Aging, Growth and the Allocation of Public Expenditures on Health and Education

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Abstract

In this paper, we develop an overlapping generations endogenous growth model in which both public education and health are sources of growth by affecting the accumulation rate of the human capital stock and the savings rate over life expectancy. We first find that dynamic complementarities of public expenditures lead to minimum threshold levels of public education and health expenditures that ensure sustainable growth. Considering endogenous fertility, we then study the process of aging and its effect on endogenous government policy. We show how governments can use the allocation of public expenditures as an alternative policy instrument to maximize growth without increasing the tax rate.

Keywords: Demographics, Fiscal Policy and Taxation; Public Health and Education Expenditures; Human capital; Endogenous Growth.

JEL classification: O11, O41, H55, E62.

1. Introduction

Improvements in health and education are essential factors for economic growth and development. The dramatic decline in both mortality and fertility rates has led to an aging population and to increasing pressure for public

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1For a review of the positive correlation between the mortality rate and the increased fertility rate of the population during demographic transition, see, among others, Easterlin (1996) (Chap. 6). In addition, Lorentzen et al. (2008) considered adult mortality to provide empirical evidence for the strong positive relation between adult mortality and the fertility rate.

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spending on health. In turn, aging has created concerns about a decrease in the retirement period or an increase in the tax rate to finance higher health expenditures (Shelton, 2008). Furthermore, many studies examined the impact of changes in longevity on economic growth. In particular, the literature on longevity and growth has studied the effect of mortality decline on fertility choices and on the return of private investment in education (Easterlin, 1996; De-La-Croix and Licandro, 1999; Kalemni-Oscan et al., 2000; Soares, 2006). Other studies have shown how the mortality rate jointly affects fertility choice, savings, private education, and economic growth (Blackburn and Cipriani, 2002; Zhang and Zhang, 2005). Although demographic changes affect the structure of the population, individual choices and the tax base of the economy, little work has been done on the endogenous response of government policy and the associated effects on economic growth.  

In this paper, we investigate the long-run growth properties and fiscal policy implications of aging in the form of increasing longevity and decreasing fertility. We show how the government can use the allocation of existing government revenues in health and education under the financial pressure of population aging instead of increasing the tax rate or decreasing the retirement period. The mechanism that drives our results is that increasing longevity not only affects the fertility rate in the economy and the return of private investment in education, as in, among others, Zhang et al. (2003), Doepke (2004), Zhang and Zhang (2005), Soares (2006), Hazan and Zoabi (2006), but it also endogenously affects the tax base of the economy and the relative return of public health and education investment on economic growth.

Several empirical facts have motivated our analysis. Developing countries tend to be characterized by a positive relation between improvements in health and economic growth. Growth stagnation is often linked to a lack of sufficient public policies on health and education (Rivera and Currais, 1999; World-Bank, 2003). To this end, in many countries public expenditures on education and health constitute the greatest part of social spending,

\[\text{In a seminal paper on exogenous longevity and endogenous fertility choice, Zhang et al. (2003) studied the effect of mortality decline not only on saving, growth and investment in education but also on the endogenous tax rate determined by a median voter mechanism.}\]

\[\text{Empirical time series analyses by Arora (2001) and Schultz (1997) have provided evidence of a strong relationship between health, productivity and growth.}\]
which is constantly increasing. Furthermore, a closer inspection of the data shows that the growth impact of health capital decreases at relatively large endowments of health stock (Schultz, 1997; Kwabena and Wilson, 2004). Intuitively, increased spending on health may negatively affect the growth rate of the economy because it crowds out other public activities, such as investment in education, for which the relative return in terms of growth can increase.

Motivated by the aforementioned stylized facts, we build an Overlapping Generations (OLG) endogenous growth model that relates to the literature on human capital formation, demographics, health, public policy and economic growth. Human capital formation builds upon the works of Glomm and Ravikumar (1992), Glomm and Ravikumar (1997), Blankenau and Simpson (2004), Blankenau et al. (2007) and Yew and Zhang (2009), in which public investment in human capital, the allocation of agents’ time on education and learning-by-doing increase the quality (productivity) of the labor force and, thus, the growth rate of the economy. Additionally, following Chakraborty (2004) and Bhattacharyya and Qiao (2007), we assume that public health expenditures positively affect life expectancy. In this setup, we also embed the analyses of Zhang et al. (2001), Zhang et al. (2003) and Zhang and Zhang (2005) on the effect of changes in longevity on fertility choice, private education and savings. Then, in a unified framework, we examine the subsequent feedback effects of aging on the endogenous allocation of public health and education expenditures and the tax rate, focusing, following Blankenau and Simpson (2004), on long-run growth.

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4 For instance, public expenditures accounted for 84.5% of total health expenditure in the United Kingdom, 83.3% in Sweden and 77.4% in Japan (Gomez et al., 2001). A number of countries in 2000 had public education shares close to 100%, such as Norway (98.7%), Turkey and Portugal (98.6% each), Finland (98%) and Sweden (97%) (Osang and Sarkar, 2007). Of the 36 OECD and non-OECD countries covered in this study, 19 countries (53%) financed at least 90% of their overall educational expenditures through public spending in 2000 (Osang and Sarkar, 2007).

5 In a similar vein, Zhang and Zhang (2005) and Soares (2005, 2006) found that a decline in population mortality and fertility rates increases the return of private investment in education. In this paper, in addition to the effect of longevity on fertility (quantity), we are interested in the impact of longevity on the relative return of public expenditures on health and education (relative quality of public expenditures).

6 Blankenau and Simpson (2004) study the effect of public education expenditures on economic growth, under the long-run growth maximizing objective. These authors study
Our major findings can be summarized as follows. First, we find that threshold levels of government taxation and public expenditures for health and education are necessary for the existence of a balanced growth path. In particular, the government should provide a minimum level of public health expenditure to ensure that the effects of public education expenditures on productivity are not eliminated by the low level of life expectancy. Second, we find that the shares of health and education expenditures in total public spending are crucial for the design of long-run fiscal policies under demographic changes. Rising longevity affects the relative return of investing in health and education and, at the same time, declining fertility affects the population structure and thus the tax base of the economy. We find that due to the change in relative returns of health and education on growth, the government can use the allocation of existing public revenues to improve the long-run growth rate of the economy, without necessarily increasing the tax rate.

