A Simple Model of a Monetary Union*

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February 12, 2010

Abstract

This paper explicitly models strategic interaction between two independent national fiscal authorities and a single central bank in a simple New Keynesian model of a monetary union. Closed analytical solutions for the policy instruments are computed for several strategic games. In this set-up the analytical results of a monetary leadership regime coincides with the one of a Nash game. The analytical results depend highly non-linear on parameters of the model. Thus, impulse response graphs to various shocks are discussed. It depends on the shock (symmetric or asymmetric) whether the policy regime of fiscal leadership performs better than the one of a Nash game.

JEL-Classification: E62, E63, F33

Keywords: Monetary Union, Fiscal Policy, Non-coordination

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*I would like to thank participants of the 13th International Conference on Macroeconomic Analysis and International Finance 2009, Rethymno, Crete, the Scottish Economic Society Annual Conference 2009 and the Nagoya-Freiburg Joint Seminar 2008, Nagoya, Japan for valuable comments which helped to improve a previous version of this paper.

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1 Introduction

The New Keynesian approach is widely used in macroeconomics to analyze monetary policy. There is hardly any disagreement about the role and functions of monetary policy in this consensus approach.$^1$ This kind of model has been criticized for many missing elements and work is still in progress to overcome these criticism. But one of the main weaknesses of the New Keynesian models is the absence of an explicit role for fiscal policy.$^2$ This is at odds with the large size of the public sector in modern economics, and the increasing role governments play for the solution of the current financial crisis and the related great recession.$^3$ Christiano et al. (2009) and Woodford (2010) among others start to analyze the increasing role of fiscal policy due to the 2007-2009 crisis. Monetary policy is ineffective in two cases, first, if the economy is in a liquidity trap and second, if a monetary union is hit by asymmetric shocks which cancel on the union level. This implies that fiscal policy might play a more prominent role.

With the formation of the European Monetary Union (EMU) in 1999 macroeconomic conditions have changed for all EU member countries. By handing over monetary policy to an independent and unique central bank for all members national governments cannot use monetary policy as a stabilization tool any longer. On matters of fiscal policy each member country is assumed to maintain the autonomy of its own fiscal policy, but the Stability and Growth pact (SGP) with its thresholds for debt-GDP ratio of 60% and deficit-GDP ratio of 3% was intended to discipline national governments to ensure monetary stability. Thus, having to obey the fiscal constraints of the SGP the stabilization role of fiscal policy is hampered. As a consequence a very important question concerning the optimal monetary-fiscal policy regime in the current policy framework of the EMU is whether there is still a role for fiscal policies to serve as potential stabilization tools of cyclical fluctuations.

However, fiscal policy is conducted to serve national interests rather than union interest. But in a highly integrated region like a monetary union the stabilization of asymmetric shocks might lead to spill-overs on other members. This raises the question whether a coordination of fiscal policies is more likely to be welfare enhancing. Moreover, it might be possible for fiscal authorities to strengthen their policy tools if the governments coordinate.

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$^1$During the last years there has been an extensive research agenda concerning monetary policy in a closed economy within the framework of a micro-founded dynamic general equilibrium model, as can be found, for example, in the well-known books by Woodford (2003) or Galí (2008).

$^2$Moreover, different conclusions about the stabilisation effects of fiscal policy can be found in the literature. But mainstream contributions assume that the Ricardian equivalence hypothesis holds which supports the limited use of fiscal policy.

$^3$The recession of the Japanese economy of the 1990s where a zero interest rate prevailed for many years and with an ineffective monetary policy is another example when fiscal policy was called for help (Bernanke, 2003, Krugman, 2004).
This paper tries to give answers to these questions. To be more precise, different strategic games between two fiscal authorities and the common central bank are studied with the help of a simple consensus model which builds on advances made in the analysis of monetary policy, namely the New Keynesian model with imperfect competition, price rigidities and rational expectations.

The focus is on a two-country model of a monetary union. Each country is hit by cost-push shocks or deficit spending shocks. National governments set government spending financed by lump-sum taxes. Foremost the paper abstracts from debt dynamics. The outcome of different kind of games played by the fiscal and monetary authorities are explored. It is assumed that governments and the central bank maximize appropriate welfare functions in order to set their instruments optimally, but policy authorities do not follow rules for fiscal and monetary policy. Moreover, all policy makers cannot commit to future policies on assumption. Therefore, the focus is on discretionary policy making. In contrast to the current research it is allowed for non-coordination focusing on a Nash equilibrium. This result is compared to different other possible strategic interactions between independent national fiscal policies and a centralized monetary policy.

Previous literature on this issue assumes coordination of both monetary and national fiscal policies, i.e. the existence of a supra-national authority which decides how to set instruments. Both monetary policy and fiscal policy within New Keynesian models for one single country are considered by Schmitt-Groh and Uribe (2006) or Benigno and Woodford (2003) among others. In the field of open economy macroeconomic literature focusing on currency areas modeled with the help of general dynamic equilibrium models with micro-foundation, Benigno (2004) investigates in a two-country model how monetary policy should be conducted in a monetary union which is hit by asymmetric shocks across regions. Beetsma and Jensen (2004) and Beetsma and Jensen (2005) extend the above two-country model including fiscal policy in form of government spending as an active stabilization tool. They analyze the performance of several monetary and fiscal policy rules in a currency union where all three authorities coordinate their policies in order to maximize union-wide welfare. Ferrero (2009) uses a similar set-up but in his model exogenous government spending is financed by distortionary taxes and riskless bonds. He determines optimal fiscal and monetary rules when policy is conducted in a coordinated fashion. Galí and Monacelli (2008) discuss the benchmark case of coordination when the monetary union consists of a continuum of small open economies. Forlati (2009)...

Lambertini (2007) analyzes optimal fiscal policy (i.e. solution under commitment of the Ramsey problem) in a monetary union when the central bank follows an interest rate rule. In this set-up governments set labor income taxes and issue public debt to finance stochastic government spending. It is optimal for governments to run a deficit and raise income taxes. Public debt is not inflated away by monetary policy as the central bank follows a Taylor rule,
but follows a random walk.\(^4\).

