Entry deterrence under scope economies *

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Abstract

In this paper we develop a model where the incumbent may expand to a second market so as to signal the existence of scope economies and deter potential entry. We show that the incumbent only expands to another market when scope economies are large enough. Thus expansion is indeed a signal of larger economies of scope and for certain parameter values it leads to entry deterrence.

We characterize the unique PBE for the various parameter values and show that the PBE may involve accommodation, entry deterrence or a mixed strategy equilibrium.

Keywords: Scope economies; signalling; entry deterrence.

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1 Introduction

In this paper we study entry deterrence when the incumbent benefits from scope economies if he expands to another product’s market. The paper shows that in the presence of scope economies deterring entry may be welfare improving since it increases efficiency.

We consider a two-period model where the incumbent’s degree of scope economies is private information. In the first period, facing potential entry, the incumbent decides whether or not to expand to a second market. The entrant observes the incumbent’s choice and decides whether to enter or not in the first market, after updating his beliefs about the magnitude of the scope economies. If entry occurs, firms compete in quantities. We characterize the equilibrium of this dynamic game and explore the welfare effects of entry deterrence with scope economies.

Limit pricing is one form of predation involving signaling in incomplete information contexts. The first predation signaling model was developed by Milgrom and Roberts (1982). In this model, a potential entrant has imperfect knowledge about the incumbent’s production cost and the incumbent exploits this uncertainty by setting low prices, in order to make the entrant believe that entry is unprofitable.

The limit pricing strategy is usually claimed to have negative welfare effects, as it hinders competition, even though consumers benefit from temporarily low prices. Subsequent works have dealt with limit pricing in various contexts. For instance, Cabral and Riordan (1997) argue that, in the presence of learning economies, driving rivals out of the market or preventing the entry may allow achieving higher efficiency levels, and thus benefit consumers.

In Milgrom and Roberts (1982) model the incumbent operates in a single market. Most firms, however, operate in several markets, especially when there are economies of scope to be exploited, which is the case we intend to address. An example of a limit pricing model with multimarket firms is Pires and Jorge (2008), which addresses the discriminatory pricing policy of an incumbent that wants to deter entry. This is a case of multiple signals. The authors show that being a multimarket incumbent facilitates entry deterrence. Other authors, such as Bagwell and Ramey (1988), have also explored the use of multiple signals to deter entry. They extend Milgrom and Roberts (1982) model by allowing firms to use price and advertising as potential signals.

Scope economies are usually related with the existence of inputs that may be shared among two or more production processes. These may be physical inputs, or «intangible» ones, such as, for instance, a given technology, managerial experience or a good sales team. Scope economies may arise through the fixed cost component of the multiproduct cost function (e.g. Röller and Tombak, 1990) and/or through the variable cost component (e.g. Dixon, 1994, who presents a model with deseconomies of scope). In the present paper we consider that scope economies
impact on variable costs. An example of scope economies impacting through fixed costs is umbrella branding, in which brand extension allows quality signaling and thus achieving marketing economies (e.g. Choi, 1998; Cabral, 2000 and 2009).

Common examples of industries where economies of scope are relevant include telecommunications (share of inputs between long and short-distance calls, in the cellular market and even with the cable TV market, etc), transportation (share of inputs between several routes in the airline industry or by railway companies), software (share of expertise between different programs or versions), the pharmaceutical industry (share of knowledge and/or components), etc.\textsuperscript{1} As Cantos-Sánchez \textit{et al.}(2003) point out, in the presence of scope economies regulatory measures aimed at one of the markets may affect competition in the other(s), and thus the overall welfare effect must be considered. This is actually taken into account in the current paper.

Even though firms may decide to expand to a second market just to profit from economies of scope and thus decrease unitary costs, this expansion may also be used, in certain cases, to prevent possible rivals from entering in the first market, because the incumbent becomes more efficient.\textsuperscript{2} In the following sections we explore this strategy.

2 The model and some preliminary computations

Consider a two-period model where a monopolist incumbent, firm $I$, faces a potential entrant, firm $E$. In the first period the incumbent operates only in market $A$ and decides whether to expand to a new independent market, market $B$, where the firm would be a monopolist. The products sold in markets $A$ and $B$ can be jointly produced and there is economies of scope. The degree of economies of scope is given by $\theta \in [0, 1]$. The marginal costs are equal to $c \in (0, 1)$ when a single product is produced and equal to $\theta c$ when the two products are produced. So the lower is $\theta$, the stronger is the degree of scope economies. This degree is private information. The entrant believes that $\theta$ is uniformly distributed on $[0, 1]$ and these beliefs are assumed to be common knowledge.

In the first period, the incumbent decides whether to expand to market $B$ (this decision is contingent on $\theta$, the type of firm $I$). Firm $E$ does not know the incumbent’s type, but observes the expansion decision. In the second period, after observing the incumbent’s expansion decision, the entrant updates his beliefs concerning the degree of economies of scope of the incumbent and decides whether to enter in market $A$ with an homogenous product (this decision is contingent

\textsuperscript{1}See, among others, Kessides and Willig(1995) for an explanation of the existence of economies of scope in rail operations, and Banker \textit{et al.} (1998) for evidence on scope economies in the U.S. telecommunications industry.

\textsuperscript{2}Expanding to several markets may also be a strategy of spatial preemption, by occupying the product spectrum so as to leave no niche for the entrant(s) (Schmalensee, 1978, Eaton and Lipsey, 1979).
on whether \( I \) expands to \( B \) or not). If firm \( E \) enters, the two firms decide simultaneously their quantities.

Let \( f^I \) and \( f^E \) be the entry costs of firm \( I \) (in market \( B \)) and firm \( E \) (in market \( A \)), respectively and let \( c \) be the marginal costs of firm \( E \) (and of firm \( I \) when a single product is produced). The second period profits are discounted by \( \delta \in (0, 1] \).