Some aspects of our work should be stressed in comparison with the existing literature on demographics and public policy. First, we show how the allocation of existing revenues between health and education is important for the emergence of growth stagnation expanding the set of mechanisms through the complementary relation of public expenditures on growth. Second, we provide a unified framework that stresses the importance of the endogenous composition of productive government expenditures (among others Barro, 1990; Devarajan et al., 1996; Barro and Sala-i-Martin, 1992; Agenor, 2010) under the effect of demographic changes on public health, human capital, fertility, longevity and in turn, on economic growth (Doepke, 2004; Chakraborty, 2004; Zhang and Zhang, 2005; Hazan and Zoabi, 2006; Soares, 2006). In our framework, the Barro (1990) taxation rule is suboptimal and depends not only on production technology (as in Blankenau and Simpson, 2004) but also on demographic parameters through the composition of public spending. We establish that when the population ages, savings increase, making public investment in education more productive (relative to health) in terms of growth due to higher learning-by-doing and the higher stock of human capital. As the return of public investment in education increases, the role of tax structure and technology parameters on this relation. In our paper, we also consider the role of demographic parameters and the endogenous composition of government expenditures between health and education.
the government can change the allocation of existing revenues for education and increase the growth rate and output (tax base). In turn, a lower tax rate will be required for financing public expenditures on health.

The rest of the paper is structured as follows. Section 2 establishes and solves the optimization problem of households and firms. Section 3 presents the competitive decentralized equilibrium and studies the existence and properties of the equilibrium growth rate. Section 4, investigates the growth-maximizing fiscal policies. Finally, Section 5 concludes the paper and discusses further research directions.

2. The model

This section presents an OLG model that builds upon Glomm and Ravikumar (1997), Blankenau and Simpson (2004), Chakraborty (2004) and Zhang and Zhang (2005). The main features of the model are as follows: (a) the agents have finite and uncertain life-time horizons, (b) they are endowed with a given amount of time allocated to work, rearing their children and providing for their education, (c) the productivity of labor depends on education through time allocation by parents, public expenditures on education and learning-by-doing and (d) public health expenditures affect the health stock in the economy that in turn, positively affects life expectancy.

2.1. Demand side

Consider an economy with overlapping generations of identical $N$ agents that face a finite lifetime horizon and consume a single good. Agents learn when young and work in middle age with a certain lifetime horizon.\footnote{Following Zhang et al. (2001) and Zhang and Zhang (2005), we assume a fixed working period. We focus on other instruments to mitigate the financing pressure in aging economies rather than the standard increase (reduction) in working (retirement) period.} Old agents face a probability of being alive at the retirement period, given by $\phi(m_t)$, which depends on the health stock in the economy, $m_t$, and has the following properties:

**Assumption 1.** $\phi(m_t) \in [0, 1]$ is continuous, $\phi(0) = 0$, $\lim_{m \to \infty} \phi(m_t) = 1$.

**Assumption 2.** $\phi' \equiv \frac{\partial \phi(m_t)}{\partial m_t} > 0$. 

\[\text{Assumption 1. } \phi(m_t) \in [0, 1] \text{ is continuous, } \phi(0) = 0, \lim_{m \to \infty} \phi(m_t) = 1.\]

\[\text{Assumption 2. } \phi' \equiv \frac{\partial \phi(m_t)}{\partial m_t} > 0 .\]
**Assumption 3.** \( \phi'' \equiv \frac{\partial^2 \phi(m_t)}{\partial (m_t)^2} \leq 0 \).

Assumption 1 comes from the definition of a probability function.\(^8\) Assumptions 2 and 3 posit that the health stock in the economy affects positively the probability of being alive in the retirement period at a non-increasing rate, following, among others, Chakraborty (2004).\(^9\)

Each working agent, \(i\), is endowed with one unit of time, supplies labor, \(L_{it}\), rears children and provides for their education. Following Becker et al. (1990) and Zhang and Zhang (2005) we assume that rearing a child requires \(\sigma > 0\) constant units of time. In turn, the agent’s labor income, \(I_L\), in the working period is given by:

\[
I_L \equiv w_t(1 - \sigma n_t - e_t n_t)
\]

where \(w_t\) denotes the wage rate, \(n_t\) denotes the number of children at period \(t\) and \(e_t\) denotes time allocation for the education of children.

Because the lifetime horizon after retirement is uncertain, we assume (similarly, among others, to Zhang and Zhang, 2005) the existence of an actuarially fair annuity market, which maps savings to investment in physical capital, \(K\), for production in the next period. In particular, the agents save part of their labor income to annuity assets, \(s_t\), and, assuming that the private insurance market is competitive, they obtain a return for a unit of saving equal to \(r_{t+1}\) where \(r\) denotes the return on annuity assets. In the event of being alive, the agent’s income in the retirement period, \(I_R\), is given by:

\[
I_R \equiv s_t + r_{t+1}s_t
\]

The objective of the agent is to maximize intertemporal utility, given by

\[
U(c_t, c_{o,t+1}, n_t) = \ln c_t + \rho \phi \ln c_{o,t+1} + \eta \ln n_t h_{t+1}
\]

where \(c_t\) denotes consumption in the working period, \(t\), \(c_{o,t+1}\) denotes consumption in the retirement period and \(h_{t+1}\) denotes the human capital of

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\(^8\)The zero lower bound of the probability function is used for analytical tractability. Our results are robust with the use of a non-zero lower bound and are available upon request.

\(^9\)A non-convex production function implies that the effect of health on life expectancy is higher for low health endowments (Kwabena and Wilson, 2004; Zhang and Zhang, 2005) and is more consistent with the upper bound limit of the probability function.
children. Parameters $\rho \in (0, 1)$ and $\eta > 0$ correspond to the rate of time preference and the preference for an additional child, respectively.\textsuperscript{10} Following Glomm and Ravikumar (1992), Glomm and Kaganovich (2008) and Yew and Zhang (2009), the human capital production of individuals depends positively on parents’ allocation of time, on the human capital stock of parents, $h_t$, and on average human capital expenditures provided by the government, $\bar{H}_E = \frac{H_E}{N_t}$. It is given by

$$h_{t+1} = v(e_t h_t)^{\mu} (\bar{H}_E)^{1-\mu}$$

where $v > 0$ denotes an exogenous parameter for the production of human capital and $\mu \in (0, 1)$ is a parameter that measures the intensity of private and public provision in the accumulation of human capital.

