Buyer power and capacity consolidation

Preliminary version, please do not quote

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Abstract: This paper shows, in the first place, that market power in a final market confers buyer power in intermediate markets, and leads to quantity discounts for large buyers. Then it is also shown that buyer power gives incentives to create a dominant firm downstream through the acquisition of capacity of already existing firms, even if its owners must be paid the higher profits they anticipate should they remain in the industry.


Keywords: Intermediate markets; Capacity acquisition;

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1. Introduction

It is frequent to argue informally that large firms obtain better deals from input suppliers than smaller firms. There has been recently some theoretical, experimental and empirical analysis about this issue (see the survey in Snyder (2005)). To summarize, it is not clear from what we presently know whether some firms have more countervailing power than others with respect to suppliers, whether this additional strength against suppliers can be explained in terms of the buyer's size (a concept itself that requires some clarification) and which could be in any case the ultimate source for the existence of this countervailing power.

Below a theory of countervailing power is proposed. It is argued that size per se (more specifically a large capacity of production) do not guarantee a firm to receive discounts from suppliers (when compared with the wholesale prices paid by other buyers). For a firm to obtain better deals, size must be coupled with market power in the final market. Hence, two firms with similar capacity of production that operate in two different markets will not necessarily receive the same discounts from suppliers. More generally, the analysis in this paper shows that size confers countervailing power when it allows the buyer to improve the revenues generated by its outside options in case of disagreement with the seller.

It is then seen that buyer power is a powerful incentive to consolidate capacity in a market. Unfortunately, the discounts that a large firm obtains do not translate into lower prices for consumers: consolidation not just increases countervailing power, but leads to an increase in market power.

Recent empirical papers (Sorensen, 2003; Ellison and Snyder, 2001; and Chipty and Snyder, 1999) suggest that size alone is not enough to obtain better deals from suppliers. It is only when other factors hold, that buyer size can help to obtain such better deals. In the hospital-insurer bargaining process, for instance, Sorensen (2003) points out that the size of payers (as measured by a payer’s total payments to hospitals in a particular market) appears to affect payer bargaining power, although taken in isolation, it cannot explain why some insurers get much better deals than others. The impact of a payer’s willingness and/or the differential
ability of insurance companies to channel patients to lower-priced hospitals seem to be much more relevant. Likewise, Ellison and Snyder (2001) examined the wholesale market for antibiotics and found that large buyers obtain a price discount in comparison to small buyers, but only for antibiotics with expired patents, for which there are substitution opportunities, so that there is some competition among sellers in the market. Beyond these empirical papers, our results suggest that one should also consider industries where suppliers treat small and major customers separately (possibly reorganizing the sales force once large customers appear), and then check whether large customers receive price discounts, and whether separate treatment leads to higher prices (and more effective exploitation) for individual customers.

2. The Model

Consider an industry with input suppliers (upstream firms) and retailers (downstream firms). Downstream, total capacity of production is $X$. From one unit of capacity, at most one unit of output can be obtained. There is a dominant firm $D$ that owns capacity $Z$ and a fringe with remaining capacity $X-Z$. All along the paper, total capacity is fixed; the next section analyzes the effect of changes on $Z$ on the wholesale prices that $D$ pays to $U$, whereas Section 4 discusses the possibility that $D$ emerges from a process of capacity consolidation.

Upstream, there is one super-competitive supplier ($U$ hereafter) able to produce an essential input for the downstream industry at costs $C_U(Q)$, with $C'_U > 0$ and $C_U \geq 0$. There is also an alternative supply of the input at constant marginal cost $c_A$. We discuss both the case when efficiency requires that $U$ serves all the market, $C_U(X) < c_A$, and the alternative possibility that $U$ efficiently serves only part of the industry, $C_U(0) = 0$ and $C'_U(0) < c_A < C_U(X)$. The input is transformed without additional production costs into a final product in a one-to-one basis by downstream firms, which also sell the product to final consumers.

Finally, the final product is homogeneous and the market demand is given by $P(Q)$, with $P'(Q) < 0$. 
Before production takes place, upstream and downstream firms of the industry set vertical contracts that establish the terms under which the input is transferred. Hence the sequence of events goes as follows: The upstream firm U offers bilateral input contracts to downstream firms. DF and fringe firms choose a supplier. Finally there is product market competition.

I will assume that \( P(X) > c_A \); hence fringe firms always produce, and the existence of an alternative supply limits the price U can charge them to \( w = c_A \). For U and D, I assume along the paper that they set a supply contract that maximize their joint profits. Hence I assume that U can price discriminate; moreover I assume that U has all the bargaining power in the intermediate market (of course limited in the contracts that will be accepted by downstream firms to the existence of an alternative source of the input) and offers take-it-or-leave-it contracts to retailers.

In equilibrium U offers to each downstream firm a supply contract that maximize their joint profits. Specifically, U offers a two-part tariff to DF, which may accept or reject the deal. In the case of refusal, the large firm buys the input from the competitive suppliers at price \( w = c_H \) per unit of input, which is the unit price that downstream fringe firms always pay (below it is shown that this feature comes endogenously for a price-taking firm in the final market). In equilibrium the alternative input supply is never used, but its existence affects the split of profits between U and downstream firms.

Throughout the paper, the following assumptions are done:

**Assumption 1.** The cost function of the upstream supplier satisfies \( 0 \leq C'(X) < c_H \); \( C'' \geq 0 \).

**Assumption 2.** Demand is downward sloping, \( P' < 0 \), and total downstream capacity \( X \) satisfies \( c_H < P(X) \).

**Assumption 3.** For any pair \( (q,s) \) that satisfies \( 0 \leq q \leq Q \), \( 0 \leq s \) and \( Q + s \leq X \), \( P''(Q)q + 2P'(Q) - C''(q + s) < 0 \) holds.
Assumption 1 establishes that it is efficient that U serves the all market. Along the paper I will frequently analyze the particular case of assumption 1 in which the upstream firm has constant marginal costs, \( C' = c_L \), with \( c_L < c_H \). Assumption 2 guarantees full employment of the existing capacity by fringe firms. Assumption 3 guarantees that U and DF’s optimizing problems in the quantities they set are strictly concave, so that we may look at first order conditions to find the optimal levels of production.

Maximize joint profits

\[
\text{Max } P(X - Z + q_D)q_D + c_A Q_u - C_U(q_U + Q_u) - c_A(q_D - q_U) \quad \text{s. t. } 0 \leq q_D \leq Z, \\
0 \leq q_U \leq q_D \text{ and } 0 \leq Q_U \leq X - Z.
\]

If \( C_U'(X) < c_A \), then U serves all the industry, \( Q_U = X - Z \) and \( q_U = q_D \), and there exists a level of D’s capacity \( Z_u \) such that \( q_D < Z \Leftrightarrow Z > Z_u \), where \( Z_u < X \) if \( X > Q_U^{\text{mon}} \), where

\[
Q_U^{\text{mon}} = \arg \max P(Q)Q - C_U(Q).
\]

If \( C_U'(X) > c_A \), define \( Z_c \) as D’s capacity that satisfies \( C_U(X - Z_c + q_c) = c_A \), where \( q_c \) is the level of production that D chooses at marginal cost \( c_A \). If \( Z < Z_c \), then U produces until

\[
C_U'(Q) = c_A \text{ and serves retailers at a wholesale price } w = c_A; \text{ D uses all its capacity only if } Z < Z_A, \\
\text{defined by the condition } P'(X)Z_A + P(X) - c_A = 0. \text{ U serves all the industry, } Q_U = X - Z \text{ and } q_U = q_D \text{ only if } Z \text{ is above } Z_c. \text{ It is immediate that } Z_A < Z_c.
\]

How to implement the efficient level of production. A two part tariff can do the trick,

\[
T(q) = T + wq. \text{ Show that } w = c_A \text{ if } Z < Z_u \text{ or } Z < Z_c. \text{ If } C'' > 0,
\]
For marginal cost of production $c$, DF’s profits, gross of any fixed fee if $U$ is chosen as supplier, are

$$
\pi^{DF}(Z,c) \equiv \underset{q}{\text{Max}} \left[P(X - Z + q) - c\right]q \quad \text{s. t.} \quad 0 \leq q \leq Z.
$$

Denote as $q(Z,c)$ the level of output $q$ that solves the former problem. Hence the final price is $P((X - Z) + q(Z,c))$. I show below that DF obtains better deals from $U$ than a fringe firm as long as he has market power downstream, that is, as long as he may find profitable to restrict output in order to increase final prices. But when does a firm really has incentives to restrict output? For such an incentive to exist, it must possess a sufficiently high level of capacity, as the next lemma shows. Define as $Q^{\text{mon}}(c)$ the optimal level of production of an unconstrained monopolist.