We assume identical demands in the two markets. The inverse demand function is given by:

\[
p = 1 - q
\]

where \( p \) is the price and \( q \) is the total quantity sold in the market.

Considering the previous assumptions, let us present the second period profits under the various scenarios. Under monopoly, if \( I \) does not expand to market \( B \), it is easy to show that his profit is given by:

\[
\Pi^m_A(c) = \frac{(1 - c)^2}{4}
\]

where the non-negativity constraint on quantity implies \( c < 1 \). On the other hand, if \( I \) expands to market \( B \), his post-expansion profits in each market are given by:

\[
\Pi^m_A(\theta, c) = \Pi^m_B(\theta, c) = \frac{(1 - c\theta)^2}{4}
\]

The non-negativity constraint on quantity implies \( c\theta < 1 \), which is implied by \( c < 1 \) and \( \theta \in [0, 1] \).

The profits under duopoly depend on whether firm \( I \) expands or not to market \( B \). If firm \( I \) does not expand, then we have a symmetric duopoly in market \( A \) and profits are given by:

\[
\Pi_A(c) = \Pi_E(c) = \frac{(1 - c)^2}{9}
\]

On the other hand, if firm \( I \) expands to market \( B \), the duopoly in market \( A \) is asymmetric (firm \( I \) has marginal costs \( \theta c \), while firm \( E \) has marginal costs \( c \)). The equilibrium profits are given by:

\[
\Pi_A(\theta, c) = \frac{(1 - c(2\theta - 1))^2}{9}
\]

\[
\Pi_E(\theta, c) = \frac{(1 - c(2 - \theta))^2}{9}
\]

\[
\Pi_B(\theta, c) = \frac{(1 - c\theta)^2}{4}
\]

The non-negativity constraint on the quantity of the incumbent is verified by \( c, \theta < 1 \). The non-negativity constraint on the quantity of the entrant further implies that \( \theta > \frac{2c - 1}{c} \). So the previous expression for the equilibrium profits are only relevant for \( \theta > \max \left[ 0, \frac{2c - 1}{c} \right] \). When
$c > \frac{1}{2}$, for values of $\theta < \frac{2c - 1}{c}$ the equilibrium is $q^E = 0$ and $\Pi_A^E(\theta, c) = 0$, whereas the incumbent’s profits are $\Pi_A^I(\theta, c) = \Pi_A^m(\theta, c)$.

Note that $\Pi_A^E(\theta, c)$ is increasing with $\theta$ in the relevant range (where quantities are positive) and that for $\theta = 1$ (no economies of scope) we are in the symmetric case and thus $\Pi_A^E(1, c) = \Pi_A^m(c)$. On the contrary, $\Pi_A^m(\theta, c)$ and $\Pi_A^I(\theta, c)$ are decreasing with $\theta$.

### 3 Optimal strategy of the entrant

In this section we analyze the optimal strategy of the entrant. The entrant’s strategy is contingent on whether the incumbent expands or does not expand to market $B$. To simplify the exposition we will assume that when the entrant is indifferent between entering or not, he enters. However under indifference any decision is optimal (entering, not entering or following any mixed strategy between entering or not).

When $I$ does not expand to $B$, the optimal entry decision is the following one:

**Lemma 1** If firm $I$ does not expand to $B$, then $E$ should enter in market $A$ iff

$$\Pi_A^E(c) = \frac{(1 - c)^2}{9} \geq f^E.$$ 

**Proof.** If $I$ does not expand to $B$, when $E$ enters there is a duopoly with symmetric cost and post-entry profits are given by $\Pi_A^E(c)$. As a consequence, entry in market $A$ is optimal as long as $\Pi_A^E(c) \geq f^E$. ■

Since the entrant’s profits when the incumbent benefits from scope economies are lower than when he doesn’t, another immediate result is:

**Lemma 2** If it optimal for the entrant not to enter in market $A$ when $I$ does not expand to $B$, then, regardless of beliefs, it is also optimal not to enter when $I$ expands to $B$.

**Proof.** Not entering when $I$ does not expand can only be optimal for $f^E > \Pi_A^E(c) = \frac{(1 - c)^2}{9}$. Since $\Pi_A^E(c) \geq \Pi_A^E(\theta, c)$ for all $\theta \in [0, 1]$ (equality holds for $\theta = 1$) it follows that $f^E > \Pi_A^E(c) \Rightarrow f^E > \Pi_A^E(\theta, c)$ for all $\theta \in [0, 1]$. Thus, regardless of the entrant’s beliefs about $\theta$, it is optimal not to enter when $I$ expands to $B$. ■

The previous result does not depend on the entrant’s beliefs. However, in general, when $I$ expands to market $B$ the optimal decision for the entrant depends on his beliefs about the degrees of economies of scope of the incumbent. Let us assume that the entrant believes that the incumbent’s types who expand to market $B$ are the ones with larger economies of scope (latter on we will see that these beliefs are consistent with the incumbent’s optimal strategy).
If the entrant believes that \( I \) expands if and only if \( \theta \leq \bar{\theta} \) where \( \bar{\theta} \in (0, 1) \), then the posterior beliefs following \( I \)'s expansion to \( B \) should be that \( \theta \) is uniformly distributed on \( [0, \bar{\theta}] \). Under these circumstances the optimal decision of the entrant is:

**Lemma 3** When \( I \) expands to \( B \), if the entrant believes that \( \theta \) is uniformly distributed on \( [0, \bar{\theta}] \) where \( \bar{\theta} \in (0, 1) \), and \( \bar{\theta} \leq \frac{2c-1}{c} \) then the entrant should not enter in market \( A \). Moreover, if \( \bar{\theta} > \frac{2c-1}{c} \) the entrant should enter in market \( A \) when \( I \) expands to \( B \) iff:

\[
E_{\theta} \left[ \Pi^E(\theta, c) | \theta \sim U[0, \bar{\theta}] \right] \geq f^E \iff \int_{\max[0, \frac{2c-1}{c}]}^{\bar{\theta}} \frac{(1-c(2-\theta))^2}{9} \frac{1}{\theta} \ d\theta \geq f^E.
\]