2.2. Production side

We assume the existence of $N$ perfectly competitive homogeneous firms that seek to maximize profits. Each firm $i$, uses capital, $K_{it}$, and labor, $L_{it}$, under the following production technology:

$$Y_{it} = K_{it}^\omega (A_t L_{it})^{1-\omega}$$

where $0 < \omega < 1$ denotes the share of physical capital in the production function, $Y_{it}$ denotes individual output, and $A_t$ denotes labor productivity. We assume that labor productivity depends on the stock of human capital and on knowledge through average learning by doing. In particular, labor productivity in our model is determined by

$$A_t = \frac{K_t^\beta H_t^{(1-\beta)}}{L_t}$$

where $H_t$ denotes the aggregate stock of human capital, $K_t$ is the aggregate capital stock, and $\beta$ measures the intensity of each type of knowledge on labor productivity.\textsuperscript{11} Assuming a large number of agents, firms do not take

\textsuperscript{10}See Zhang et al. (2001), Zhang and Zhang (2005) for a similar specification of the utility function.

\textsuperscript{11}Equation (6) is in the spirit of Romer (1986), where each individual firm benefits from an increase in economy-average labor productivity by a rise in the human or physical capital stock available in the economy.
into account aggregate quantities when maximizing their profits. The first-order conditions of the after-tax profits maximization problem of firms are then given by

\[ r_t = (1 - \tau)\omega K_{it}^{1-\omega} L_{it}^{1-\omega} A_t^{1-\omega} \]  
\[ w_t = (1 - \tau)(1 - \omega)K_{it}^{\omega} L_{it}^{-\omega} A_t^{1-\omega} \]

where equations (7) and (8) state that the marginal productivity of capital and the marginal productivity of labor have to equal their factor prices. Also, \( \tau \) denotes a flat tax rate on output.

2.3. Government

The presence of public goods and learning-by-doing externalities justifies policy intervention in this model. The government spends on health and education to enhance the accumulation of human capital and the health stock. The accumulation rate of human capital is affected by the public sector size by a flat tax rate on output and by altering the allocation of public expenditures between education and health.

More specifically, we assume that the government levies a flat tax rate, \( \tau \), where \( 0 < \tau \leq 1 \), on income to finance government revenues through a balanced budget given by

\[ H_t^E + H_t^M = \tau Y_t \]  

Government revenues, \( \tau Y_t \), are used for the provision of public education and health expenditures, given by \( H_t^E \) and \( H_t^M \), respectively. To ease exposition, we parameterize public health expenditures, \( H_t^M \), as a share of tax revenues, denoted by \( \psi \in [0, 1] \). The internal allocation of government expenditures can then be written as

\[ H_t^E = (1 - \psi)\tau Y_t \]  
\[ H_t^M = \psi \tau Y_t \]

Hence, in addition to \( \tau \), \( \psi \) comprises a policy instrument of the government.

The aggregate health stock and human capital stocks in the economy are formed with the following technologies:

\[ m_{t+1} = \xi \frac{H_t^M}{Y_t} + (1 - \delta_m) m_t \]
\[ H_{t+1} = v(c_tH_t)^\mu (H_E^E)^{1-\mu} \] (13)

The equation (12) shows that the social health stock of the economy is affected positively by the share of income devoted to health, \( H_{t}^M / Y_t \), where \( \xi > 0 \) and \( v > 0 \) denote the technology that is applied to investment in health and education respectively, and \( \delta_m \in (0, 1) \) denotes the natural decay rate (biological deterioration) of the health stock. The justification behind (12) stems from the strong correlation between public health as a share of output and the health stock of the economy, as found by Barro (1996), Schultz (1997), Rivera and Currais (1999), and Osang and Sarkar (2007). Intuitively, the formulation of health status by (12) implies that output has a positive effect on health through the provision of health expenditures and a negative effect through activities that are products of output, such as pollution and obesity. An alternative interpretation of (12) is to consider \( m_t \) as the health infrastructure (e.g., hospitals) where health expenditures have to increase in proportion with output to increase the health stock in the economy because of congestion caused by the aggregate economic activity, \( Y_t \) (see Barro, 1990).\(^{12}\)

Equation (13) follows from the aggregation of (4) and, according to Glaum and Ravikumar (1992), Glaum and Ravikumar (1997) and Blanke- nau et al. (2007) demonstrates that public expenditures on education, \( H_E^E \), have a positive impact on the accumulation of aggregate human capital stock, \( H_t \).

2.4. The Household’s Problem

The agent’s problem is to choose \( c_t, n_t, e_t, s_t \) to maximize intertemporal utility (3) subject to the resource constraints given by

\[ w_t(1 - \sigma n_t - e_t n_t) = c_t + s_t \] (14)

and

\[ \phi c_{o,t+1} = s_t + r_{t+1} s_t \] (15)

Equation (14) states that agents allocate their labor income to consumption and savings, \( I_L = c_t + s_t \), and equation (15) states that the fraction of retired

\(^{12}\)A similar production function of health stock in a continuous time model is provided by Agenor (2010) and in an OLG setup by Aisa and Pueyo (2004) and Aisa and Pueyo (2006).
persons who are alive consume income from the savings of their working period, $I_R = \phi c_{o,t+1}$.\footnote{In other words, retired agents with probability $1 - \phi$ consume nothing, and with probability $\phi$ consume $\phi c_{o,t+1}$. Following Zhang and Zhang (2005), equation (15) implies that the wealth of agents who unexpectedly die is redistributed to other agents.}

The interior solution of the above maximization problem is given by the following optimal allocations:

$$n^*_t = \frac{\eta \mu}{[1 + \phi(m_t) \rho + \eta] e^*_t} \quad (16)$$

$$s^*_t = \frac{\phi(m_t) \rho}{1 + \phi(m_t) \rho + \eta} w_t \quad (17)$$

$$e^*_t = \frac{\mu \sigma}{(1 - \mu)} \quad (18)$$

where $n^*_t$ denotes the optimal number of children that parents have to rear. This is positively related to their taste for children in the utility function, negatively related to the time allocation to education and negatively related to the probability of being alive in the retirement period. These effects reflect the quantity-quality trade-off regarding children faced by the middle-aged agents and show that the rate of fertility in absolute terms falls with the decrease in the rate of mortality, thus triggering population aging.\footnote{These results coincide with empirical facts on the demographic transition in the last century in which a fall in the mortality rate has been accompanied by a fall in the fertility rate (see, among others Lorentzen et al., 2008; Easterlin, 1996).}

Equation (18) shows that time to education increases with the intensity of private education in the human capital production function and with the cost of per child rearing time. Finally, equation (17) determines the optimal allocation for asset holdings as a function of the wage rate and the model parameters. This equation shows that the savings of the economy depend linearly and positively on the wage rate, negatively on the taste for children in the utility function and positively on the length of life. An increase in the survival probability positively affects savings because agents store more capital for their greater lifetime.