**Lemma 1** Under assumption 3a:

(i) If $X < Q^{\text{mon}}(c)$, for any level $Z$ of capacity DF uses all his capacity. Formally,

$$
X < Q^{\text{mon}}(c) \Rightarrow \forall Z \in [0,X], q^{DF}(Z,c) = Z.
$$

(ii) When $Q^{\text{mon}}(c) < X < P^{-1}(c)$, DF uses all his capacity $Z$ only if it is lesser than a capacity level $Z(c)$, where this cut-off value $Z(c)$ satisfies $0 < Z(c) < X$; formally,

$$
Q^{\text{mon}}(c) < X < P^{-1}(c) \Rightarrow \left\{ \exists Z(c) \in (0,X) \text{ s.t.} \begin{array}{l} q(Z,c) = Z \iff Z \leq Z(c) \text{ and} \end{array} \begin{array}{l} q(Z,c) < Z \iff Z > Z(c) \end{array} \right\}.
$$

The threshold value $Z(c)$ is decreasing in $c$. Furthermore, when $Z > Z(c)$, total production $X - Z + q(Z,c)$ is strictly decreasing in both $Z$ and $c$. 

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From Lemma 1, a downstream firm does not have market power when its level of capacity \( Z \) is below \( Z(c_H) \). Note that the Lemma makes clear that we do not need to think on a competitive industry or a fringe as a continuum of atomistic firms. Anyway we will maintain all along the paper this interpretation.

**Example 1. Linear demand.** Let demand be \( P(Q) = A - Q \); when marginal costs are \( c \), a firm has market power when its capacity \( Z \) is above \( Z(c) = A - X - c \). Note that the threshold level \( Z(c) \) is decreasing in \( X \).

**Example 2. Constant elasticity demand.** Let demand be \( P(Q) = AQ^{-\gamma} \), where \( \gamma \in (0,1) \). Now the threshold level is \( Z(c) = \frac{X}{\gamma} \left( 1 - \frac{c}{P(X)} \right) \). This threshold level \( Z(c) \) is now increasing in \( X \) until total capacity reaches a level \( X = \hat{X} = \left( \frac{\gamma A}{\gamma + 1 - c} \right)^{1/\gamma} \), where \( \hat{X} > Q^{\text{mon}}(c) \) and it is decreasing for higher levels of capacity.

It is clear that DF restricts output only if it has a sufficiently higher level of capacity. Fringe firms may free ride on the price increases that this implies. But it is also clear from Lemma 1 that DF would still restrict more final output if he did not reach an agreement with U and had marginal costs \( c_H \); this is something that in equilibrium is never observed, and thus fringe firms can not free ride on it. But this possibility gives buyer power to DF in intermediate markets above that of fringe firms.

U offers a two-part tariff contract for the input, \( T(q) = F + wq \), to the large firm. In such a contract, by setting the marginal wholesale price \( w = c_L \), DF has the same marginal cost than the efficient supplier, and hence chooses the level of production that maximizes joint profits, \( q(Z,c_L) \). Therefore its profits are \( \pi^{DF}(Z,c_L) \). On the other hand, the large firm has the alternative option of buying the input from competitive suppliers at wholesale price \( w = c_H \), in which case its profits amount to \( \pi^{DF}(Z,c_H) \). Finally, the efficient supplier can appropriate the increase in the large firm’s profits by imposing in the input contract a fixed fee \( F \) equal to
\[ \pi^{DF}(Z,c_L) - \pi^{DF}(Z,c_H) \], by which net profits of the large firm are in fact \[ \pi^{DF}(Z,c_H) \]. These are the profits DF takes into account when bidding for capacity.

Given that we model fringe firms as a total level of capacity, \(X-Z\), and that Assumption 1 implies that fringe firms always produce at full capacity, we need not be very specific about how fringe firms buy the input. This is indeed the case for any firm with capacity below \(Z(c_H)\). In any case, for the efficient supplier it is optimal to sell them the input at a wholesale price slightly below \(c_H\). If it turns out to be the case that the efficient supplier serves all downstream firms, then internal production is efficient. Anyway, fringe profits per unit of capacity are \(V^F = P(X) - c_H\); all along the paper \(V^F\) stands for the expected profits per unit of capacity for a fringe firm, for a given market structure.

An immediate consequence from Lemma 1 is that we may state in which cases a downstream firm obtains a discount from the supplier.

**Lemma 2.** A retailer with capacity \(Z > Z(c_H)\) pays a lower price per unit on input that a fringe firm.

The former discussion shows that we have an explicit account of buyer power: large firms (firms that possess a level of capacity \(Z\) above \(Z(c_H)\)) pay less for the input that fringe firms.

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1. We assume that the efficient supplier obtains all the rents only for the sake of simplicity. We could allow some sharing of such rents between the efficient supplier and the large downstream firm, by assuming, for instance, that profits of the large firm are given by \(\beta \Pi(c_L, f) + (1 - \beta) \Pi(c_H, f)\), with \(0 < \beta < 1\).

2. Notice, however, that the average wholesale price is not necessarily decreasing in buyer’s size. Consider for instance what happens when demand is linear and buyer’s capacity is above \(Z(c_L) = A - X - c_L\). A downstream firm with capacity \(Z\) is asked to pay a fixed fee \(\pi^{DF}(Z,c_L) - \pi^{DF}(Z,c_H)\) and in equilibrium chooses to produce \(q(Z,c_L)\). The average wholesale price actually paid is

\[
\frac{T(q(Z,c_L))}{q(Z,c_L)} = \frac{\pi^{DF}(Z,c_L) - \pi^{DF}(Z,c_H)}{q(Z,c_L)} + c_L = \frac{1}{2} \frac{(A - (X - Z) - c_L)^2 - (A - (X - Z) - c_H)^2}{(A - (X - Z) - c_L)} + c_L,
\]

which is increasing in \(Z\). This result should not come as a surprise. Although a large firm has more buyer power than small firms, it is also true that the increase in profits from producing at lower marginal costs is larger, and \(U\) may reap an important part of these additional profits through the fixed fee (it is indeed assumed in the paper that \(U\) reaps all of the additional profits, since the fixed fee is \(\pi^{DF}(Z,c_L) - \pi^{DF}(Z,c_H)\), but the basic point does not change if a more even share of these additional profits is considered).
Notice that it is not size of a buyer per se, but market share in the final market (and hence its power to influence the final price) that gives him buyer power in the intermediate market. Consider for instance a firm with total capacity $Z > Z(c_H)$ split up in several markets, and that capacity in all markets is below $Z(c_H)$. Then, this firm does not obtain discounts from the supplier.

Before entering into the analysis of the first stage (bidding for capacity) it is worth to see how aggregate downstream profits evolve as a function of the size of DF. These profits increase in the level of consolidation through both an increase in (i) the buyer power of DF against U; and (ii) in the level of collusion in the final market.

When there is only a fringe downstream (when all downstream firms have capacity below $Z(c_H)$), aggregate fringe profits are $V^F X = [P(X) - c_H]X$. When downstream there is a firm with enough capacity to influence final prices, i.e. a large firm with capacity $Z > Z(c_H)$ and a fringe firm with capacity $X-Z$, aggregate downstream profits depend on DF’s size. DF’s profits are increasing in its size:

$$\frac{\partial \pi^{DF}(Z,c_H)}{\partial Z} = -P' \left( X - Z + q(Z,c_H) \right) q(Z,c_H) = P \left( X - Z + q(Z,c_H) \right) - c_H > 0$$

(The first equality comes from the envelope theorem, the second one from the FOC of the problem $\max_q \left[ P(X-Z+q) - c \right] q$).

When $Z(c_H) < Z \leq Z(c_L)$, in equilibrium, DF uses all its capacity. Hence, final prices, fringe profits (per unit of capacity) and consumer surplus do not change. Downstream industry profits increase in $Z$ solely because DF increases its buyer power against U. Formally,

$$\frac{\partial \left[ \pi^{DF}(Z,c_H) + V^F(X-Z) \right]}{\partial Z} = P \left( X - Z + q(Z,c_H) \right) - P \left( X - Z + q(Z,c_L) \right) > 0.$$
For a larger DF, i.e. when its capacity $Z$ satisfies $Z(c_L) < Z$, a new effect comes into the scene.

Now in equilibrium DF restricts output and hence final prices are above $P(X)$. Now fringe firms have higher profits and consumers surplus is negatively affected by the existence of DF.