When \( I \) expands to \( B \), if the entrant believes that \( \theta = 0 \), then he should enter in market \( A \) iff:

\[
\Pi^E_A(0, c) = \left(1 - \frac{2c}{9} \right) \geq f^E
\]

**Proof.** When \( \bar{\theta} \leq \frac{2c-1}{c} \) the entrant’s profit in case of entry is nil, thus entry cannot be profitable. When \( \bar{\theta} > \frac{2c-1}{c} \) entry is profitable if the expected profit, given that \( \theta \) is uniformly distributed on \( [0, \bar{\theta}] \), is higher than the entry costs. Finally, when \( \Pr(\theta = 0 | I \text{ expands to } B) = 1 \), the duopoly profits are given by \( \Pi^E_A(0, c) \) and entry is profitable iff \( \Pi^E_A(0, c) \geq f^E \).

The previous lemmas show that the optimal strategy of the entrant depends on \( c \) and \( f^E \). It is interesting to characterize the entrant’s optimal strategy as a function of \( c \) and \( f^E \).

Note that the most favorable scenario for the entrant occurs when the incumbent does not expand to market \( B \), and thus \( I \) does not benefit from economies of scope. In this case, the two firms have symmetric costs and the post-entry profits when \( E \) enters are given by \( \Pi^E_A(c) \). It is immediate that if \( f^E > \Pi^E_A(c) = \frac{(1-c)^2}{9} \) the entrant does not enter even if \( I \) does not expand to \( B \). Since \( E \) never wants to enter, entry is blockaded and consequently the incumbent can behave as a monopolist (there is no credible threat of entry).

On the other hand, the least favorable scenario for the entrant is when the incumbent expands to market \( B \) only if economies of scope are maximal (\( \Pr(\theta = 0 | I \text{ expands to } B) = 1 \)). In this case, the entrant’s profits are (asymmetric duopoly):

\[
\Pi^E_A(0, c) = \frac{(1 - 2c^2)}{9}
\]

If \( E \) wants to enter in this case, \( E \) will always enter. This happens if \( f^E < \Pi^E_A(0, c) \).

Figure 1 shows the set of points in the space \((c, f^E)\) where the optimal decision of firm \( E \) is independent of \( I \)'s expansion strategy. For \( f^E > \Pi^E_A(c) \) firm \( E \) never enters no matter if \( I \) expands or not to market \( B \), thus entry is blockaded. For \( f^E < \Pi^E_A(0, c) \) firm \( E \) always enters in market \( A \).

Between the two curves the \( E \)'s optimal decision depends on the expansion strategy of firm \( I \) and on the entrant’s beliefs after observing entry.
Figure 1: Set of \((c, f^E)\) where \(E\) never enters and where \(E\) always enters in market \(A\).

4 Optimal strategy of the incumbent

In this section we derive the optimal strategy of the incumbent taking into account the expected strategy of the entrant. Obviously the optimal strategy depends on whether the entrant never enters in market \(A\), always enters in market \(A\) or enters if and only if \(I\) does not expand to \(B\) (later on we will also consider the case where \(E\) follows a mixed strategy whenever \(I\) expands to \(B\)). However we will show that the optimal strategy is always of the cut-off type: incumbent’s types with \(\theta\) below or equal to certain cut-off value expand to market \(B\), whereas incumbent’s types with \(\theta\) higher than the cut-off value do not expand to \(B\).

4.1 No threat of entry in Market \(A\)

A monopolist incumbent with no threat of entry would enter market \(B\) if and only if

\[
f^{I} \leq \delta(\Pi^{m}_A(\theta, c) - \Pi^{m}_A(c)) + \delta \Pi^{m}_B(\theta, c).
\]  

(1)

Notice that the right hand side of the previous expression is decreasing with \(\theta\). This implies that if the previous condition is satisfied for \(\theta = \tilde{\theta}\) then it will also be satisfied for all \(\theta < \tilde{\theta}\) and, conversely, if the condition is not satisfied for \(\theta = \tilde{\theta}\), then it will also not be satisfied for \(\theta > \tilde{\theta}\). This suggests that the optimal strategy of the incumbent is of the cut-off type:

**Lemma 4** Suppose that the incumbent expects that \(E\) never enters in market \(A\). For given \(c, \delta\) and \(f^{I} \leq \frac{\delta(1+2c-c^2)}{4}\) there exists a cut-off value \(\theta^* \in [0, 1]\) such that if \(\theta \leq \theta^*\) the incumbent expands to market \(B\) while if \(\theta > \theta^*\) the incumbent does not expand to market \(B\). The value of
\( \theta^* \) depends on \( c, \delta \) and \( f^I \) as follows:

\[
\theta^* = g(\delta, c, f^I) = \begin{cases} 
\frac{\sqrt{2\delta - \sqrt{\delta(1-c)^2 + 4f^I}}}{c\sqrt{2\delta}} & \text{if } f^I \in \left( \frac{\delta(1-c)^2}{4}, \frac{\delta(1+2c-c^2)}{4} \right) \\
1 & \text{if } f^I \leq \frac{\delta(1-c)^2}{4}
\end{cases}
\]

On the other hand, if \( f^I > \frac{\delta(1+2c-c^2)}{4} \) then the incumbent does not expand to \( B \) for all \( \theta \in [0, 1] \).