The main contribution of this paper is to characterize the solution to the problem in analytical form. In order to find analytical closed solutions to different policy regimes (benchmark scenario of full coordination, the case of non-coordination Nash regime, or fiscal leadership and monetary leadership regimes) a simple consensus model of the New Keynesian framework is used. The set-up is kept as simple as possible since more dynamics in a model make it harder to find analytical solutions. Moreover, this model better helps to identify and understand transmission mechanisms of different policies.

The key findings of this paper are the following: First, it is noteworthy that the analytical results in case of monetary leadership coincide with the ones for the Nash regime of non-coordination. Second, as in a closed economy, monetary policy focuses on aggregate shocks and faces the same trade-off between inflation rate stabilization and output gap stabilization on the union level in all regimes. But interest rates are set differently as the central bank takes into account how aggregate fiscal policy reacts to the shocks. Third, results of the instruments and country-specific output gap and inflation rate depend on the country size in the case of the Nash game and the fiscal leadership game.

Though the most simplest form of equations has been chosen some of the analytical results are no longer straightforward and depend non-linearly on parameters of the model. To visualize the results the model is calibrated and and impulse responses to the underlying shocks are plotted. It depends on the shock whether the policy regime of fiscal leadership performs better than the one of a Nash game. Moreover, in the case of symmetric shocks, impulse responses for the country-specific output gap and inflation rate to a cost-push are identical for all policy regimes if both countries are equally sized. Only the reactions of the policy instruments differ. This result still holds in the case of the benchmark case and the non-coordination case if the countries are of different size.

The rest of the paper is organized as follows. Section 2 introduces the key equations of the economic model. The next section 3 introduces the policy problem of the economy. In section 4 the case of joint coordination is analyzed whereas in sections 5 to 7 the different policy regimes are analyzed and analytical results are obtained. Section 8 compares welfare effects of the various regimes. Impulse response functions to the different shocks are depicted in section 9. The last section concludes.

\(^4\)Older contributions do not assume fully micro-founded welfare criteria, like Dixit and Lambertini (2003) or Uhlig (2003) among others
2 A consensus model

In this section a standard-version of the New Keynesian model like the one in Clarida et al. (1999) is laid out. The core of the model can be summarized in three equations, a New Keynesian IS-curve, a dynamic Phillips curve and an equation which describes the behavior of the policy maker, usually a Taylor rule for the central bank. As the aim of the paper is the analysis of discretionary policy-making which is not based on the following of rules the latter equation will be replaced by an optimal response of the policy maker in the section about policy making.

A monetary union consists of two countries, the H(ome) and the F(oreign) country. The economic conditions of each member country \( j = H, F \) of the monetary union can be described by an aggregate demand (IS-) curve and an aggregate supply (or Phillips) curve which are given by:

\[
x_j = E_t x_{t+1}^j - \varphi(\bar{\iota}_t - E_t \pi_{t+1}^j) + g_t^j \quad \text{(1)}
\]

\[
\pi_j = \beta E_t \pi_{t+1}^j + \lambda x_t^j + u_t^j \quad \text{(2)}
\]

All variables should be read as deviations from their respective values at an efficient steady state.

County-specific demand is related inversely to the nominal interest rate \( \bar{\iota} \) which is set by the monetary policy and which is the same for both countries. So, real interest rates differ due to possible different inflation expectations. \( g_t^j \) is a demand shock. Clarida et al. (1999) interpret this as government spending or tax cuts. In this paper a slightly different interpretation is used. The variable \( g_t^j \) should be read as government deficits, so an increase in \( g_t^j \) is due to either an increase in governments spending or a tax cut. Governments try to run a balanced budget, but as in Uhlig (2003) the assumption is imposed that governments are hit by fiscal shocks \( \epsilon_t^j \) to their budget constraint. Though the model is dynamic in nature, debt dynamics are not included at this stage because introducing debts makes the set-up more complicated with the results that it is harder to obtain closed analytical results. The role of \( \epsilon_t^j \) gets clear when introducing policy objectives.

Inflation dynamics are driven by forward-looking elements, the output gap and a country-specific cost-push shock \( u_t^j \). Both shocks \( \epsilon_t^j \) and \( u_t^j \) follow AR(1)-processes, i.e. \( u_t^j = \rho^j u_{t-1}^j + \epsilon_t^j \) with \( \rho^j \in (0, 1) \). \( \epsilon_t^j \) is i.i.d. with \( E_{t-1} \epsilon_t^j = 0 \).

Before proceeding the following useful notation for a generic variable \( x \) is introduced. With \( x^W \equiv n x^H + (1-n) x^F \) the aggregate level of a variable, i.e. a weighted average of the Home and the Foreign variable is denoted, whereas \( x^R \equiv x^F - x^H \) denotes the relative level. Using
this notation a variable for the *H*ome respectively for the *F*oreign country can be expressed as:

\[ x^H = x^W - (1 - n)x^R \quad (3) \]

\[ x^F = x^W + nx^R. \quad (4) \]

### 3 The policy problem

Two independent governments and an independent monetary authority, a central bank in the monetary union are considered. As policy authorities do not set their instruments by following a rule, but choose their instruments optimally by maximizing an appropriate welfare criterion, a loss function to each of the three players is assigned.

These will be postulated *ad hoc* in this paper in contrast to the current approach in research of deriving a welfare criterion based on a second-order Taylor approximation of a weighted average of utilities of all consumers in the union. In general, this leads to a quadratic loss function that measures welfare losses of deviations from an efficient steady state.

The *common central bank* focuses on aggregate variables of the monetary union and thus includes the aggregate inflation rate \( \pi^W_t \) and the aggregate output gap \( x^W_t \) in the central bank’s loss function. Instrument is the nominal interest rate.

\[
\max_{\bar{\iota}_t} -\frac{1}{2} \left( \alpha(x^W_t)^2 + (\pi^W_t)^2 \right)\tag{5}
\]

where \( \alpha > 0 \) denotes the relative weight of inflation over output.\(^5\)

In contrast, the *fiscal authorities* focus on national output, are not concerned about inflation, but, as in Uhlig (2003), they include deviations of government spending \( g^j_t, j = H, F \) from the shock \( \varepsilon \) into their loss function. Instrument is government spending \( g^j_t, j = H, F \). They finance this stream of public spending \( g^j_t, j = H, F \) for goods produced in the own country (complete home bias) by collecting lump-sum taxes so that government budget balances each period.