Fringe profits: $V^F = P(X - Z + q(Z,c_L)) - c_H$.

$$\frac{\partial V^F}{\partial Z} = -P'(X - Z + q(Z,c_L)) \left(1 - \frac{\partial q(Z,c_L)}{\partial Z}\right) = -\frac{(P')^2}{2P' + P''q} > 0.$$ 

Consumer surplus decreases, as total output decreases:

$$\frac{\partial (X - Z + q(Z,c_L))}{\partial Z} = \left(1 - \frac{\partial q(Z,c_L)}{\partial Z}\right) = -\frac{P'}{2P' + P''q} < 0$$

Aggregate downstream profits increase in $Z$ as follows:

$$\frac{\partial \{\pi(X) + V^F(Z)(X-Z)\}}{\partial Z} = -P'(X - Z + q(Z, c_H)) q(Z, c_H) + \frac{\partial V^F(Z)}{\partial Z}(X-Z) - V^F(Z) =$$

$$= P(X - Z + q(Z, c_H)) - P(X - Z + q(Z, c_L)) - P' \left(1 - \frac{\partial q}{\partial Z}\right)(X-Z) > 0$$

For $Z(c_H) < Z \leq Z(c_L)$, there is a redistribution of profits from $U$ to downstream firms, but consumers are unaffected. When $Z(c_L) < Z$, there is moreover an increase in final prices (a more collusive outcome).

3. Creation of a dominant firm downstream through capacity acquisition from already established firms

I will discuss in this section the possibility that a dominant firm downstream (DF from now on) is created through acquisition of capacity $Z$ from fringe firms.

Consider the following sequence of events:
**First stage:** An outside player DF bids for retail capacity $Z$. If DF does not monopolize the final market (namely if $Z < X$), there subsists a fringe with capacity $X - Z$.

**Second stage:** Having observed $Z$, the upstream firm U offers bilateral input contracts to downstream firms.

**Third stage:** DF and fringe firms choose a supplier; then there is product market competition.

In the third stage, DF’s profits from acquiring a level of capacity $Z$ from the fringe are given by

$$\pi_{DF}^Z(Z, c_L) - bZ,$$

where the first term are the (anticipated) net profits after a successful bargaining process with U and $b$ is the public bid for each unit of capacity, i.e., the price to pay for it. I assume unconditional offers.

Under rational expectations, fringe firms correctly infer the final spot price. More precisely, they infer from $b$ the level of capacity that DF acquires, $Z$, and may evaluate the impact caused by $Z$ on final prices (through the effect of $Z$ on the expected level of production of the large firm). Fringe firms assume that DF produces at low costs, that its level of production is $q(Z, c_L)$, and hence

$$p^e = P(X - Z + q(Z, c_L)).$$

Thus, in order to obtain a level of capacity $Z$, DF must offer a bid price
Lemma 3. Bids and the acquisition process

The rational expectations equilibrium outcome of the acquisitions process:

For \( b < P(X) - c_H \), the bid fails.

For \( b = P(X) - c_H \), any amount of capacity \( Z \) satisfying \( Z \in [0, Z(c_L)] \) is acquired.

For \( b \in \left( P(X) - c_H, P\left( Q^{\text{mon}}(c_L) \right) - c_H \right) \), the amount of capacity \( Z \) acquired is the one that satisfies \( b = P(X - Z + q(Z, c_L)) - c_H \).

For \( b \geq P\left( Q^{\text{mon}}(c_L) \right) - c_H \), all the existing capacity is acquired, \( Z = X \).

The lemma states the equilibrium outcome of stage 1, not an equilibrium of fringe firms’ strategies. DF chooses the level of capacity \( Z^{DF} \) that solves the following problem:

\[
\max_{Z} \Pi^{DF}(Z) = \pi^{DF}(Z, c_L) - V^{F} Z \quad \text{s. t. } 0 \leq Z \leq X.
\]

From the envelope theorem, the derivative of DF’s profits with respect to \( Z \) is equal to

\[
\frac{d\Pi^{DF}(Z)}{dZ} = \frac{\partial \pi^{DF}(Z, c_L)}{\partial Z} - V^{F} - \frac{dV^{F}}{dZ} Z.
\]

In (.), the difference, \( \frac{\partial \pi^{DF}(Z, c_L)}{\partial Z} - V^{F} \), reflects the difference in buyer power against U that DF has when compared with a fringe firm. He acquires an additional unit of capacity, whereas the second term, \( V^{F} \). The difference in buyer power can be written as:
\[ \frac{\partial \pi^{DF}(Z,c_2)}{\partial Z} - V^F = P(Q(Z,c_H)) - P(Q(Z,c_L)) \]

The difference \( P(Q(Z,c_2)) - P(Q(Z,c_1)) \) accounts for the difference in buying power between DF and the fringe. \( P(Q(Z,c_L)) \) is the final price in equilibrium, and hence the final price that fringe firms expect; if a fringe firm do not reach an agreement with U, it may turn to the alternative input supply, but the final price does not change. DF may also turn to the alternative input supply, but in this case he would (possibly) restrict output, and hence final prices would increase to \( P(Q(Z,c_H)) \). This out-of-equilibrium increase in final prices is not observed (nor expected) by fringe firms, and hence they can not free ride on it.

The last term in (.), \( \frac{dV^F}{dZ} \), is the impact of a large firm in final prices. Fringe firms free ride if \( q<Z \).

The third term, \( \frac{dV^F}{dZ} = \frac{dp^e}{dZ} \), is the marginal change in fringe profits. DF may restrict output in order to increase final prices even if his marginal costs are \( c_1 \), and the fringe free rides on it. Hence this term reflects the change in the acquisition cost of capacity from the change in market performance that the increase in DF’s size may imply.

For low level of \( Z \), there is no difference in buyer power. Hence \( \frac{d\Pi^{DF}(Z)}{dZ} = 0 \) in this interval of capacity. For intermediate level of capacity, there is a dif in buyer power but \( d\Pi^{DF}(Z) = 0 \), hence \( \frac{d\Pi^{DF}(Z)}{dZ} = P(Q(Z,c_H)) - P(Q(Z,c_L)) > 0 \) and it is profitable to buy capacity; the

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3 For a careful discussion of the issues at hand, see lemma 2 in Burkart, Gromb and Panunzi (JPE, 1998) in a takeover bidding process scenario.
4 The FOC of the large firm is \( P(X - Z + q(Z,c))q(Z,c) + P(X - Z + q(Z,c)) - c = 0 \). Hence \( \frac{d\pi(Z,c)}{dZ} = -P(X - Z + q(Z,c))q(Z,c) = P(X - Z + q(Z,c)) - c \).
only term in (.) is the difference in buyer power. Hence it is immediate that capacity acquisition is profitable.

When large levels of $Z$ there is an effect on $V$. Define $E$ as the degree of concavity of the residual inverse demand, 

$$
E = \frac{P'(X - Z + q)}{P''(X - Z + q)} q.
$$

**Assumption 4.** $E$ is non-increasing in $q$.

The linear and constant elasticity demands are examples of demand functions that satisfy assumption 4. When a downstream firm has capacity $Z \geq Z(c_H)$ and hence may restrict output, assumption 4 implies that, when we evaluate $E$ at $q(c)$,

$$
\frac{\partial E}{\partial c} = \frac{\partial E}{\partial q} \frac{\partial q(c)}{\partial c} \geq 0.
$$

The next proposition shows that buying power in intermediate markets makes valuable to acquire capacity downstream.

**Proposition 1.** As long as $X$ is bigger than $Q^{\text{mon}}(c_H)$, capacity acquisition downstream is strictly profitable. Only when total capacity satisfies $Q^{\text{mon}}(c_H) < X < Q^{\text{mon}}(c_L)$ there is complete monopolization downstream, that is, DF buys all the existing capacity to the fringe. Otherwise DF buys a level of capacity $Z^{DF}$ that satisfies $0 < Z^{DF} = Z(c_L) < X$.

**Corollary** The process of consolidation is not harmful for consumers, because in equilibrium DF uses all his capacity: $q(Z^{DF}, c_L) = Z^{DF}$.