**Proof.** A monopolist incumbent with no threat of entry would enter market \( B \) if and only if condition (1) holds. Substituting the values of the profits, the condition is equivalent to:

\[
f^I \leq \frac{\delta (1 + 2c - 4c\theta - c^2 + 2c^2\theta^2)}{4} \tag{2}
\]

For \( f^I \leq \frac{\delta(1-c)^2}{4} \) it is easy to verify that \( \theta = 1 \) satisfies the previous condition, thus \( \theta^* = 1 \). On the other hand, for \( f^I > \frac{\delta(1+2c-c^2)}{4} \) the previous condition is not satisfied even for \( \theta = 0 \), implying that no type of incumbent wants to expand to market \( B \). Finally, for \( f^I \in \left( \frac{\delta(1-c)^2}{4}, \frac{\delta(1+2c-c^2)}{4} \right) \) condition (2) holds in equality for

\[
\theta^* = \frac{\sqrt{2\delta - \sqrt{\delta(1-c)^2 + 4f^I}}}{c\sqrt{2\delta}}
\]

Thus the incumbent enters iff \( \theta \leq \theta^* \). ■

Figure 2 shows the optimal expansion decision as a function of the entry costs, \( f^I \), and the degree of economies of scope, \( \theta \), under no threat of entry (in the figure \( \delta \) and \( c \) are fixed).

![Figure 2: Optimal expansion decision with no threat of entry.](image)

To summarize, if \( f^I > \frac{\delta(1+2c-c^2)}{4} \) then no type of incumbent expands to market \( B \). On the other hand, if \( f^I \leq \frac{\delta(1+2c-c^2)}{4} \) then the optimal expansion decision is of the cut-off type. Below \( \theta^* \) it is optimal to expand, above \( \theta^* \) it is optimal not to expand. In other words, the types who expand are the ones with higher economies of scope (lower \( \theta \)).
4.2 Threat of entry and entry deterrence

Let us now study the optimal strategy of the incumbent when he expects that \( E \) does not enter if he expands to market \( B \) but enters otherwise. Given the entrant’s expected strategy, the condition for expanding to market \( B \) to be optimal for \( I \) is:

\[
f^I \leq \delta (\Pi^m_A(\theta, c) - \Pi_A^I(c)) + \delta \Pi^m_B(\theta, c)\tag{3}\]

Since the right hand side of the previous condition is decreasing with \( \theta \), it is easy to show that the incumbent follows a cut-off strategy:

**Lemma 5** Suppose that the incumbent expects that \( E \) does not enter in market \( A \) if and only he expands to \( B \). For given \( c, \delta \) and \( f^I \leq \frac{\delta (4c-2c^2+7)}{18} \) there exists a cut-off value \( \theta^{**} \in [0,1] \) such that if \( \theta \leq \theta^* \) the incumbent expands to market \( B \) while if \( \theta > \theta^{**} \) the incumbent does not expand to market \( B \). The value of \( \theta^{**} \) depends on \( c, \delta \) and \( f^I \) as follows:

\[
\theta^{**} = h(\delta, c, f^I) = \begin{cases} 
3\sqrt{5-\sqrt{2}\sqrt{\delta(1-c)^2+9f^I}}/3c\sqrt{\delta} & \text{if } f^I \in \left(\frac{7\delta(1-c)^2}{18}, \frac{\delta (4c-2c^2+7)}{18}\right] \\
1 & \text{if } f^I \leq \frac{7\delta(1-c)^2}{18}
\end{cases}
\]

On the other hand, if \( f^I > \frac{\delta (4c-2c^2+7)}{18} \) then the incumbent does not expand to \( B \) for all \( \theta \in [0,1] \).

**Proof.** Substituting the equilibrium profits in condition (10) we conclude that expansion to \( B \) is optimal as long as:

\[
f^I \leq \frac{\delta \left(4c + 9c^2\theta^2 - 18c\theta - 2c^2 + 7\right)}{18}\tag{4}\]

For \( f^I \leq \frac{7\delta(1-c)^2}{18} \) it is easy to verify that \( \theta = 1 \) satisfies the previous condition, therefore \( \theta^* = 1 \). On the other hand, for \( f^I > \frac{\delta (4c-2c^2+7)}{18} \) the previous condition is not satisfied even for \( \theta = 0 \), implying that no type of incumbent wants to expand to market \( B \). Finally, for \( f^I \in \left(\frac{7\delta(1-c)^2}{18}, \frac{\delta (4c-2c^2+7)}{18}\right] \) condition (4) holds in equality for

\[
\theta^{**} = \frac{3\sqrt{5-\sqrt{2}\sqrt{\delta(1-c)^2+9f^I}}}{3c\sqrt{\delta}}
\]

Thus the incumbent enters iff \( \theta \leq \theta^{**} \). ■

It is interesting to compare the cut-off values of \( \theta \) in the entry deterrence case with the cut-off values in the blockaded entry case. Note that, for given \( \delta, \theta \) and \( c \), the RHS of condition (10) is higher than the RHS of condition (1) since \( \Pi^m_A(c) < \Pi^m_B(c) \). This implies that the cut-off level, \( \theta^{**} \), below which entrance in market \( B \) occurs when expansion deters entry is higher than the cut-off level when there is no threat of entry, i.e., \( \theta^{**} > \theta^* \).
Figure 3 shows how the expansion decision depends on the entry costs, $f^I$, and the degree of scope economies, $\theta$, for given values of the remaining parameters $(c, \delta)$. For values of $\theta > \theta^{**}$ the incumbent does not want to expand to market $B$ even if by doing so it deters entry in market $B$. For $\theta^* < \theta \leq \theta^{**}$ the incumbent wants to expand to market $B$ if that deters entry in market $A$ but would not enter in market $B$ if there was no threat of entry. Finally, for $\theta \leq \theta^{**}$ the incumbent wants to expand to market $B$ both in the case where expansion deters entry as well as in the case where there is no threat of entry. The shaded area corresponds to the case where expansion to market $B$ is just to deter entry (it would not occur under no threat of entry). Thus the shaded area is a region of «strategic expansion».