3. The Competitive Decentralized Equilibrium

We can now define the competitive decentralized equilibrium in our model and present the solution for the aggregate economy and its equilibrium dy-
3.1. Definition of equilibrium and market clearing

**Definition 1.** The competitive decentralized equilibrium (CDE) of the economy is defined for the exogenous policy instruments $\tau$ and $\psi$, factor prices $r_t$ and $w_t$, and aggregate allocations $K_t$, $H_t$, $H^F_t$, $H^M_t$, $L_t$ such that:

i) Individuals solve their intertemporal utility maximization problem by choosing $c_t$, $s_t$, $n_t$, and $e_t$ given the policy instruments and factor prices.

ii) Firms choose $L_{it}$ and $K_{it}$ to maximize their profits, given factor prices and aggregate allocations.

iii) All markets clear. The market clearing condition for the capital market is given by

\[ K_{t+1} = s_tN_t \]  

(19)

iv) The government budget constraint holds.

The competitive equilibrium in the aggregate economy is defined by (i)-(iii) under aggregation given by the following conditions:

\[ \sum_{i=1}^{N_t} K_{it} = K_t, \quad \sum_{i=1}^{N_t} L_{it} = L_t, \quad L_t = (1 - \sigma n_t - e_t n_t)N_t \]  

(20)

Equation (19) shows that in equilibrium net investment in capital has to equal aggregate savings, which are determined by the aggregate purchase of annuity assets.

3.2. Competitive Equilibrium and Balanced Growth Path

Using the equilibrium conditions of aggregation for a symmetric equilibrium, $N_tK_t = K_t$, $N_tL_{it} = L_t$, combining equations (8), (17), (18) and (16) and substituting (19), we obtain the accumulation rate of the aggregate physical capital stock in equilibrium, $g_K$, as follows:

\[ g_K \equiv \frac{K_{t+1} - K_t}{K_t} = \frac{\phi(m_t)\rho(1 - \omega)(1 - \tau)}{1 + \phi(m_t)\rho}\left(\frac{H_t}{K_t}\right)^{(1 - \alpha)} - 1 \]  

(21)

where $\alpha \equiv \omega + \beta(1 - \omega)$.

Using (11) and (12), the rate of change of the social health stock in equilibrium, $g_m$, is given by

\[ g_m \equiv \frac{m_{t+1} - m_t}{m_t} = \xi\psi\tau - \delta_m \]  

(22)
Substituting (10) and (18) in (13) and using the equilibrium conditions and the production technology, we obtain the accumulation of aggregate human capital stock, \( g_H \), as follows:

\[
g_H \equiv \frac{H_{t+1} - H_t}{H_t} = v((1 - \psi)\tau)^{1 - \mu} \left( \frac{\mu\sigma}{1 - \mu} \right)^\mu \left( \frac{H_t}{K_t} \right)^{-\alpha(1 - \mu)} - 1 \quad (23)
\]

Equations (21), (22) and (23) characterize the dynamics of the competitive equilibrium in the aggregate economy which in turn determine the equilibrium growth rate.

The Balanced Growth Path (BGP) is defined as a state in which all the variables of the economy grow at a constant rate. In this economy, at the BGP, the aggregate equilibrium growth rates of human and private capital have to grow at the same rate, i.e., \( \frac{H_{t+1} - H_t}{H_t} = \frac{K_{t+1} - K_t}{K_t} = g \), while the health stock of the economy has to grow at a constant zero rate. This result is easily obtained by investigating the equilibrium growth rates of these variables separately. In particular, if the health stock of the economy grows at a constant non-negative rate, \( \gamma \), we get \( \lim_{t \to \infty} (m_t) = \infty \), and the right-hand side of (22) converges to \(-\delta_m\); therefore, a positive constant growth rate is not attainable. Thus, for the health stock to follow a BGP, its growth rate has to be zero. The steady-state health stock in equilibrium is given by:

\[
\tilde{m} = \frac{\xi \psi \tau}{\delta_m} \quad (24)
\]

Because the aggregate health stock is constant in the long run, from (23) we find that for the human capital stock to grow at a constant rate, this growth rate has to equal the growth rate of the private capital stock to eliminate diminishing returns to capital. This condition also satisfies equation (21). Thus, a necessary condition for a balanced growth path in our economy is that \( g_H = g_K = g \) and \( g_m = 0 \).

Given the above result for the BGP we can now derive the equilibrium growth rate of the economy. Rearranging (21) we find that at the BGP for the long-run ratio of human to physical capital stock, \( \tilde{z} \), is given by

\[
\tilde{z} = \left( \frac{\tilde{g} + 1}{\Psi} \right)^{\frac{1}{1 - a}}
\]

where \( \Psi = \frac{\phi(\xi \psi \tau)\rho(1 - \omega)(1 - \tau)}{1 + \phi(\xi \psi \tau)\rho} \) is a function of the model parameters and policy
instruments. Substituting (24) and (25) in (23), we can then obtain the long-run growth rate, \( \tilde{g} \), in our economy as

\[
\Phi(\tilde{g}) \equiv \tilde{g} - v \left( (1 - \psi) \tau \right)^{1-\mu} \left( \frac{\mu \sigma}{(1 - \mu)} \right)^{\mu} \left( \frac{\tilde{g} + 1}{\psi} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} + 1 \tag{26}
\]

where the solution of this continuous function for \( \tilde{g} > 0 \), such that \( \Phi(\tilde{g}) = 0 \), determines the existence and the properties of the equilibrium long-run growth rate, \( \tilde{g} \), given by the following proposition:

**Proposition 1.** There exists a unique and strictly positive long-run equilibrium growth rate, \( \tilde{g} > 0 \), iff

\[
v \left( (1 - \psi) \tau \right)^{1-\mu} \left( \frac{\mu \sigma}{(1 - \mu)} \right)^{\mu} \left( \frac{1 + \phi(\frac{\tilde{g} \tau}{\delta m}) \rho}{\phi(\frac{\tilde{g} \tau}{\delta m}) \rho (1 - \omega)(1 - \tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} > 1
\]

for any parameter values in their assumed domain and some values of the policy instruments, and it is given by the fixed point in (26), \( \tilde{g} : \Phi(\tilde{g}) = 0 \).

