Venture Capital, Patents and Innovation

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Abstract

This paper provides a new channel through which venture capital promote start-ups in the sense that by gaining a private benefit from start-up financing without patent protection, the venture capital is more willing to finance entrepreneur at the beginning than other investors. Based on a double moral hazard model, we find that comparing to the non-VC investors, the willingness to invest is higher for venture capitalists, it mitigates the credit constraints of entrepreneur and thus facilitate the startup of entrepreneur and foster their productive innovation activities.
1 Introduction

Does venture capital foster productive innovations? If so, how? Besides advising and management benefits, what’s the advantage of venture capital financing in high-tech industries with innovation? Does patent protection always serve innovators positively? By answering these questions, our paper poses a novel channel through which venture capital promote startups: without patent protection, venture capitalists could gain a private benefit from expropriation or knowledge transmission from financed portfolio startups; thus it lowers down the willingness to fund of venture capitalists, softs the credit constraints of entrepreneurs, and therefore fosters entrepreneurs’ innovation activities.

There has been substantial empirical literature exploring the impact of venture capital on innovation. By investigating 20 U.S. manufacturing industries between 1965 and 1992, Kortum and Lerner(2000) reveals that the venture capital is three times as potent as the corporate R&D and accounts for 8% of industrial innovations in the decade ending in 1992. However, limited by the industry data, Kortum and Lerner(2000) fails to distinguish the ex-ante selection effect from causal effect of venture capital on innovation. By employing individual firm data over different developed countries, Engel and Keilbach(2007), Caselli, Gatti and Perrini(2008), Haeussler, Harhoff and Muller(2009) and Peneder(2010) were able to find out that innovative firms have a higher probability of being venture funded. In other words, the selection effect does exist. Furthermore, Da Rin and Penas(2007) show that venture capital favors the build-up of absorptive capacity and results in a more permanent in-house R&D efforts. Although not immediately overwhelming, this evidence suggests that the causal effect of venture capital in innovation also exists. But overall, there is very little evidence on how venture capital affects innovation.

Besides considering the ex-ante screening effect, the theoretical literature on venture capital primarily illustrates two mechanisms whereby venture capitalists affect the performance of their portfolio firms: monitoring and intervention\(^1\), which alleviates potential moral hazard problems.

\(^1\)See, for example, Dessi (2005) and Holmstrom and Tirole (1997).
on the side of entrepreneur and the provision of advice and support which helps performance directly. Both mechanisms could apply to innovation. However, while there is growing evidence of the role played by venture capitalists in helping to commercialize innovations, as well as their role in helping to recruit key personnel and replace founders with new CEOs, there is very little direct evidence showing that they play an important role in fostering innovation. Our paper is going to fill this gap by providing a possible mechanism in which venture capital influence innovation.

The remaining part is organized as follows. Section 2 introduces the basic model setting. Section 3 simplifies the model as a benchmark. Section 4 and 5 consider the complete model and discusses the implications. Section 6 concludes.

2 The Model

2.1 Project

Consider an entrepreneur (start-up firm) endowed with an innovative investment project. The project requires a contractible initial investment \( I \) (money) and two unobservable (and a fortiori noncontractible) efforts by the entrepreneur: \( e_1 \) to transform an innovative idea into a valuable innovation ("innovative effort"); \( e_2 \) to make the project succeed ("commercialization effort"). The two efforts are sequential. The project is risky and the first, innovative effort may result in the development of a valuable innovation or it may fail. For simplicity, assume that the probability of a valuable innovation being developed is given by \( e_1 \). If there is such an innovation (and in the absence of expropriation, see below), the project will finally succeed with probability \( p + e_2 \), where \( p > 0 \) captures the positive impact of innovative success, while \( e_2 \) captures the additional value of the entrepreneur’s effort following the innovation. Thus to make things reasonable, there


\[ \text{3 Colombo, Grilli and Piva (2006), Gans, Hsu and Stern (2002), Hsu (2006).} \]

\[ \text{4 Hellmann and Puri (2002).} \]
must be $e_1, e_2, p + e_2 < 1$. If there is no (valuable) innovation\(^5\), the project’s success probability is reduced; for simplicity, we assume it is equal to zero. If the project succeeds, it yields verifiable returns $R$; if it fails, it yields nothing ($R > 0$). All agents are risk neutral and protected by limited liability. For simplicity, $R$ and $I$ is normalized to be smaller than unit one.

Assume the entrepreneur has no initial monetary wealth, and needs to raise finance from outside investors. At date 0, assuming outside funding has been obtained and the project has been undertaken, the entrepreneur chooses his innovative effort level $e_1$, where $0 \leq e_1 < 1$. The cost of effort is given by $c(e_1) \equiv \frac{1}{2}e_1^2$.

If he is successful in developing a valuable innovation, the entrepreneur can apply for patent protection. We assume that the patent application will be approved with probability $\beta$, or rejected with probability $1 - \beta$. The exogenous parameter $\beta$ captures differences in the efficiency of the patenting process across industries and/or countries, as well as differences in the potential for patenting products and processes with different characteristics. If the patent is granted, the innovation is under legal protection and no one can expropriate the idea and damage the commercial value of the project\(^6\); however, if the patent is not granted, the entrepreneur faces the danger that the innovation might be expropriated by another firm, or that another firm might develop an equivalent innovation independently. We assume that this happens with probability $\alpha$. If the entrepreneur obtains the funds to undertake the project from a venture capitalist, there is also a possibility of expropriation by the venture capitalist.

Indeed, the venture capitalist (henceforth VC) would find it much easier to expropriate the innovation than any outsider, since he interacted closely and repeatedly with the entrepreneur, and had privileged access to information, throughout the time in which the innovation was being developed. Expropriation by the venture capitalist may take different forms. The VC could sell knowledge to another start-up or established firm - although the difficulties of selling knowledge

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\(^5\)For expositional convenience, we will refer to this as the "no innovation" outcome, but it should be understood throughout as capturing also the possibility of a worthless innovation.

\(^6\)In reality, the extent to which patent protection prevents expropriation will depend on product characteristics and on the breadth of the patent, and will vary across industries. We assume for simplicity that obtaining a patent gives complete protection; industry differences in the effectiveness of patenting can be captured partly through differences in $\beta$. 

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are well known (refs). Another form of expropriation seems more likely and easier for venture capitalists. The VC could communicate knowledge along with helpful advice to another firm it is already funding, so that the other firm’s probability of success is increased while decreasing the chances of success of the expropriated entrepreneur.

We therefore model expropriation very simply as follows. If the VC transfers knowledge about the innovation to another firm, the latter makes a gain with expected value \( G \), while the innovating firm’s probability of success decreases to \( k(p + e_2) \) (where \( 1 > k > 0 \)). Expropriation by the VC cannot be the object of a straightforward sale. For this reason, we assume that the resulting benefit to the VC will be lower than the expected gain to the firm receiving the knowledge: denote it by \( \lambda G \), with \( \lambda < 1 \).

For simplicity, we assume that when another firm develops an equivalent innovation independently or expropriates the entrepreneur’s innovation, it also reduces the entrepreneur’s probability of success to \( k(p + e_2) \). This happens during the second period with probability \( \alpha \) in the absence of a patent, unless the innovation has already been expropriated by the VC. In the latter case, we assume, for simplicity again, that further expropriation (or development of an equivalent innovation) will not further reduce the entrepreneur’s probability of success.

After learning the result of the patent application, the entrepreneur chooses his second effort level \( e_2 \), where \( 0 \leq e_2 < 1 - p \), and the cost of effort is given by \( c(e_2) \equiv \frac{1}{2} e_2^2 \). This second effort is needed to make the project succeed: the innovation has to be commercialized, key strategic decisions have to be made, new personnel may need to be recruited, and so on.

To summarize, expropriation by the VC means that the project will succeed with probability \( k(p + e_2) \), and the VC enjoys a private benefit of value \( \lambda G \). We will assume that \( (1 - \beta) \lambda G < I \), implying that the expected private benefit from expropriation on its own would never be sufficient to induce the venture capitalist to fund the project: he also needs a share of the project’s financial returns. If the VC does not expropriate, or the entrepreneur obtained financing from non-VC investors, the innovation (in the absence of patent protection) will be expropriated (or
an equivalent innovation developed)\(^7\) with probability \(\alpha\).