Proposition 1 tells that the possibility to have more buying power against U gives incentives to become a DF downstream for any level of capacity already existing, but only in some cases the creation of a monopoly downstream will be observed. However, in equilibrium the only effect of the creation of a DF downstream is a redistribution of rents from U to DF. In any case DF finds optimal to buy a level of capacity such that in equilibrium he do not restrict output and as a consequence final prices remain as when there was only a fringe downstream.
The result is in some sense in accordance with the results obtain in Kamien and Zang (.) in an oligopoly model; firms that do not sell their capacity free ride from DF; to buy capacity becomes too expensive. The addition of an intermediate market with the appearance of a new effect, the impact of size on buyer power, surprisingly does not change their main result: a large firm is created, but it is too expensive to create with the contention to create a more collusive final market. As a consequence we obtain the striking result that in equilibrium we do not observe an increase in final prices.

Unfortunately for welfare purposes, for proposition 1 to obtain it is crucial to assume that there is only one agent that buys capacity downstream. As the next section shows, if there is more than one outside player interested in reaping the rents that consolidation downstream gives (through the increase in buying power in intermediate market that it implies), consolidation may become harmful for consumers.

4. Consolidation by established fringe firms

In section 2 it is shown that a large firm obtain better deals from U. It is not clear that a large firm obtains larger profits (per unit of capacity) that a fringe firm. Buyer power in the intermediate market must be balanced with the fact that a large firm (may) restrict output and hence increase final prices; this is a policy that benefits more fringe firms that the large firm (the standard free rider effect).

In the previous section, any process of consolidation came from outsiders acquiring capacity of already established fringe firms. In this section I analyze the incentives of established atomistic firms to begin a process of capacity consolidation. The process may take any form that leads members to restrict output in the mutual interest of members. It may come as the result of a merger process, as a process of cartelization (Madhavan et al., Cave and Salant (1987, 1995)), etc. In any case membership to the “coalition” of producers is voluntary.
Members of a consolidated firm obtain profits \( \Pi^C = \frac{\pi^{DF}(Z, c_H)}{Z} \), per unit of capacity, whereas independent firms\(^5\) obtain profits \( V^F = P(Q(Z, c_L)) - c_H \). Compared with the process of consolidation in section 3, now the relevant issue is the change in average profits. It is in the interest of insiders to incorporate new members to the coalition if \( \Pi^C \) increases in \( Z \). On the other hand, outsiders want to enter into the coalition if \( \Pi^C > V^F \).

Now, it is immediate from the analysis in section 3 that:

1. For \( Z \leq Z(c_H) \), \( \Pi^C = P(X) - c_H = V^F \). For \( Z(c_H) < Z \leq X \), \( \Pi^C \) is increasing in \( Z \).

2. Fringe profits do not change as long as the coalition’s size satisfies \( Z \leq Z(c_L) \), and these profits are \( V^F = P(X) - c_H \). For \( Z(c_H) < Z \leq X \), \( V^F \) is increasing in \( Z \).

Hence it is immediate that established firms have incentives to consolidate. For levels of consolidation below \( Z(c_L) \), the only effect of consolidation is the increase in buyer power against \( U \) of the consolidated firm. Once consolidation is above \( Z(c_L) \), however, independent firms may free ride on any reduction of production accorded by members of the coalition.

Anyway \( \Pi^C \) and \( V^F \) are continuous functions of \( Z \). Hence now the level of consolidation is above \( Z(c_L) \).

**Proposition 2.** A coalition is always created, and its size exceeds \( Z(c_L) \), \( Z^C > Z(c_L) \). As a consequence, \( Q^* < X \).

Does the possibility of consolidation lead to a monopoly?

**Lemma 4.** If A4 is satisfied, then for \( Z(c_L) < Z \leq X \), \( \Pi^C > V^F \Rightarrow \frac{\partial \Pi^C}{\partial Z} < \frac{\partial V^F}{\partial Z} \).

---

\(^5\) If \( X \) is sufficiently large, then more than just one coalition could take place. The analysis could be easily extended to this case.
The process of consolidation may be total or partial.

Example 1 (continued) linear demand, plot of $\Pi^C$ and $V^F$.

For a DF with capacity $Z$ that satisfies $Z(c_L) = A - X - c_L < Z \leq X$:

$$\Pi^C = \frac{1}{4} \left( A - (X - Z) - c_H \right)^2 / Z$$

$$V^F = \frac{1}{2} (A - (X - Z) + c_L) - c_H$$

$$0 < \frac{\partial \Pi^C}{\partial Z} = \frac{1}{4} \left( 1 - \left( \frac{A - X - c_H}{Z} \right)^2 \right) < \frac{\partial V^F}{\partial Z} = \frac{1}{2}$$

$$\Pi^C = V^F \quad \text{at} \quad Z^* = (c_H - c_L) + \sqrt{\left( c_H - c_L \right)^2 + \left( A - X - c_H \right)^2} < X\quad \text{whenever total capacity} \ X$$

satisfies $X > \frac{1}{2} \left( \frac{(A - c_H)^2}{(A - c_H) - (c_H - c_L)} \right)$.

5. Commitment problems of a raider

In previous sections, I have assumed just one period where acquisitions are made. There is a commitment problem if we consider further periods of acquisitions. Let $\pi^{DF}_i$ design DF’s profits in period $i$, $i=1, 2, Z$, DF’s capacity in period $i$, etc., and let $\delta$ be the discount factor. A bid $b_1$ in period 1 leads to the acquisition of a level of capacity $Z_1$ that satisfies

$$b_1 = V(Z_1) + \delta V(Z_2^\epsilon), \quad (.)$$

where $Z_2^\epsilon$ denotes the expected level of capacity for DF in period 2DF (I am abusing notation here, given that $Z_2^\epsilon$ is a function of the level of capacity already acquired in period 1). In period 2, DF may acquire further capacity $Z_2 - Z_1$ at a cost.
In the REE, $Z^e = Z$. DF has net profits

$$\Pi(Z, Z') = \pi^D_{1} (Z, c_H) - b_1 Z + d \{ \pi^D_{2} (Z, c_H) - b_2 (Z - Z') \}$$

where $b_1$ and $b_2$ are given by (.) and (.) respectively. In period 2, DF chooses to acquire further capacity $Z_2 - Z_1$ solving the problem:

$$\max_{Z_2} \Pi(Z_2, Z_1) = \pi^D_{2} (Z_2, c_H) - V(Z_2)\{Z_2 - Z_1\}$$

As a result, $Z_2 = Z_2(Z_1)$. Assume perfect foresight by fringe owner, and hence $Z^e = Z_2(Z_1)$.

A revealed preference argument shows that $Z^a_1 < Z^b_1 \Rightarrow Z^a_2 \leq Z^b_2$. In period 1,

$$\Pi(Z, Z') = \pi^D_{1} (Z, c_H) - V(Z)\{Z - Z'\} + d \{ \pi^D_{2} (Z, c_H) - V(Z)\{Z\} \}$$

$$\frac{\partial \Pi^D (Z_1, Z_2)}{\partial Z_1} = \frac{\partial \pi^D (Z_1, c_H)}{\partial Z_1} - \frac{\partial V(Z_1)\{Z_1\}}{\partial Z_1} - \frac{\partial V(Z_2)\{Z_1\}}{\partial Z_1} \frac{\partial Z_2}{\partial Z_1}$$

When $Z_1$ takes values on the interval $[0, Z(c_H)]$, net profits at t=1 are zero (the raider does not gain buyer power compared with fringe firms, and hence profits are at best equal than if the raider does not acquire capacity at t=1 (and it is strictly worse off if $0 < Z_1 \Rightarrow Z > Z(c_L)$ from proposition 1). With a linear demand it turns out that capacity acquisition at t=1 below $Z(c_H)$ is innocuous, because $Z^*_2 = Z(c_L)$.  

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\[ Z(c_L) < Z_1 \Rightarrow Z_2 > Z_1 \] and profits are decreasing.

With a linear demand, when \( Z_i \) takes values on the interval \( [Z(c_L), X] \), then

\[ Z_2^* = \min\{Z_1 + c_H - c_L, X\}. \]

With a linear demand, the optimal level of capacity at \( t=1 \) is

(i) \[ Z_1 \text{ takes any value on the interval } [0, Z(c_H)], \text{ or} \]

(ii) \[ Z_1 = Z(c_L) \]

The issue at hand: the same problem that faces a durable-good monopolist: a commitment problem. Here, the raider benefits from a regulation that bans any level of consolidation above \( Z(c_L) \).

6. Demand uncertainty

This section deals with the incentives to acquire capacity when there is demand uncertainty. Let \( P(Q, \theta) \) be the inverse demand function and \( \theta \) the uncertainty parameter of the demand.

Parameter \( \theta \) is distributed in \( [\theta, \bar{\theta}] \) with \( \theta < \bar{\theta} \).