![Figure 3: Comparison of cut-off values under no threat of entry and under entry deterrence.](image)

The intuition for the result is the following one. When expansion leads to entry deterrence the benefit of expanding to market $B$ is equal to the profit in market $B$ plus the benefit of being a monopolist with marginal costs $\theta c < c$ instead of a duopolist with costs $c$. On the other hand, the benefit of expanding under no threat of entry is equal to the profit in market $B$ plus the increase in the monopoly profit when costs drop from $c$ to $\theta c$. Since expansion is more profitable under the threat of entry, expansion will be optimal for lower economies of scope (higher $\theta$).

### 4.3 Entry accommodation

If the incumbent cannot avoid entrance in market $A$ ($E$ enters even if $I$ expands to $B$) his decision of expanding to market $B$ or not is based on:

$$f^I \leq \delta(\Pi_A^I(\theta, c) - \Pi_A^I(c)) + \delta\Pi_B^m(\theta, c)$$

(5)

Since the right hand side of the previous condition is decreasing with $\theta$, it is easy to show that the incumbent follows a cut-off strategy:
Lemma 6 Suppose that the incumbent expects that \( E \) enters in market \( A \) regardless of his expansion decision. For given \( c, \delta \) and \( f^I \leq \frac{\delta(9+16c)}{36} \) there exists a cut-off value \( \theta' \in [0, 1] \) such that the incumbent expands to market \( B \) iff \( \theta \leq \theta' \). The value of \( \theta' \) depends on \( c, \delta \) and \( f^I \) as follows:

\[
\theta' = \begin{cases} \frac{17\sqrt{\delta} + 8c\sqrt{\delta} - 2\sqrt{225f^I + 16\delta(1-c)^2}}{25c\sqrt{\delta}} & \text{if } f^I \in \left(\frac{\delta(1-c)^2}{4}, \frac{\delta(9+16c)}{36}\right) \\ 1 & \text{if } f^I \leq \frac{\delta(1-c)^2}{4} \end{cases}
\]

On the other hand, if \( f^I > \frac{\delta(9+16c)}{36} \) then the incumbent does not expand to \( B \) for all \( \theta \in [0, 1] \).

Proof. Substituting the equilibrium profits in condition (5) we conclude that expansion to \( B \) is optimal as long as:

\[
f^I \leq \frac{\delta (9 - 25c\theta + 16c)(1 - c\theta)}{36} \quad (6)
\]

For \( f^I \leq \frac{\delta(1-c)^2}{4} \) it is easy to verify that \( \theta = 1 \) satisfies the previous condition, thus \( \theta' = 1 \). On the other hand, for \( f^I > \frac{\delta(9+16c)}{36} \) the previous condition is not satisfied for \( \theta = 0 \), implying that no type of incumbent wants to expand to market \( B \). Finally, for \( f^I \in \left(\frac{\delta(1-c)^2}{4}, \frac{\delta(9+16c)}{36}\right) \) condition (6) holds in equality for

\[
\theta' = \frac{17\sqrt{\delta} + 8c\sqrt{\delta} - 2\delta\sqrt{225f^I + 16\delta(1-c)^2}}{25c\sqrt{\delta}}
\]

Thus the incumbent enters iff \( \theta \leq \theta' \). ■

Note that, comparing with the case where expansion to \( B \) deters entry, the RHS of the previous expression is clearly lower (as \( \Pi^I_A (\theta, c) < \Pi^I_A (\theta, c) \)). Thus the cut-off value \( \theta' \) below which expansion occurs is smaller than the cut-off level in the previous case, \( \theta' < \theta^{**} \).

5 Perfect Bayesian equilibrium

Having described the optimal strategies of firms \( I \) and \( E \), we are now ready to characterize the PBE of the game. We restrict our analysis to the cases where \( f^I \leq \frac{\delta(4c-2c^2+7)}{18} \) and \( \frac{(1-2c)^2}{9} < f^E \leq \frac{(1-c)^2}{9} \). When \( f^I > \frac{\delta(4c-2c^2+7)}{18} \) the incumbent would never expand to market \( B \) and thus economies of scope would be irrelevant. Moreover, to describe the PBE when entry is blockaded or when entry always occurs regardless of the beliefs is trivial considering the analysis in the two previous sections.

Let \( f^E_d \) be the entrant’s entry cost such that the entrant is indifferent between entering and not entering in market \( A \), when the incumbent expands to \( B \) and \( E \) believes that \( \theta \) is uniformly distributed on \([0, \theta^{**}]\), where \( \theta^{**} > 0 \); that is:

\[
\int_{\max[0, \frac{\theta^{**}}{\theta^{**}+1}]}^{\theta^{**}} \Pi^E(\theta, c) \frac{1}{\theta^{**}} d\theta = f^E_d.
\]
Obviously, for \( f^E > f^E_d \) and the aforementioned beliefs the entrant does not enter in \( I \) when \( I \) expands to \( B \).

Similarly, let \( \bar{f}^E_a \) be \( E \)'s entry costs such that the entrant is indifferent between entering and not entering in market \( A \), when the incumbent expands to \( B \) and \( E \) believes that \( \theta \) is uniformly distributed in \([0, \theta']\), where \( \theta' > 0 \); that is:

\[
\int_{\max[0, \frac{2c-1}{9}]}^{\theta'} \Pi^E(\theta, c) \frac{1}{\theta'} d\theta = \bar{f}^E_a.
\]

Note that, for \( f^E < f^E_d \) the entrant enters in market \( A \) when \( I \) expands to \( B \) if he believes that \( \theta \) is uniformly distributed on \([0, \theta']\).

