Optimal Income Taxation and Public Provision of Productive Inputs

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Abstract

We characterize optimal non-linear income tax and optimal public provision of a productive input when gross earnings are unobservable and determined by exogenous capability and endogenous input investment. The public provision is welfare improving with respect to pure non-linear taxation only if input is a sufficiently strong complement of capability in earnings function. In this case, the optimal public provision scheme (whether a pure scheme based on opting-out or topping up mechanisms or a mixed one) depends on the structure of preferences and technology. If input is an economic substitute of capability (which happens also when capability and input are technologic complements), and heterogeneity across classes is sufficiently strong, the pure opting-out scheme is optimal. Conversely, when input and capability are economic complements, the pure topping up is optimal. While, in general, usual results are obtained as regards the optimal tax schedules, when preferences and technology are such that the scope of the opting-out public provision pillar is limited, a marginal subsidy of high-capability labor supply is found to be optimal.

Keywords: In-kind redistribution; Public provision of private goods; Opting out; Topping up; Non-linear income tax

JEL classification: H42, H21

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1 Introduction

The second-best analysis of the public provision of private goods has challenged the traditional first-best view about redistribution, forging a new argument to justify public social services (Balestrino, 1999, 2000). Whenever household economic condition is imperfectly verified and affects the demand of some goods, in-kind transfers (or quotas on consumption) of these goods Pareto-dominate cash as a redistribution tool (Nichols and Zeckhauser, 1982; Guesnerie and Roberts, 1984). Efficiency gains are driven by higher costs of opportunistic behaviors in taking up subsidies (Blackorby and Donaldson, 1988) and in paying taxes (Guesnerie and Roberts, 1984), which in turn are determined by specific public provision rules.

The conventional wisdom of the considered literature is that social services are consumption goods. However, the economic nature of some publicly provided goods is related more to production than to consumption (Balestrino, 1999, p. 346). Education and healthcare primarily affect household production capacity (i.e., human capital), while their features as consumption goods are relatively less relevant. Also, services such as childcare and elderly-care can be more convincingly modeled as inputs, letting households fully exploit their potential income capacity. Extending the analysis by ?, we investigate the implications of this production view of social services in terms of redistribution, optimal provision schemes and optimal taxation.

As benchmark we consider a simple second-best framework with endogenous labor supply and nonlinear income taxation. Household income is the product of wage and labor supply. Following Boadway and Marchand (1995, p. 51-55), wage is modeled as an increasing function of two factors: exogenous wealth (or ability) and a productivity-enhancing input (e.g., education, healthcare, childcare or elderly-care) that can be provided by gov-

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1 A remarkable exception is Boadway and Marchand (1995, p. 51-55), who prove that the universal and unconditional public provision of education, affecting the marginal productivity of households, can relax the information constraint on taxation. However, they do not focus on the assessment of the optimal provision mechanism.
ernment and by a market of competitive firms.

Inherited capability represents a composite asset summing up physical, financial, human and social “exogenous” factors, which determine household potential productivity; input represents any service or intermediate good that improves, in different ways, household potential earnings. Depending on the assumed nature of the productivity-enhancing technology, input can be complementary to or substitute of exogenous capability. When household capacity to exploit potential earnings is strengthened by input (e.g., childcare or higher education) then it complements wealth. Conversely, input substitutes capability when it affords households with more of the same factors constituting their production capacity (e.g., social network through schooling and basic living conditions through social housing or health prophylaxis).

Taking pure taxation as a benchmark, the public provision of input reduces the efficiency cost of redistribution in two ways. First, the public provision may alleviate tax distortion on household investment choices by forcing households to use more input (Boadway and Marchand, 1995; Cremer and Galvari, 1997). Second, if transfers of input can be targeted to the poor more effectively than cash, then they improve the redistribution capacity of public policies by reinforcing self-selection mechanisms (Besley and Coate, 1991; Munro, 1992; Blomquist and Christiansen, 1995).

Government can implement public provision policies in two ways:

1. by supporting private expenditure on social services through conditional transfers (e.g., vouchers, tax allowances); such a policy is also called a topping-up scheme given that it amounts to publicly providing a given quantity (or quality) of a social service to everybody allowing for private supplementing;

2. by providing public services (through different institutional arrangements) as an alternative to private services; such policies - also called opting-out schemes - leave households free to choose private or public services, though in the latter case private
supplementing is not possible.\footnote{For example, children attending a public school cannot attend, at the same time, a private one. In other cases, private supplementing may be legally forbidden. It is worth remarking that, by means of conditional grants, topping-up mechanisms can also be implemented in sectors where using different services at the same time is not \textit{technologically} feasible (Blomquist and Christiansen, 1995, pp. 564-5).}

These forms of public provision may coexist, giving rise to mixed schemes. For example, households opting for a public school automatically give up tax credits for private schools; hence, savings on tax credits implicitly finance public school expenditure. Also, an increase of tax allowances for households’ expenditure on private schools implicitly reduces the additional transfer (with respect to tax allowances) underlying free public schools. Therefore, abstracting from differences in the quality of public and private services, real-world public social programs can generally be represented by a two-pillar provision scheme:

1. the first (topping-up) pillar affords all households with some input that can be privately supplemented;

2. the second (opting-out) pillar provides some additional input as an alternative to private supplementing.

In this framework, we find that public provision is \textit{always} welfare improving. However, optimal provision schemes depend on two structural features of the economy: the complementarity or substitutability of input and capability in income production and the balance between households’ production heterogeneity and government’s preference for redistribution. When households’ heterogeneity is strong enough (as compared to government’s preference for redistribution), the optimal provision scheme is made by a single pillar: a pure opting-out mechanism, when input and capability are complementary, or a pure topping-up mechanism, when input substitutes capability. Whenever households’ heterogeneity is relatively weak, it is optimal to provide all households with the same amount of input (without private supplementing). Finally, when the balance between households’ heterogeneity and preference for redistribution is intermediate, and input is
complementary to capability, a full-fledged two-pillar scheme is optimal. [REVISE THIS PARAGRAPH]

Our results complement the main findings of the literature on public provision of private goods, providing a somewhat different perspective. We consider social programs characterized by different and potentially coexisting provision rules, generalizing the approach of Blomquist and Christiansen (1998a), who contrast pure topping-up and opting-out schemes. Also, our work highlights that opting-out schemes cannot implement the public provision of goods satisfying households’ basic needs.3

However, the main difference with the literature regards the effect of households’ heterogeneity on the scope for public provision. Consistent to the literature, we find that the degree of heterogeneity affects the optimal structure of social programs. Nevertheless, we show that the public provision of input plays a tax-correction role for any specification of technology, particularly when input demand does not depend on household private capability. This is at odds with the results highlighting that there is no scope for the public provision of a given commodity whenever households’ heterogeneity has no effect on its demand (Balestrino, 1999, 2000).

The reason for this theoretical divergence is that, in our model, government provides an input that directly corrects tax distortion on household production effort. When household production function is separable, input demand is independent of household capability, and its public provision becomes a perfect tool for correcting tax distortion: in this case, public provision without private supplementing implements the first-best outcome. Conversely, in models with public provision of consumption goods, the government provides such goods to influence the productive effort of individuals (i.e., labor supply) indirectly. Therefore, once this indirect link is broken (i.e., when, by separability of utility function, the demand

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3In our model, the input satisfies basic needs when it substitutes capability. This - under our assumptions - also implies that the former is inferior. Although evidence highlighted that primary healthcare in developing countries seems to be an inferior good (7, p. 306), normality of social services is the most common and intuitive case. As discussed in the concluding remarks, our main results can be generalized to models with endogenous labor supply and non-linear taxation, in which the publicly-provided input is a normal good.
of the considered commodity does not depend on leisure), there is no more scope for public provision (Blomquist and Christiansen, 1998a, p. 405).

The paper is organized as follows: [COMPLETE HERE].

2 The Model

The economy is populated by a large number (a unit measure) of households. The productivity of household \( i \) depends on exogenous individual capability \( \theta_i \) and on investment in input \( q_i \): 
\[
w_i = w(\theta_i, q_i),
\]
where \( w(., .) \) is strictly increasing and twice differentiable in both arguments, and concave in \( q \). In our benchmark case, we assume that \( \lambda \in (0, 1) \) households have low capability (\( \theta_i = \underline{\theta} \)), while the others have high capability (\( \theta_i = \overline{\theta} \)).

Also, we assume constant returns to scale production technology, competitive labor market (hence, gross wage is equal to individual productivity), and unitary price of investment and consumption.

The government maximizes the sum of households utilities, but it can only observe gross income \( y_i = w_i \cdot l_i \), conversely gross wage rate \( w_i \) and household labor supply \( l_i \) are not observed. Government can observe after-tax income \( x_i \), but not the exact amount of private consumption \( c_i \) and private investment in input \( q_i^m \).