The order of moves assumed for the full game is the same in previous sections, with the following additions: demand uncertainty is solved after both the capacity acquisition stage; moreover the reasoning is stated as if the state of demand is revealed before input contracts are established, and hence industry participants know DF’s power for manipulating final prices, but with risk-neutral agents, we could equally assume that the state of demand is revealed afterwards.

Stage 1. The dominant firm offers a bid \( b \) for capacity \( Z \).

Demand uncertainty is revealed.
Stage 2. The dominant firm deals with the upstream supplier. In case of disagreement, the dominant firm is restricted to use the alternative source of input.

Stage 3. The dominant firm decides how much output to produce.

Define $Z(\theta, c)$ as the level of capacity for which DF uses all its capacity only if $Z \leq Z(\theta, c)$. Assume that $P_\theta(Q, \theta) > 0$ and that $\forall q \leq Q$, $P_{\theta q}(Q, \theta)q + P_\theta(Q, \theta) > 0$. The second condition guarantees that $Z(\theta, c)$ is increasing in $\theta$. Assume further that fringe firms have strictly positive profits for any level of demand, costs and consolidation, $P(X, \theta) > c_H$, and assume moreover $Z(\theta, c_L) < Z(\theta, c_H)$ and $Z(\theta, c_L) < X$. The first one says that when DF chooses a capacity ... $Z < Z(A_1, c_1)$, the second one says discards monopolization in equilibrium, as $Q^{mom}(A_2, c_1) < X$.

It is immediate to see that $\frac{\partial P(q(s, c, \theta) + m, \theta)}{\partial c} > 0$. We will assume that

$$\frac{dP(q(s, c, \theta) + m, \theta)}{d\theta} = P_q \frac{\partial q(s, c, \theta)}{\partial \theta} + P_\theta > 0. \tag{6}$$

It is immediate to see that DF’s profits are increasing in $Z$ for $0 < Z < Z(\theta, c_H)$ and decreasing in $Z$ for $Z(\theta, c_L) < Z < X$.

DF solves

$$\max_{Z} \mathbb{E}(\pi(Z, c_H)|\theta) - V(Z)$$

where

---

6 This is indeed the case, for instance, for linear demand $P(Q, \theta) = \theta - Q$ and constant-elasticity demand $P(Q, \theta) = \theta Q^{-\gamma}$ with $\gamma \in (0,1)$.
with $\theta^{off}$ being the demand state for which $Z = Z(\theta^{off}, c_H)$. DF obtain discounts only when $\theta \in [\theta, \theta^{off})$, since otherwise $Z < Z(\theta, c_H)$.

Fringe firms ask for a payment equal to the expected market profits, $V(Z) = \int_{\theta}^{\theta^{off}} V(Z|\theta) f(\theta)d\theta$,

where we define $V(Z|\theta) = P(X - Z + q(Z, c_L, \theta), \theta) - c_H$. We define $\theta^{eq}$ as the demand state for which $Z = Z(\theta^{eq}, c_L)$. DF restrict output only when $\theta \in [\theta, \theta^{eq})$, since otherwise $Z < Z(\theta, c_L)$. Hence $V(Z|\theta) = P(X, \theta) - c_H$ for $\theta \in [\theta^{eq}, \theta]$. Hence fringe’s expected market profits per unit of capacity are

$$V(Z) = \int_{\theta}^{\theta^{eq}} (P(X - Z + q(Z, c_L, \theta), \theta) - c_H) f(\theta)d\theta + \int_{\theta^{eq}}^{\theta} (P(X, \theta) - c_H) f(\theta)d\theta$$

Notice that $\theta^{eq} < \theta^{off}$. Proof:

It is immediate to expand the analysis in section 3 and Proposition 1 to obtain the following results:

For $Z \in (Z(\theta, c_H), Z(\theta, c_L))$, $\theta^{eq} = \theta$ and $\frac{\partial}{\partial Z} [E(\pi(Z, c_H)|\theta) - V(Z)|Z] > 0$. This is a subset of capacity levels for which DF increases its buyer power without distorting final prices. Hence

$$\frac{\partial}{\partial Z} [E(\pi(Z, c_H)|\theta) - V(Z)|Z] = \int_{\theta}^{\theta^{eq}} (P(X - Z + q(Z, c_H, \theta), \theta) - P(X, \theta)) f(\theta)d\theta > 0$$
For \( Z \in \left[ Z(\bar{\theta},c_L),X \right) \), \( \theta^{eq} = \bar{\theta} \) and \( \hat{\beta}[E(\pi(Z,c_H)\theta)-V(Z)Z] < 0 \). For these levels of capacity, in all demand states DF restrict production to manipulate final prices. Hence we may immediately apply Proposition 1 to show that for any demand state \( \theta \in [\theta_{\bar{\theta}}] \),

\[
\hat{\beta}\left[\pi(Z,c_H)\theta-V(Z)Z\right] < 0, \text{ and hence } \int_{\theta}^{0} \hat{\beta}\left[\pi(Z,c_H)\theta-V(Z)Z\right] f(\theta)d\theta < 0.
\]

The interesting case is when \( Z \in \left[ Z(\bar{\theta},c_L), Z(\bar{\theta},c_L) \right) \): DF may want to increase buyer power and the expense of rising the payment for capacity.

\[
\lim_{Z \to Z(\bar{\theta},c_L)} \frac{\partial}{\partial Z} \left[ \pi(Z,c_H)\theta - V(Z)Z \right] > 0 \Rightarrow Z^* > Z(\bar{\theta},c_L).
\]

Example with a linear demand. \( Z \in \left( Z(\bar{\theta},c_L), Z(\bar{\theta},c_L) \right) \), with \( Z(\bar{\theta},c_L) = \bar{\theta} - X - c_L \) and \( Z(\bar{\theta},c_L) = \bar{\theta} - X - c_L \).

\[
\frac{\partial}{\partial Z} \left[ \pi(Z,c_H)\theta - V(Z)Z \right] = \frac{(c_H - c_L)^2}{4(\theta_H - \theta_L)} > 0 \text{ if } \theta_H - \theta_L > c_H - c_L \text{ and }
\]

\[
\frac{\partial}{\partial Z} \left[ \pi(Z,c_H)\theta - V(Z)Z \right] = \frac{2(c_H - c_L)(\theta_H - \theta_L)}{4} > 0 \text{ if } \theta_H - \theta_L < c_H - c_L
\]

Hence it is in DF’s interest to acquire capacity above \( Z(\bar{\theta},c_L) \), and in equilibrium DF manipulates final prices.

Some parametric examples with a linear demand:

With \( \theta_L = 100, \theta_H = 140, c_L = 0, c_H = 20, X = 75 \),

\( Z(\bar{\theta},c_L) = 25 < Z(\theta_H,c_L) = 45 < Z(\bar{\theta},c_L) = 65 \) and \( Z^* = 36,86 \in \left( Z(\theta_L,c_L), Z(\theta_H,c_L) \right) \). At
\( Z^* \), there are consumer surplus losses for demand states below \( \theta = 111.86 \) (i.e. for 29.65% of the demand states, the lowest ones) and CS losses amount to 15% at the lowest demand state.

With \( \theta_L = 110, \theta_H = 140, c_L = 0, c_H = 20, X = 85 \), \( Z(\theta, c_L) = 25 < Z(\theta_H, c_H) = 35 < Z(\theta, c_L) = 55 \) and \( Z^* = 36.8 \in (Z(\theta_H, c_H), Z(\theta_H, c_L)) \). At \( Z^* \), there are consumer surplus losses for demand states below \( \theta = 121.8 \) (i.e. for almost 40% 39.66% of the demand states, the lowest ones) and CS losses amount to 13.4% at the lowest demand state.

With \( \theta_L = 110, \theta_H = 120, c_L = 0, c_H = 20, X = 70 \), we have
\[
Z(\theta_L, c_H) = 20 < Z(\theta_L, c_L) = 30 < Z(\theta_H, c_L) = 40 \quad \text{and} \quad Z^* = 44.14 \in (Z(\theta_L, c_L), Z(\theta_H, c_L)).
\]
At \( Z^* \), there are consumer surplus losses for demand states below \( \theta = 114.14 \) (i.e. for 41.4% of the demand states, the lowest ones) and CS losses amount to 5.8% at the lowest demand state.

7. Strictly convex costs of production

There is an upstream firm (U) with increasing and strictly convex costs \( C(Q), C', C'' > 0 \). As before, there is a valuable alternative source of the input (valuable in the sense of assumption 2: \( c_H < P(X) \)).