\(^5\)Usually in this set-up from a fully micro-founded model, the weights depend on the parameters of the model. For example, the weight on output gap fluctuations is the ratio \( \lambda/\epsilon \), where \( \epsilon \) is the price elasticity that price setting firms face (Woodford, 2003).
\[
\max_{g_t^i} -\frac{1}{2} \left( (x_t^i)^2 + \theta (g_t^i - \varepsilon_t^i)^2 \right)
\] 

As all three policy makers do not coordinate in setting their instruments the timing of policy actions is relevant for the analysis. Nash, leadership and coordinated equilibria are analyzed.

Arguments for and against each kind of game can be derived. A Nash equilibrium in a monetary union seems plausible as actual policy reveals that policy authorities hardly coordinate. Rather they set their instruments independently of each other.

If one is convinced of the argument that time-lags in fiscal policy between recognition of a shock and hence a necessity to implement a new policy action and the implementation itself, are longer than in monetary policy, a Stackelberg game with the central bank as a leader (in setting its instruments) and the two governments reacting to the monetary decision is the appropriate framework to analyze stabilization policy.

Finally, the argument that monetary policy reacts on the decisions made by fiscal policy to stabilize inflation is in favor of a leadership concept with the governments as first mover.

All policy plans are compared to the optimal policy plan which a single social planner implements. The benevolent social planner chooses all fiscal and monetary instruments in order to maximize union-wide welfare.

This paper describes the results of discretionary policy, i.e. under assumption no policy authority can commit to future policy choices. Optimal policy under discretion describes best reality, as no major central bank makes any kind of binding commitment (Clarida et al., 1999). Regarding governments even within the limits imposed by the SGP fiscal policy is conducted in a discretionary manner.

4 Benchmark: Joint coordination

Before analyzing all different kind of strategic interaction the benchmark of full coordination of all three policy makers has to be explored. On assumption both governments and the central bank set their instruments to minimize welfare losses of a representative agent of the monetary union as if there is one single authority maximizing welfare of a representative household living in the monetary union assuming that it is a single country where the household faces an (aggregated) inflation rate \( \pi_t^W \) and an (aggregated) output gap \( y_t^W \). But the single authority takes into account that the two regions may be hit by inflation differential, i.e. the authority takes relative inflation rates \( \pi_t^R \) into account. Fluctuations in the terms of trade lead to fluctuations in production across countries as relative prices allocate resources within the
union (Ferrero, 2009) or (Benigno, 2004). Moreover, deviations of aggregate government spending $g^W_t$ due to a shock $\varepsilon^W_t$ generate losses. Instruments are the nominal interest rate $\bar{i}$, aggregated government spending $g^W_t = ng^H_t + (1-n)g^F_t$ and differential government spending $g^R_t = g^F_t - g^H_t$.

To summarize, the single authority wants to stabilize the economy of the whole monetary union according to the following welfare criterion

$$\max_{\bar{i}, g^W_t, g^R_t} \bar{\iota}_t - \frac{1}{2} \left( \alpha (x^W_t)^2 + (\pi^W_t)^2 + \theta (g^W_t - \varepsilon^W_t)^2 + \mu (\pi^R_t)^2 \right)$$

(7)

where $\alpha$, $\theta$, and $\mu$ are positive constants denoting the relative weight of output, government spending and the relative inflation rate.

Constraints are given by the aggregate and relative demand and Phillips curves

$$x^W_t = E_t x^W_{t+1} - \varphi (\bar{i} - E_t \pi^W_{t+1}) + g^W_t$$

(8)

$$\pi^W_t = \beta E_t \pi^W_{t+1} + \lambda x^W_t + u^W_t$$

(9)

$$\pi^R_t = \beta E_t \pi^R_{t+1} + \lambda x^R_t + u^R_t$$

(10)

$$x^R_t = E_t x^R_{t+1} + \varphi E_t \pi^R_{t+1} + g^R_t$$

(11)

Note that (8) is used to calculate the nominal interest rate

$$\bar{i} = \frac{1}{\varphi} \left[ -x^W_t + E_t x^W_{t+1} + \varphi E_t \pi^W_{t+1} + g^W_t \right]$$

(12)

First-order conditions with respect to $\bar{i}_t$, $g^W_t$, and $g^R_t$ are given by

$$0 = \alpha x^W_t + \lambda \pi^W_t$$

$$0 = \alpha x^W_t + \lambda \pi^W_t + \theta (g^W_t - \varepsilon^W_t)$$

$$0 = \mu \lambda \pi^R_t$$

The first optimality condition leads to the well-known trade-off between stabilizing the inflation rate and the output gap. Assuming that $u^W_t$ follows an AR(1) process with coefficient $\rho^W_u$ the results are given by

$$\pi^W_t = \alpha \varphi u^W_t$$

(13)

$$x^W_t = -\lambda \varphi u^W_t$$

(14)
Together with the second equation, the first one yields
\[ g_t^W = \varepsilon_t^W \]  
and hence with equation (12) the optimal nominal interest rate given by
\[ i_t = \gamma \pi_t \rho_u \alpha q u_t^W + \frac{1}{\varphi} \varepsilon_t^W \]  
where \( q = \frac{1}{\lambda^2 + \alpha (1 - \beta \rho)} \), and \( \gamma = 1 + \frac{(1 - \rho_u^W) \lambda}{\rho_u^W} \)

Simplifying further leads to\(^6\)
\[ i_t = \frac{q}{\varphi} \left[ \lambda (1 - \rho_u^W) + \alpha \varphi \rho_u^W \right] u_t^W + \frac{1}{\varphi} \varepsilon_t^W \]  
\[ = \frac{\lambda (1 - \rho_u^W) + \alpha \varphi \rho_u^W}{\varphi (\lambda^2 + \alpha (1 - \beta \rho_u^W))} u_t^W + \frac{1}{\varphi} \varepsilon_t^W \]

The third first-order condition results in equal inflation rates of both countries, i.e. \( \pi_t^H = \pi_t^F = \pi_t^W = \alpha q u_t^W \). This result and the Phillips curves (2) determine the output gaps of both countries which are given by
\[ x_t^H = \frac{1}{\lambda} \left[ (1 - \beta \rho_u^W) \alpha q u_t^W - u_t^H \right] \]  
\[ x_t^F = \frac{1}{\lambda} \left[ (1 - \beta \rho_u^W) \alpha q u_t^W - u_t^F \right] \]