The input is also publicly provided. The government supplies a uniform quantity \( q_f \) through a first pillar (topping up scheme) independently of households’ consumption and investment choices. Then, a supplementary quantity of input \( q^s \) is provided to individuals opting for a second public pillar, and accepting not to privately top-up the public provision. Individuals opting out of the second pillar can privately supplement the first pillar input provision (with \( q_i^m \)).

The government budget constraint is 
\[
\lambda \cdot (\overline{y} - \overline{x}) + (1 - \lambda) \cdot (\overline{y} - \overline{x}) \geq q_f^f + q^s \cdot I,
\]
where: \( \overline{y} \), \( \overline{x} \), \( \overline{y} \), and \( \overline{y} \) are gross and net incomes of high-capability and low-capability households, respectively; and \( I \in [0, 1] \) is the share of population covered by the second-pillar public provision of input.
The utility function of the generic household is $U(c_i, l_i)$, strictly increasing and concave in private consumption, and strictly decreasing and concave in labor supply. Moreover, consumption and leisure are normal goods. By $l_i = \frac{y_i}{w_i}$, $c_i = x_i - q_i^m$ (for households opting out of the second-pillar provision), and $c_i = x_i$ (for households opting for the second-pillar provision), the generic utility of opting-out households is $U(x_i - q_i^m, \frac{y_i}{w_i(q_i^f + q_i^m)})$ and the utility of opting-in households is $U(x_i, \frac{y_i}{w_i(q_i^f + q_i^m)})$.

The timing of the model reads as follows. The government determines the non-linear income tax and the first- and second-pillar public provision of input, then households choose gross- and net-of-tax incomes and whether to opt out of the second-pillar provision, and in this case the amount of private supplement of the first-pillar provision.

2.1 Household’s choices and Policy regimes

In this section we consider the effect of government policies on the household’s behavior. Taking as given gross and net incomes, we first analyze input private demand of opting-out households, then household decision to opt out or to accept the second-pillar provision. Finally, we discuss the behavior of households with different capabilities, reacting to different tax schedules.

2.1.1 Private demand of input

Let us assume that a generic household with capability $\theta$ decided to opt out. Taking as given gross and net incomes, let $V(x, y, q^f, \theta) \equiv \max_{q^m} U(x - q^m, \frac{y_i}{w_i(q_i^f + q_i^m)})$. If $q^m > 0$, by comparative statics of the first order condition with respect to $q$

$$-(U_c + U_l \cdot \frac{y}{w^2} \cdot w_q) = 0$$

it is possible to show that - under the assumed well-behaved preferences and technology - the private demand of input, $q^m(x, y, q^f, \theta)$, is increasing in net income ($\frac{dq^m}{dx} \in (0, 1)$)
and in gross income \( (\frac{dq^m}{dy} \in (0, \frac{dx}{dy} |V)) \), and it is decreasing in first-pillar public provision - though there isn’t complete crowding out: \( \frac{dq^m}{dq} \in (-1, 0) \). The behavior of private input demand with respect to exogenous ability is such that:

1. if input and capability are strong technologic complements (say, there is some \( w_{q\theta}^+ > 0 \) such that \( w_{q\theta} > w_{q\theta}^+ \)), then private input is an economic complement of capability \( (\frac{dq^m}{d\theta} > 0) \);

2. if input and capability are strong technologic substitutes (say, there is some \( w_{q\theta}^- < 0 \), such that \( w_{q\theta} < w_{q\theta}^- \)), then private input is an economic strong substitute of capability \( (\frac{dq^m}{d\theta} < -\frac{w_{q\theta}}{w_{q\theta}}) \);

3. if capability and input are weak technologic complements or substitutes (say, \( w_{q\theta} \in (w_{q\theta}^-, w_{q\theta}^+) \)), then private input is an economic substitute of capability \( (\frac{dq^m}{d\theta} \in (-\frac{w_{q\theta}}{w_{q\theta}}, 0)) \).

It is worth remarking that to observe economic complementarity between capability and private demand for input, other things equal, we need strong technologic complementarity. In the other cases, the effect of capability on the private demand for input is negative: high-capability households would - other things equal - optimally demand less input. In the Appendix, we show that the Single Crossing Property (SCP)

\[
\frac{d}{d\theta}(\frac{dx}{dy} |V) = \frac{\partial}{\partial \theta}(\frac{dx}{dy} |V) + \frac{\partial}{\partial q}(\frac{dx}{dy} |V) \cdot \frac{dq^m}{d\theta} < 0
\]

taking into consideration the effect of \( \theta \) on private input demand, is never satisfied in the case of strong substitututability between input and capability (case 2), therefore we will exclude such a case in our analysis.\(^5\)

\(^4\)Remark that \(-\frac{w_{q\theta}}{w_{q\theta}}\) is the marginal rate of technologic substitution keeping constant the level of \( w \).\(^5\)Moreover, such in such a case the total effect of exogenous capability on individual productivity is negative and the very concept of high- and low- capability could be questioned.
2.1.2 Opting out choice

We consider now the opting out choice. The opting-in condition of the generic household can be written as

\[ q_s(x, y, q^f, \theta) = \{ q' \in \mathbb{R}_+ \mid U(x, y, w(\theta, q^f + q')) = V(x, y, q^f, \theta) \} \]

(2)

that is increasing in \( x \) and \( y \), and decreasing in \( q^f \) (namely, \( \frac{dq_s}{dq^f} \in (-1, 0) \)). The effect of \( \theta \) on \( q_s \) is such that:

- if capability and input are strong technologic complements (say, there is a \( w_{q\theta} > 0 \), such that \( w_{q\theta} > w_{q^f} \)), then \( \frac{dq_s}{d\theta} > 0 \);
- if capability and input are weak technologic complements or substitute (say \( w_{q\theta} < w_{q^f} \)), then \( \frac{dq_s}{d\theta} < 0 \).

In the Appendix, we show that the SCP is always satisfied for opting-in households, namely

\[ \frac{d}{d\theta} \left( \frac{dx}{dy} \bigg|_V \right) = \frac{\partial}{\partial \theta} \left( \frac{dx}{dy} \bigg|_V \right) < 0. \]

2.1.3 First best benchmark and mimicking behaviors

In first best, government observes exogenous capabilities of individuals and can implement optimal lump sum taxation. In this case, cash redistribution is superior to in-kind transfers. Thus, in first best there is no role for the public provision of input, and without loss of generality we put \( q^f = q^s = 0 \). The first best optimization conditions imply non-distorting optimal taxation, namely

\[ \frac{dx}{dy} \bigg|_V = -\frac{U_i}{U_c \cdot w} = 1 \]

\(^6\)Such a choice is observed by the government, hence the most general tax schedule could depend also on it. However, at the optimum households with the same exogenous capability choose either to opt in or to privately supplement, therefore the case for a general tax schedule depending on individual choice to opt for the second-pillar provision happens only out of the tax equilibrium.
for all $\theta$. Moreover, by individual optimization (1) also $\frac{u}{y-u-q} = 1$, for all $\theta$.

The first best allocation may be incentive-incompatible. However, as usual, poor households have no incentive to mimic rich households, while the reverse can happen, namely when redistribution is sufficiently large. In the following, we will assume that this is the case.

2.1.4 Policy regimes

In the considered setting, we have four possible of policy regimes depending on government policies and structural parameters (Greco, 2011):

**PT** in the pure taxation regime, the level of $q_f$ and $q_s$ are such that no individual is constrained by the first pillar and no individual opt for the second pillar public provision, hence $q^* < \min\{\underline{q^*}, \overline{q^*}\}$ (where $\underline{q^*} \equiv q^*(\underline{x}, \underline{y}, q_f, \underline{\theta})$ and $\overline{q^*} \equiv q^*(\overline{x}, \overline{y}, q_f, \overline{\theta})$);

**INC** in the inclusive regime, the level of $q_f$ and $q_s$ are such that all individuals are constrained by the first pillar or opt for the second pillar public provision, hence $q^* > \max\{\underline{q^*}, \overline{q^*}\}$;

**DM** we may have two discriminating regimes, depending on the type (high- or low-capability) of households with higher minimum public provision inducing them to opt in the second pillar scheme:

**DML** when $\underline{q^*} \leq q^* \leq \overline{q^*}$, low-capability households opt for the second-pillar provision while high-capability households opt out;

**DMH** when $\underline{q^*} \geq q^* \geq \overline{q^*}$, high-capability households opt for the second-pillar provision while low-capability households opt out.
3 Optimal Taxation and Public Provision

The government may choose the relevant policy regime, by setting appropriate tax schedule and public provision levels for the first and second pillar. However, the nature of the prevailing discriminating regime also depends on structural features such as individual preferences and technology. Passing from one policy regime to the other introduces, in the two-class setting, discontinuities in the structure of government objective and constraints, therefore we will firstly find optimal solutions within each policy regime, then we will analyze global optima.

3.1 Discriminating Regimes

In this section, we analyze optimal solutions in the discriminating regimes. First, we consider the case in which, given the first-pillar public provision and the optimal tax schedule, low-capability households opt for the second-pillar public provision, while high-capability ones opt out. In the following subsection, we will focus on the other case in which high-capability households opt for the second-pillar provision, while low-capability households opt out.