**Proof.** See Appendix 1. 

Notice that our economy will exhibit a zero equilibrium growth rate if the necessary and sufficient parametric condition of Proposition 1 holds with equality. A crucial remark is that the policy instruments, \( \tau \) and \( \psi \), are critical not only for the quantitative determination of the equilibrium growth rate but also for the existence of a non-negative BGP. In fact, a direct consequence of Proposition 1 is that there exists a subset in the domain of the policy instruments that forms a set of sufficient values for sustainable growth. The following corollaries formalize this point.

**Corollary 1.** Given the parametric characteristics of the economy, there exists a range \( (\hat{\psi}, \check{\psi}) \in (0, 1) \) of public health expenditures as a share of total public expenditures that has to be implemented for Proposition 1 to hold. The maximum, \( \hat{\psi} \), and minimum, \( \check{\psi} \), threshold levels of the health expenditures share in total public spending are given by

\[
v \left( (1 - x) \tau \right)^{1-\mu} \left( \frac{\mu \sigma}{(1 - \mu)} \right)^{\mu} \left( \frac{1 + \phi(\frac{\tilde{g} \tau}{\delta m}) \rho}{\phi(\frac{\tilde{g} \tau}{\delta m}) \rho (1 - \omega)(1 - \tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1 \text{ where } x = \hat{\psi}, \check{\psi}
\]

\[
(27)
\]
Proof. See Appendix 2.

Corollary 1 establishes the threshold levels of public expenditures on health and education for the economy to exhibit endogenous growth with a positive long-run growth rate. These threshold levels arise from the complementarities in the growth process between education expenditures on human capital accumulation and health expenditures on the saving rate (through the survival probability). Intuitively, an increase in investment in education will raise human capital accumulation and output. If health expenditures remain constant, their share in the total output will fall, and by (22), less health stock will be formed. The lower level of health stock will lead to lower life expectancy, lower saving rate and lower tax base and, in turn, to a decrease in human capital accumulation. At low health stock levels, the increase in human capital accumulation will be absorbed by the decrease in life expectancy, and thus the economy will exhibit zero or negative growth unless a minimum amount of public health expenditures takes place. This result is consistent with empirical evidence by Baldacci et al. (2008) and Rivera and Currais (1999), in which a lack of sufficient health expenditures (medical stock) reduces the favorable growth effects of education in developing countries through low school enrollments, disabilities and epidemic diseases.

A by-product of our analysis is that a similar methodology applies for the lower and the upper bound of the government size required to ensure positive growth. It is straightforward to show that Proposition 1 imposes boundaries for the tax rate (because it does not hold for $\tau = 0$ or $\tau = 1$) that are determined endogenously by the structural characteristics of the economy. Thus, a change in the population structure or the technological characteristics of the economy can expand or restrict the range of values of these policy instruments. The following Corollary to Proposition 1 determines the boundaries for the government size.

Corollary 2. Given the parametric characteristics of the economy, there exists a range of values for the tax rate on income, $\tau < \bar{\tau} < \underline{\tau}$, that has to be implemented by the government for Proposition 1 to hold. The maximum, $\bar{\tau}$, and minimum, $\underline{\tau}$, threshold levels of the government size are given by:
\[ v ((1 - \psi)j)^{1-\mu} \left( \frac{\mu \sigma}{(1 - \mu)} \right)^\mu \left( \frac{1 + \phi(\frac{\xi \psi j}{\delta m}) \rho}{\phi(\frac{\xi \psi j}{\delta m}) \rho (1 - \omega)(1 - j)} \right)^{-\frac{\alpha(1-\rho)}{1-\alpha}} = 1 \text{ where } j = \bar{\tau}, \tau \]

\begin{equation}
(28)
\end{equation}

**Proof.** See Appendix 3.

Corollary 2 sets the upper and lower bound for the government size. Sustainable growth cannot be attained if the government size exceeds \( \bar{\tau} \) because of the high distortion to private savings and low physical capital accumulation. In addition, the tax rate has to be greater than \( \tau \) for the government to finance expenditures on health and education that drive growth in this economy. These maximum threshold levels depend on population aging determined by a positive change in the probability of living in the retirement period, \( \phi \). A rise in fertility can generate fiscal limitations (lower tax base in the next period because of the lower labor force), while population aging can expand the range of feasible taxation on output depending on the actual level of the tax rate and the composition of public expenditures.

Our findings seem to conform to the view that development traps can be the result of the lack of sufficient health and education expenditures and the associated government size that guarantee positive growth. In the following section, by endogenizing government policy aiming at growth-maximization, we will show how government can use the composition of public expenditures and government size not only to preserve positive growth but also to enhance growth under demographic changes.

4. **Growth-maximizing policies**

In this section, we will analyze growth-maximizing fiscal policy rules to assess the impact of taxation and the contribution of the components of public expenditures on long-run growth.

4.1. **Growth-maximizing policy rules**

**Definition 2.** Growth-maximizing policies in the competitive equilibrium of the aggregate economy are given under Definition 1 when the government acts as a Stackelberg player and chooses \( \tau^* \) and \( \psi^* \) to maximize the long-run growth rate of the economy by taking into account the aggregate maximizing
behavior of the competitive equilibrium, and when the government budget constraints, feasibility and technological conditions are met.