With the help of the demand curves (1) the governmental instruments can be determined
\[ g_t^H = \frac{1}{\lambda} \left[ (1 - \rho_u^W) u_t^W - (1 - \rho_u^H) u_t^H \right] + \varepsilon_t^W \]  
\[ g_t^F = \frac{1}{\lambda} \left[ (1 - \rho_u^W) u_t^W - (1 - \rho_u^F) u_t^F \right] + \varepsilon_t^W \]

To conclude, in the benchmark case the nominal interest rate helps to stabilize the trade-off between the output gap and the aggregate inflation rate due to the aggregate output shock. Moreover, the nominal interest rate is set taking into account the fiscal deficit spending shock \( \varepsilon_t^W \). Aggregate government spending is set to stabilize this fiscal shock, but does not react to the aggregate cost-push shock which is in line with the case in the closed economy. But in contrast, relative government spending \( g_t^R \) reacts to the relative cost-push shock, but not to aggregate cost-push shock as can be seen by substracting (38) from (38). Country-specific government spending take into account the shock which hits the economy as well as the shock which hits the neighbor.

\(^6\)For all algebraic derivations refer to appendix A.
5 Simultaneous decisions of all policy-makers

In this section the targeting rules for both governments and the central bank are derived assuming that all three policy makers do not coordinate and set their instrument independently of the others.

In this case the timing of the events is as follows. Considering discretionary policy expectations of the private sector on inflation have been made beforehand, i.e. expectations $E_t \pi^j_{t+1}$, $j = H, F$ are given for all policy makers. The monetary union is hit by the cost-push shocks $u^j_t$ and the fiscal shocks $\varepsilon^j_t$, $j = H, F$ at the same time. The policy makers observe these shocks, then they set their instruments.

The problem of the central bank is to maximize (5) with respect to the interest rate taking as given government spending of the home and the foreign country. The results are given by the trade-off between stabilizing the aggregate inflation rate and the aggregate output gap (13) and (14).

Optimal responses of fiscal policies are the results of optimizing (6) subject to the structural equations (1) and (2). For $j = H, F$ these are given by

$$0 = x^j_t + \theta (g^j_t - \varepsilon^j_t) \iff g^j_t = -\frac{1}{\theta} x^j_t + \varepsilon^j_t \quad (22)$$

Combining this result for $j = H, F$ and inserting (14) leads to aggregate government spending

$$g^W_t = -\frac{1}{\theta} x^W_t + \varepsilon^W_t = \frac{\lambda}{g} u^W_t + \varepsilon^W_t \quad (23)$$

In contrast to the benchmark scenario in which aggregate government spending just focuses on government spending shocks (see equation (15)), in this case fiscal policy also reacts to aggregate cost-push shocks.

Using (12) the nominal interest rate is given by\(^7\)

$$\bar{i}_t = \frac{\lambda + \lambda \theta (1 - \rho^W_u) + \alpha \varphi \theta^2 \rho^W_u}{\varphi^2 (\lambda^2 + \alpha (1 - \beta^W_u))} u^W_t + \frac{1}{\varphi} \varepsilon^W_t \quad (24)$$

In the next step the results for relative variables are computed. Together with the results for the aggregate variables derived above and with the help of the relations for the home and foreign variables (??) and (??) country-specific government spending, output gaps and inflation rates can be found (Aoki’s method).

\(^7\)Algebraic derivation can be found in appendix B.
Optimal fiscal responses (22) lead to

$$g_t^R = -\frac{1}{\theta} x_t^R + \varepsilon_t^R$$

This is inserted into equation (11). The result forms together with (10) a system of two equations in two unknowns $\pi_t^R$ and $x_t^R$ with forward-looking rational expectations which is solved using the method of undetermined coefficients. As a result the relative output gap and relative inflation rate are given by

$$x_t^R = c^N_\varepsilon \varepsilon_t^R + c^N_u u_t^R$$

$$\pi_t^R = d^N_\varepsilon \varepsilon_t^R + d^N_u u_t^R$$

with the coefficients spelled out in the appendix B.

With these equations relative government spending can be computed

$$g_t^R = \frac{\theta - c_\varepsilon^N}{\theta} \varepsilon_t^R - \frac{1}{\theta} c^N_u u_t^R$$

$$= \frac{(1 - (1 + \beta + \lambda \varphi) \rho^R_\varepsilon + \beta (\rho^R_\varepsilon)^2) \theta}{1 + \theta - \rho^R_\varepsilon (\beta + \theta + (\beta + \lambda \varphi - \beta \rho^R_\varepsilon) \theta)} \varepsilon_t^R + \frac{-(\varphi \rho^R_\varepsilon)}{1 + (1 + \rho^R_\varepsilon + \lambda \varphi \rho^R_\varepsilon) \theta + \beta \rho^R_\varepsilon (1 + \theta - \rho^R_\varepsilon \theta)} u_t^R$$

This equation, together with (23), implies the following results for home and foreign government spending which are a combination of the following shocks

$$g_t^H = g_t^W - (1 - n) g_t^R$$

$$= \frac{\lambda \theta}{\theta} u_t^W + \varepsilon_t^W - (1 - n) \left[ \frac{\theta - c_\varepsilon^N}{\theta} \varepsilon_t^R - \frac{1}{\theta} c^N_u u_t^R \right]$$

(25)

$$g_t^F = g_t^W + n g_t^R$$

$$= \frac{\lambda \theta}{\theta} u_t^W + \varepsilon_t^W + n \left[ \frac{\theta - c_\varepsilon^N}{\theta} \varepsilon_t^R - \frac{1}{\theta} c^N_u u_t^R \right]$$

(26)

Inflation in the home resp. foreign country are given by

$$\pi_t^H = \alpha u_t^W - (1 - n) \left[ d^N_\varepsilon \varepsilon_t^R + d^N_u u_t^R \right]$$

(27)

$$\pi_t^F = \alpha u_t^W + n \left[ d^N_\varepsilon \varepsilon_t^R + d^N_u u_t^R \right]$$

(28)

*Refer to Appendix B for details.
For output the following equations can be derived

\[ x_t^H = -\lambda q u_t^W - (1 - n) \left[ c_{c_t}^N c_{c_t}^R + c_{w_t}^N u_t^R \right] \]  
\[ x_t^F = -\lambda q u_t^W + n \left[ c_{c_t}^N c_{c_t}^R + c_{w_t}^N u_t^R \right] \]

This implies that not only the country specific shocks play a role in setting the instrument to stabilize the country economy but as well spill over effects are observable due to the common interest rate.