2.2 Investors

In our model the main difference between raising finance from a venture capitalist and raising it from other investors is due to the venture capitalist’s connections with other firms, and in particular the fact that the VC can transfer knowledge relatively easily to another firm it is funding. As we shall see, this brings potential advantages and disadvantages. To focus on the tradeoff between these costs and benefits, we abstract from other roles played by venture capitalists, such as monitoring or screening, which have been studied in a number of contributions to the literature on venture capital. For the same reason, we assume that venture capitalists, just like other investors, are competitive and do not earn any rents.

2.3 Contract Design

Contracts specify the investor’s (venture capitalist’s) financial contribution at the beginning (\(I\)), and the following contractual terms:

- a reward \(R_c\) for the entrepreneur at the intermediate stage, paid if, and only if, he has succeeded in developing a valuable innovation;

- a sharing rule for final returns, contingent on whether the innovation at the intermediate stage was patented. We denote these as \((R^P_c, R - R^P_c)\) and \((R^I_c, R - R^I_c)\).

2.4 Time line

The model has two periods and three dates, 0, 1, 2. At the beginning of the first period (date 0), the contract is designed, the investment \(I\) is made to undertake the project, and the entrepreneur chooses effort level \(e_1\) to develop the idea into an innovation. At the end of the first period (date 1), the innovation is generated with probability \(e_1\), and it is patented with probability \(\beta\). If the

\(^7\)In the remainder of the paper, for expositional convenience, these two possibilities will be referred to jointly as "expropriation" by another firm, occurring with probability \(\alpha\), since they have the same impact on the (original) innovating firm.
innovation is not patented, and the project has been funded by a venture capitalist, the latter chooses whether to expropriate the innovation or not. The entrepreneur then chooses effort level $e_2$ to make the project succeed. In the absence of patent protection, the innovation may be expropriated by another firm with probability $\alpha$ during the second period. At the end of the second period (date 2), the outcome of the project is realized.

The timeline of the model is shown in figure 1. The model is also illustrated in figure 2.

### 3 Why sequential efforts matter: A simplified model

To see why the fact that the entrepreneur’s effort is valuable in both stages matters, we can briefly consider a simplified model without second-stage effort. The picture of the model is in figure 3. The entrepreneur chooses innovative effort $e$, a valuable innovation is developed with probability $e$ and patented with probability $\beta$, and the project succeeds with probability $p$ in the most favorable case (innovation, no expropriation).

It is straightforward to verify that the optimal contract in this simplified setting rewards the entrepreneur at the intermediate stage if, and only if, he succeeds in developing a valuable innovation, while the VC receives the project’s final returns. The intuition for this result is simple: since the entrepreneur only provides effort in the first stage, and his effort only affects the outcome at the intermediate stage, it is efficient to reward him on the basis of the intermediate outcome (innovation or no innovation). And since the VC receives the final project returns, he fully internalizes the costs and benefits for the project of patenting or expropriating any innovation,
thereby making the efficient decisions at the second stage.

The result is summarized in the following Proposition.

**Proposition 1** By setting $R_e = \gamma pR$, where $\gamma = [(1 - \beta)\alpha k + [1 - \alpha(1 - \beta)]]$, the entrepreneur will exert social optimal effort level as $e_F = \gamma pR$. All the surplus $\frac{1}{2}(\gamma pR)^2$ goes to the entrepreneur; and the VC will transfer the knowledge to another firm if and only if $\lambda G > (1 - \alpha)(1 - k)pR$, which is also social optimal.

**Proof.** See Appendix. $\blacksquare$

An interesting feature of the simplified model without second-period effort is that the entrepreneur will always (weakly) prefer to raise financing from a venture capitalist rather than another investor. The reason is that the VC will always make the efficient decisions ex post, including the decision to transfer knowledge to another firm it is funding, when this is efficient. A non-VC investor, on the other hand, lacks the ability to transfer knowledge in this way.
However, with a second stage effort of entrepreneur, our problem becomes more realistic and meaningful since it’s the entrepreneur who enforce the project till the final stage in reality. In this case, the entrepreneur cannot be rewarded right after the innovation stage, otherwise, he has no incentive to exert effort thereafter. Also by offering part of the final return to the entrepreneur, the venture capitalist cannot recover all the returns in the end from his decision of expropriation. Thus his decision will not be first-best. Even when the social optimal condition for expropriation is not met, as long as the venture capitalist’s personal incentive constraint is satisfied, he will choose to expropriate. We will consider this problem step by step in the following.

4 Sequential efforts: complete information

As we describe above, adding the second stage effort of the entrepreneur, we can explore more interesting results from the model (Figure 2). Thereafter, from now on, we will consider the case in which the entrepreneur’s effort is required at the innovation stage and also at the further
development and commercialization stage. To build the intuition for the results, we first examine the first best case when all actions by the contracting parties are observable and contractible (efforts, patenting and expropriation by the VC). Expropriation by another firm, on the other hand, is not contractible. This will provide a useful benchmark for the analysis that follows.

Since actions by the contracting parties (the entrepreneur and the investor or venture capitalist) are contractible, we can, without loss of generality, focus on the case where the project is funded by a venture capitalist.

When allowing venture capitalist to transfer knowledge to another firm, the objective function of this program becomes:

$$\max_{e_1, e_2^p, e_2^i} V(e_1, e_2^p, e_2^i) = e_1\beta(p + e_2^p)R + e_1(1-\beta)\left[\lambda G + k(p + e_2^i)R\right] - \frac{1}{2}e_1^2 - \frac{1}{2}e_1\beta(e_2^p)^2 + (1-\beta)(e_2^i)^2$$

- which gives us

$$e_1^{FB} = \beta pR + (1-\beta)(kpR + \lambda G) + \frac{1}{2}\beta R^2 + \frac{1}{2}(1-\beta)k^2 R^2$$

$$e_2^p^{FB} = R$$

$$e_2^i^{FB} = kR$$

where $e_1$ is the first-stage effort level, $e_2^p$ is the second-stage effort level if patented, $e_2^i$ is the second-stage effort level if not patented. Thus the overall revenue from this project can be deducted by plug effort levels in $V(e_1, e_2^p, e_2^i) = \frac{1}{2}(e_1^{FB})^2$

- When expropriation of VC is not allowed, the objective function of this program becomes:

$$\max_{\dot{e}_1, e_2^p, e_2^i} V(\dot{e}_1, e_2^p, e_2^i) = \dot{e}_1\beta(p + e_2^p)R + \dot{e}_1(1-\beta)(1-\alpha + \alpha k)(p + e_2^i)R - \frac{1}{2}e_2^2 - \frac{1}{2}e_1\beta(e_2^p)^2 + (1-\beta)(e_2^i)^2$$
which gives us

\[ \hat{e}_1^{FB} = \beta pR + (1 - \beta)zpR + \frac{1}{2}(1 - \beta)z^2R^2 \]

\[ (e^P_2)^{FB} = R \]

\[ (\hat{e}_2^I)^{FB} = zR \]

where \( z \equiv 1 - \alpha + \alpha k \).

The reason we use the same notation for \( e^P_2 \) in these two cases while different notations for \( e_1, e^I_2 \), is because in both cases with patent protection, entrepreneur always choose the second-stage effort \( e^P_2 \) to maximize the same objective function \( \max R(p + e^P_2) - \frac{1}{2}(e^P_2)^2 \), thus the optimal effort level is the same; while to exert effort for the first stage and the second stage without patent protection, the objective function changes as venture capitalist chooses to expropriate or not. The overall revenue from this project is thus given by \( V(\hat{e}_1, e^P_2, \hat{e}_2^I) = \frac{1}{2}(\hat{e}_1^{FB})^2 \).

- Comparing the case of expropriation with the case of no expropriation, we find that when \( V(e_1, e^P_2, e^I_2) > V(\hat{e}_1, e^P_2, \hat{e}_2^I) \), expropriation by venture capitalists is favorable. This inequality refers to the threshold value for \( \lambda G \) : as long as \( \lambda G > (\lambda G)^* \), expropriation is of benefit for the project and should be encouraged, where \( (\lambda G)^* \equiv pR(z - k) + \frac{1}{2}R^2(z^2 - k^2) \).