The problem of long-run growth-maximizing policies can be formulated as

$$\max \bar{g}(\tau, \psi, z) = \phi(\frac{\xi\psi}{\delta_m})\rho(1 - \omega)(1 - \tau)\left(\frac{1}{1 + \phi(\xi\psi/\delta_m)}\right)(z)^{1-\alpha} - 1$$

subject to the decentralized equilibrium response by the private agents given by

$$\phi(\frac{\xi\psi}{\delta_m})\rho(1 - \omega)(1 - \tau)\left(\frac{1}{1 + \phi(\xi\psi/\delta_m)}\right) - v\left((1 - \psi)\tau\right)^{1-\mu}\left(\frac{\mu\sigma}{(1 - \mu)}\right)^\mu(\mu^\mu) - 1 = 0$$

The solution and properties of this problem are given by the following proposition:

**Proposition 2.** The growth maximizing tax rate, $\tau^*$, and share of total public expenditures to health, $\psi^*$, under endogenous longevity to public health expenditures is given by the following system of equations

$$\tau^* = \frac{1 - a}{1 - a\psi^*} > 0 \quad (29)$$

$$\psi^* = \frac{\Omega - a}{a(\Omega - 1)} > 0 \quad (30)$$

where $\Omega(\tau^*, \psi^*, \xi, \rho, \delta_m, \cdot) = \frac{\phi(\xi\psi/\delta_m)(1 + \phi(\xi\psi/\delta_m))\rho}{\phi(\xi\psi/\delta_m)}$ is a function of model parameters and policy instruments. The growth maximizing tax rate is higher than the Barro (1990) optimal taxation rule, $\tau^* > 1 - a$, and depends positively on health expenditures as a share of total government revenues.

**Proof.** See Appendix 4.

Futhermore, the growth-maximizing physical to human capital stock is given by
\[ z^* = v^{1-a\mu} \left( (1 - \psi^*) \tau^* \right)^{1-a\mu} \left( \frac{1+\phi(\frac{\xi\psi^*\tau^*}{\delta_m})}{1+\phi(\frac{\xi\psi^*\tau^*}{\delta_m}) \frac{\mu}{(1-\mu)^\mu}} \right) \]  

4.2. The effect of population aging on growth-maximizing policies

In this section we numerically solve the nonlinear system (29), (30), and (31). In particular, we study the effect of parameter values that trigger population aging through our framework on the growth-maximizing tax rate and the allocation of government expenditures. We assume as sources of population aging a positive change in the health technology parameter, \( \xi \), or
a fall in the rate of biological deterioration, $\delta$.\textsuperscript{15} Concerning parameterization, the values of the parameters are in line with those in the literature or are chosen to yield plausible values for the fertility rate and survival probability. For the probability of surviving, we use a concave function, $\phi(m) = \frac{m}{1+m}$, that satisfies the Assumptions 1-3.\textsuperscript{16}

In Tables 1 and 2, we report the numerical results of the growth rate, the fertility rate, the probability of surviving and the endogenous policy parameters for varying values of parameters $\xi$ and $\delta$ in ranges, $0.2 < \xi < 2$, and $0.08 < \delta < 0.12$, respectively. Our results show that an increase in the technology of the health stock accumulation or a decline in the rate of biological deterioration leads to an increase in life expectancy and a decline in the fertility rate, thus, triggering population aging. In turn, the government shifts the allocation of expenditures from health to education, leading to an increase in the growth rate. The higher growth rate implies a higher tax base and results in a decrease in the growth-maximizing tax rate.

Intuitively, an increase in the survival probability during the retirement period (population aging because the fertility rate decreases) has two effects on private agents’ decisions. First, agents in the working period reduce the time of rearing children and increase work to increase consumption in the longer retirement period (see (16)). Second, they overaccumulate private capital through higher savings (see (17)) for their greater lifetime. The overaccumulation of private capital is suboptimal because agents do not internalize the favorable effects of knowledge spillovers, which increases the marginal social value of private capital and in turn raises the growth-maximizing tax rate (“taxation” effect). This effect is reinforced by the fact that fewer children form a lower tax base in the next period. However, at the same time, population aging affects the relative return of investing in health and education. Aging decreases the relative return of investing in health because the higher accumulation of physical capital stock by the rise in savings increases the productivity of labor, $A$. In turn, the return on productivity of investing in education increases because of the higher associated learning-by-doing effect. Because of this, by reallocating expenditures from health to education, the government can enhance the tax base of the economy through higher

\textsuperscript{15}For the role of such changes on health parameters in causing population aging, see Aisa and Sanso (2006).

\textsuperscript{16}Our numerical results are very robust to many other specifications of the probability function and are available upon request.
growth and, following (29), reduce the government size. In other words, the additional tax revenues finance public education at a greater proportion (“re-allocation” effect) and reduce the need for higher taxes.

A direct policy implication of the above analysis is that countries facing fiscal limitations can re-allocate the existing public resources by steering expenditures towards (away from) education and away from (towards) health in response to a decline (rise) in the mortality rate. The numerical results presented above, show that as the population ages, countries should target low tax rates accompanied by a higher share of public expenditure on education.

5. Concluding remarks

According to the aim of the paper, we explored the steady state and fiscal policy implications of public health and education expenditures in an endogenous growth model under the presence of population aging driven by a decline in mortality and fertility rate. We showed that a minimum amount of public expenditures has to be devoted to health and education for the economy to attain positive long-run growth. This result conforms to the empirical findings provided by Rivera and Currais (1999) and Baldacci et al. (2008), who have argued that a minimum level of health and education provision for the labor force is necessary to maintain continuous growth.

We also found that the allocation of public expenditures between health and education is a critical determinant of the taxation rule. The share of health expenditures to total public expenditures positively affects the optimal government size due to the change in the marginal cost of public funds. A fall in the mortality rate not only affects fertility and the tax base of the economy but also causes a change in the relative return of public investment in health and education on the growth rate. A by-product of our results is that the policy change in the efficient allocation of resources induced by a decline in the mortality rate in the retirement period counteracts the positive effect on the tax rate, because the return on health and education on growth changes. Hence, in the presence of population aging, the government can now benefit from the optimal re-allocation of public expenditures between education and health. According to Corolaries 1 and 2, the efficient composition of public expenditures becomes more important in the presence of binding financial constraints such as those posed by the International Monetary Fund and the World Bank to less developed economies, which limit the provision of expenditures for social services.
We believe that this framework and the endogenous policy mechanism provide many additional research directions. A straightforward extension is to consider different social security regimes, and mainly a pay-as-you go system (as in Zhang et al., 2001). A pay-as-you go system would provide an additional channel for the fertility choice of agents in the model and the efficiency of the tax base. Furthermore, the negative effect of income on the health stock in the economy can be explicitly modelled by introducing the law of motion for pollution or the obesity level in the economy as positive functions of income. Such an extension would enrich our analysis by addressing other issues, such as policies for environmental sustainability and reduction in obesity rates. Lastly, welfare maximization policies can also be considered, although in this OLG setup, political economy issues arise regarding the generation that will be better off.