Comparing these results with the benchmark results: everything is much more complicated ... comparing this with simple static model... everything is much more complicated...

6 Fiscal leadership

In the policy regime of fiscal leadership the timing of events can be described as follow. First expectations are formed, then the shocks occur. Government spending are set by both countries simultaneously. Finally the central bank sets the nominal interest rate.

Solving this problem backwards gives in the final stage the usual trade-off between stabilizing aggregate output gap and aggregate inflation rate as given by (13) and (14). The nominal interest rate is set according to equation (12). Fiscal policy takes this nominal inflation rate into account. Thus governments optimize (6) with respect to the following IS-equation

\[ x_t = E_t x_{t+1} + \varphi E_t \pi_t - \varphi \pi_t + g_t \]

resulting in the following optimal responses

\[ 0 = (1 - n)x_t^H + \theta (g_t^H - \varepsilon_t^H) \iff g_t^H = -\frac{1}{\theta} (1 - n)x_t^H + \varepsilon_t^H \]

\[ 0 = n x_t^F + \theta (g_t^F - \varepsilon_t^F) \iff g_t^F = -\frac{1}{\theta} n x_t^F + \varepsilon_t^F \]

These two equations imply for relative government spending

\[ g_t^R = -\frac{1}{\theta} (n x_t^F - (1 - n)x_t^H) + \varepsilon_t^F \]

\[ = -\frac{1}{\theta} \left[ (2n - 1)x_t^W + (2n(n - 1) + 1)x_t^R \right] + \varepsilon_t^R \]
where in the last equation (??) and (??) have been inserted.

Aggregate government spending is given by

\[
g^w_t = -\frac{1}{\theta} \left[ 2n(1-n)x^W_t + (2n-1)n(1-n)x^R_t \right] + \varepsilon^W_t
\]

Inserting the relation ship for relative optimal responses of governments into the relative IS-
curve (11) and using the relative Phillips curve (10), analogue to the previous case this results
in a system of linear difference equations with forward looking rational expectations which
can be solved with the method of undetermined coefficients as is done in appendix C.

Thus relative inflation and output are given by

\[
x^R_t = c^F \varepsilon^R_t + c^F u^R_t + c^F w^W_t
\]

\[
\pi^R_t = d^F \varepsilon^R_t + d^F u^R_t + d^F w^W_t + \varepsilon^R_t
\]

where \(c^F, d^F, c^F, d^F, c^F,\) and \(d^F\) are spelled out in the appendix.

This implies the following results

\[
g^w_t = -\frac{1}{\theta} \left[ -2n(1-n)\lambda qu^W_t + (2n-1)n(1-n) \left( c^F \varepsilon^R_t + c^F u^R_t + c^F w^W_t \right) \right] + \varepsilon^W_t
\]

\[
g^R_t = -\frac{1}{\theta} \left[ -(2n-1)\lambda qu^W_t + (2n(n-1)+1) \left( c^F \varepsilon^R_t + c^F u^R_t + c^F w^W_t \right) \right] + \varepsilon^R_t
\]

Country specific governments spending are given by

\[
g^H_t = -\frac{1}{\theta} \left[ -\lambda qu^W_t - (1-n)^2 \left( c^F w^W_t + c^F \varepsilon^R_t + c^F u^R_t \right) \right] + \varepsilon^W_t - (1-n)\varepsilon^R_t (31)
\]

\[
g^F_t = -\frac{1}{\theta} \left[ -n\lambda qu^W_t + n^2 \left( c^F \varepsilon^R_t + c^F u^R_t + c^F w^W_t \right) \right] + \varepsilon^W_t + n\varepsilon^R_t (32)
\]

Inflation in the home resp. foreign country are given by

\[
\pi^H_t = \alpha qu^W_t - (1-n) \left[ d^F \varepsilon^R_t + d^F u^R_t + d^F w^W_t \right] (33)
\]

\[
\pi^F_t = \alpha qu^W_t + n \left[ d^F \varepsilon^R_t + d^F u^R_t + d^F w^W_t \right] (34)
\]

For output the following equations can be derived
\[ x_t^H = -\lambda q u_t^W - (1 - n) \left[ c_{\varepsilon}^F \varepsilon_t^R + c_u^F u_t^R + c_{uw}^F u_t^W \right] \] (35)

\[ x_t^F = -\lambda q u_t^W + n \left[ c_{\varepsilon}^F \varepsilon_t^R + c_u^F u_t^R + c_{uw}^F u_t^W \right] \] (36)

Discussing the results: It is obvious that the results differ from the ones of the other policy regimes, all depend on the country size...

Concerning parameters: Looking at the coefficients (spelled out in the appendix) of the fiscal leadership, the following points can be observed: If \( n \) is equal to 1/2 then, \( c_{uw}^F = d_{uw}^F = 0 \). If \( n \) is equal to zero or one, then \( d_u^F = d_u^N, c_u^F = c_u^N, c_{\varepsilon}^F = c_{\varepsilon}^N, \) and \( d_{\varepsilon}^F = d_{\varepsilon}^N \)

7 Monetary leadership

In the policy regime of monetary leadership by assumption the central bank first sets the nominal interest rate, then both governments react simultaneously and set government spending. As in the previous case this game is solved by backward induction. First, considering the problem of the governments, leads to the optimal responses (22). Combining both equations to aggregate government spending \( g_t^W \) results in (23). This is inserted into the aggregate IS-curve (8) to yield

\[ x_t^W = \frac{\theta}{\theta + 1} \left[ E_t x_{t+1}^W - \varphi (\bar{\varepsilon} - E_t \pi_{t+1}^W) + \varepsilon_t^W \right] \]

This is the constraint the central bank takes into account when minimizing the monetary loss function (5). As the constant \( \frac{\theta}{\theta + 1} \) is just a scaling factor the optimal response of the monetary authority is the same as in the benchmark case, and both the scenarios of Nash and fiscal leadership. As a result, optimal aggregate government spending is given as in the case of the Nash game by (23). As a consequence, the results of the case of monetary leadership are the same as in the case of simultaneous decision of all policy makers.