Comparing to the case without second stage effort, as long as \( \lambda G > pR(z - k) \), expropriation is favorable, in current situation, expropriation demands higher private benefit (greater than \( pR(z - k) + \Delta \), where \( \Delta \equiv \frac{1}{2}R^2(z^2 - k^2) \), and is positive). The reason for this divergence lies in that the effort level is distorted with the existence of expropriation. Thus a higher benefit from expropriation is demanded to recover the efficiency loss from the effort distortion.
5 Sequential efforts: Incomplete information

We now turn to the analysis of optimal contracts in the presence of asymmetric information. We begin by assuming that external finance is raised from investors and/or financial intermediaries with no opportunities for knowledge transfer.

5.1 Financing with knowledge transfer ruled out

Suppose knowledge transfer by investors is ruled out. If the entrepreneur succeeds in developing a valuable innovation but does not obtain a patent, there remains the possibility of expropriation by another firm, with probability $\alpha$. The entrepreneur will take this into account in choosing his effort level in the second period. When the innovation is granted a patent, second-period effort, $e_2^P$, is such that $e_2^P \in \arg\max_{e_2} (p + e_2)R_c^P - \frac{1}{2}e_2^2$. The first order condition yields

$$e_2^P = R_c^P. \quad (1)$$

When the innovation is not granted a patent, on the other hand, second-period effort, $e_2^I$, is given by $e_2^I \in \arg\max_{e_2}(1 - \alpha)(p + e_2)R_c^I + \alpha k(p + e_2)R_c^I - \frac{1}{2}e_2^2$. The first order condition yields

$$e_2^I = [1 - \alpha(1 - k)]R_c^I \equiv zR_c^I. \quad (2)$$

The entrepreneur’s effort in the first period, $e_1$, is determined by the incentive compatibility condition:

$$e_1 \in \arg\max_{e_1} e_1R_c + e_1\beta(p + e_2^P)R_c^P + e_1(1 - \beta)(p + e_2^I)zR_c^I - \frac{1}{2}e_1^2 - \frac{1}{2}e_1[\beta(e_2^P)^2 + (1 - \beta)(e_2^I)^2] \quad (3)$$

which may be written as follows:
\[ e_1 \in \arg \max_{e_1} e_1 R_e + e_1 \beta p R_e^p + e_1 (1 - \beta) p z R_e^f \]
\[ -\frac{1}{2} e_1^2 + \frac{1}{2} e_1 [\beta (e_2^p)^2 + (1 - \beta) (e_2^f)^2] \]

yielding the first order condition:

\[ e_1 = R_e + p (\beta e_2^p + (1 - \beta) e_2^f) + \frac{1}{2} [\beta (e_2^p)^2 + (1 - \beta) (e_2^f)^2] \]

The optimal contract maximizes the entrepreneur’s expected utility subject to the investors’ participation constraint, as well as the entrepreneur’s incentive compatibility and limited liability constraints:

\[ \max_{R_e, R_e^i, R_e^f} U = \frac{1}{2} e_1^2 \]
\[ \text{s.t.} \quad e_2^p = R_e^p, \ e_2^f = z R_e^f, \]
\[ e_1 = R_e + p (\beta e_2^p + (1 - \beta) e_2^f) + \frac{1}{2} [\beta (e_2^p)^2 + (1 - \beta) (e_2^f)^2] (IC_e) \]
\[ e_1 \{ (p + e_2^p) (R - R_e^p) + (1 - \beta) (p + e_2^f) z (R - R_e^f) - R_e \} \geq I, (PC_1) \]
\[ R_e \geq 0, R_e^i \geq 0, R_e^f \geq 0, (LL_e) \]

The solution to this problem, \( P1 \), is described by the following result:

**Proposition 2** When the entrepreneur raises external finance from non-VC investors (no knowledge transfer), the optimal contract sets \( R_e = 0, R_e^p \geq \frac{R - p}{2}, R_e^f \geq \frac{R - p / z}{2} \) and \( R_e^p > R_e^f \); As \( k \) increases or \( \alpha \) decreases, the difference between \( R_e^p \) and \( R_e^f \) decreases. The initial investment level \( I \) must be smaller than \( I^* \) (definition see appendix), which is determined by \( R, p, z, \beta \).

**Proof.** See the Appendix. ■
5.2 Knowledge transfer: the role of venture capitalists

Now consider the case where external finance is provided by a venture capitalist, who can expropriate the innovation ex post if it is not protected by a patent. If the VC chooses not to expropriate, this option cannot be better than funding by non-VC investors, from the entrepreneur’s point of view. We can therefore focus on the case where the VC does expropriate the innovation in the absence of a patent. The entrepreneur’s problem becomes:

\[
\max_{R_e, R^P_e, R^I_e} U = e_1 R_e + e_1 \beta (p + e^P_2) R^P_e + e_1 (1 - \beta) k (p + e^I_2) R^I_e - \frac{1}{2} e_1^2 - \frac{1}{2} e_1 \beta (e^P_2)^2 - \frac{1}{2} e_1 (1 - \beta) (e^I_2)^2 = \frac{1}{2} e_1^2
\]

\[\begin{align*}
\text{s.t.} & \quad e^P_2 = R^P_e, e^I_2 = k R^I_e, \\
& \quad e_1 = R_e + \beta (p + e^P_2) R^P_e + (1 - \beta) k (p + e^I_2) R^I_e - \frac{1}{2} \beta (e^P_2)^2 - \frac{1}{2} (1 - \beta) (e^I_2)^2 \\
& \quad = R_e + \beta p R^P_e + (1 - \beta) k p R^I_e + \frac{1}{2} \beta (e^P_2)^2 + \frac{1}{2} (1 - \beta) (e^I_2)^2, (IC_e) \\
& \quad e_1 \{ \beta (p + e^P_2) (R - R^P_e) + (1 - \beta) [\lambda G + k (p + e^I_2) (R - R^I_e)] - R_e \} \geq 1, (PCVC) \\
& \quad R_e \geq 0, R^I_e \geq 0, R^P_e \geq 0, (LL_e) \\
& \quad \lambda G + k (p + e^I_2) (R - R^I_e) > [1 - \alpha (1 - k)] (p + e^I_2) (R - R^I_e), (ICVC)
\end{align*}\]

The solution to this problem, \( P2 \), is described by the following result:

**Proposition 3** When the entrepreneur raises external finance from VC-investors, the optimal contract sets \( R_e = 0, R^P_e \geq \frac{R - p}{2}, R^I_e \geq \frac{R - p/k}{2} \) and \( R^P_e > R^I_e \). As \( k \) increases, the difference between \( R^P_e \) and \( R^I_e \) decreases. The initial investment level \( I \) must be smaller than \( I^{**} \) (definition see appendix), which is determined by \( R, p, z, \beta \) and \( I^{**} > I^* \).

**Proof.** See the Appendix. \( \blacksquare \)
5.3 Venture capital and innovation

When will the entrepreneur prefer to raise external finance from a venture capitalist? What are the implications for innovation? We can now address these questions. Comparing the entrepreneur’s problem when raising external finance from VC and non-VC investors shows two key differences: when a valuable innovation is developed but is not protected by a patent, the probability of success of the project decreases from $z \equiv 1 - \alpha (1 - k)$ to $k$ under VC financing, because of expropriation by the VC. At the same time, the VC (unlike non-VC investors) obtains a private benefit $B = \lambda G$. The effects on first-period effort work in opposite directions, as can be seen from problem $P2$ above. The decrease in the probability of success is equivalent to a reduction in $k$ in problem $P2$: this reduces effort directly (as is clear from the entrepreneur’s incentive compatibility condition) and indirectly, by making it more difficult to satisfy the participation constraint. The private benefit $B$, on the other hand, relaxes the participation constraint: a higher value of $B$ therefore makes it possible to increase $R^P_e$ and $R^I_e$ without violating the participation constraint, thereby increasing effort.