It should be noted that \( f^E_d < \Pi^E_A(c) = \frac{(1-c)^2}{9} \). Since \( \Pi^E(\theta, c) < \Pi^E_A(c) \) for all \( \theta < 1 \), the expected profit conditional on \( \theta \leq \theta^{**} \) is necessarily below \( \Pi^E_A(c) \). Moreover since \( \Pi^E(\theta, c) \) is increasing with \( \theta \) and \( \theta' < \theta^{**} \) then the expected profit conditional on \( \theta \leq \theta^{**} \) cannot be lower than the expected profit conditional on \( \theta \leq \theta' \), thus \( \bar{f}^E_a \leq f^E_d \) (and \( \bar{f}^E_a < f^E_d \) when \( \theta' < \theta^{**} \)). Finally, \( \bar{f}^E_a > \Pi^E(0, c) = \frac{(1-2c)^2}{9} \) if \( \theta' > 0 \) since \( \Pi^E(\theta, c) \) is increasing with \( \theta \).

The next proposition describes the PBE when the incumbent’s entry costs are low.

**Proposition 1** For given \( \delta, c \), \( f^E \) and \( f^I \leq \delta(1-c)^2 \) there exists a unique PBE. In this PBE the incumbent expands to \( B \) for all \( \theta \in [0, 1] \) and, when \( I \) expands to \( B \), \( E \) believes that \( \theta \) is uniformly distributed on \([0, 1] \). On the other hand, the entrant’s equilibrium strategy depends on \( f^E \) as follows:

1. For \( \frac{(1-2c)^2}{9} < f^E \leq \bar{f}^E_a = f^E_d \) the entrant enters in \( A \) regardless of \( I \)'s expansion decision.
2. For \( \bar{f}^E_a = f^E_d < f^E \leq \frac{(1-c)^2}{9} \) the entrant enters in market \( A \) if and only if the incumbent does not expand to \( B \).

**Proof.** We need to check that the incumbent’s strategy is optimal given the entrant’s strategy, that the entrant’s strategy is optimal given beliefs and that beliefs are consistent with Bayes rule and the incumbent’s equilibrium strategy. When \( f^I < \delta(1-c)^2 \) lemmas 5 and 6 imply that \( \theta^{**} = \theta' = 1 \) and thus the optimal strategy of the incumbent is to expand for all \( \theta \in [0, 1] \), regardless of the entrant’s strategy. Since all incumbent’s types expand to \( B \), expansion to \( B \) is not informative about \( \theta \), thus posterior beliefs should be equal to the prior beliefs that \( \theta \) is uniformly distributed on \([0, 1] \). Given these beliefs, the optimality of the entrant’s strategy follows from lemmas 1 - 3. Note that \( \bar{f}^E_a = f^E_d = \int_{\max[0, \frac{2c-1}{9}]}^{1} \frac{(1-c(2-\theta))}{9} d\theta \) since \( \theta^{**} = \theta' = 1 \).

When the incumbent’s entry costs are low, the incumbent expands to market \( B \) independently of his degree of economies of scope. On the other hand, the entrant’s optimal strategy depends
on his entry costs. For low $f^E$ the entrant always enters and thus the PBE involves entry accommodation. For higher values of $f^E$, the entrant enters if and only if $I$ does not expand. In this case, expansion to $B$ leads to entry deterrence.

Let us now describe the PBE for intermediate values of $f^I$:

**Proposition 2** For given $\delta, c$, $f^E$ and \( \frac{\delta(1-c)^2}{4} < f^I < \frac{\delta(9+16c)}{36} \) there exists a unique PBE. Moreover, for each $\delta$ and $c$ the PBE depends on $f^I$ and $f^E$ as follows:

1. For $\frac{1-2c}{9} < f^E \leq \bar{f}^E_a$ the incumbent expands to market $B$ iff $\theta \leq \theta'$; if $I$ expands to $B$, $E$ believes that $\theta$ is uniformly distributed on $[0, \theta']$; and the entrant enters in market $A$ regardless of $I$’s expansion decision.

2. For $\bar{f}^E_d \leq f^E \leq \frac{(1-c)^3}{9} f^E$ the incumbent expands to market $B$ iff $\theta \leq \theta^{**}$; if $I$ expands to $B$, $E$ believes that $\theta$ is uniformly distributed on $[0, \theta^{**}]$; and the entrant enters in market $A$ if and only if the incumbent does not expand to $B$.

3. Finally, for $\bar{f}^E_a < f^E < \bar{f}^E_d$ the incumbent expands to market $B$ iff $\theta \leq \theta''$ where $\theta''$ is such that

\[
f^E = \int_{\max[0, \frac{2c-1}{c}]}^{\theta''} \frac{(1-c(2-\theta))^2}{9} \theta'' d\theta; \tag{7}\]

if $I$ expands to $B$, $E$ believes that $\theta$ is uniformly distributed on $[0, \theta'']$; and the entrant enters in market $A$ if $I$ does not expand to $B$ and enters in market $A$ with probability $\beta$ if $I$ expands to $B$, where $\beta$ is the solution to:

\[
f^I = \delta(\beta \Pi^I_A(\theta'', c) + (1-\beta)\Pi^P_A(\theta'', c) - \Pi^I_A(c)) + \delta \Pi^P_B(\theta'', c). \tag{8}\]

**Proof.** When $\frac{\delta(1-c)^2}{4} < f^I < \frac{\delta(9+16c)}{36}$ lemmas 5 and 6 imply that the incumbent follows a cut-off strategy for all the possible strategies of the entrant and that $0 \leq \theta' < 1$ ($\theta^{**}$ is equal to 1 for $\frac{\delta(1-c)^2}{4} < f^I \leq \frac{7\delta(1-c)^2}{18}$). Considering this, the proofs of cases 1 and 2 are immediate consequences of lemmas 1-3 and lemmas 5 and 6.