Table 1 gives an overview of all analytical results.

8 Welfare effects

In this section welfare effects under the various policy regimes are computed.
<table>
<thead>
<tr>
<th>Fiscal leadership</th>
<th>Nash/Monetary leadership</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} u + M^{n_{b^0} - = \lambda x} ]</td>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} u + M^{n_{b^0} - = \lambda x} ]</td>
</tr>
<tr>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} u + M^{n_{b^0} - = \lambda x} ]</td>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} u + M^{n_{b^0} - = \lambda x} ]</td>
</tr>
<tr>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} (u - 1) - M^{n_{b^0} - = \lambda x} ]</td>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} (u - 1) - M^{n_{b^0} - = \lambda x} ]</td>
</tr>
<tr>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} (u - 1) - M^{n_{b^0} - = \lambda x} ]</td>
<td>[ M^{n_{m^0} + \lambda h_{n^0} + \lambda^2 n^0} (u - 1) - M^{n_{b^0} - = \lambda x} ]</td>
</tr>
</tbody>
</table>

Table 1: Summary of analytical results
Assuming that the economy is initially in its steady state and assuming that all equilibrium sequences are covariance stationary the loss function can be computed as

\[ L = -E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \alpha (x_t^W)^2 + (\pi_t^W)^2 \right) \right\} = -\frac{1}{1-\beta} (\text{Var} \pi + \alpha \text{Var} x) \] (37)

\( \text{Var} x \) denotes the unconditional variance (here, conditional on the information available at the beginning of period 0) of \( \pi_t \).

9 Simulating the model

9.1 Calibration

To visualize the dynamic behavior of the model impulse response functions to different shocks (especially symmetric and asymmetric) are depicted in this section. Therefor the following values are assigned to the parameters: The discount factor \( \beta \) is set to 0.99. Following Galí (2008) the parameter \( \varphi \) is set to be equal to 1 and the parameter \( \lambda \) is set to be equal to 0.0425.\(^9\) The monetary policy sets the weight on output stabilization to 0.5. In contrast, governments want to stabilize output and put more weight on output stabilization (\( \theta = 0.1 \)). Shocks are assumed to be highly persistent with a coefficient \( \rho \) of 0.9.

As the results depend on the choice of \( n \) as is shown next various values for the country size are assumed.

The parameters are summarized in table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>( \beta ) 0.99</td>
</tr>
<tr>
<td></td>
<td>( \varphi ) 1</td>
</tr>
<tr>
<td></td>
<td>( \lambda ) 0.0425</td>
</tr>
<tr>
<td>Relative weight in the loss function of the central bank</td>
<td>( \mu ) 0.5</td>
</tr>
<tr>
<td>Relative weight in the loss function of governments</td>
<td>( \theta ) 0.1</td>
</tr>
<tr>
<td>Home country size</td>
<td>( n ) 0.5</td>
</tr>
<tr>
<td>AR term of cost-push shock</td>
<td>( \rho^W ) 0.9</td>
</tr>
</tbody>
</table>

\(^9\)As Galí (2008) shows \( \lambda = \frac{(1-\theta)\alpha(1-\theta)\lambda}{(1-\alpha)(1-\theta)\lambda} \) where all the parameters are deep structural ones coming from a proper microfoundation. Taking over the values Galí (2008) assigns to all these parameters, namely \( \alpha = 1/3, \epsilon = 6, \) and \( \theta \lambda = 2/3 \), implies the given value for \( \lambda \).
9.2 Country size matters - analytical results are useful

As the analytical results suggest the policy instruments depend on the country size $n$ among other parameters in some cases. In other words, changing $n$ results in setting the instruments differently. As an example impulse response functions of the interest rate and home and foreign government spending to a cost-push shock which hits the home country are depicted for different values of $n$ in figure 1. These are computed for the scenario of uncoordinated policy (Nash game). In all three cases the nominal interest rate reacts positively to the shock, but is set higher the higher the home country size is as this country is hit by the shock. From the point of view of the foreign country, fiscal policy reacts to the shock in the neighbor country more aggressively the bigger the home country is though the shock has not hit the foreign country. The reaction of the home government spending alters with the size of $n$ from a negative reaction (small country size) to a positive one (big country size).

In the same line, figure 2 shows the extreme results of the three policy instruments in the scenario of fiscal leadership when the monetary union is hit by asymmetric fiscal shocks.
9.3 Asymmetric shocks, equal country size

Impulse response functions of the policy instruments to asymmetric cost-push shocks $u_t^H = -u_t^F$ are given by figure 3. On assumption both countries are of the same size $n = 1/2$. In this case the weighted average of both shocks is equal to zero and the relative shock is given by $u_t^R = -2u_t^H$. As for the central bank the whole monetary union is not hit by any shock monetary policy does not react by changing the nominal interest rate in each of the strategic games. The same results holds for aggregate government spending, as this instruments reacts to the weighted average of the home and foreign cost push and fiscal shock. This implies that home government spending and foreign government spending react inversely. In the benchmark case of full coordination the reaction is less strong than in the other to cases which are under the given calibration close to each other. But setting all instruments simultaneously implies a reaction of government spending which is less than in the case of fiscal leadership and thus generating losses to a lesser extent.

Figure 4 depicts the impulse response functions of the country-specific inflation rate and output gap. Under the benchmark there is no reaction in both the home and foreign inflation rate which depends in general on the weighted average of the shocks. But an increase of the home inflation rate of about 8% in case of fiscal leadership and a little less than this value in
case of the Nash setting can be observed. In the benchmark case the shock is reflected in the reaction of the output gaps (a negative output gap in the case of the home country). But in the other two games there is in small reaction to a positive output gap. The foreign variables reflect the home variables inversely.

### 9.4 Symmetric shocks, equal country size

This section discusses the impulse response functions in the case that two countries are of the same size and are both hit by symmetric shocks, i.e. \( u_t^H = u_t^F = u_t^W \) and \( u_t^R = 0 \). As can be seen in figure 5 there is a reaction of the nominal interest rate in each of the three cases analyzed. The monetary policy has to increase the interest rate in all three settings with the highest reaction in the case of the Nash game and the lowest in the case of full coordination. Both countries are hit by the same cost-push shock and thus set their instruments in the same way with no reaction in the benchmark case. When policy instruments are set simultaneously
the reaction is stronger than in the case of fiscal leadership. This result is opposite to the one described above of asymmetric shocks.