There is therefore a threshold value for $B$, call it $B^*$, such that for $B > B^*$, first-period effort $e_1$ is higher with VC financing. Since the entrepreneur’s expected utility is a strictly increasing function of $e_1$, VC financing will be strictly preferred in this case. The opposite will be true for $B < B^*$. The threshold value $B^*$ is determined in the Appendix.

**Proposition 4** Under the condition that $\lambda G > (1 + \alpha)(1 - k)(p + z R^I_e)R$, the entrepreneur prefers to raise external finance from VC rather than non-VC. The entrepreneur exerts effort higher with VC and also gets paid higher than with non-VC.

**Proof.** See the Appendix. ■

This condition is a sufficient condition for the results. It implies that as long as the private benefit is large enough such that greater than $(1 + \alpha)(1 - k)(p + z R^I_e)R$, with the effort level $(\hat{e}_1, \hat{e}_2^P, \hat{e}_2^I)$\(^8\) induced by the contract $(\hat{R}_c^P, \hat{R}_c^I, \hat{R}_c = 0)$ with VC which is the same as that

\(^8\)The value of $\hat{e}_1, \hat{e}_2^P, \hat{e}_2^I$ is defined in the appendix.
induced by the optimal contract \((\hat{R}_e^p, \hat{R}_e^f, \hat{R}_e = 0)\) with non-VC, the participation constraint of VC becomes slack. The slack participation constraint implies that by increasing the payment to the entrepreneur in each case until the participation constraint is binding, the effort level of the entrepreneur is increased, thus the expected utility of the entrepreneur increases and the expected profit of the VC remains break even.

A number of implications then follow from our analysis:

(i) venture capital can foster innovation by enabling capital-constrained innovative entrepreneurs essentially to "collateralize" potential knowledge transfers occurring in the absence of patent protection. Ex post, when a valuable innovation is developed but does not obtain patent protection, the venture capitalist can benefit by transferring knowledge to another firm it is funding. Ex ante, this relaxes the venture capitalist’s participation constraint, making it possible to offer a higher share of success returns to the entrepreneur. This in turn elicits higher entrepreneurial effort, which increases the probability of developing a valuable innovation.

To our knowledge, this channel, through which venture capital can encourage innovation, has not been explored in the existing literature.

(ii) if entrepreneurs with potentially worthwhile innovative projects can all obtain the form of external finance they prefer, there is no reason to expect to find a positive association between venture backing and innovative success. However, if some of those that would prefer venture funding are unable to obtain it and secure non-venture funding instead, we would expect to observe a positive association between venture funding and innovation.

(iii) Venture capitalists tend to finance projects with higher potential innovation. \(\lambda G\) which measures the expropriation value of venture capitalists also denotes the value of innovation. we can imagine that the more innovative idea leads to the higher expropriation value. And thus the entrepreneur with higher potentially innovative projects can be more easily financed by VC since they bring more private benefit to VC if expropriation.

This need not translate into a positive association between venture funding and ultimate project success, since this will also depend on development and commercialization efforts.
5.3.1 The role of $\beta$

The parameter $\beta$ which captures differences in the efficiency of the patenting process and the propensity of patent granting across industries and countries must closely related with the contract of our model and the behavior of agents. Investigation of $\beta$ not only provides us policy insights about patenting process, but also make us more clear of the role of venture capitalists. The effect of $\beta$ lies in two aspects: on the one hand, the entrepreneur will exert more effort at the first stage if the probability of being patented is high; on the other hand, higher $\beta$ leaves venture capitalists less possibility to expropriate, thus the channel that VC affects innovation we describe above tends to disappear. This first effect is positive while the second one is negative. The overall effect of $\beta$ depends on which effect dominates the other one. Thus it gives us the following proposition.

Proposition 5 Only when $\beta < \frac{1}{2}$ and $(\hat{R}_e^p + 2p)\hat{R}_e^p > (1 + \frac{1}{1 - 2p})(\hat{R}_e^l + 2p)\hat{R}_e^l$, increasing $\beta$ is benefit for the entrepreneur financing; otherwise, increasing $\beta$ is detrimental to entrepreneur financing, it will decrease the possibility of expropriation by venture capitalists and thus reduce the chances of being financed by VC.

Proposition 5 indicates that When $\beta < \frac{1}{2}$ and $(\hat{R}_e^p + 2p)\hat{R}_e^p > (1 + \frac{1}{1 - 2p})(\hat{R}_e^l + 2p)\hat{R}_e^l$, the effect of $\beta$ on inducing first-stage effort is greater than the effect of reducing the probability of VC’s expropriation. When $\beta \geq \frac{1}{2}$, or when $\beta < \frac{1}{2}$ and $(\hat{R}_e^p + 2p)\hat{R}_e^p < (1 + \frac{1}{1 - 2p})(\hat{R}_e^l + 2p)\hat{R}_e^l$, the effect of decreasing the probability for VC to expropriate overcomes the effect of inducing the first-stage effort. The conditions for the overall effect of $\beta$ is positive have to consider two aspects: one is that $\beta$ has to be small such that the marginal effect for increasing patenting probability is large, the other one is that the payment to the entrepreneur when patent granted has to be large enough comparing to the payment when patent failed. Only when these two conditions are satisfied simultaneously, increasing the patenting probability will induce higher level of first-stage effort and thus improve the expected utility of the entrepreneur.
6 Conclusion

This paper provides a new channel through which venture capital promote starts-ups in the sense that by gaining a private benefit from start-up financing without patent protection, the venture capital is more willing to finance entrepreneur at the beginning than other investors. Based on a double moral hazard model, we find that comparing to the non-VC investors, the willingness to invest is higher for venture capitalists, it mitigates the credit constraints of entrepreneur and thus facilitate the startup of entrepreneur and foster their productive innovation activities.

7 Reference


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8 Appendix

8.1 Proof of proposition 1:

In the simplified model, without the second-stage effort, the incentive compatibility constraint of the entrepreneur is to choose the first-stage effort level \( e \) such that his expected net return is maximized, which is denoted as \( IC_e \); the optimal contract problem is to set \( R_e \) such that maximize the revenue of the entrepreneur conditional on that the participation constraint of the venture capitalist and the incentive compatibility constraints of the entrepreneur and the VC is satisfied:

\[
\begin{align*}
\max_{R_e} U &= R_e - \frac{1}{2}e^2 \\
\text{s.t. } e &\in \arg\max_{\tilde{e}} \tilde{e} R_e - \frac{1}{2}\tilde{e}^2 \ (IC_e); \\
e\beta p R + e(1 - \beta)[akp R + (1 - \alpha)p R] - e R_e &\geq I \ (PC_{VC}); \\
e\beta p R + e(1 - \beta)[akp R + (1 - \alpha)p R] - e R_e &< e\beta p R + e(1 - \beta)(kp R + \lambda G) - e R_e \ (IC_{VC}).
\end{align*}
\]

The participation constraint of venture capitalist must be binding, thus plug it into the objective function, we can find that the objective function of the entrepreneur is equivalent to the social value function, i.e., \( e\beta p R + e(1 - \beta)[akp R + (1 - \alpha)p R] - I - \frac{1}{2}e^2 \). From \( IC_e \), we can get \( e = R_e \), and \( IC_{VC} \) implies that \( akp R + (1 - \alpha)p R < kp R + \lambda G \Rightarrow (1 - \alpha)(1 - k)p R < \lambda G \). As long as expropriation is efficient from the social point of view, which implies that \( IC_{VC} \) is satisfied, then the venture capitalist will transfer the knowledge to other firms, and the optimal contract for this model is to set \( R_e = e = [(1 - \beta)ak + [1 - \alpha(1 - \beta)]p] R \equiv \gamma p R \). The social surplus is \( e R_e - \frac{1}{2}e^2 = \frac{1}{2}e^2 = \frac{1}{2}(\gamma p R)^2 \), where \( \gamma = [(1 - \beta)ak + [1 - \alpha(1 - \beta)]]. \)