6. Appendix

Appendix 1. Proof of Proposition 1

The properties of \( \Phi(\tilde{g}) = \tilde{g} - v ((1 - \psi)\tau)^{1-\mu} \left( \frac{\mu\sigma}{(1-\mu)} \right)^\mu \left( \frac{\tilde{g} + 1}{\tilde{g}} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} + 1 \) are given as follows:

1. Continuous in \( \tilde{g} \) from the addition of continuous functions.

2. \( \Phi(0) = -v ((1 - \psi)\tau)^{1-\mu} \left( \frac{\mu\sigma}{(1-\mu)} \right)^\mu \left( \frac{1}{\phi(\frac{1}{\delta_m})} \right)^\mu(1-\omega) - \frac{\alpha(1-\mu)}{1-\alpha} + 1 \)

3. \( \frac{\partial\Phi}{\partial\tilde{g}} > 0 \) for \( \tilde{g} > 0 \)

4. \( \lim_{\tilde{g}\to\infty} \Phi(\tilde{g}) = \infty \)

Under 1-4 there exists a unique fixed point \( \tilde{g} > 0 \) that solves \( \Phi(\tilde{g}) \), iff \( \Phi(0) < 0 \) which implies \( v ((1 - \psi)\tau)^{1-\mu} \left( \frac{\mu\sigma}{(1-\mu)} \right)^\mu \left( \frac{1}{\phi(\frac{1}{\delta_m})} \right)^\mu(1-\omega) - \frac{\alpha(1-\mu)}{1-\alpha} > 1 \), for any parameter value in its assumed domain and some values of the policy instruments.

Appendix 2. Proof of Corollary 1

From Proposition 1, for the economy to attain sustainable growth the following necessary and sufficient condition must hold
\[ v((1 - \psi)\tau)^{1-\mu} \left( \frac{\mu \sigma}{(1-\mu)} \right)^\mu \left( \frac{(1+\phi(\xi \psi \tau)\rho)}{\phi(\xi \psi \tau)\rho(1-\omega)(1-\tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} > 1. \]

However, for \( \psi = 1 \) value, because \( 0 > 1 \) implies a contradiction. Thus, Proposition 1 holds only below a maximum threshold level, \( \hat{\psi} \),

where \( v((1 - \hat{\psi})\tau)^{1-\mu} \left( \frac{\mu \sigma}{(1-\mu)} \right)^\mu \left( \frac{(1+\phi(\xi \hat{\psi} \tau)\rho)}{\phi(\xi \hat{\psi} \tau)\rho(1-\omega)(1-\tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1 \) given the parameter values. In the same vein, for \( \psi = 0 \) Proposition 1 implies \( 0 > 1 \Rightarrow \) which does not hold. Thus, Proposition 1 holds above a minimum threshold level \( \tilde{\psi} \), where \( v((1 - \tilde{\psi})\tau)^{1-\mu} \left( \frac{\mu \sigma}{(1-\mu)} \right)^\mu \left( \frac{(1+\phi(\xi \tilde{\psi} \tau)\rho)}{\phi(\xi \tilde{\psi} \tau)\rho(1-\omega)(1-\tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1 \).

**Appendix 3. Proof of Corollary 2**

From Proposition 1, for the economy to attain sustainable growth the following necessary and sufficient condition must hold:

\[ v((1 - \psi)\tau)^{1-\mu} \left( \frac{\mu \sigma}{(1-\mu)} \right)^\mu \left( \frac{(1+\phi(\xi \psi \tau)\rho)}{\phi(\xi \psi \tau)\rho(1-\omega)(1-\tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} > 1. \]

However, for \( \tau = 1 \) the necessary and sufficient condition does not hold, for any other parameter value, because \( 0 > 1 \) implies a contradiction. Thus, Proposition 1 holds only below a maximum threshold level \( \bar{\tau} \) where

\[ v((1 - \psi)\tau)^{1-\mu} \left( \frac{\mu \sigma}{(1-\mu)} \right)^\mu \left( \frac{(1+\phi(\xi \psi \tau)\rho)}{\phi(\xi \psi \tau)\rho(1-\omega)(1-\tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1 \] given other parameter values. In the same vein, Proposition 1 implies for \( \tau = 0 \) that \( 0 > 1 \Rightarrow \) which is a contradiction because the necessary condition does not hold. Thus, Proposition 1 holds above a minimum threshold level \( \tilde{\tau} \), at which

\[ v((1 - \psi)\tau)^{1-\mu} \left( \frac{\mu \sigma}{(1-\mu)} \right)^\mu \left( \frac{(1+\phi(\xi \psi \tau)\rho)}{\phi(\xi \psi \tau)\rho(1-\omega)(1-\tau)} \right)^{-\frac{\alpha(1-\mu)}{1-\alpha}} = 1. \] Thus, when Proposition 1 holds, it holds in a range of tax rate with minimum, \( \tau \), and maximum, \( \bar{\tau} \), threshold levels of income tax rate.

**Appendix 4. Proof of Proposition 2.**
We need to maximize
\[
\tilde{g}(\tau, \psi, z) = \frac{\phi(\xi_{\psi}^\tau \delta_m \rho(1 - \omega)(1 - \tau) (z)^{(1-\alpha)}}{1 + \phi(\xi_{\psi}^\tau \delta_m \rho)} - 1 \quad (A1)
\]
subject to the decentralized equilibrium response by the private agents given by
\[
\frac{\phi(\xi_{\psi}^\tau \delta_m \rho(1 - \omega)(1 - \tau)}{1 + \phi(\xi_{\psi}^\tau \delta_m \rho)} - v ((1 - \psi)\tau)^{1-\mu} \left(\frac{\mu\sigma}{(1 - \mu)}\right)^\mu (z)^{a\mu - 1} = 0 \quad (A2)
\]

After some algebra, equation (A2) gives the solution for \(z\) as a function of \(\tau, \psi\):
\[
z(\tau, \psi) = v \frac{1}{1-a\mu} ((1 - \psi)\tau)^{1-\mu} \left(\frac{\Psi}{\left(\frac{\mu\sigma}{(1 - \mu)}\right)^\mu}\right) \quad (A3)
\]
where \(\Psi = \frac{\phi(\xi_{\psi}^\tau \delta_m \rho(1 - \omega)(1 - \tau)}{1 + \phi(\xi_{\psi}^\tau \delta_m \rho)}\).