3.1.1 DML: low-capability households opt in

In this discriminating regime, the low-ability opt in, the high-ability opt out, thus the high-capability households mimicking low-capability are forced to opt for the second pillar
public provision. Therefore the maximization problem of the government is

\[
\max_{(x, y, \overline{x}, \overline{y}, q^f, q^s)} \lambda \cdot U(x, \frac{y}{w(\overline{x}, q^f + q^s)}) + (1 - \lambda) \cdot V(\overline{x}, \overline{y}, q^f, \overline{y})
\]

s.t.:

\[
\lambda \cdot (y - x) + (1 - \lambda) \cdot (\overline{y} - \overline{x}) - q^f - \lambda \cdot q^s \geq 0
\]

\[
V(\overline{x}, \overline{y}, q^f, \overline{y}) \geq U(x, \frac{y}{w(\overline{x}, q^f + q^s)})
\]

\[
q^s(\overline{x}, \overline{y}, q^f, \overline{y}) \geq q^s
\]

\[
q^f \geq 0
\]

\[
q^s(\overline{x}, \overline{y}, q^f, \overline{y}) \geq 0
\]

By the optimization conditions of the program (3) we have

**Proposition 1** If low-capability households opt in for lower second-pillar public provision, then the optimal discriminating policy, \( \{x^*, y^*, \overline{x}^*, \overline{y}^*, q^f, q^s\} \), is such that:

1. if exogenous capability and input are economic substitutes, and the scope for the second-pillar provision is

   (a) wide enough (i.e., \( q^s(\overline{x}, \overline{y}, q^f, \overline{y}) > q^{**} \)):
   
   • the optimal taxation schedule never distorts high-capability labor supply and the optimal marginal distortion of low-capability labor supply has the same shape as in the pure taxation case;
   
   • moreover the optimal public provision scheme is pure opting-out involving over-investment for low-capability households (i.e., \( q^{f*} = 0 \), and \( q^{**} > q^{m} \));

(b) not wide enough (i.e., \( q^s(\overline{x}, \overline{y}, q^f, \overline{y}) = q^{**} \)):
   
   • the high-capability labor supply is optimally distorted by a marginal subsidy and the low-capability labor supply distortion has the same shape as in the
pure taxation case;

• moreover, the optimal public provision scheme mixes topping-up and opting-out (i.e., \(q_f^* > 0\), and \(q_s^* > 0\));

2. if exogenous capability and input are economic complements, and

• the optimal taxation schedule never distorts high-capability labor supply, and the marginal tax on low-capability labor supply is featured by an additional term with respect to the pure taxation case;

• moreover the public provision scheme is pure topping-up (i.e., \(q_f^* > 0\), and \(q_s^* = 0\)).

We now deepen our investigation by a numerical characterization of the considered setting. We consider a standard functional form of the utility function (Heatcote et al., 2009):

\[
U(c, l) = c^{1-\rho - 1} - \frac{l^{1+k}}{1+k}
\]

The parameter \(\rho\) represents the coefficient of relative risk-aversion (as baseline case, we take \(\rho = 3\)) and \(\frac{1}{k}\) is the Frisch elasticity of labor supply. In particular, we set \(k = 2\) in order to account for the standard compensated elasticity of labor estimated in literature, 0.33 (HERE SOME CITATION). Consistently with the literature on Mincer’s earning functions (CITATION HERE?), we use the following Cobb-Douglas specification of individual earning function

\[
w(\theta, q_f^m) = \theta^\alpha \cdot (q_f^m + q_m^m)^\beta
\]

Using a log-transformation, it is easy to see that \(\alpha\) and \(\beta\) represent the estimated coefficients of a Mincer’s regression with regressors \(\theta\) and \(q\), respectively. However, we also check the robustness of our results using a linear earning function (i.e., with \(\theta\) and \(q\) technologic substitutes).
Table 1 reports our benchmark parametrization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.2</td>
</tr>
<tr>
<td>$k$</td>
<td>2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>3</td>
</tr>
</tbody>
</table>

In the benchmark specification, we assume that $\theta$ and $q$ have the same weight. In the benchmark case, the discriminating regime is DML (i.e., low-capability households opt in for lower second-pillar provision).

Table 2 summarizes the results featuring the optimal discriminating policy.

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Benchmark</th>
<th>$\alpha = 0.4$, $\beta = 0.6$</th>
<th>$k = 6$</th>
<th>$\theta = 5$</th>
<th>$\lambda = 0.55$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.380</td>
<td>0.229</td>
<td>0.562</td>
<td>0.099</td>
<td>1.176</td>
</tr>
<tr>
<td>$x$</td>
<td>0.614</td>
<td>0.571</td>
<td>0.605</td>
<td>0.724</td>
<td>1.481</td>
</tr>
<tr>
<td>$\bar{\gamma}$</td>
<td>2.898</td>
<td>3.810</td>
<td>2.270</td>
<td>3.872</td>
<td>1.823</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.320</td>
<td>3.281</td>
<td>1.801</td>
<td>3.168</td>
<td>0.701</td>
</tr>
<tr>
<td>$q'$</td>
<td>0.155</td>
<td>0.050</td>
<td>0.145</td>
<td>0.030</td>
<td>0.235</td>
</tr>
<tr>
<td>$q^*$</td>
<td>0.035</td>
<td>0.087</td>
<td>0.136</td>
<td>0.020</td>
<td>0.014</td>
</tr>
<tr>
<td>Welfare</td>
<td>$-0.960$</td>
<td>$-1.124$</td>
<td>$-0.975$</td>
<td>$-0.470$</td>
<td>$-3.833$</td>
</tr>
</tbody>
</table>

Heckman et al. (2006) show that inherited abilities and family background (our exogenous capability) have similar weight than schooling in the determination of individual wage.
As expected both taxation and public provision are used as redistribution tools. In the benchmark case, $\theta$ and $q$ are economic complements (HERE SOME CHECK OF THIS) and as expected the optimal tax schedule does not distort high-capability households (Figure 1), while the marginal tax on low-capability households is positive (Figure 2). HERE WE SHOULD CHECK - WITH THE SAME PARAMETERS - IF THE OPTIMAL MARGINAL TAX ON LOW-CAPABILITY HOUSEHOLD IN THE PURE TAXATION REGIME IS LOWER THAN IN THIS CASE (I.E., IF THE SLOPE IS STEEPER IN PURE TAXATION THAN IN THIS CASE).

We now consider an alternative specification of the earning function, $w(\theta, q^f + q^m) = \alpha \cdot \theta + \beta \cdot (q^f + q^m)$. In this case, capability and input are economic substitutes (HERE CHECK THIS). With the benchmark parametrization of the Table 1, the optimal discriminating regime is $\{y, \underline{x}, \bar{y}, \bar{x}, q^f, q^s\} = \{0.364, 0.791, 1.721, 1.294, 0, 0.0002\}$. Consistently with Proposition 1, we find that the optimal public provision scheme is pure opting out, given that the scope for the second pillar is sufficiently wide (i.e., $q^f = 0$, and
Figure 2: DML: WHICH SUB-CASE? High-capability household

$q^*(x, y, q^f, \theta) = 0.0006 > q^{**} = 0.0002$.

Keeping the latter specification, we assume now that $\theta = 2$. With this change, there is not enough scope for the second pillar public provision, hence both pillars are active and the optimal tax schedule for high-capability households involves a marginal subsidy (Figure 4).
Figure 3: DML: WHICH SUB-CASE? Low-capability household

Figure 4: DML: WHICH SUB-CASE? High-capability household
3.1.2 DMH: high-capability households opt in

We now consider the other discriminating case, where high-capability households opt-in for lower level of $q^s$ than low-capability. The maximization problem of the government is

$$\max_{(x, y, \overline{y}, q^f, q^s)} \lambda \cdot V(x, y, q^f, \overline{\theta}) + (1 - \lambda) \cdot U(\overline{x}, \overline{y}, q^f + q^s)$$

s.t.:

$$\lambda \cdot (y - x) + (1 - \lambda) \cdot (\overline{y} - \overline{x}) - q^f - (1 - \lambda) \cdot q^s \geq 0$$

$$U(\overline{x}, \overline{y}, q^f + q^s) \geq V(x, y, q^f, \overline{\theta})$$

$$q^s(x, y, q^f, \overline{\theta}) \geq q^s$$

$$q^s \geq q^s(\overline{x}, \overline{y}, q^f, \overline{\theta})$$

$$q^f \geq 0$$

$$q^s(x, y, q^f, \overline{\theta}) \geq 0$$

By the first order conditions of (4), we get the following

**Proposition 2** If high-capability households opt in for lower second-pillar public provision, then the optimal discriminating policy replicates the optimal policy under the pure taxation regime. In particular, first- and second-pillar public provision does not distort high-capability households.

3.2 Welfare Analysis

Which policy (among the regime-specific optima) is globally optimal? In this section, we first provide a qualitative answer to this question, then we characterize the optimality of alternative regime by means of numerical specifications of our general setting.