8.2 Proof of proposition 2:

The entrepreneur’s problem is:
\[
\begin{align*}
\max_{R_e, r_e^I, r_e^P} U &= e_1 \left\{ R_e + \beta R_e^P (p + e_2^P) + (1 - \beta)(p + e_2^I) z R_e^I \right\} - \frac{1}{2} \epsilon_1^2 - \frac{1}{2} e_1 \left[ \beta (e_2^P)^2 - (1 - \beta)(e_2^I)^2 \right] \\
\text{s.t.} \quad e_2^P = R_e^P, e_2^I = z R_e^I, \\
& \quad e_1 = R_e + p (\beta e_2^P + (1 - \beta) e_2^I) + \frac{1}{2} \beta (e_2^P)^2 + (1 - \beta)(e_2^I)^2 (IC_e) \\
& \quad e_1 \left\{ \beta (p + e_2^P) (R - R_e^P) + (1 - \beta)(p + e_2^I) z (R - R_e^I) - R_e \right\} \geq I (PC_e) \\
& \quad R_e \geq 0, R_e^I \geq 0, R_e^P \geq 0, (LL_e)
\end{align*}
\]

To implement the first-best effort levels in the second period, \( e_2^P = R_e \) and \( e_2^I = z R_e \), would require setting \( R_e^P = R_e^I = R \), which would violate the investors’ participation constraint. Thus effort levels will be lower than the first-best levels, and the investors’ participation constraint will hold as an equality. Plug the \( PC_e \) into the objective function, it can be reduced to \( \frac{1}{2} e_1^2 \).

The problem then becomes

\[
\begin{align*}
\max_{R_e, r_e^I, r_e^P} U &= \frac{1}{2} e_1^2 \\
\text{s.t.} \quad e_2^P = R_e^P, e_2^I = z R_e^I, \\
& \quad e_1 = R_e + p (\beta R_e^P + (1 - \beta) z R_e^I) + \frac{1}{2} \beta (R_e^P)^2 + (1 - \beta)(z R_e^I)^2 \\
& \quad e_1 \left\{ \beta (p + R_e^P) (R - R_e^P) + (1 - \beta)(p + z R_e^I) z (R - R_e^I) - R_e \right\} = I \\
& \quad R_e \geq 0, R_e^I \geq 0, R_e^P \geq 0, (LL_e)
\end{align*}
\]

Suppose \( R_e > 0 \) at the optimum. Then it would be feasible to decrease \( R_e \) by \( dR_e \) without violating the entrepreneur’s limited liability constraint. Consider corresponding increases in \( R_e^I \) and \( R_e^P \), denoted by \( dR_e^I \) and \( dR_e^P \), that would keep first-period effort unchanged. From the entrepreneur’s first-period incentive compatibility constraint, we know that these must satisfy the condition:

\[
\]
\[ dR_e + \beta(p + R^P_e) dR^P_e + (1 - \beta)z [p + zR^I_e] dR^I_e = 0 \]

Take the differential of the LHS of the participation constraint then gives

\[ e_1 \left[ -dR_e + \beta(R - p - 2R^P_e) dR^P_e + (1 - \beta)z(R - p - 2zR^I_e) dR^I_e \right] \]

\[ = e_1 \left[ \beta(R - R^P_e) dR^P_e + (1 - \beta)z(R - zR^I_e) dR^I_e \right] \]

\[ = e_1 \left[ \beta(R - R^P_e) dR^P_e + (1 - \beta)z^2(R - R^I_e) dR^I_e \right] > 0 \]

Thus it is possible to relax the participation constraint without affecting the entrepreneur’s expected utility. But then we cannot be at the optimum. Hence at the optimum we must have \( R_e = 0 \).

The entrepreneur’s problem becomes \((P1*)\):

\[
\max_{R_e, R^P_e, R^I_e} U = \frac{1}{2} e_1^2 \\
\text{s.t.} \quad e_1 = \beta px + (1 - \beta) py + \frac{1}{2} \beta x^2 + \frac{1}{2} (1 - \beta) y^2 \quad (ICC_e) \quad (7)
\]

\[
e_1 [\beta(p + x)(R - x) + (1 - \beta)(p + y)(zR - y)] = I \quad (PC_{UC}) \quad (8)
\]

\[
x \geq 0, y \geq 0 \quad (9)
\]

where \( x \equiv R^P_e \) and \( y \equiv zR^I_e \).

Rearrange equation (7) as

\[
\frac{(x + p)^2}{(2e_1 + p^2)/\beta} + \frac{(y + p)^2}{(2e_1 + p^2)/(1 - \beta)} = 1. \quad (10)
\]
Also rearrange equation (8) as

\[
\beta(x - \frac{R - p}{2})^2 + (1 - \beta)(y - \frac{zR - p}{2})^2 = \frac{\beta}{4}(R + p)^2 + \frac{1 - \beta}{4}(zR + p)^2 - I \tag{11}
\]

By defining \( I^* \equiv \frac{\beta}{4}(R + p)^2 + \frac{1 - \beta}{4}(zR + p)^2 \), equation 16 can be simplified as:

\[
\beta(x - \frac{R - p}{2})^2 + (1 - \beta)(y - \frac{zR - p}{2})^2 = I^* - \frac{I}{e_1} \tag{12}
\]

It’s clear that the problem \((P1^*)\) is equivalent to choosing \(e_1\) as large as possible such that the values of \(x, y\) which uniquely define \(e_1\) satisfy equation (10) and (12). And equation 15 and 17 tell us that the possible point set of \((x, y)\) form two ellipses with respect to these two equations: one with center coordinate \((-p, -p)\), height \(2\sqrt{\frac{2e_1 + p^2}{\beta}}\), and width \(2\sqrt{\frac{2e_1 + p^2}{\beta}}\); the other one with center coordinate \(\left(\frac{R - p}{2}, \frac{zR - p}{2}\right)\), height \(2\sqrt{\frac{I^* - \frac{I}{e_1}}{1 - \beta}}\), and width \(2\sqrt{\frac{I^* - \frac{I}{e_1}}{\beta}}\).

From equation 16, we know that \(I^* - \frac{I}{e_1} \geq 0\). Thus

\[
1 > e_1 \geq \bar{e}_1 \equiv \frac{I}{I^*}. \tag{13}
\]

For any given value of \(I, R, p, \beta, \alpha, k\), the optimal value of effort level \(e_1\) must lie in the above interval. Thus the width of the ellipse 15 must lie in \([2\sqrt{\frac{2e_1 + p^2}{\beta}}, 2\sqrt{\frac{2e_1 + p^2}{\beta}}]\); and the height of the ellipse 15 must lie in \([2\sqrt{\frac{2e_1 + p^2}{\beta}}, 2\sqrt{\frac{2e_1 + p^2}{\beta}}]\). Similarly, the width of the ellipse 17 must lie in the interval \([0, 2\sqrt{\frac{I^* - \frac{I}{e_1}}{\beta}}]\); and the height of the ellipse 17 must lie in the interval \([0, 2\sqrt{\frac{I^* - \frac{I}{e_1}}{\beta}}]\).

The optimal contract for problem \((P1^*)\) must be the tangency point of ellipse 15 and 17 which lies in area C in Figure 4.

Also from inequality (13), we have \(1 > \bar{e}_1\), which implies that the initial investment level of the project \(I\) must be smaller than the value \(I^* = \frac{\beta}{4}(R + p)^2 + \frac{1 - \beta}{4}(zR + p)^2\).

Below we will prove that in optimality, \(x, y\) must satisfies that \(x \geq \frac{R - p}{2}, y \geq \frac{zR - p}{2}\). We will prove it through two steps: (1) \(x < \frac{R - p}{2}\) is not the optimal; (3) \(y < \frac{zR - p}{2}\) is not the optimal.

Suppose the optimal solution for this problem lies in point \((\hat{x}, \hat{y})\), where \(\hat{x} < \frac{R - p}{2}\). Then
consider point \((\tilde{x}, \tilde{y})\), where \(\tilde{x} = R - p - \tilde{x}\). Plug \((\tilde{x}, \tilde{y})\) into \(PC_{VC}\), the equality still holds. However, we can achieve a higher effort level through \(ICC_{e}\), and thus increase the utility of entrepreneur. Thus point \((\tilde{x}, \tilde{y})\) is not optimal. Similarly arguments can be achieved for the case when \(\tilde{y} < \frac{zR - p}{2}\) by choosing \(\tilde{y} = zR - p - \tilde{y}\).