Assume moreover that \( C'(x) < c_H \), so that we may say that U is more efficient than the competitive supply (maybe this is the only local producer, and transport costs explain its comparative efficiency). Hence cost efficiency requires U being the sole source of the input. In equilibrium, it is indeed the case that U squeezes out other suppliers.

U may charge \( w = c_H - \varepsilon \) to fringe firm. For DF, a two-part tariff \( T(q) = T + wq \) must satisfy \( T \leq \pi^{DF}(Z, w) - \pi^{DF}(Z, c_H) \). Hence U’s problem when offering a contract to DF is
Max \( T + wq(w) + c_H Q - C(Q + q(w)) \) where \( q(w) = \arg \max_{q \leq Z} P(X - Z + q)q - wq \) and subject to \( T \leq \pi^{DF}(Z, w) - \pi^{DF}(Z, c_H) \) and \( Q \leq X - Z \).

U sets \( T^* = \pi^{DF}(Z, w) - \pi^{DF}(Z, c_H) \). Through the choice of \( w \), U indirectly controls DF's choice of \( q \), hence U solves the following problem:\(^7\)

Max \( P(X - Z + q)q + c_H Q - C(Q + q) \) subject to \( q \leq Z \) and \( Q \leq X - Z \).

Note that assumption 2, \( P(X) > c_H \), implies the full use of the fringe capacity. Then \( C'(X) < c_H \) and the convexity of the cost function immediately imply that U serves to all the fringe, \( Q^* = X - Z \). Then, U chooses \( w \) in the two-part tariff so that DF chooses later on a level of production \( q^{DF} \) that satisfies

\[
P'(X - Z + q^{DF})q^{DF} + P(X - Z + q^{DF}) - C'(X - Z + q^{DF}) = 0
\]

(Assume \( P'q + P' \cdot C' < 0 \)). Then there is a level of capacity \( Z^F > Z(c_H) \) such that DF restricts sales whenever its capacity satisfies \( Z > Z^F \). Formally, \( q^{DF} = Z \) if \( Z \leq Z^F \) and \( q^{DF} < Z \) otherwise, where \( Z^F \) is defined by condition \( P'(X)Z^F + P(X) - C'(X) = 0 \).

It is immediate to see that \( C'(X) < c_H \) implies \( Z^F > Z(c_H) \). Then, for \( Z \) that satisfy \( 0 < Z < Z^F \), the analysis is just as in section 3. For \( Z \) above \( Z^F \):

\(^7\) In order to implement the optimal output \( q^* \), U sets the marginal part of the two-part tariff \( w \) at

\[
w = P'(X - Z + q^*)q^* + P(X - Z + q^*)
\]
Evaluate \( \frac{\partial}{\partial Z} \left[ \pi^D \left( Z, c_H \right) - V^F (Z) \right] \) at \( Z = Z^F \):

\[
\left. \frac{\partial}{\partial Z} \left[ \pi^D \left( Z, c_H \right) - V(Z) \right] \right|_{Z = Z^F} = \ldots = P(\pi - Z + q^D \left( Z^F, c_H \right)) - P(\pi) - \frac{\partial V}{\partial Z} Z = \ldots = \left\{ P(\pi - Z + q^D \left( Z^F, c_H \right)) - P(\pi) - \pi^C(X) \right\} \frac{P'(X)}{P^C(X)Z^F + 2P(X) - C^C(X)} = \ldots
\]

Compare with \( C'(X) = c_L \), in which case \( Z^F = Z(c_L) \). Then

\[
0 < \frac{P'(X)}{P^C(X)Z^F + 2P(X) - C^C(X)} < \frac{P'(X)}{P^C(X)c_L + 2P(X)} \quad \text{and a consequence the following result is obtained.}
\]

\[
\text{Lemma 5.} \quad \left. \frac{\partial}{\partial Z} \left[ \pi^D \left( Z, c_H \right) - V(Z) \right] \right|_{Z = Z^F} > \left. \frac{\partial}{\partial Z} \left[ \pi^D \left( Z, c_H \right) - V(Z) \right] \right|_{Z = Z(c_L)}.
\]

In some cases, we may have \( \left. \frac{\partial}{\partial Z} \left[ \pi^D \left( Z, c_H \right) - V(Z) \right] \right|_{Z = Z^F} > 0 \), and hence it may be in DF's interest to consolidate capacity to the point that as a consequence production is reduced.

Consider the following example: \( P(Q) = A - Q \) and \( C(Q) = \frac{\lambda}{2} Q^2 \). Then assumptions 2 and \( C'(X) < c_H \) take the form \( A - X - c_H > 0 \) and \( \lambda X < c_H \) respectively, and we obtain
\[ Z^F = A - (1 + \lambda)X \]. The optimal production is implemented with a two-part tariff \[ T(q) = T + \lambda q \text{ where } w = \frac{\lambda}{2 + \lambda} (A + X - Z). \]

\[
\left. \frac{\partial^2 \pi_{DF}(Z, c_H) - V(Z)Z}{\partial Z} \right|_{Z=Z^F} = \ldots = \frac{1}{2(2 + \lambda)} \{ \lambda(c_H - \lambda X) - 2(A - X - c_H) \}.
\]

When \( c_H < P(X) \) and \( C'(X) = c_H \),\[
\left. \frac{\partial^2 \pi_{DF}(Z, c_H) - V(Z)Z}{\partial Z} \right|_{Z=Z^F} = \frac{A - X - c_H}{2 + \lambda} < 0.
\]

When \( c_H = P(X) \) and \( C'(X) < c_H \),\[
\left. \frac{\partial^2 \pi_{DF}(Z, c_H) - V(Z)Z}{\partial Z} \right|_{Z=Z^F} = \frac{\lambda}{2(2 + \lambda)} (c_H - \lambda X) > 0.
\]

In general, for \( Z \geq Z^F \),\[
\left. \frac{\partial^2 \pi_{DF}(Z, c_H) - V(Z)Z}{\partial Z} \right|_{Z=Z^F} = \frac{\partial^2 \pi_{DF}(Z, c_H) - V(Z)Z}{\partial Z} = \frac{1}{2} \{ 2(2 + \lambda)k_H - \lambda(A + X) - (2 - \lambda)Z \}.
\]

**Proposition 3.** Assume a linear demand \( P(Q) = A - Q \) and a cost function \( C(Q) = \frac{\lambda}{2} Q^2 \); assume further that \( \forall Q \leq X \ P(Q) - c_H > 0 \) and \( C'(Q) < c_H \).

Then there is a constellation of parameters for which the dominant firm chooses a level of capacity \( Z^* > Z^F \), and hence in equilibrium total production is reduced, \( Q^* < X \).

For parameters \( \lambda < 2 \) and \( \frac{1}{2 + \lambda} \{ \lambda^2 X + 2(A - X) \} < c_H \), the dominant firm chooses a level of consolidation \( Z^* \) that satisfies \( Z^F < Z^* = \min \left\{ X, \frac{(2 + \lambda)k_H - \lambda(A + X)}{2 - \lambda} \right\} \).

For parameters \( \lambda > 2 \) and \( \frac{1}{2(2 + \lambda)} \{ \lambda^2 X + (2 + \lambda)A \} < c_H \), the dominant firm chooses a level of consolidation \( Z^* = X \).
For the remaining constellation of parameters, if \( \lambda < 2 \) and \( c_H < \frac{1}{2 + \lambda} \left\{ 2^2 X + 2(A - X) \right\} \), or \( \lambda > 2 \) and \( c_H < \frac{1}{2(2 + \lambda)} \left\{ 2^2 X + (2 + \lambda)A \right\} \), the dominant firm chooses \( Z^* = Z^F \) and hence all the capacity is used in equilibrium, \( Q^* = X \).