**Remark:** That when DML is relevant, the optimal discriminating regime may (in principle) replicate the optimal pure taxation one, however this does not happen (i.e., the optimal public pro-
VISION DISTORTS INDIVIDUAL CHOICES), THEREFORE THE OPTIMAL DISCRIMINATING POLICY INVOLVES AN HIGHER WELFARE THAN THE OPTIMAL PURE TAXATION ONE. WHEN DMH IS RELEVANT, THE OPTIMAL DISCRIMINATING POLICY JUST REPLICA TED THE PURE TAXATION REGIME ALLOCATION, THEREFORE THE OPTIMAL POLICIES UNDER THE TWO REGIMES ARE EQUIVALENT ON THE WELFARE POINT OF VIEW.

THUS, IN THE DML CASE, WE NEED TO CONTRAST THE OPTIMAL DISCRIMINATING POLICY WITH THE OPTIMAL INCLUSIVE POLICY, AND IN THE DMH CASE, WE NEED TO CONTRAST THE OPTIMAL PURE TAXATION POLICY WITH THE OPTIMAL INCLUSIVE POLICY. [I GUESS INCLUSIVE REGIME IS ALWAYS SUBOPTIMAL...]

[HERE NUMERICAL ANALYSIS: WELFARE COMPARISON OF POLICY REGIMES]

4 Extension: Multi-class Economy

[HERE ANALYSIS OF THE CASE WHERE THE EXOGENOUS capability IS A CONTINUOUS VARIABLE ON A FINITE SUPPORT]

5 Conclusions

[INCOMPLETE]
1 Technical Appendix

1.1 Household’s input demand

Let us consider the behavior of households opting out of the second pillar. (To save space we omit household’s index, \(i\)). Taking as given the government’s tax policy and the first-piller public provision of investment \((q^f)\) as well as the generic household’s choice about labor supply (hence, the net - \(x\) - and gross - \(y\) - incomes), the optimal private investment of the household is determined by the program

\[
\max_{q^m} U(x - q^m, \frac{y}{w(\theta, q^f + q^m)}) \quad \text{s.t.} \quad q^m \geq 0
\]

The first order condition is

\[
\frac{dU}{dq^m} = \phi = -U_{cc} - U_{cl} \cdot \frac{y}{w^2} \cdot w_q = 0 \quad (5)
\]

By concavity of \(U(.,.)\) (hence, \(U_{cc} + 2 \cdot U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 < 0\)) and \(w(.,.)\) (hence, \(w_{qq} \leq 0\)), the second order condition with respect to \(q^m\) is satisfied

\[
\frac{d^2U}{dq^m^2} = \phi_{qq} = U_{cc} + 2 \cdot U_{cl} \cdot \frac{y}{w^2} \cdot w_q + U_{ll} \cdot (\frac{y}{w^2} \cdot w_q)^2 +
\]

\[
+U_{li} \cdot (\frac{2 \cdot y}{w^3} \cdot w_q^2 - \frac{y}{w^2} \cdot w_{qq}) < 0 \quad (6)
\]

1.1.1 Comparative statics

We now characterize the shape of \(q^m(x, y, q^f, \theta)\). Applying the implicit function theorem to the first order condition with respect to \(q^m\), we know that

\[
\frac{dq^m}{dz} = -\frac{\phi_z}{\phi_{qq}}
\]
where $z \in \{x, y, q', \theta\}$. By weak concavity of $w(\cdot, \cdot) \in q$, we know that $\phi_{q''} < 0$, thus the effect of $z$ on $q''(x, y, q', \theta)$ depends on the differential of the first order condition, $\phi_z$. We summarize all results in the following Lemmas.

**Lemma 3** Private input demand increases in net income less than one-to-one: $\frac{dq^m}{dx} \in (0, 1)$.

**Proof.** $\frac{dq^m}{dx} > 0$ if and only if $\phi_x = -U_{cl} - U_{cl} \cdot \frac{w}{w_q} \cdot w_q > 0$. By (5), $\frac{U}{U_{cl}} = \frac{w}{w_q} \cdot w_q$, thus $\phi_x = -U_{cl} + \frac{U}{U_{cl}} \cdot U_{cl} > 0$ if and only if leisure is a normal good. Moreover, $\frac{dq^m}{dx} < 1$ if $\phi_x < -\phi_q$ or $(-U_{cl} + \frac{U}{U_{cl}} \cdot U_{ll}) \cdot \frac{w}{w_q} \cdot w_q - U_l \cdot (2 \cdot \frac{w}{w_q} \cdot w_q^2 - \frac{w}{w_q} \cdot w_{qq}) > 0$, that is satisfied if consumption is a normal good (hence, $-U_{cl} + \frac{U}{U_{cl}} \cdot U_{ll} > 0$).

**Lemma 4** Private input demand increases in gross income less than the marginal rate of substitution between net and gross income: $\frac{dq^m}{dy} \in (0, \frac{dy}{dy} |V|)$.

**Proof.** $\frac{dq^m}{dy} > 0$ if $\phi_y = -U_{cl} \cdot \frac{1}{w} - U_{ll} \cdot \frac{w}{w_q} \cdot w_q - U_l \cdot \frac{w_q}{w_q} > 0$. By (5), $\frac{U}{U_{cl}} = \frac{w}{w_q} \cdot w_q$, thus $\phi_y = \frac{1}{w} \cdot \left(-U_{cl} + \frac{U}{U_{cl}} \cdot U_{ll} - U_l \cdot \frac{w_q}{w_q} \right) > 0$ that is true if consumption is a normal good (hence, $-U_{cl} + \frac{U}{U_{cl}} \cdot U_{ll} > 0$). Moreover, $\frac{dq^m}{dy} = \frac{w}{w_q} \cdot \alpha_y$ or - by the first order conditions for $q''$ and consumption-labor choices - $\frac{dq^m}{dy} = \frac{dy}{dy} |V| \cdot \alpha_y$, where

$$\alpha_y = \frac{U_{cl} \cdot \frac{w}{w_q} \cdot w_q + U_{ll} \cdot (\frac{w}{w_q} \cdot w_q)^2 + U_l \cdot \frac{w_q}{w_q} \cdot w_q^2}{U_{cc} + 2 \cdot U_{cl} \cdot \frac{w}{w_q} \cdot w_q + U_{ll} \cdot (\frac{w}{w_q} \cdot w_q)^2 + U_l \cdot (2 \cdot \frac{w}{w_q} \cdot w_q^2 - \frac{w}{w_q} \cdot w_{qq})} < 1$$

if $U_{cc} + U_{cl} \cdot \frac{w}{w_q} \cdot w_q + U_l \cdot (\frac{w}{w_q} \cdot w_q^2 - \frac{w}{w_q} \cdot w_{qq}) < 0$, that - by the first order condition - is satisfied if leisure is a normal good (hence, $U_{cc} - \frac{U}{U_{cl}} \cdot U_{cl} < 0$) and $w(\cdot, \cdot)$ is concave in $q$.

**Lemma 5** The first-pillar public provision crowds partially out private input demand: $\frac{dq^m}{dq} \in (-1, 0)$.

**Proof.** $\frac{dq^m}{dq} < 0$ if $\phi_{q'} = \frac{w}{w_q} \cdot w_q \cdot (U_{cl} + \frac{w}{w_q} \cdot w_q \cdot U_{ll}) + U_l \cdot (2 \cdot \frac{w}{w_q} \cdot w_q^2 - \frac{w}{w_q} \cdot w_{qq}) < 0$. By (5), $\phi_{q'} = \frac{w}{w_q} \cdot w_q \cdot (U_{cl} - \frac{U}{U_{cl}} \cdot U_{ll}) + U_l \cdot (2 \cdot \frac{w}{w_q} \cdot w_q^2 - \frac{w}{w_q} \cdot w_{qq}) < 0$, that is true if
consumption is a normal good (hence, \( U_{cl} - \frac{U_{cl}}{U_l} \cdot U_{ll} < 0 \)). Moreover, \( \frac{d\theta}{dq} > -1 \) if \( \phi_q > \phi_{qm} \) or \( U_{cc} + \frac{w}{w_q} \cdot w_q \cdot U_{cl} < 0 \), that is satisfied if leisure is a normal good (hence, \( U_{cc} - \frac{U_{cc}}{U_l} \cdot U_{cl} < 0 \)).

Let us remark that, for any given level of household’s wage (e.g., \( w' \)), the marginal rate of technical substitution between input and capability that keeps constant wage is given by

\[
\frac{d\theta}{dq} = -\frac{w_q}{w_q} \cdot w' < 0.
\]

**Lemma 6** Private input is an economic

- **complement of capability** (i.e., \( \frac{d\theta}{dq} > 0 \)), if input and capability are strong technologic complements (i.e., \( w_q > w_q^+ > 0 \));

- **strong substitute for capability** (i.e., \( \frac{d\theta}{dq} < -\frac{w_q}{w_q} \)), if input and capability are strong technologic substitutes (i.e., \( w_q < w_q^- < 0 \));

- **substitute for capability** (i.e., \( \frac{d\theta}{dq} \in (-\frac{w_q}{w_q}, 0) \)), if capability and input are weak technologic complements or substitutes (i.e., \( w_q \in (w_q^-, w_q^+) \)).