In case 3 one can show that there cannot exist a PBE where the entrant follows a pure strategy when $I$ expands. If $E$ never enters when he observes $I$ expanding to market $B$, then types $\theta \in [0, \theta^{**}]$ would expand to market $B$. However, considering the posterior beliefs, the entrant would be better off by entering as $f^E < \bar{f}^E_d$, a contradiction. Similarly, if $E$ always enters when $I$ expands to $B$, only types $\theta \in [0, \theta']$ want to expand to market $B$, but then it would be optimal for $E$ not to enter as $f^E > \bar{f}^E_a$, a contradiction. Thus, if $\bar{f}^E_a < f^E < \bar{f}^E_d$ there does not exist a PBE where $E$ follows a pure strategy when $I$ expands to market $B$. 

13
Let us now check the mixed strategy PBE. In order for it to be optimal for $E$ to follow a mixed strategy when $I$ expands to market $B$, firm $E$ has to be indifferent between entering and not entering. That is, $\theta''$ has to be such that condition (7) holds.

Considering the optimal strategy of firm $E$ (entering when $I$ does not expand to $B$, entering with probability $\beta$ when $I$ expands to $B$), firm $I$ should expand to market $B$ if and only if:

$$f^I \leq \delta(\beta \Pi_A^I(\theta, c) + (1 - \beta)\Pi_A^m(\theta, c) - \Pi_A^I(c)) + \delta \Pi_B^m(\theta, c) \quad (9)$$

Thus if $\beta$ is the solution to equation (8), then type $\theta''$ will be indifferent between expanding or not to market $B$ while types $\theta < \theta''$ strictly prefer to expand to $B$. Thus it is optimal for $I$ to expand to $B$ for $\theta \leq \theta''$. Finally, the belief that $\theta$ is uniformly distributed on $[0, \theta'']$ is consistent with the cut-off strategy of the incumbent. ■

When the incumbent’s entry costs are intermediate the PBE may involve entry deterrence, entry accommodation or the entrant playing a mixed strategy when the incumbent expands to $B$. It is worthwhile to explore how the mixed strategy PBE changes with $f^E$. When $f^E$ decreases, the value of $\theta''$ that satisfies condition (7) has to decrease in order to maintain the equality (in the PBE less incumbent’s types expand to $B$). Moreover, since the RHS of condition (8) is decreasing with $\beta$ and with $\theta''$, when $\theta''$ decreases, $\beta$ has to increase in order to maintain the equality. As a consequence, the lower is $f^E$, the higher has to be the probability of the entrant entering in market $A$ when $I$ expands to $B$. When $f^E$ decreases to values close to $f^E_a$, $\theta'' \rightarrow \theta'$ and $\beta \rightarrow 1$. On the other hand, when $f^E$ tends to $\frac{f^E_c}{\delta}$ the cut-off level $\theta''$ converges to $\theta^{**}$ and $\beta \rightarrow 0$.

Finally, the next proposition describes the PBE when the incumbent’s entry costs are high (but not so high that $I$ never wants to expand to $B$):

**Proposition 3** For given $\delta, c$, $f^E$ and $\frac{\delta(9 + 16c)}{36} < f^I < \frac{\delta(4c - 2c^2 + 7)}{18}$ there may exist multiple PBE.

1. For $\frac{(1-2c)^2}{9} < f^E \leq \frac{f^E_c}{\delta}$ there are the following PBE:

   (a) The incumbent does not expand to $B$ for all $\theta \in [0, 1]$ and the entrant enters in market $A$ regardless of $I$’s expansion decision. The entrant’s equilibrium strategy can be sustained by the belief, when $I$ expands to $B$, that $\theta$ is uniformly distributed on $[0, \theta^{**}]$.\(^3\)

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\(^3\)Note that these are off-the-equilibrium path beliefs, since in equilibrium no incumbent type is expected to expand. Off-the-equilibrium path beliefs are unrestricted and it is possible to find other beliefs that support this PBE outcome, but these beliefs satisfy the intuitive criterion.
(b) The incumbent expands to market B iff \( \theta \leq \theta'' \) where \( \theta'' \) is such that

\[
f^E = \int_{\max[0, \frac{2c-1}{c}]}^{\theta''} \frac{(1 - c(2 - \theta))^2}{9} \frac{1}{\theta''} d\theta;
\]

if I expands to B, E believes that \( \theta \) is uniformly distributed on \([0, \theta'']\); and the entrant enters in market A if I does not expand and enters in market A with probability \( \beta \) if I expands to B, where \( \beta \) is the solution to:

\[
f^I = \delta(\beta \Pi_A(\theta'', c) + (1 - \beta)\Pi_A(\theta'', c) - \Pi_A(c)) + \delta \Pi_B(\theta'', c).
\]

2. For \( f^E_d < f^E \leq \frac{(1-c)^2}{9} \) the incumbent expands to market B iff \( \theta \leq \theta^{**} \); if I expands to B, E believes that \( \theta \) is uniformly distributed on \([0, \theta^{**}]\); and the entrant enters in market A if and only if the incumbent does not expand to B.

**Proof.** When \( \frac{\delta (9+16c)}{36} < f^I < \frac{\delta (4c-2c^2+7)}{18} \) lemma 5 implies that \( \theta^{**} > 0 \) and lemma 6 implies that if I expects E to always enter then I does not expand for all \( \theta \in [0, 1] \).

To prove 1.a we just need to note that, given E’s strategy, not expanding to B is indeed optimal for all \( \theta \in [0, 1] \). Moreover, when the incumbent does not expand to B, it is optimal for E to enter by lemma 1 and, given beliefs, it is also optimal to enter as \( f^E \leq f^E_d \).

The proof of 1.b and 2 are similar to the proofs of cases 3 and 2 in the previous proposition, respectively.

For given \( \delta \) and \( f^I \), one can find the set of values in the space \((c, f^E)\) which are compatible with entry deterrence, entry accommodation or with a mixed strategy PBE. Figure 4 illustrates the case where \( f^I = 0 \). In this case, proposition 1 applies for all \( c \in (0, 1) \) and thus, for \( \Pi_A^E(0, c) < f^E \leq \Pi_A^E(c) \), either we have entry accommodation or entry deterrence. The figure shows in light grey the set of values of \( c \) and \( f^E \) which are compatible with an entry deterrence PBE. Below \( f^E_d \) the entrant always enters.