8. Vertical Integration

In this section, I consider the incentives \( U \) has to buy capacity downstream in order to (partially or totally) vertically integrate the industry. \( U \) offers fringe firms to buy their capacity at a price \( b \) equal to their forgone profits. From Assumption 2, final prices are always above \( c_H \); hence fringe firms that do not sell their capacity to \( U \) do not leave the market, because they may buy the input to the competitive supply; and indeed in equilibrium, \( U \) serves the remaining fringe because it is in her interest to undercut the competitive supply and still supply input to the fringe at a wholesale price \( w = c_H - \varepsilon \). In order to convince fringe firms, \( U \) must compensate the owners of capacity their forgone profit, that is, the bid must satisfy \( b = p^e - c_H \). Rational expectations imply that firms in the fringe correctly infer the final price and hence \( b = P\left((X - Z) + q^u(c_L)\right) - c_H \). Hence the acquisition problem for \( U \) is

\[
\max_Z \Pi^U(Z) = \pi^u(Z, c_L) - bZ + (c_H - c_L)(X - Z) \quad \text{s.t.} \quad 0 \leq Z \leq X
\]

where \( \pi^u(Z, c_L) \) are the profits that \( U \) obtains in the final market from her capacity downstream, \( \pi^u(Z, c_L) = \left\{ P\left((X - Z) + q^u(c_L)\right) - c_L \right\} q^u(c_L) \). Remember that \( q^u(c_L) \leq Z \), and the inequality is strictly satisfied if \( Z(c_L) \leq Z \leq X \), that is, \( U \) do not employ all her capacity if it is sufficiently large. Hence \( U \) can not obtain additional profits through vertical integration, and her profits are even reduced if \( Z \) satisfies \( Z(c_L) < Z \leq X \) or there is any positive, even small,
fixed cost of vertical integration (say, financial costs, or costs of managing a vertical structure).

Hence the upstream firm never has an incentive to buy capacity to the fringe. When $P(X) > c_H$, the existence of a competitive supply implies that $U$ can at most extract a rent $c_H - c_L$ to fringe firms. The only incentive to vertically integrate, absent as they are countervailing power incentives present in horizontal mergers, is to acquire market power in the final market. But fringe firms free ride on any price increase in the final market, and this makes too expensive (and unprofitable indeed) to buy capacity downstream. The problem of a vertically integrated firm would be

$$\pi^U(Z, c) \equiv \max_{q, Q} P(X - Z + q)q + c_H Q - C(Q + q) \quad \text{s.t.} \quad 0 \leq q \leq Z \text{ and } 0 \leq Q \leq X - Z$$

It is immediate that $Q^* = X - Z$ whenever $c_H > C'(X)$.

And $q^* < Z$ whenever $Z > Z(c)$, defined from $P'(X)Z(c) + P(X) - c = 0$. In this case, $q^*$ satisfies the F.O.C.,

$$P'(X - Z + q^*)q^* + P(X - Z + q^*) - C'(X - Z + q^*) = 0.$$ 

I assume the S.O.C. is satisfied, $P''q + 2P' - C'' < 0 \quad \forall Z, q \leq Z$.

For, $Z > Z(c)$, the acquisition problem for $U$ is

$$\max_{Z} \pi^U(Z, c) - V(Z)Z,$$ 

where as usual $V(Z)$ are the expected profit of fringe firms, $V(Z) \equiv P(X - Z + q^*) - c_H$. The derivative of problem (**) is

$$\frac{\partial \pi^U(Z, c)}{\partial Z} - V'(Z)Z - V''(Z) = ... = P' \left(1 - \frac{\partial q^*}{\partial Z}\right) \frac{\partial^2 \pi^U(Z, c)}{\partial Z^2} < 0.$$
Incentives to vertically integrate may appear, even if $P(X) > c_H$, when $U$ is not the only bidder for capacity downstream. When there is also an outside bidder for capacity downstream (i.e., when there is the possibility of creation of a DF downstream), $U$ has an incentive to vertically integrate in order to impede the existence of a downstream firm with buying power.

**Proposition 4.** $U$ and DF have the same willingness to pay for a level of capacity $Z$ from the fringe.

One may then wonder if a DF must fear that $U$ will buy capacity downstream in order to foreclosure DF. The next proposition shows that this is not the case.

**Proposition 5.** Assume that there is a DF downstream with capacity $Z > Z(c_H)$. Then the upstream firm does not vertically integrates through acquisition of capacity to the fringe.

Proposition 5 shows that $U$ prefers not to integrate vertically, once a DF have been created. First at all, it is too expensive to buy capacity to fringe firms that free ride on any increase in final prices, and secondly, since ex-post $U$ has incentives to foreclosure DF, which implies that ex-post $U$ does not increase ex-ante profits of U-DF relationship.

But $U$ may vertically integrate in order to prevent the creation of a dominant firm downstream. Consider the following order of moves (1) $U$ acquires $Z^U \geq 0$, (2) DF acquires $Z^D \geq 0$.

**Proposition 6.** $U$ has incentives to vertically integrate in order to prevent the creation of a DF.
If U and DF simultaneously buy capacity (with $X$ large enough for U and DF bidding for separate capacity) then 2 qualitatively different kind of equilibrium may emerge. In one equilibrium we observe the emergence of a large DF with buyer power, $Z^{DF} = Z(c_L), Z^U = 0$. Other equilibria have the feature that U vertically integrates and forecloses the emergence of a downstream firm with buyer power, $Z^{DF} \in [0, Z(c_H)], Z^U \in [Z^{for}, Z(c_L)]$, where $Z^{for}$ is defined as the level of U’s capacity that satisfies: $Z^U \geq Z^{for}$, $w(Z^U) = c_H$, i.e. U does not offers discounts to any downstream firm.

For the proof, reaction functions:

$$Z^U = R(Z^{DF}) = \begin{cases} [0, Z(c_L)] & \text{if } Z^{DF} \leq Z(c_H) \\ 0 & \text{if } Z(c_H) < Z^{DF} \end{cases}$$

$$Z^{DF} = R(Z^U) = \begin{cases} Z(w(Z^U)) & \text{if } 0 \leq Z^U \leq Z^{for} \\ [0, Z(c_H)] & \text{if } Z^{for} \leq Z^U \leq Z(c_L) \\ [0, Z(c_H, Z^U)] & \text{if } Z(c_L) < Z^U \end{cases}$$

where we define $Z(c_H, Z^U)$ as the level of DF’s capacity such that, if U’s capacity satisfies $Z(c_L) < Z^U$, then DF do not restricts output when its capacity is below $Z(c_H, Z^U)$. 


Appendix

Proof of lemma 1. Consider first the unconstrained problem $\max_q \left[ P(X - Z + q) - c \right] q$.

Assumption 3a guarantees that the function to maximize is strictly concave in $q$. The unique solution $q(c)$ of this problem is the one that satisfies the first order condition

$$P'(X - Z + q(c))q(c) + P(X - Z + q(c)) - c = 0$$

Consider now the constrained problem $\max_q \left[ P(X - Z + q) - c \right] q$ subject to $0 \leq q \leq Z$.

(i) When $X < Q^{mon}(c)$, profits are strictly increasing at $q = Z$ for any level of capacity $Z$: $P'(X)X + P(X) - c > P'(Q^{mon}(c))Q^{mon}(c) + P(Q^{mon}(c)) - c = 0$. The first statement in the lemma follows immediately.

(ii) When $X > Q^{mon}(c)$: First define $Z(c)$ as the level of capacity $Z$ that satisfies $P'(X)Z(c) + P(X) - c = 0$. This level of capacity is well defined: the function $f(Z) = P'(X)Z + P(X) - c$ satisfies $f(0) > 0$ (since $P(X) - c > 0$ by Assumption …) and $f(X) < 0$ (since $X > Q^{mon}(c)$) and hence continuity implies that there is a $Z \in (0, X)$ that satisfies $f(Z) = 0$. When $Z \leq Z(c)$, it is optimal to use all the capacity, as profits are increasing at the level of output $Z$: $P'(X)Z + P(X) - c > P'(X)Z(c) + P(X) - c = 0$. When $Z(c) < Z \leq X$, instead, it is better to restrict output: the function $g(q) = P'(X - Z + q)q + P(X - Z + q) - c$ satisfies $g(0) = P(X - Z) - c > 0$ and $g(Z) < 0$ (since $g(Z) = P'(X)Z + P(X) - c < P'(X)Z(c) + P(X) - c = 0$) and hence continuity implies that there is a level of production $q \in (0, Z)$ that satisfies $g(q) = 0$ (which is indeed the level of production $q(c)$ that satisfies the F.O.C. of the unconstrained problem)
For the asserted comparative statics, notice first that \( \frac{\partial Z(c)}{\partial c} = \frac{1}{P'} < 0 \). On the other hand, the assertion on total production \( X - Z + q(Z,c) \) comes easily from the first order condition

\[
P'(X - Z + q(Z,c))q(Z,c) + P(X - Z + q(Z,c)) - c = 0
\]

and the implicit theorem:

\[
\frac{\partial}{\partial c} [X - Z + q(Z,c)] = -\frac{P'}{2P'P''_c q} < 0 \quad \text{and}
\]

\[
\frac{\partial}{\partial c} [X - Z + q(Z,c)] = \frac{1}{2P'P''_c q} < 0. \quad \blacksquare
\]