**Proof.**

\[
\phi_\theta = \frac{y}{w^2} \cdot w_q \cdot (U_{cl} + \frac{y}{w^2} \cdot w_q \cdot U_{ll}) + U_l \cdot \left( \frac{y}{w^3} \cdot w_q \cdot w_q - \frac{y}{w^2} \cdot w_q \right) \tag{7}
\]

By (5), \( U_{cl} + \frac{w}{w_q} \cdot w_q \cdot U_{ll} = U_{cl} - \frac{U_{cl}}{U_l} \cdot U_{ll} < 0 \) if consumption is normal. (7) is negative (or positive) if and only if \( w_q \in (w_q^-, w_q^+) \) (or \( w_q \in (w_q^+, w_q^-) \)), where

\[
w_q^+ = 2 \cdot \frac{w_q \cdot w_q}{w} + \frac{U_{cl} - \frac{U_{cl}}{U_l} \cdot U_{ll}}{U_l} > 0. \tag{8}
\]

Moreover, \( \frac{d\theta}{dq} = -\frac{w_q}{w_q} \cdot \alpha_\theta \), where

\[
\alpha_\theta \equiv \frac{U_{cl} \cdot \frac{w}{w_q} \cdot w_q + U_{ll} \cdot (\frac{w}{w_q} \cdot w_q)^2 + U_l \cdot (2 \cdot \frac{w}{w_q} \cdot w_q^2 - \frac{w}{w_q} \cdot w_q \cdot w_q)}{U_{cc} + 2 \cdot U_{cl} \cdot \frac{w}{w_q} \cdot w_q + U_{ll} \cdot (\frac{w}{w_q} \cdot w_q)^2 + U_l \cdot (2 \cdot \frac{w}{w_q} \cdot w_q^2 - \frac{w}{w_q} \cdot w_q \cdot w_q)} < 1
\]
if $U_{cc} + U_l \cdot \frac{w}{w^2} \cdot (\frac{w_q}{w_{qq}} \cdot w_{q\theta} - w_{qq}) < 0$ or $w_{q\theta} > w_{q\theta}^-$ where

$$w_{q\theta}^- = \frac{w_q}{w_q^*} \cdot \left(-\frac{w^2}{y} \cdot \frac{U_{cc}}{U_l} + w_{qq}\right) < 0. \tag{9}$$

1.1.2 Single Crossing Property

The effect of $\theta$ on the marginal rate of substitution between net and gross income depends, in this setting, also on the reaction of $q^m$ to such a change (Boadway and Marchand, 1995).

**Lemma 7** The single crossing property

$$\frac{\partial^2 x|V}{\partial q \partial \theta} = \frac{\partial^2 x|V}{\partial \theta} + \frac{\partial^2 x|V}{\partial q} \cdot \frac{dq}{d\theta} < 0 \tag{10}$$

is satisfied if and only if $w_{q\theta} > w_{q\theta}^*$. Moreover, the single crossing property is violated whenever $w_{q\theta} < w_{q\theta}^-$. 

**Proof.** Remark that

$$\frac{\partial^2 x|V}{\partial \theta} = -\frac{w_q}{w_{q}^*} \cdot \frac{U_l}{U^2_{l}} \cdot \frac{U_c U_{cl} + U^2_{cl} U_{ll} - U_{c} \cdot \frac{w_q}{w}}{U_{c} \cdot \frac{w_q}{w}} < 0$$

and

$$\frac{\partial^2 x|V}{\partial q} = -\frac{U_l}{U^2_{l}} \cdot \left(U_{cc} - 2 \cdot \frac{U_{c} \cdot U_{cl} + U^2_{cl} \cdot U_{ll} - U_{c} \cdot \frac{w_q}{w}}{U_{c} \cdot \frac{w_q}{w}}\right) < 0$$

then, (10) is satisfied if and only if

$$\frac{dq}{d\theta} > -\frac{\partial^2 x|V}{\partial \theta} \frac{\partial^2 x|V}{\partial q}$$
that is equivalently written as $\beta > \alpha_\theta$, where

$$\beta \equiv \frac{-\frac{U}{U_l} \cdot U_{cl} + \frac{U^2}{U_l} \cdot U_{ul} - U_c \cdot \frac{w}{w}}{U_{cc} - 2 \cdot \frac{U}{U_l} \cdot U_{cl} + \frac{U^2}{U_l} \cdot U_{ul} - U_c \cdot \frac{w}{w}} \in (0, 1)$$

under the assumption that $c$ is normal and $U(\cdot, \cdot)$ is strictly concave), and boils down to

$$w_{q\theta} > w_{q\theta}^*$$

with

$$w_{q\theta}^* \equiv \frac{w_q \cdot w_{q\theta}}{w} - \beta \cdot \left(\frac{w_q}{w} - w_{q\theta}\right)$$

Remark that when $w_{q\theta} < w_{q\theta}^*$, then $\alpha_\theta \geq 1$, hence $\beta > \alpha_\theta$ - and (10) - is violated. ■

1.2 Minimum second-pillar provision

The minimum second-pillar provision inducing the household with capability $\theta$ to opt in is

$$q^*(x, y, q^f, \theta) \equiv \{q' \in \mathcal{R}_+ \mid U(x, y, q^f, \theta) = V(x, y, q^f, \theta)\} \quad (11)$$

Let $U^s \equiv U(x, y, q^f, \theta)$ and $U^m \equiv \max_q q^m U(x - q^m, y, q^f + q^m)$. Thus, as far as

$q^m(x, y, q^f, \theta) > 0, U^s_c < U^m_c, |U^s_l| > |U^m_l|$, and $q^m(x, y, q^f, \theta) > q^*(x, y, q^f, \theta) > 0$ (hence, $w^m > w^*$, where $w^* \equiv w(\theta, q^f + q^*)$ and $w^m \equiv w(\theta, q^f + q^m(x, y, q^f, \theta))$). Moreover, in this case, changes in net and gross incomes affect both sides of (11). Thus,

$$\frac{d q^s}{dx} = \left(\frac{U^m}{U^s_c} - 1\right) \cdot \frac{w^*}{\frac{U^m}{w^*}} > 0$$

and

$$\frac{d q^s}{dy} = \frac{w^*}{y \cdot w^s} \cdot (1 - \frac{U^m}{U^m}) > 0$$
Given that $\eta \equiv \frac{U_m}{U_s} < 1$,

$$\frac{dq^*}{dq^m} = -(1 - \eta \cdot \frac{w_{q}^{m}}{w_{q}^{s}}) \in (-1, 0)$$

Given $\eta$, the sign of

$$\frac{dq^*}{d\theta} = -\frac{w_{q}^{s}}{w_{q}^{s}} (1 - \eta \cdot \frac{w_{q}^{m}}{w_{q}^{s}})$$

depends on the technical complementarity/substitutability between $q$ and $\theta$, namely $\frac{dq^*}{d\theta} < 0$ if and only if $1 - \eta \cdot \frac{w_{q}^{m}}{w_{q}^{s}} > 0$ or

$$\frac{w_{q}^{s}}{w_{q}^{s}} - \eta \cdot \frac{w_{q}^{m}}{w_{q}^{s}} = \frac{w_{q}^{m}}{w_{q}^{m}} \cdot (1 - \eta) - \int_{q^{m}}^{q^{s}} \left( \frac{w_{q}^{s}}{w} - \frac{w_{q}^{s} \cdot w_{q}^{s}}{w^{2}} \right) \cdot dq > 0$$

Thus, given $\eta$, a sufficient condition for $\frac{dq^*}{d\theta} < 0$ (or $\frac{dq^*}{d\theta} > 0$) is that $w_{q}^{s} < w_{q}^{++}$ (or $w_{q}^{s} > w_{q}^{++}$), where

$$w_{q}^{++} = \frac{w_{q} \cdot w_{q}}{w} + \frac{w_{q}^{m} \cdot 1 - \eta}{q^{m} - q^{s}} \cdot w > 0.$$  

1.2.1 Single Crossing Property for opting-in households

Opting-in households do not privately demand any input, thus only the direct effect of $\theta$ on the marginal rate of substitution between net and gross income is relevant; hence

$$\frac{\partial z}{\partial q} |_{U} = \frac{\partial z}{\partial q} |_{U} < 0$$

as shown in Lemma 7.