The results for the country-specific inflation and output gap in the case of symmetric shocks are astonishing as they are exactly the same no matter how the central bank and both governments interact. Moreover, the movements resemble those reactions in the case of the closed economy. In the benchmark case of full coordination a positive cost-push shock leads to a trade-off between the inflation rate and the output gap. The cost-push shock increases the inflation rate due to the Phillips curve relationship directly. Indirectly the shock increases the conditional expectations of the future interest rate through the AR(1)-structure of the shock. This implies a decrease in the output gap. As a result the central bank increases the nominal interest rate. The economy is stabilized with the help of the monetary instrument. Almost the same mechanism holds in the case of simultaneous setting of all instruments. As governments increase their spending the central bank has to react with a higher interest rate.
Figure 5: Impulse Responses of policy instruments in case of symmetric shocks

than in the benchmark case.

Figure 6: Impulse Responses of country-specific inflation and output gap in case of symmetric shocks
This result still holds even for various values of $n$. In the benchmark case of full coordination the impulse response functions for all variables do not depend on the country size $n$. In case of non-coordinated policies the outcomes are the same for all values of $n$. In this case the impulse response functions for the inflation rates and the output gaps are exactly the same as in the benchmark scenario. But the interest rate reacts stronger as both home and foreign government spending react to the shock. The fiscal leadership regime yields exactly the same impulse response functions for both inflation rates as in the other two scenarios. But the results for all other scenarios vary with the country size.

10 Conclusion

A simple model of a two-country monetary union based on the New Keynesian framework is set up. Three policy makers, i.e. two governments and one single central bank decide over policy variables to stabilize the economy. Authorities do not coordinate but set their instruments in an uncoordinated manner or in some kind of policy regime where one (group) of the policy makers reacts to the policy decision made by the other one before. Closed analytical solutions for all variables (specially for the policy instruments) are derived for all different strategic scenarios. Moreover, the different outcomes are evaluated by comparing welfare effects. Finally, to visualize the dynamic behavior of the variables the model is simulated and impulse response functions in the case of asymmetric and symmetric shocks are depicted.

Though the structure of the model is quite simple, analytical results are no longer straight-forward in some of the strategic games (specially the regime of fiscal leadership). The closed analytical results depend on the various constants of the underlying model in a non-linear way. As is shown, the direction of the effect of a shock depends on the calibration of the parameters, in particular on the country size. But nonetheless, results are comparable across the different regimes.

(tbc)
References


A Benchmark

Output gaps of both countries are given by equations (38) and (38)

\[ x_t^H = \frac{1}{\lambda} \left[ (1 - \beta \rho^W_u) \alpha q u^W_t - u^H_t \right] \]

\[ x_t^F = \frac{1}{\lambda} \left[ (1 - \beta \rho^W_u) \alpha q u^W_t - u^F_t \right] \]

With the help of the demand curves (1) the governmental instruments can be determined

\[ g_t^H = \frac{1}{\lambda} \left[ (1 - \beta \rho^W_u) \alpha q(1 - \rho^W_u) u^W_t - (1 - \rho^W_u) u^H_t \right] + q \lambda (1 - \rho^W_u) u^W_t - \varepsilon^W_t \]

\[ g_t^F = \frac{1}{\lambda} \left[ (1 - \beta \rho^W_u) \alpha q(1 - \rho^W_u) u^W_t - (1 - \rho^F_u) u^F_t \right] + q \lambda (1 - \rho^W_u) u^W_t - \varepsilon^W_t \]

B Nash

B.1 Algebraic derivation of results

Using (12) the nominal interest rate is given by

\[ i_t = \frac{q}{\varphi} \left[ \lambda(1 - \rho^W_u + \frac{1}{\varphi}) + \alpha \varphi \rho^W_u \right] u^W_t + \frac{1}{\varphi} \varepsilon^W_t \]

\[ = \frac{\gamma \rho^W_u \alpha q u^W_t}{\varphi} + \frac{1}{\varphi} \lambda q u^W_t + \frac{1}{\varphi} \varepsilon^W_t \]

\[ = \lambda + \lambda \theta(1 - \rho^W_u) + \alpha \varphi \theta \rho^W_u \frac{1}{\varphi^2 + \alpha(1 - \rho^W_u)} u^W_t + \frac{1}{\varphi} \varepsilon^W_t \]

Relative government spending

\[ g_t^R = -\frac{1}{\theta} x_t^R + \varepsilon_t^R \]

\[ = -\frac{1}{\theta} \left[ c_e \varepsilon_t^R + c_u u_t^R \right] + \varepsilon_t^R \]

\[ = \frac{\theta - c_e}{\theta} \varepsilon_t^R - \frac{1}{\theta} c_u u_t^R \]

\[ = \frac{(1 - (1 + \beta + \lambda \varphi) \rho^R_q + \beta(\rho^R_u) \theta)}{1 + \theta - \rho^R_e(\beta + \theta + (\beta + \lambda \varphi - \beta \rho^R_e) \theta)} \varepsilon_t^R + \frac{-(\varphi \rho^R_u)}{1 + (\varphi \rho^R_u) + \lambda \varphi \rho^R_q \theta + \beta \rho^R_u(1 + \theta - \rho^R_u \theta)} u_t^R \]
B.2 Undetermined coefficients

\[ x_t^R = \frac{\theta}{\theta + 1} \left[ E_t x_{t+1}^R + \varphi E_t \pi_{t+1}^R + \varepsilon_t^R \right] \]
\[ \pi_t^R - \lambda x_t^R = \beta E_t \pi_{t+1}^R + u_t^R \]

Assuming that

\[ x_t^R = c^N_x \varepsilon_t^R + c^N_u u_t^R \]
\[ \pi_t^R = d^N_x \varepsilon_t^R + d^N_u u_t^R \]

where \( c^N_x, d^N_x, c^N_u, d^N_u \) are the coefficients to be determined.