Now in the \((x, y)\) coordinate plain system, consider the point \((\frac{R - p}{2}, \frac{zR - p}{2})\). If we let \(x = \frac{R - p}{2}, y = \frac{zR - p}{2}\), \(ICC_{e}\) implies \(e_1 = \)

The Lagrangian function for the problem can then be written as:

\[
L = \frac{1}{2}e_1^2 + \mu\{e_1[\beta(p + x)(R - x) + (1 - \beta)(p + y)(zR - y)] - 1\}
\]

where \(\mu\) is the lagrangian multiplier. Thus:

\[
\frac{\partial L}{\partial x} = e_1 \frac{\partial e_1}{\partial x} + \mu\left\{\frac{\partial e_1}{\partial x}[\beta(p + x)(R - x) + (1 - \beta)(p + y)(zR - y)] + e_1[\beta(R - p - 2x)]\right\}
\]

where

\[
\frac{\partial e_1}{\partial x} = \beta(p + x)
\]

\[
\frac{\partial L}{\partial y} = e_1 \frac{\partial e_1}{\partial y} + \mu\left\{\frac{\partial e_1}{\partial y}[\beta(p + x)(R - x) + (1 - \beta)(p + y)(zR - y)] + e_1[(1 - \beta)(zR - p - 2y)]\right\}
\]

where

\[
\frac{\partial e_1}{\partial y} = (1 - \beta)(p + y)
\]
The first-order conditions are given by:

\[
\frac{\partial L}{\partial x} = e_1 \beta(p + x) + \mu \{ \beta(p + x)[\beta(p + x)(R - x) + (1 - \beta)(p + y)(zR - y)] \\
+ e_1[\beta(R - p - 2x)] \} = 0
\]

\[
\frac{\partial L}{\partial y} = e_1(1 - \beta)(p + y) + \mu \{(1 - \beta)(p + y)[\beta(p + x)(R - x) + (1 - \beta)(p + y)(zR - y)] \\
+ e_1[(1 - \beta)(zR - p - 2y)] \} = 0
\]

Let \( w \equiv \beta(p + x)(R - x) + (1 - \beta)(p + y)(zR - y) \). Then:

\[
e_1 \beta(p + x) = -\mu \{ \beta(p + x)w + e_1[\beta(R - p - 2x)] \}
\]

\[
e_1(1 - \beta)(p + y) = -\mu \{(1 - \beta)(p + y)w + e_1[(1 - \beta)(zR - p - 2y)] \}
\]

\[
\frac{(p + x)}{(p + y)} = \frac{(p + x)w + e_1(R - p - 2x)}{(p + y)w + e_1(zR - p - 2y)}
\]

\[
(p + x)(p + y)w + (p + x)e_1(zR - p - 2y) = (p + y)(p + x)w + (p + y)e_1(R - p - 2x)
\]

\[
(p + x)(zR - p - 2y) = (p + y)(R - p - 2x)
\]

\[
x(p + zR) = y(p + R) + pR(1 - z)
\]
\[ x = y \frac{(p + R)}{(p + zR)} + \frac{pR(1 - z)}{(p + zR)} \]

where \( x \equiv R_e^P \) and \( y \equiv zR_e^I \), so that

\[ R_e^P = R_e^I \frac{z(p + R)}{(p + zR)} + \frac{pR(1 - z)}{(p + zR)} \quad (14) \]

Let \( R_e^P = aR_e^I + b \). Substituting into the binding participation constraint:

\[ e_1 \{ \beta(p + aR_e^I + b)(R - aR_e^I - b) + (1 - \beta)(p + zR_e^I)z(R - R_e^I) \} = I \]

where

\[ e_1 = \beta p(aR_e^I + b) + (1 - \beta)pzR_e^I + \frac{1}{2} \beta(aR_e^I + b)^2 + \frac{1}{2}(1 - \beta)(zR_e^I)^2 \]

This determines \( R_e^I \).

The expression (14) can be rewritten as

\[ R_e^P = R_e^I \frac{z(p + R)}{(p + zR)} + R - R_e^I \frac{z(p + R)}{(p + zR)} = aR_e^I + R(1 - a) \]

We can easily find that \( a < 1 \). And whatever the value \( a \) is, the line \( R_e^P = aR_e^I + R(1 - a) \) go across the point \((R, R)\) in the \((R_e^P, R_e^I)\) coordinate system. Thus between the interval \((0, R)\), \( R_e^P \) is always greater than \( R_e^I \). As \( a \) goes up (\( \alpha \) increases or \( k \) increases, i.e., \( z \) increases) to 1, the difference between \( R_e^P \) and \( R_e^I \) decreases.

### 8.3 Proof of proposition 3:

The entrepreneur’s problem is:
The first-best effort levels in the second period would be those that maximize the expected returns from the project; i.e. $e_2^e = R, e_1^l = kR$. It is not possible to implement these without violating the VC’s participation constraint. We therefore know that the participation constraint will be binding at the optimum.

The entrepreneur’s problem becomes:

$$\begin{align*}
\max_{R_e, R_l^e, R_p^e} \quad & U = e_1 R_e + e_1 \beta (p + e_2^p) R_p^e + e_1 (1 - \beta) k (p + e_2^l) R_l^e - \frac{1}{2} e_1^2 - \\
& \quad \frac{1}{2} e_1 \beta (e_2^p)^2 - \frac{1}{2} e_1 (1 - \beta) (e_2^l)^2 = \frac{1}{2} e_1^2 \\
\text{s.t.} & \quad e_2^e = R_e, e_1^l = kR_l^e, \\
& \quad e_1 = R_e + \beta (p + e_2^p) R_p^e + (1 - \beta) k (p + e_2^l) R_l^e - \frac{1}{2} \beta (e_2^p)^2 - \frac{1}{2} (1 - \beta) (e_2^l)^2 \\
& \quad = R_e + \beta p R_p^e + (1 - \beta) k p R_l^e + \frac{1}{2} \beta (e_2^p)^2 + \frac{1}{2} (1 - \beta) (e_2^l)^2, (ICC_e) \\
& \quad e_1 \beta (p + e_2^p) (R - R_e^p) + (1 - \beta) [\lambda G + k (p + e_2^l) (R - R_l^e)] - R_e \geq I, (PC_VC) \\
& \quad R_e \geq 0, R_l^e \geq 0, R_p^e \geq 0, (LL_e) \\
& \quad \lambda G + k (p + e_2^l) (R - R_l^e) > [1 - \alpha (1 - k)] (p + e_2^l) (R - R_l^e), (IC_VC)
\end{align*}$$

We can now proceed as in the proof of Proposition 2. Suppose $R_e > 0$ at the optimum. Then it would be feasible to decrease $R_e$ by $dR_e$ without violating the entrepreneur’s limited
liability constraint. Consider corresponding increases in $R'_c$ and $R'_p$, denoted by $dR'_c$ and $dR'_p$, that would keep first-period effort unchanged. From the entrepreneur’s first-period incentive compatibility constraint, we know that these must satisfy the condition:

$$dR_e + \beta(p + R'_p)dR'_p + (1 - \beta)k(p + kR'_l)dR'_l = 0$$

Taking the differential of the LHS of the participation constraint then gives

$$e_1\{ -dR_e + \beta(R - p - 2R'_e)dR'_e + (1 - \beta)k(kR - p - 2kR'_e)dR'_e \}$$

$$= e_1\{ \beta(R - R'_e)dR'_e + (1 - \beta)k(kR - kR'_e)dR'_e \}$$

$$= e_1\{ \beta(R - R'_e)dR'_e + (1 - \beta)k^2(R - R'_e)dR'_e \} > 0$$

Thus it is possible to relax the participation constraint without affecting the entrepreneur’s expected utility. But then we cannot be at the optimum. Hence at the optimum we must have $R_e = 0$.