**Proof of lemma 2.** DF pays \( \frac{\pi^{\text{DF}}(Z,c_L) - \pi^{\text{DF}}(Z,c_H)}{q(Z,c_L)} + c_L \) per unit of input, whereas a fringe firm pays \( w = c_H \). Hence DF buys cheaply the input if the inequality

\[
\frac{\pi^{\text{DF}}(Z,c_L) - \pi^{\text{DF}}(Z,c_H)}{q(Z,c_L)} + c_L < c_H
\]

is satisfied. This inequality can be written as

\[
[P(X - Z + q(Z,c_L)) - c_H]q(Z,c_L) < \pi^{\text{DF}}(Z,c_H). \quad \text{Hence a firm pays less than } c_H \text{ only if}
\]

\[
q(Z,c_L) \in \arg\max_q [P(X - Z + q) - c_H]q. \quad \text{From lemma 1, we know that a firm with capacity}
\]

\[
Z \leq Z(c_H) \text{ always employs all its capacity, } q(Z,c_L) = Z = \arg\max_q [P(X - Z + q) - c_H]q; \text{ as a consequence it is immediate the firm pays for the input a total amount } c_H Z \text{ to } U. \text{ When a downstream firm possesses capacity } Z \text{ above } Z(c_H), \text{ instead, it restricts production when its marginal costs are high; hence the optimal response to a residual demand } X - Z \text{ is}
\]

\[
q(Z,c_H) < q(Z,c_L) \leq Z. \quad \text{Hence the inequality above is satisfied when } Z > Z(c_H). \quad \blacksquare
Proof of Proposition 1. When Z takes values on the interval \([Z(c), X]\), the problem (1) has an interior solution. DF produces \(q(X, Z, c)\), the quantity that solves

\[
P'(X - Z + q)q + P(X - Z + c) - c = 0. \tag{A.1}
\]

\[
\frac{d\Pi^{DF}(Z)}{dZ} = \frac{\partial \pi(X, Z, c)}{\partial Z} - \left( \frac{p^e + dp^e}{dZ} Z \right) = \tag{A.2}
\]

\[
= -P'(X - Z + q(Z, c_H))q(Z, c_H) - \left( P(X - Z + q(Z, c_L)) - c_H \right)
\]

\[
+ \left( 1 - \frac{\partial q(Z, c_L)}{\partial Z} \right)P'(X - Z + q(Z, c_L))Z = \tag{A.3}
\]

\[
= P(X - Z + q(Z, c_H)) - P(X - Z + q(Z, c_L))
\]

\[
- \left( 1 - \frac{\partial q(Z, c_L)}{\partial Z} \right) \left( P(X - Z + q(Z, c_L)) - c_L \right) \frac{Z}{q(Z, c_L)} < \tag{A.4}
\]

(for \(Z > Z(c_L)\) \(q(Z, c_L) < Z(c_L)\))

\[
< P(X - Z + q(Z, c_H)) - P(X - Z + q(Z, c_L))
\]

\[
- \left( 1 - \frac{\partial q(Z, c_L)}{\partial Z} \right) \left( P(X - Z + q(Z, c_L)) - c_L \right) < \tag{A.5}
\]

\[
(P(Q(Z, c_L)) > P(X) > c_H)
\]

\[
< P(X - Z + q(Z, c_H)) - P(X - Z + q(Z, c_L)) - \left( 1 - \frac{\partial q(Z, c_L)}{\partial Z} \right)(c_H - c_L) = \tag{A.6}
\]

\[
= \int_{c_L}^{c_H} \left\{ P'(X - Z + q(Z, s)) \frac{\partial q(Z, s)}{\partial c} - \left( 1 - \frac{\partial q(Z, c_L)}{\partial Z} \right) \right\} ds = \tag{A.7}
\]

\[
\int_{c_L}^{c_H} \frac{1}{2 + E(s)} - \frac{1}{2 + E(c_L)} ds < 0 \tag{A.8}
\]

as long as \(\frac{\partial E(c)}{\partial c} > 0 \ \forall c > c_L\).
Proof of Lemma 4. \[ \frac{\partial \Pi^C}{\partial Z} = \frac{1}{Z} \left( \frac{\partial \pi^{DF}(Z,c_H)}{\partial Z} - \Pi^C \right) < \frac{\partial V^F}{\partial Z} \iff \frac{\partial \pi^{DF}(Z,c_H)}{\partial Z} - \Pi^C < \frac{\partial V^F}{\partial Z} Z \, . \]

We are assuming that \( \Pi^C > V^F \). On the other hand, in the appendix it is shown that for levels of capacity \( Z \) above \( Z(c_L) \), assumption 5 is a sufficient condition for

\[ \frac{\partial \pi^{DF}(Z,c_H)}{\partial Z} - V^F < 0 \, .\]

Hence:

\[ \frac{\partial \pi^{DF}(Z,c_H)}{\partial Z} - \Pi^C < \frac{\partial \pi^{DF}(Z,c_H)}{\partial Z} - V^F \leq \frac{\partial V^F}{\partial Z} Z \, . \]

\[ \blacksquare \]

Proof of Proposition 4. U’s willingness to pay for \( Z \) is the difference between profits when vertically integrated, \( \Pi^V \), and profits when there is a DF downstream, \( \Pi^U \). U compares

\[ \Pi^V \equiv \pi(Z,c_L)+(c_H-c_L)(X-Z)-b^U \]

(where the first term are the profits that U could obtain from operating a capacity \( Z \) downstream; and the second term are the profits from supplying the input to the remaining firms), and

\[ \Pi^U \equiv \pi(Z,c_L)-\pi(Z,c_H)+(c_H-c_L)(X-Z) \, . \]

(where the first two terms are the rent that U can obtain from DF, and the last term is as above the profits from supplying the input to the remaining firms). The difference amounts to

\[ \Pi^V - \Pi^U \equiv \pi(Z,c_H)-b^U \, . \]

Thus U is willing to bid at most \( b^U \leq \pi(Z,c_H) \). But \( \pi(Z,c_H) \) is just the expected profits of an outside player and thus what he is willing to pay for capacity \( Z \), \( b^{DF} \leq \pi(Z,c_H) \).

\[ \blacksquare \]

Proof of Proposition 5. Ex-post behavior of the upstream firm: whenever U has some capacity \( Z^U > 0 \), U sets a two-part tariff to DF, \( T(q) = T + wq \), where \( w > c_L \). Ex-ante, U’s profits from acquiring capacity \( Z^U \) are

\[ (P-c_L)q^U - V^F Z^U + \max \{ \Pi^{DF}(w)-\Pi^{DF}(c_H),0 \} + (w-c_L)q^{DF} + (c_H-c_L)(X-Z^{DF}-Z^U) \, . \]
where \( q^U \leq Z^U \). In a rational-expectations equilibrium, capacity costs \( V^F = P - c_H \) (fringe firms ask for a payment equivalent to expected profits from maintaining activity). Hence the former expression of profits can not exceed

\[
\text{Max}\{\Pi^{DF}(w) - \Pi^{DF}(c_H), 0\} + (w - c_L)q^{DF} + (c_H - c_L)(X - Z^{DF}).
\]

But these profits, whenever \( w > c_L \) (and this is indeed the case when \( Z^U > 0 \)), are below those U obtains without vertical integration, \( \Pi^{DF}(c_L) - \Pi^{DF}(c_H) + (c_H - c_L)(X - Z^{DF}) \) ■

**Proof of Proposition 6.** Assume U has acquired \( Z^U \) such that whenever \( Z^{DF} > Z(c_H) \), then \( w = c_H \). Then DF would suffer losses acquiring capacity above \( Z(c_H) \). U can achieve this outcome with \( Z^U > Z(c_L) \). Its profits (net of the costs of acquiring capacity \( V^F = P(X) - c_H \)) are \( \Pi^U = (c_H - c_L)X \), that are larger that those obtained when U does not integrate vertically and DF acquires \( Z^{DF} = Z(c_L) \), \( \Pi^{DF} = \Pi^{DF}(c_L) - \Pi^{DF}(c_H) + (c_H - c_L)(X - Z^{DF}) \) ■
References

Cave and Salant, *American Economic Review*.


Figure 1. Price distortions when DF chooses $Z^\ast$ with demand uncertainty, with parameters

$$\theta_L = 100, \theta_H = 140, c_L = 0, c_H = 20, X = 75,$$

compared with competitive prices.

Figure 2. CS losses when DF chooses $Z^\ast$ with demand uncertainty, with parameters

$$\theta_L = 100, \theta_H = 140, c_L = 0, c_H = 20, X = 75,$$

in % with respect to CS with a competitive fringe.