1.3 Incentive Compatibility of First Best Allocations

Given individual optimization of private input investment, first best allocations can be characterized by the maximization of a social welfare function under government’s budget
constraint:

\[
\max_{x,y,z} \lambda \cdot V(x, y, 0, \theta) + (1 - \lambda) \cdot V(\bar{x}, \bar{y}, 0, \bar{\theta}) \\
\text{s.t. } \lambda \cdot (y - x) + (1 - \lambda) \cdot (\bar{y} - x) \geq 0 \quad (\mu)
\]

by the first order conditions, the following optimization conditions arise

\[
\frac{dx}{dy} \bigg|_{V = 1}
\]

(and, by individual optimization, also \(\frac{w}{y_w} = 1\), for all \(\theta \in \{\theta, \bar{\theta}\}\).

Lemma 8 The first best allocation is incentive-compatible for low-capability households.

Proof. Let \(T\) be the first best income transfer received by low-capability households, thus by government budget constraint, \(x = y + T\) and \(\bar{x} = \bar{y} - \lambda \cdot T\). The first best allocation is incentive compatible for low-capability individuals if

\[
U(y + T - q^m(y + T, y, 0, \theta)) + \frac{w(\theta, q^m(y + T, y, 0, \theta))}{\bar{y} - \lambda \cdot T} \geq U(y - \frac{\lambda}{1 - \lambda} \cdot T - q^m(y - \frac{\lambda}{1 - \lambda} \cdot T, y, 0, \theta)) + \frac{\bar{y} - \lambda \cdot T}{\bar{y} - \lambda \cdot T} \cdot w(\theta, q^m(y, y, 0, \theta)) \quad (12)
\]

Given that \(q^m(x, y, 0, \theta)\) is the optimal quantity of input demanded by low-capability households under \(\{x, y\}\) - hence, the first order condition (1) - is always satisfied - we apply the envelope theorem and write (12) as

\[
\int_{\bar{y}}^{y} \tilde{U}_c(T) \cdot (1 + \frac{\tilde{U}_t(T)}{\tilde{U}_c(T)} \cdot \tilde{w}(T)) \cdot dy - \int_{\frac{\lambda}{1 - \lambda} \cdot T}^{T} \tilde{U}_c(T) \cdot d\tau \leq 0 \quad (13)
\]
where

\[
\begin{align*}
\hat{U}_c(T) & \equiv \frac{\partial}{\partial c} U(y + T - q^m(y + T, y, 0, \theta), \frac{y}{w(\bar{\theta}, q^m(y + T, y, 0, \theta))}) \\
\hat{U}_l(T) & \equiv \frac{\partial}{\partial l} U(y + T - q^m(y + T, y, 0, \theta), \frac{y}{w(\bar{\theta}, q^m(y + T, y, 0, \theta))}) \\
\hat{w}(T) & \equiv w(\bar{\theta}, q^m(y + T, y, 0, \theta)) \\
\end{align*}
\]

for all \( y \in [\tilde{y}, \bar{y}] \) and all \( \tau \in [-\frac{\lambda}{1-\lambda} \cdot T, T] \). By the SCP, if \( y < \tilde{y} \) (or \( y > \bar{y} \)) \( 1 + \frac{\hat{U}_l(T)}{\hat{U}_c(T) \cdot \hat{w}(T)} < 0 \) (or \( 1 + \frac{\hat{U}_l(T)}{\hat{U}_c(T) \cdot \hat{w}(T)} > 0 \)), thus (13) is always satisfied with strict inequality.

As regards the first best tax schedule for households with high exogenous capability, we cannot conclude anything, given that - by the same kind of arguments used in Lemma 8:

\[
U(\tilde{y} - \frac{\lambda}{1-\lambda} \cdot T - q^m(\tilde{y} - \frac{\lambda}{1-\lambda} \cdot T, \tilde{y}, 0, \bar{\theta}), \frac{\tilde{y}}{w(\bar{\theta}, q^m(\tilde{y} - \frac{\lambda}{1-\lambda} \cdot T, \tilde{y}, 0, \bar{\theta}))}) + \\
- U(y + T - q^m(y + T, y, 0, \bar{\theta}), \frac{y}{w(\bar{\theta}, q^m(y + T, y, 0, \bar{\theta}))}) = \\
\int_{\frac{\tilde{y}}{y}}^{\tilde{y}} \hat{U}_c(-T) \cdot (1 + \frac{\hat{U}_l(-T)}{\hat{U}_c(-T) \cdot \hat{w}(-T)}) \cdot dy - \int_{\frac{\lambda}{1-\lambda} \cdot T}^{T} \hat{U}_c(\tilde{y}) \cdot d\tau
\]

In our analysis, we assume that the tax schedule for high-ability households is incentive incompatible. The same arguments hold also when we consider the public provision of private input. In the inclusive case, the SCP holds and all the above arguments apply, given that individuals do not control \( q \) any more. In the discriminating case, the low-ability households opting for the second-pillar provision receive at least the opt-out utility, thus the incentive constraint is satisfied. Also high-ability households opting for the second pillar provision receive at least their opting-out utility, which may reduce the incentive problem characterizing second best redistribution.
1.4 Optimal Tax and Public Provision

In the following, the optimization problems under different policy regimes are considered.

1.4.1 Pure Taxation Regime

The government’s program is

$$\max_{\{x,y,\bar{x}\}} \lambda \cdot V(x, y, q^f, \theta) + (1 - \lambda) \cdot V(\bar{x}, q^f, \bar{\theta})$$

subject to

$$\lambda \cdot (y - x) + (1 - \lambda) \cdot (\bar{y} - \bar{x}) \geq 0 \quad (\mu)$$

$$V(\bar{x}, q^f, \bar{\theta}) \geq V(x, y, q^f, \theta) \quad (\nu)$$

Thus, the Lagrangian is

$$\mathcal{L} = \lambda \cdot V + (1 - \lambda) \cdot \bar{V} + \mu \cdot [\lambda \cdot (y - x) + (1 - \lambda) \cdot (\bar{y} - \bar{x})] + \nu \cdot (V - \bar{V})$$

where $V \equiv V(x, y, q^f, \theta)$, $\bar{V} \equiv V(\bar{x}, q^f, \bar{\theta})$, and $\hat{V} \equiv V(x, y, q^f, \theta)$. The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x} = \lambda \cdot V_x - \lambda \cdot \mu - \nu \cdot \hat{V}_x = 0 \quad (14)$$

$$\frac{\partial \mathcal{L}}{\partial y} = \lambda \cdot V_y + \lambda \cdot \mu - \nu \cdot \hat{V}_y = 0 \quad (15)$$

$$\frac{\partial \mathcal{L}}{\partial \bar{x}} = (1 - \lambda) \cdot \bar{V}_x - (1 - \lambda) \cdot \mu + \nu \cdot \bar{V}_x = 0 \quad (16)$$

$$\frac{\partial \mathcal{L}}{\partial \bar{y}} = (1 - \lambda) \cdot \bar{V}_y + (1 - \lambda) \cdot \mu + \nu \cdot \bar{V}_y = 0 \quad (17)$$

By (16) and (17), $-\bar{V}_x = \frac{dx}{dy} |_{\bar{\psi}} = 1$. By (14) and (15), after some algebra we obtain

$$\frac{dx}{dy} |_{\bar{\psi}} = 1 - \frac{\frac{\hat{V}_x}{V_x} \cdot \frac{dx}{dy} |_{\bar{\psi}}}{1 - \frac{\hat{V}_y}{V_y} \cdot \frac{dx}{dy} |_{\bar{\psi}}} \cdot (\frac{dx}{dy} |_{\bar{\psi}} - \frac{dx}{dy} |_{\bar{\psi}})$$
where \( \frac{dx}{dy} |_V = -\frac{V_y}{V_x} \) and \( \frac{dx}{dy} |_\hat{V} = -\frac{\hat{V}_y}{\hat{V}_x} \). By (14), \( 1 - \frac{\hat{V}_x}{V_x} \cdot \frac{\hat{V}_y}{V_y} > 0 \), and by the SCP, \( \frac{dx}{dy} |_V - \frac{dx}{dy} |_\hat{V} > 0 \), hence \( \frac{dx}{dy} |_V < 1 \) (the optimal distortion implies a positive marginal tax on low-ability labor supply).

### 1.4.2 Inclusive Regime

Let \( q = q^f + q^s \) be the total provision of public good. Under this regime the optimization problem is

\[
\max_{\{x, y, \varpi, q\}} \lambda \cdot U(x, \frac{y}{w(\theta, q)}) + (1 - \lambda) \cdot U(\bar{x}, \frac{\bar{y}}{w(\bar{\theta}, \bar{q})})
\]

s.t.

\[
\lambda \cdot (y - x) + (1 - \lambda) \cdot (\bar{y} - \bar{x}) - q \geq 0 \quad (\mu)
\]

\[
U(\bar{x}, \frac{\bar{y}}{w(\bar{\theta}, \bar{q})}) \geq U(x, \frac{y}{w(\theta, q)}) \quad (\nu)
\]

\[
q \geq q_{\max} \quad (\eta)
\]

where \( q_{\max} \equiv \{q^f + q^s \mid \max\{q^s(x, y, q^f, \theta), q^s(\varpi, \bar{y}, q^f, \bar{\theta})\} \leq q^s\} \).