The entrepreneur’s problem becomes:

$$\max_{R_e, R'_l, R'_p} U = \frac{1}{2}x^2$$

$$e_1 = \beta px + (1 - \beta)py + \frac{1}{2}(1 - \beta)y^2 \quad (IC_{W})$$

$$e_1[\beta(p + x)(R - x) + (1 - \beta)(\lambda G + (p + y)(kR - y))] = I \quad (PC_{VC})$$

$$x \geq 0, y \geq 0 \quad (17)$$

$$\lambda G \geq (1 - \alpha)(1 - k)(p + y)(R - y/k) \quad (IC_{VC}) \quad (19)$$

where $x \equiv R'_e$ and $y \equiv kR'_l$. To begin with, we ignore the feasible constraints (18) and check it afterwards.
Rearrange equation (16) as
\[
\frac{(x + p)^2}{(2e_1 + p^2)/\beta} + \frac{(y + p)^2}{(2e_1 + p^2)/(1 - \beta)} = 1. \tag{20}
\]
Also rearrange equation (17) as
\[
\beta(x - \frac{R - p}{2})^2 + (1 - \beta)(y - \frac{kR - p}{2})^2 = \frac{\beta}{4}(R + p)^2 + \frac{1 - \beta}{4}(zR + p)^2 + \lambda G(1 - \beta) - \frac{I}{e_1} \tag{21}
\]
By defining \( I^* \equiv \frac{\beta}{4}(R + p)^2 + \frac{1 - \beta}{4}(zR + p)^2 + \lambda G(1 - \beta) \), equation 25 can be simplified as:
\[
\beta(x - \frac{R - p}{2})^2 + (1 - \beta)(y - \frac{kR - p}{2})^2 = I^* - \frac{I}{e_1} \tag{22}
\]
It’s clear that the problem \( (P2\ast) \) is equivalent to choosing \( e_1 \) as large as possible such that the values of \( x, y \) which uniquely define \( e_1 \) satisfy equation (20) and (22). And equation 26 and 27 tell us that the possible point set of \( (x, y) \) form two ellipses with respect to these two equations: one with center coordinate \((-p, -p)\), height \(2\sqrt{\frac{2e_1 + p^2}{1 - \beta}}\), and width \(2\sqrt{\frac{2e_1 + p^2}{1 - \beta}}\); the other one with center coordinate \(\left(\frac{R - p}{2}, \frac{kR - p}{2}\right)\), height \(2\sqrt{\frac{I^* - I}{1 - \beta}}\), and width \(2\sqrt{\frac{I^* - I}{1 - \beta}}\).

From equation 27, we know that \( I^* - \frac{I}{e_1} \geq 0 \). Thus
\[
1 > e_1 \geq \frac{I}{I^*} \equiv \frac{I}{I^*}. \tag{23}
\]
For any given value of \( I, R, p, \beta, \alpha, k \), the optimal value of effort level \( e_1 \) must lie in the above interval. Thus the width of the ellipse 26 must lie in \(2\sqrt{\frac{2e_1 + p^2}{1 - \beta}}, 2\sqrt{\frac{2e_1 + p^2}{1 - \beta}}\); and the height of the ellipse 26 must lie in \(2\sqrt{\frac{\frac{I^* - I}{1 - \beta}}}, 2\sqrt{\frac{\frac{I^* - I}{1 - \beta}}\). Similarly, the width of the ellipse 27 must lie in the interval \(0, 2\sqrt{\frac{\frac{I^* - I}{1 - \beta}}\); and the height of the ellipse 27 must lie in the interval \(0, 2\sqrt{\frac{\frac{I^* - I}{1 - \beta}}\).

The optimal contract for problem \( (P2\ast) \) must be the tangency point of ellipse 26 and 27 which lies in area D in Figure 5.

Also from inequality (23), we have \(1 > \frac{I}{I^*}, \) which implies that the initial investment level of
the project $I$ must be smaller than the value $I^{**} = \frac{\beta}{4}(R + p)^2 + \frac{(1-\beta)}{4}(zR + p)^2 + \lambda G(1-\beta)$.

Below we will prove that in optimality, $x, y$ must satisfies that $x \geq \frac{R-p}{2}$, $y \geq \frac{kR-p}{2}$. We will prove it through two steps: (1) $x < \frac{R-p}{2}$ is not the optimal; (3) $y < \frac{kR-p}{2}$ is not the optimal.

Suppose the optimal solution for this problem lies in point $(\hat{x}, \hat{y})$, where $\hat{x} < \frac{R-p}{2}$. Then consider point $(\check{x}, \hat{y})$, where $\check{x} = R - p - \hat{x}$. Plug $(\check{x}, \hat{y})$ into $PC_{VC}$, the equality still holds. However, we can achieve a higher effort level through $ICC_{c}$, and thus increase the utility of entrepreneur. Thus point $(\check{x}, \hat{y})$ is not optimal. Similarly arguments can be achieved for the case when $\check{y} < \frac{kR-p}{2}$ by choosing $\check{y} = kR - p - \check{y}$.

The Lagrangian function for the problem can then be written as:

$$L = \frac{1}{2}e_{1}^{2} + \mu\{e_{1}[\beta(p + x)(R - x) + (1 - \beta)(\lambda G + (p + y)(kR - y))] - I\}$$

where $\mu$ is the lagrangian multiplier. Thus:

$$\frac{\partial L}{\partial x} = e_{1}\frac{\partial e_{1}}{\partial x} + \mu\{e_{1}[\beta(p + x)(R - x) + (1 - \beta)(\lambda G + (p + y)(kR - y))] + e_{1}[\beta(R - p - 2x)]\}$$

where

$$\frac{\partial e_{1}}{\partial x} = \beta(p + x)$$

$$\frac{\partial L}{\partial y} = e_{1}\frac{\partial e_{1}}{\partial y} + \mu\{e_{1}[\beta(p + x)(R - x) + (1 - \beta)(\lambda G + (p + y)(kR - y))] + e_{1}[(1 - \beta)(kR - p - 2y)]\}$$

where
The first-order conditions are given by:

\[
\frac{\partial e_1}{\partial y} = (1 - \beta)(p + y)
\]

\[
\frac{\partial L}{\partial x} = e_1\beta(p + x) + \mu\{\beta(p + x)[\beta(p + x)(R - x) + (1 - \beta)(\lambda G + (p + y)(kR - y))]

+ e_1[\beta(R - p - 2x)]\} = 0
\]

\[
\frac{\partial L}{\partial y} = e_1(1 - \beta)(p + y) + \mu\{(1 - \beta)(p + y)[\beta(p + x)(R - x) + (1 - \beta)(\lambda G + (p + y)(kR - y))]

+ e_1[(1 - \beta)(kR - p - 2y)]\} = 0
\]

Let \( w \equiv \beta(p + x)(R - x) + (1 - \beta)(\lambda G + (p + y)(kR - y)) \). Then:

\[e_1\beta(p + x) = -\mu\{\beta(p + x)w + e_1[\beta(R - p - 2x)]\}\]

\[e_1(1 - \beta)(p + y) = -\mu\{(1 - \beta)(p + y)w + e_1[(1 - \beta)(kR - p - 2y)]\}\]

\[
\frac{(p + x)}{(p + y)} = \frac{(p + x)w + e_1(R - p - 2x)}{(p + y)w + e_1(kR - p - 2y)}
\]

\[(p + x)(p + y)w + (p + x)e_1(kR - p - 2y) = (p + y)(p + x)w + (p + y)e_1(R - p - 2x)\]

\[(p + x)(kR - p - 2y) = (p + y)(R - p - 2x)\]

where $x \equiv R_e^P$ and $y \equiv kR_e^I$, so that

$$R_e^P = R_e^I \frac{k(p + R)}{(p + kR)} + \frac{pR(1 - k)}{(p + kR)} \quad (24)$$

Let $R_e^P = cR_e^I + d$. Substituting into the binding participation constraint:

$$e_1[\beta(p + cR_e^I + d)(R - cR_e^I - d) + (1 - \beta)[\lambda G + (p + kR_e^I)k(R - R_e^I)]] = 1$$

where

$$e_1 = \beta p(cR_e^I + d) + (1 - \beta)pkR_e^I + \frac{1}{2} \beta(cR_e^I + d)^2 + \frac{1}{2}(1 - \beta)(kR_e^I)^2$$

This determines $R_e^I$.