Then, substituting (A3) on (A1), we obtain
\[
\tilde{g}(\tau, \psi) = \Psi^{(1-a\mu)} v^{(1-a\mu)} (1-\mu)\mu^{(1-a\mu)} (1-\psi)\tau^{(1-a\mu)} \quad (A4a)
\]

Then, to maximize growth, we take the first order conditions, which are given by
\[
\frac{\partial \tilde{g}(\tau, \psi)}{\partial \tau} = 0, \quad \frac{\partial \tilde{g}(\tau, \psi)}{\partial \psi} = 0
\]

In particular, we have

Derivative with respect to \(\tau\)

\[
\frac{\partial \tilde{g}(\tau, \psi)}{\partial \tau} = 0 \Rightarrow a(1-\mu)\Psi^{(1-a\mu)}(1-\psi)\tau^{(1-a\mu)} + (1-\mu)(1-a)\Psi^{(1-a\mu)}(1-\psi)^{1-a\mu} = 0 \Rightarrow \quad (A4a)
\]
\[
a(1-\mu)\Psi^{-1}\Psi + (1-\mu)(1-a)\tau^{-1} = 0
\]

22
Derivate with respect to $\psi$

$$\frac{\partial \tilde{g}(\tau, \psi)}{\partial \psi} = 0 \Rightarrow a(1-\mu) \Psi^{1-a\mu} - (1-\mu)(1-a)(1-\psi)^{-1} = 0$$

where $\Psi_\tau$ and $\Psi_\psi$ denote partial derivatives of $\Psi$ with respect to $\tau$ and $\psi$:

$$\Psi_\tau = \frac{\partial \Psi}{\partial \tau} = \rho(1 - \omega) \left( \frac{(1-\tau) \dot{\phi} \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \xi_{\psi} \delta_m}{(1 + \phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \rho)^2} - \frac{\phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \xi_{\tau} \delta_m}{1 + \phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \rho} \right)$$

$$\Psi_\psi = \frac{\partial \Psi}{\partial \psi} = \rho(1 - \omega)(1 - \tau) \left( \frac{\dot{\phi} \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \xi_{\tau} \delta_m}{(1 + \phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \rho)^2} \right)$$

Then, by rewriting (A4b) as $a(1-\mu)\Psi^{-1} = \frac{(1-\mu)(1-a)(1-\psi)^{-1}}{\Psi_\psi}$ and plugging into (A4a), after some algebra we obtain

$$(1-\tau) = \frac{\phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) (1 + \phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \rho)}{\phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \left( \frac{\xi_{\tau}}{\delta_m} \right)}$$

Then, substituting in explicitly $\Psi_\psi$ on (A4b), it can be rewritten as

$$a(1-\mu)\rho(1-\omega)(1-\tau) \left( \frac{\phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \frac{\xi_{\tau}}{\delta_m}}{(1 + \phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \rho)^2} \right) - (1-\mu)(1-a)(1-\psi)^{-1} = 0$$

$$a(1-\mu) \left( \frac{\phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \frac{\xi_{\tau}}{\delta_m}}{(1 + \phi \left( \frac{\xi_{\psi \tau}}{\delta_m} \right) \rho)^2} \right) - (1-\mu)(1-a)(1-\psi)^{-1} = 0$$

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and substituting equation (A5) gives (29) as follows:

\[
\frac{a(1-\mu)(\phi(\xi\psi\tau_{\delta_m})\xi_{m})}{\phi(\xi\psi\tau_{\delta_m})(1+\phi(\xi\psi\tau_{\delta_m})\rho)} - (1 - \mu)(1 - a)(1 - \psi)^{-1} = 0 \Rightarrow
\]

Then, to obtain (30) we substitute (29) on (A5) which after some algebra gives

\[
\psi^* = \frac{\Omega - a}{a(\Omega - 1)} > 0
\]

where \(\Omega(\tau^*, \psi^*, \xi, \rho, \delta_m, \ldots) = \frac{\phi(\xi\psi\tau^*\rho(1-\omega)(1-\tau^*)}{\phi(\xi\psi\tau_{\delta_m})\xi_{m}}\).
Then, it is straightforward to prove that the growth maximizing tax rate, \( \tau^* \), is higher than the Barro (1990) optimal taxation rule, \( \tau_{Barro} = 1 - a \), for any positive value of \( \psi \) and depends positively on health expenditures as a share of total government revenues. In particular, \( \tau^* = \frac{1-a}{1-a\psi^*} > (1-a) \Rightarrow \frac{1}{1-a\psi^*} > 1 \Rightarrow 1 > 1 - a\psi^* \Rightarrow a\psi^* > 0 \) and because \( a > 0 \) then it requires \( \psi^* > 0 \) which holds for a positive growth rate as given by proposition 1. Additionally, \( \frac{\partial \tau^*}{\partial \psi^*} = \frac{(1-a)a}{(1-a\psi^*)^2} > 0 \), which implies that the growth maximizing tax rate depends positively on health expenditures as a share of total government revenues.

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**References**


Table 1. Changes in $\xi$, aging and the growth-maximizing allocation

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Notes: $a = 0.5$, $\beta = 0.3$, $v = 0.2$, $\rho = 0.99$, $\delta_m = 0.04$, $\eta = 0.6$, $\mu = 0.5$, $\sigma = 0.1$.

Table 2. Changes in $\delta_m$, aging and the growth-maximizing allocation

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Notes: $a = 0.5$, $\beta = 0.3$, $v = 0.2$, $\rho = 0.99$, $\xi = 1$, $\eta = 0.6$, $\mu = 0.5$, $\sigma = 0.1$. 