The Lagrangian is

\[
\mathcal{L} = \lambda \cdot \underline{U} + (1 - \lambda) \cdot \bar{U} + \mu \cdot [\lambda \cdot (y - x) + (1 - \lambda) \cdot (\bar{y} - \bar{x}) - q] + \nu \cdot (\bar{U} - \hat{U}) + \eta \cdot (q - q_{\max})
\]

where \( \underline{U} \equiv U(x, \frac{y}{w}), \bar{U} \equiv U(\bar{x}, \frac{\bar{y}}{w(\bar{\theta}, \bar{q})}), \hat{U} \equiv U(x, \frac{y}{w(\theta, q)}), w \equiv w(\theta, q), \) and \( \underline{w} \equiv w(\bar{\theta}, \bar{q}) \). The first
order conditions are

\[ \frac{\partial L}{\partial x} = \lambda \cdot U_c - \lambda \cdot \mu - \nu \cdot \hat{U}_c = 0 \] (18)
\[ \frac{\partial L}{\partial y} = \lambda \cdot \frac{U_l}{w} + \lambda \cdot \mu - \nu \cdot \hat{U}_l \] (19)
\[ \frac{\partial L}{\partial x} = (1 - \lambda + \nu) \cdot U_c - (1 - \lambda) \cdot \mu = 0 \] (20)
\[ \frac{\partial L}{\partial y} = (1 - \lambda + \nu) \cdot \frac{U_l}{w} + (1 - \lambda) \cdot \mu = 0 \] (21)
\[ \frac{\partial L}{\partial q} = -\lambda \cdot \frac{U_l}{w} \cdot \frac{y}{w} - (1 - \lambda + \nu) \cdot \frac{\hat{U}_l}{w} \cdot \frac{y}{w^2} \cdot \frac{\hat{y}}{\bar{w}} - \mu + \nu \cdot \hat{U}_l \cdot \frac{y}{w} \cdot \bar{w} + \eta = 0. \] (22)

By (20) and (21), \( \frac{dx}{dy} |_{\bar{U}} = -\frac{U_l}{U_x \cdot w} = 1. \) By (18) and (19), after some algebra we obtain that \( \frac{dx}{dy} |_{\bar{U}} = -\frac{U_l}{U_x \cdot w} < 1. \)

[HERE A LEMMA ON THE OPTIMAL UNIFORM PROVISION OF INPUT].

1.4.3 Discriminating Regimes

We consider first the case of low-ability in the second pillar and high-ability out. The optimization program of the government is

\[ \max_{(x, y, \bar{x}, q, q^f, q^s)} \lambda \cdot U(x, \frac{y}{w(q^f + q^s)}) + (1 - \lambda) \cdot V(x, y, q^f, \bar{y}) \]

s.t. :

\[ \lambda \cdot (y - x) + (1 - \lambda) \cdot (\bar{y} - \bar{x}) - q^f - \lambda \cdot q^s \geq 0 \] (\( \mu \))
\[ V(x, y, q^f, \bar{y}) \geq U(x, \frac{y}{w(q^f + q^s)}) \] (\( \nu \))
\[ q^s(x, y, q^f, \bar{y}) \geq q^s \] (\( \eta \))
\[ q^f \geq 0 \] (\( \varphi \))
\[ q^s \geq q^s(x, y, q^f, \bar{y}) \] (\( \psi \))

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hence, the corresponding Lagrangian is

\[
\mathcal{L} = \lambda \cdot \mathcal{U} + (1 - \lambda) \cdot \mathcal{V} + \mu \cdot [\lambda \cdot (y - \bar{y}) + (1 - \lambda) \cdot (\bar{y} - \mathcal{U}) - q^f - \lambda \cdot q^s] + \\
+ \nu \cdot (\mathcal{V} - \hat{\mathcal{V}}) + \bar{\eta} \cdot (\mathcal{V}^s - q^s) + \eta \cdot (q^s - \bar{q}^s) + \varphi \cdot q^f + \bar{\varphi} \cdot \bar{q}^s
\]

where \( \mathcal{U} \equiv U(x, \frac{y}{w}), \mathcal{V} \equiv V(\bar{x}, \bar{y}, q^f, \bar{q}), \hat{\mathcal{U}} \equiv U(x, \frac{\hat{y}}{\hat{w}}), \) and \( \hat{\mathcal{V}} \equiv \hat{w}(\bar{\theta}, q^f + \hat{q}^s) \). The first order conditions are

\[
\frac{\partial \mathcal{L}}{\partial x} = \lambda \cdot \mathcal{U}_x - \lambda \cdot \mu - \nu \cdot \hat{\mathcal{U}}_x - \eta \cdot q^s = 0 \\
\frac{\partial \mathcal{L}}{\partial y} = \lambda \cdot \mathcal{U}_y + \lambda \cdot \mu - \nu \cdot \hat{\mathcal{U}}_y - \eta \cdot q^s = 0 \\
\frac{\partial \mathcal{L}}{\partial \mathcal{V}_x} = (1 - \lambda + \nu) \cdot \mathcal{V}_x - (1 - \lambda) \cdot \mu + (\bar{\eta} + \varphi) \cdot \mathcal{V}^s = 0 \\
\frac{\partial \mathcal{L}}{\partial \mathcal{V}_y} = (1 - \lambda + \nu) \cdot \mathcal{V}_y + (1 - \lambda) \cdot \mu + (\bar{\eta} + \varphi) \cdot \mathcal{V}^s = 0 \\
\frac{\partial \mathcal{L}}{\partial q^f} = (1 - \lambda + \nu) \cdot \mathcal{V}_q + \nu \cdot \hat{\mathcal{U}}_q \cdot \frac{\hat{y}}{\hat{w}^2} \cdot \hat{w}_q - \lambda \cdot \mathcal{U}_q \cdot \frac{y}{w^2} \cdot w_q - \mu + (\bar{\eta} + \varphi) \cdot \mathcal{V}^s_q - \eta \cdot q^s + \varphi = 0 \\
\frac{\partial \mathcal{L}}{\partial q^s} = -\lambda \cdot \mathcal{U}_s \frac{y}{w^2} \cdot w_q - \mu \cdot \lambda + \nu \cdot \hat{\mathcal{U}}_s \frac{y}{w^2} \cdot \hat{w}_q - \bar{\eta} + \bar{\eta} = 0
\]

By (26) and (27), we get

\[
\frac{dx}{dy} \bigg|_{\mathcal{V}} = 1 + \frac{\bar{\eta} + \varphi}{1 - \lambda + \nu} \cdot \frac{\mathcal{V}^s_x + \mathcal{V}^s_y}{\mathcal{V}_x}
\]

where \( \frac{dx}{dy} \bigg|_{\mathcal{V}} = -\mathcal{V}_y \cdot \mathcal{V}_x \); thus, high-capability labor income is not distorted at the margin only if \( \bar{\eta} = \varphi = 0 \), hence only if \( q^s(\bar{x}, \bar{y}, q^f, \bar{q}) > q^s \geq 0 \); otherwise - when the upper constraint to the second-pillar public provision is binding or the first-pillar public provision is sufficiently high that any second-pillar provision induces households to opt in, the high-capability labor income can be optimally distorted with a marginal subsidy.
By (24) and (25), we get
\[
\frac{dx}{dy} \bigg|_{U} = 1 - \frac{\nu}{\lambda} \cdot \frac{\hat{U}_c}{\hat{U}_c} \left( \frac{dx}{dy} \bigg|_{\hat{U}} \right) + \frac{\eta}{\lambda} \cdot \frac{q_s^* + q_q^*}{1 - \nu \cdot \frac{\hat{U}_c}{\hat{U}_c}} \tag{31}
\]
where \(\frac{dx}{dy} \bigg|_{\hat{U}} = -\frac{\hat{U}_l}{\hat{U}_c \cdot \hat{w}}\) and \(\frac{dx}{dy} \bigg|_{\hat{U}} = -\frac{\hat{U}_l}{\hat{U}_c \cdot \hat{w}}\); in this case, the low-capability labor income can be taxed with an heavier marginal tax rate if the lower constraint to the second-pillar public provision is binding (when \(\eta > 0\), necessarily \(q_s = q_s(x, y, q^f, \theta)\)). Conversely, when \(\eta = 0\) we have \(q_s > q_s(x, y, q^f, \theta)\).

Substituting (26) and (28) in (29), and observing that \(V_x = V_{q^f}\), we get
\[
\eta - \eta \cdot (1 + q_s^*) + \varphi = (\eta + \varphi) \cdot (q_s^* - q_q^*) \geq 0 \tag{32}
\]
if \(q^f > 0\), then \(\varphi = 0\), consequently \(\eta > 0\) and \(\eta = 0\). Therefore, when the first-pillar is active \((q^f > 0)\), the high-capability households is subsidized at the margin.

Lemma 9 \(\varphi > 0\) (hence, \(\eta = 0\)) and \(\eta > 0\) (hence, \(\varphi = 0\)) may happen only if \(\theta\) and \(q\) are strong technologic complements (i.e., \(w_{q\theta} > w_q^+\)).