Similarly, as in the proposition 2, the expression (24) can be rewritten as

$$R_e^P = R_e^I \frac{k(p + R)}{(p + kR)} + R - R \frac{k(p + R)}{(p + kR)} = cR_e^I + R(1 - c)$$

We can easily find that $c < 1$. And whatever the value $c$ is, the line $R_e^P = cR_e^I + R(1 - c)$ go across the point $(R, R)$ in the $(R_e^P, R_e^I)$ coordinate system. Thus between the interval $(0, R)$, $R_e^P$ is always greater than $R_e^I$. As $c$ goes up ($k$ increases) to 1, the difference between $R_e^P$ and $R_e^I$ decreases.
8.4 Proof of Proposition 4:

From the proof of proposition 2, we know that there exists optimal solution for Problem 1 \((P1)\) which we denote as scheme 1 \((\hat{R}_e^P, \hat{R}_e^I, \hat{R}_e = 0)\), thus we have \(\hat{e}_2^P = \hat{R}_e^P, \hat{e}_2^I = \hat{R}_e^I, \) and \(\hat{e}_1 = \beta p \hat{R}_e^P + (1-\beta)pz \hat{R}_e^I + \frac{1}{2}\beta(\hat{R}_e^P)^2 + \frac{1}{2}(1-\beta)(z \hat{R}_e^I)^2\). Assume there is a scheme 2 \((\check{R}_e^P, \check{R}_e^I, \check{R}_e = 0)\) for the case with VC-investors, and we set \(\check{R}_e^P = \hat{R}_e^P, \check{R}_e^I = \frac{z}{k} \hat{R}_e^I\). Thus \(\check{e}_2^P = \hat{e}_2^P, \check{e}_2^I = k \hat{R}_e^I = z \hat{R}_e^I = \hat{e}_2^I, \hat{e}_1 = \hat{e}_1\) according to the incentive compatibility constraints of the entrepreneur. Then under scheme 2, the left-handside of the participation constraint of VC in \(P2\) becomes:

\[
LHS(PC_{VC}) = \hat{e}_1 \beta (p + \hat{e}_2^P)(R - \hat{R}_e^P) + \hat{e}_1(1 - \beta)[\lambda G + k(p + \hat{e}_2^I)(R - \frac{z}{k} \hat{R}_e^I)]
\]

From \(P1\), we know that under scheme 1, \(PC_I\) is binding, i.e.,

\[
\hat{e}_1 \beta (p + \hat{e}_2^P)(R - \hat{R}_e^P)^2 + \hat{e}_1(1 - \beta)z(p + \hat{e}_2^I)(R - \hat{R}_e^I) = 1
\]

Thus we have

\[
LHS(PC_{VC}) - LHS(PC_I) = \hat{e}_1(1 - \beta) \left[\lambda G + k(p + \hat{e}_2^I)(R - \frac{z}{k} \hat{R}_e^I) - z(p + \hat{e}_2^I)(R - \hat{R}_e^I)\right]
\]

\[
= \hat{e}_1(1 - \beta) \left[\lambda G + (p + \hat{e}_2^I)(kR - z \hat{R}_e^I - zR + z \hat{R}_e^I)\right]
\]

\[
= \hat{e}_1(1 - \beta) \left[\lambda G - (p + \hat{e}_2^I)(z - k)R\right]
\]

Then as long as \(\lambda G > (z - k)(p + \hat{e}_2^P)R = (z - k)(p + \hat{R}_e^I)R = (1 - \alpha)(1 - k)(p + \hat{R}_e^I)R, LHS(PC_{VC}) > LHS(PC_I) = 1\).

Under the scheme 2, the Program 2 can reach the same effort level and expected utility of the entrepreneur as optimal contract scheme 1 does in the Program 1. However, as long as \(\lambda G > (z - k)(p + \hat{R}_e^I)R\), the parcitipation constraint of VC in \(P2\) is relaxed. Thus the optimal contract of \(P2\) can be achieved by slightly increasing \(R_e^P, R_e^I\) thus increasing \(e_1, e_2, e_2\) without violating the constraints (So does \(IC_{VC}\), since \(\lambda G > (z - k)(p + \hat{R}_e^I)R > (z - k)(p + \hat{R}_e^I)(R - \hat{R}_e^I)\))

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8.5 Proof of Proposition 5:

Let’s denote $\lambda \epsilon \sigma (\rho \lambda \rho ) - \lambda \epsilon \sigma (\rho \lambda \rho ) \equiv A$, then the effect of $\beta$ can be explored from the derivative of $A$ on $\beta$, i.e.,

$$\frac{\partial A}{\partial \beta} = \left[ \frac{\partial \hat{e}_1}{\partial \beta} (1 - \beta) - \hat{e}_1 \right] [\lambda G - (p + \hat{e}_2^2)(z - k)R]$$

Since we are only interested in the case when $\lambda G > (z - k)(p + z\hat{R}_e^l)R$, thus the second term of the derivatives are positive and we can focus on the first term.

$$\frac{\partial \hat{e}_1}{\partial \beta} (1 - \beta) - \hat{e}_1 = p\lambda \epsilon \rho^p - p\lambda \rho (1 - \beta)z\lambda \rho^l + \frac{1}{2}(z\lambda \rho^l)^2 - (1 - \beta)(z\lambda \rho^l)^2$$

$$= \frac{1}{2}(1 - 2\beta)(\lambda \epsilon \rho^p + 2p)\lambda \rho^p - \frac{1}{2}(2 - 2\beta)(z\lambda \rho^l + 2p)z\lambda \rho^l$$

Notice that if $\beta \geq \frac{1}{2}$, then $1 - 2\beta \leq 0$, and above term is negative, thus $\frac{\partial A}{\partial \beta} < 0$. When $\beta < \frac{1}{2}$, if

$$(\lambda \epsilon \rho^p + 2p)\lambda \rho^p > (1 + \frac{1}{1 - 2\beta})(z\lambda \rho^l + 2p)z\lambda \rho^l$$

(25)

, then $\frac{\partial A}{\partial \beta} > 0$ for the case $\lambda G > (z - k)(p + z\hat{R}_e^l)R$. From the result of problem 1, we know that at the optimal, $\hat{R}_e^p , \hat{R}_e^l$ satisfy that $\hat{R}_e^p = a\hat{R}_e^l + b$, where $a = \frac{z(1+z)}{(p+z)R}$, $b = \frac{pR(1-z)}{(p+z)R}$, plug it into inequality (26), rearrange it, then we have

$$(a^2 - cz^2)(\hat{R}_e^l)^2 + 2(ab + ap - pcz)\hat{R}_e^l + b^2 + 2pb > 0$$

where $c \equiv 1 + \frac{1}{1 - 2\beta}$.  

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8.6 Derivation of the threshold value $B^*$

In the optimal contract with non-VC investors, described by Proposition 2, we had the following (with a slight change of notation for ease of exposition):

\[ R_c^P = azR_c^I + b; \quad a = \frac{p + R_c}{p + zR}, \quad b = \frac{pR(1 - z)}{p + zR}, \quad z \equiv 1 - \alpha(1 - k) \]

\[ E_1 M = I, M = \beta(p + azR_c^I + b)(R - azR_c^I - b) + (1 - \beta)(p + zR_c^I)z(R - R_c^I) \]

\[ E_1 = \beta p(azR_c^I + b) + (1 - \beta)pzR_c^I + \frac{1}{2}\beta(azR_c^I + b)^2 + \frac{1}{2}(1 - \beta)(zR_c^I)^2 \]

The corresponding result for the optimal contract with VC investors, described by Proposition 3, was:

\[ r_e^P = ckr_e^I + d; \quad c = \frac{p + R_c}{p + kR}, \quad d = \frac{pR(1 - k)}{p + kR} \]

\[ e_1 V = I; V = \beta(p + ckr_e^I + d)(R - ckr_e^I - d) + (1 - \beta)[B + (p + kr_e^I)k(R - r_e^I)] \]

\[ e_1 = \beta p(ckr_e^I + d) + (1 - \beta) pkr_e^I + \frac{1}{2}\beta(ckr_e^I + d)^2 + \frac{1}{2}(1 - \beta)(kr_e^I)^2 \]

The threshold $B^*$ is then defined implicitly by setting $e_1 = E_1$ and $V = M$. 

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