Plugging this into the system above, comparing the coefficients result in

\[ c^N_x = \frac{(\theta - \beta \rho^R_x \theta)}{(1 + \theta - \rho^R_x (\beta + \theta + (\beta + \lambda \phi - \beta \rho^R_x) \theta)} \]
\[ c^N_u = \frac{-\phi \rho^R_u \theta}{-1 + (-1 + \rho^R_u + \lambda \phi \rho^R_u) \theta + \beta \rho^R_u (1 + \theta - \rho^R_u \theta)} \]
\[ d^N_x = \frac{(\lambda \theta)}{(1 + \theta - \rho^R_x (\beta + \theta + (\beta + \lambda \phi - \beta \rho^R_x) \theta)} \]
\[ d^N_u = \frac{(-1 - \theta + \rho^R_u \theta)}{-1 + (-1 + \rho^R_u + \lambda \phi \rho^R_u) \theta + \beta \rho^R_u (1 + \theta - \rho^R_u \theta)} \]

C Fiscal leadership

C.1 Algebraic derivation of results

Country specific governments spending are given by
\[ g_t^H = g_t^W - (1 - n)g_t^R \]

\[ = -\frac{1}{\theta} \left[ -2n(1 - n)\lambda u_t^W + (2n - 1)n(1 - n) \left( c_e^F \varepsilon_t^R + c_a^F u_t^R + c_{uu}^F u_t^R \right) \right] + \varepsilon_t^W \\
- (1 - n) \left[ -\frac{1}{\theta} \left[ -2n(1 - n)\lambda u_t^W + (2n(n - 1) + 1) \left( c_e^F \varepsilon_t^R + c_a^F u_t^R + c_{uu}^F u_t^W \right) \right] + \varepsilon_t^R \right] \]

As \(-2n(1 - n) + (1 - n)(2n - 1)\) simplifies to \(-(1 - n)\), and as \((2n - 1)n(1 - n) - (1 - n)(2n(n - 1) + 1) = -(1 - n)^2\) this simplifies to

\[ g_t^H = -\frac{1}{\theta} \left[ -(1 - n)\lambda u_t^W - (1 - n)^2 \left[ c_{uu}^F u_t^W + c_e^F \varepsilon_t^R + c_a^F u_t^R \right] \right] + \varepsilon_t^W - (1 - n)\varepsilon_t^R \]  
(38)

\[ g_t^F = g_t^W + ng_t^R \]

\[ = -\frac{1}{\theta} \left[ -2n(1 - n)\lambda u_t^W + (2n - 1)n(1 - n) \left( c_e^F \varepsilon_t^R + c_a^F u_t^R + c_{uu}^F u_t^W \right) \right] + \varepsilon_t^W \\
+ n \left[ -\frac{1}{\theta} \left[ -2n(1 - n)\lambda u_t^W + (2n(n - 1) + 1) \left( c_e^F \varepsilon_t^R + c_a^F u_t^R + c_{uu}^F u_t^W \right) \right] + \varepsilon_t^R \right] \]

As \(-2n(1 - n) - n(2n - 1)\) simplifies to \(-n\), and as \((2n - 1)n(1 - n) + n(2n(n - 1) + 1) = n^2\) the result for foreign government spending simplifies to

\[ g_t^F = -\frac{1}{\theta} \left[ -n\lambda u_t^W + n^2 \left( c_e^F \varepsilon_t^R + c_a^F u_t^R + c_{uu}^F u_t^W \right) \right] + \varepsilon_t^W + n\varepsilon_t^R \]  
(39)

C.2 Undetermined coefficients

The optimality conditions of this problem can be written as

\[ x_t^R = \frac{\theta}{\theta + [2(n - 1) + 1]} \left[ E_t x_{t+1}^R + \varphi E_t \pi_{t+1}^R + \frac{2n - 1}{\theta} \lambda u_t^W + \varepsilon_t^R \right] \]

\[ \pi_t^R - \lambda x_t^R = \beta E_t \pi_{t+1}^R + u_t^R \]
Assuming that
\[ x_t^R = c_F^R \varepsilon_t^R + c_{u}^R u_t^R + c_{uw}^R u_t^W \]
\[ \pi_t^R = d_F^R \varepsilon_t^R + d_{u}^R u_t^R + d_{uw}^R u_t^W \]
where \( c_F^F, d_F^F, c_u^F, d_u^F, c_{uw}, \) and \( d_{uw}^F \) are the coefficients to be determined.

Plugging this into the system above, comparing the coefficients result in

\[ c_F^F = \frac{(-1 + \beta (\rho^R_u))(\theta)}{(-1 - 2(-1 + n)(-1 + \beta (\rho^R_u)) + (1 - (1 + \beta + \lambda \phi) (\rho^R_u) + \beta (\rho^R_u)^2) \theta} \]
\[ c_u^F = -\frac{\phi (\rho^R_u) \theta}{(1 + 2(-1 + n)(-1 + \beta (\rho^R_u)) + (-1 + (\rho^R_u)(1 + \beta + \lambda \phi - \beta (\rho^R_u))) \theta} \]
\[ c_{uw}^F = \frac{\lambda (-1 + 2n) (-1 + \beta (\rho^W_u))}{(\alpha + \lambda^2 - \alpha \beta (\rho^W_u))((-1 + 2(-1 + n)(-1 + \beta (\rho^W_u)) + (-1 + (\rho^W_u)(1 + \beta + \lambda \phi - \beta (\rho^W_u))) \theta} \]
\[ d_F^F = \frac{\lambda \theta}{(1 + 2(-1 + n)(-1 + \beta (\rho^R_u)) + (1 + (\rho^R_u)(1 + \beta + \lambda \phi - \beta (\rho^R_u))) \theta} \]
\[ d_u^F = -\frac{-1 - 2(-1 + n)(-1 + (\rho^R_u)) \theta}{(1 + 2(-1 + n)(-1 + \beta (\rho^R_u)) + (1 + (\rho^R_u)(1 + \beta + \lambda \phi - \beta (\rho^R_u))) \theta} \]
\[ d_{uw}^F = \frac{\lambda^2 (-1 + 2n)}{(\alpha + \lambda^2 - \alpha \beta (\rho^W_u))((-1 + 2(-1 + n)(-1 + \beta (\rho^W_u)) + (-1 + (\rho^W_u)(1 + \beta + \lambda \phi - \beta (\rho^W_u))) \theta} \]