Proof. From (24) we have
\[
\mu = \hat{U}_c \left( 1 - \frac{\nu}{\lambda} \cdot \frac{\hat{U}_c}{\hat{U}_c} \right) - \frac{\eta}{\lambda} \cdot q_s^* > 0
\]
hence, necessarily \(\alpha \equiv \frac{\nu}{\lambda} \cdot \frac{\hat{U}_c}{\hat{U}_c} \in (0, 1]\). By (29),
\[
- \left( 1 + \frac{\hat{U}_l}{\hat{U}_c \cdot \hat{w}} \cdot \frac{y}{\hat{w}} \cdot \hat{w}_q \right) = -\alpha \cdot \left( 1 + \frac{\hat{U}_l}{\hat{U}_c \cdot \hat{w}} \cdot \frac{y}{\hat{w}} \cdot \hat{w}_q \right) - \frac{\eta}{\lambda U_l} \cdot (1 + q_s^*)
\]
If \(\theta\) and \(q\) are substitutes, we have
\[
- \left( 1 + \frac{\hat{U}_l}{\hat{U}_c \cdot \hat{w}} \cdot \frac{y}{\hat{w}} \cdot \hat{w}_q \right) > - \left( 1 + \frac{\hat{U}_l}{\hat{U}_c \cdot \hat{w}} \cdot \frac{y}{\hat{w}} \cdot \hat{w}_q \right)
\]
Together
with the fact that $\alpha \in [0, 1)$, we can conclude that

$$-\frac{\eta}{\lambda U_c} \cdot (1 + q_s^*) > 0$$

Then, $\eta > 0$ never happens. ($\eta > 0$ is possible only in the case of strong complementarity between $\theta$ and $q$) [CHECK AND COMPLETE] ■

We now consider the DMH regime. In this case, government’s program is

$$\max_{\{x, y, y', q'\}} \lambda \cdot V(x, y, q', \theta) + (1 - \lambda) \cdot U(x, y', q', \theta)$$

subject to:

$$\lambda \cdot (y - x) + (1 - \lambda) \cdot (y - x) - q' - (1 - \lambda) \cdot q^s \geq 0 \quad (\mu)$$

$$U(x, y, q', \theta) \geq V(x, y', q', \theta) \quad (\nu)$$

$$q^s(x, y, q', \theta) \geq q^s \quad (\eta)$$

$$q^s \geq q^s(x, y, q', \theta) \quad (\overline{\eta})$$

$$q' \geq 0 \quad (\phi)$$

$$q^s(x, y, q', \theta) \geq 0 \quad (\overline{\phi})$$

The Lagrangian of this program is

$$L = \lambda \cdot V + (1 - \lambda) \cdot U + \mu \cdot [\lambda \cdot (y - x) + (1 - \lambda) \cdot (y - x) - q' - (1 - \lambda) \cdot q^s] +$$

$$+ \nu \cdot (U - V) + \eta \cdot (q^s - q^s) + \overline{\eta} \cdot (q^s - q^s) + \phi \cdot q' + \overline{\phi} \cdot q^s$$

where $V \equiv V(x, y, q', \theta)$, $U \equiv U(x, y', q', \theta)$, $V \equiv V(x, y, q', \theta)$, and $\overline{\theta} = w(\overline{\theta}, q'^f + q^s)$. The
first order conditions are

\[
\frac{\partial L}{\partial x} = \lambda \cdot V_x - \lambda \cdot \mu - \nu \cdot \hat{V}_x - \eta \cdot q^x + \varphi \cdot q^x = 0 (34)
\]

\[
\frac{\partial L}{\partial y} = \lambda \cdot V_y + \lambda \cdot \mu - \nu \cdot \hat{V}_y - \eta \cdot q^y + \varphi \cdot q^y = 0 (35)
\]

\[
\frac{\partial L}{\partial x} = (1 - \lambda + \nu) \cdot \bar{U}_c - (1 - \lambda) \cdot \mu - \bar{\eta} \cdot \bar{q}_x = 0 (36)
\]

\[
\frac{\partial L}{\partial y} = (1 - \lambda + \nu) \cdot \bar{U}_l + (1 - \lambda) \cdot \mu - \bar{\eta} \cdot \bar{q}_y = 0 (37)
\]

\[
\frac{\partial L}{\partial q_f} = \lambda \cdot V_{q_f} - (1 - \lambda + \nu) \cdot \bar{U}_l \cdot \frac{\eta}{\bar{w}} \cdot \bar{q}_q - \mu \cdot \bar{q}_f + (\eta + \varphi) \cdot q^f - \bar{\eta} \cdot \bar{q}_q + \varphi = 0 (38)
\]

\[
\frac{\partial L}{\partial q_s} = - (1 - \lambda + \nu) \cdot \bar{U}_l \cdot \frac{\eta}{\bar{w}} \cdot \bar{q}_q - \mu \cdot \bar{q}_s + (1 - \lambda) - \eta + \bar{\eta} = 0 (39)
\]

By (36) and (37),

\[
\frac{dx}{dy} \bigg| \bar{U} = - \frac{\bar{U}_l \cdot \frac{\eta}{\bar{w}} \cdot \bar{q}_q}{\bar{U}_c \cdot \bar{w}} = 1 - \bar{\eta} \cdot \frac{\bar{q}_q + \bar{q}_s}{(1 - \lambda + \nu) \cdot \bar{U}_c}
\]

And, by (34) and (35),

\[
\frac{dx}{dy} \bigg| V = - \frac{\bar{U}_l \cdot \frac{\eta}{\bar{w}} \cdot \bar{q}_q}{\bar{U}_c \cdot \bar{w}} = 1 - \frac{\bar{\eta} \cdot \frac{\bar{q}_q + \bar{q}_s}{1 - \nu} \cdot \frac{\bar{q}_q + \bar{q}_s}{\bar{q}_q}}{1 - \frac{\bar{\eta} \cdot \frac{\bar{q}_q + \bar{q}_s}{1 - \nu} \cdot \frac{\bar{q}_q + \bar{q}_s}{\bar{q}_q}}}
\]

Lemma 10 At the optimum, \( \bar{\eta} = 0 \).

**Proof.** Assume conversely that \( \bar{\eta} > 0 \) (hence \( \bar{\eta} = 0 \)), then necessarily \( q^s = \bar{q}^s \), hence \( -\bar{U}_c - \bar{U}_l \cdot \frac{\eta}{\bar{w}} \cdot \bar{w}_q > 0 \), by construction of \( \bar{q}^s \). By (36) and (39),

\[
- \bar{U}_c - \bar{U}_l \cdot \frac{\eta}{\bar{w}} \cdot \bar{w}_q = \frac{\eta - \bar{\eta} \cdot (1 + \bar{q}^s)}{1 - \lambda + \nu}
\]

that implies a contradiction.

By Lemma 10, the marginal tax rate on high-capability households is zero. Moreover

Lemma 11 At the optimum, also \( \eta = \varphi = 0 \).

**Proof.** As first, we show that \( \eta = 0 \). Assume by contradiction that \( \eta > 0 \), then
necessarily $q^* = q_s$. Moreover, necessarily $q^*$ is such that $-\overline{U}_c - \overline{U}_l \cdot \frac{\overline{y}}{\overline{w}} \cdot \overline{w}_q > 0$. Let $q^f$ and $q^s$ be the optimal first- and second-pillar provision at the optimum such that $\eta > 0$. It is easy to see that such a policy mix cannot be optimal. To see this we consider the following policy reform: $q^f$ increases keeping constant the overall transfer from the high-capability to the low-capability (hence, also the government’s budget is kept in equilibrium). In particular, $\frac{d\eta}{dq^f} = 1$ and $\frac{d\eta}{dq^f} = 1 - \frac{d\eta}{dq^f}$. Thus, the total welfare increases

$$\lambda \cdot (V_{x} \cdot \frac{d\eta}{dq^f} + V_{q^f}) + (1 - \lambda) \cdot (-\overline{U}_c - \overline{U}_l \cdot \frac{\overline{y}}{\overline{w}} \cdot \overline{w}_q) \cdot (1 - \frac{d\eta}{dq^f})$$

Hence, we have a contradiction. Given that necessarily $-\overline{U}_c - \overline{U}_l \cdot \frac{\overline{y}}{\overline{w}} \cdot \overline{w}_q = 0$, then by (34), (36) and (38) we also see that

$$(\eta + \overline{\eta}) \cdot (q^s - q^s_x) + \overline{\eta} \cdot \overline{q}^s - \varphi = 0$$

hence, by $\eta = \overline{\eta} = 0$ necessarily $\varphi = \varphi = 0$.

It follows that the tax schedule of low-capability households is distorted as under the optimal pure tax policy. Moreover, the public provision of input just replicates what the high-capability households would do under a pure tax regime.

References


Munro, A. (1992), Self-Selection and Optimal In-Kind Transfers, Economic Journal 102, 1184-96.