Figure 5 illustrates the case of a relatively low \( f^I \), such that for small values of \( c \) proposition 1 still applies, but for higher values of \( c \), the relevant result is proposition 2. The set of values in the space \((c, f^E)\) where the entry deterrence PBE equilibrium exists is represented in light grey. The region in dark grey is a region where there is a mixed strategy equilibrium. Below that, firm E always enters, hence we have entry accommodation. The curves indicating \( f^A \) and \( f^E_d \) depend on \( f^I \). The higher is \( f^I \) the lower are the curves.
does not enter if expands to, enters otherwise

\[ f = \mathbb{E}[\Pi^k(\theta, c)] \]

\[ \Pi^k_0(0, c) \]

\[ \Pi^k(c) \]

\[ E \text{ does not enter if } I \text{ expands to } B, \text{ enters otherwise} \]

\[ f^E \]

\[ 1 \]

\[ \frac{1}{9} \]

\[ \Pi^k_0(0, c) \]

\[ \Pi^k(c) \]

\[ f^E \]

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\[ f^E \]

\[ \frac{1}{9} \]

\[ \Pi^k_0(0, c) \]

\[ \Pi^k(c) \]
When \( I \) expands to \( B \), then \( E \) should enter iff

\[
\frac{(1 - c)(2 - \theta))^2}{9} \geq f^E
\]

Since the entrant’s profit is increasing with \( \theta \), if the previous condition holds for \( \theta = 0 \), then it will hold for all \( \theta \in [0, 1] \). Thus if \( f^E \leq \frac{(1-2c)^2}{9} \leq \frac{(1-c)^2}{9} \), then the entrant always enters, regardless of the incumbent’s expansion decision and of \( \theta \). On the other hand, when the condition does not hold for \( \theta = 1 \), then it does not hold for any \( \theta \in [0, 1] \). As a consequence for \( f^E > \frac{(1-c)^2}{9} \) the entrant never enters, regardless of the incumbent’s expansion decision and of \( \theta \). These two cases are similar to the incomplete information case.

Thus the interesting case is when \( \frac{(1-2c)^2}{9} < f^E < \frac{(1-c)^2}{9} \). In this case, when \( I \) expands to \( B \), \( E \) enters for values of \( \theta \) equal or above the value of \( \theta \in (0, 1) \) that solves:

\[
\frac{(1 - c)(2 - \theta))^2}{9} = f^E \quad \iff \quad \hat{\theta} = \frac{2c + 3\sqrt{f^E} - 1}{c}
\]

So if \( I \) expands to \( B \) and \( \theta < \hat{\theta} \) the entrant does not enter, otherwise he enters (he also enters if \( I \) does not expand to \( B \)). The cut-off value is increasing with \( f^E \); the higher is \( f^E \), the higher has to be \( \theta \) in order for entry to be profitable. Figure 6 shows the entrant’s optimal strategy in the space \((c, f^E)\). For \( \frac{(1-2c)^2}{9} < f^E < \frac{(1-c)^2}{9} \) but close to \( \frac{(1-2c)^2}{9} \) the entrant does not enter when \( I \) expands only for very small \( \theta \) (\( \hat{\theta} \) is close to zero). On the other hand, for \( f^E \) close to \( \frac{(1-c)^2}{9} \), the entrant only enters when \( I \) expands to \( B \) for \( \theta \) close to one.

![Figure 6: The entrant’s optimal strategy.](image)

When we compare Figure 6 with the incomplete information case, we see immediately that under complete information we may either have more entry or less entry than under incomplete information. For instance, comparing with the scenario of low \( f^I \), for \( \frac{(1-2c)^2}{9} < f^E < \frac{(1-c)^2}{9} \)
the entrant always enter under incomplete information but he does not always enters under complete information. The opposite happens for \( f^E > f^E_d \). In this case, there is more entry under complete information.

When the entrant always enters or when the entrant never enters, the incumbent’s optimal decision is like in the case of incomplete information. So in these cases there are no welfare differences between complete and incomplete information.

What happens when \( \frac{(1-2c)^2}{9} < f^E < \frac{(1-c)^2}{9} \)?

Let us consider an incumbent of type \( \theta < \hat{\theta} \). He knows that if he expands \( E \) does not enter while if he does not expand to \( B \) then \( E \) enters. He should expand iff

\[
f_I \leq \delta(\Pi_A^m(\theta, c) - \Pi_A^I(c)) + \delta\Pi_B^m(\theta, c)
\]

(10)

In other words, he should expand iff \( \theta \leq \theta^{**} \). What happens on the SPNE depends on the relationship between \( \hat{\theta} \) and \( \theta^{**} \).

If \( \theta^{**} < \hat{\theta} \) then for \( \theta \leq \theta^{**} \) the incumbent expands and the entrant does not enter while for \( \theta > \theta^{**} \) the incumbent does not expand and the entrant enters (this is a description of the outcome, the entrant’s equilibrium strategy depends on whether \( \theta < \hat{\theta} \) or not).

If \( \hat{\theta} < \theta' < \theta^{**} \), then incumbent types with \( \theta < \hat{\theta} \) expand to \( B \) and the entrant does not enter, for \( \hat{\theta} < \theta < \theta' \) the incumbent expands and the entrant enters, and for \( \theta > \theta' \) the incumbent does not expand and the entrant enters.

If \( \theta' < \hat{\theta} < \theta^{**} \), then incumbent types with \( \theta < \hat{\theta} \) expand to \( B \) and the entrant does not enter while for \( \theta > \hat{\theta} \) the incumbent does not expand and the entrant enters. These cases show that under complete information there cannot exist more expansion to \( B \) than under incomplete information (and in many cases there are fewer types who expand under complete information).

References


