation.

**Proof:** In a symmetric equilibrium, we can focus on an agent’s expected utility generated by that agent’s pair of efforts. This is equal to \[ a + (f_{own} + f_{other})h - \frac{a^2 + h^2}{2}. \] To find the iso-curve, we therefore set \[ a + (f_{own} + f_{other})h - \frac{a^2 + h^2}{2} = E\bar{U} \] where we define \( E\bar{U} \equiv (1/2)EV \)

(where \( EV \) represents partnership value or equivalently the expected total utility of both agents).

This equation can be rewritten as \( (a - 1)^2 + \left[ h - (f_{own} + f_{other}) \right]^2 = 1 + (f_{own} + f_{other})^2 - 2E\bar{U}. \)

From this expression, we can readily see that the family of iso-curves, each depending on different values of \( E\bar{U} \), are a series of homocentric circles centered around the point \( (1, f_{own} + f_{other}) \). Also note that the size of the circle decreases with \( E\bar{U} \) since the right-hand side of the equation is the square of the radius of the circle. Therefore, the expected total utility generated by the effort pair is decreasing in the distance between the point represented by effort pair and the effort pair represented by point PO.
References


Figure 1
Figure 2

![Diagram](https://via.placeholder.com/150)

- $\Delta l/2\theta$
- $\Delta g/2\theta$
- $f_{own} + f_{other}/2$
- $f_{own} + f_{other}$