Lifecycle savings when pensions are at risk: Theory and microeconometric evidence

Peter van Santen*

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Abstract

We construct and estimate a lifecycle consumption model in which pension benefits and lifespan are uncertain. Our theoretical model features rational individuals that work and consume or save before retirement. After retirement, they receive pension income, the level of which is uncertain from today’s perspective. Using a Constant Absolute Risk Aversion (CARA) utility function, we are able to find a closed-form expression for current consumption as a function of the expectation of pension benefits as well as the uncertainty in pension income. Intuitively, consumption increases with the expected value of pension income, and decreases with its variance; the absolute value of the marginal effect of uncertainty is larger than that of the expected value of pension benefits for realistic parameter values. Furthermore, we allow for mortality risk after retirement, and we show that consumption increases as the probability of death increases, due to impatience. We estimate the savings equation implied by the model using panel data for Dutch households. We use the answers from probabilistic survey questions to compute the expected pension income replacement rate and the variance of this replacement rate, as a measure for uncertainty. Mortality risk is elicited probabilistically as well. Our quantile regression results show that, for the higher wealth and income quantiles, savings increase with the uncertainty in pension income and decrease with mortality risk, as predicted by the theory. The results are robust to different measures for savings, household wealth and including background characteristics, including Mundlak-type fixed effects.

Keywords: Lifecycle consumption model, household savings, pension income uncertainty, displacement effect
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*Faculty of Economics and Business, University of Groningen, and Netspar. Correspondence: Rijksuniversiteit Groningen, PO Box 800, 9700 AE Groningen, the Netherlands, p.c.van.santen@rug.nl

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1 Introduction

The standard lifecycle model emphasizes the importance of saving during working life for consumption during retirement. A simple version of this model predicts that private savings decrease one-for-one with increases in pension wealth, that is, perfect displacement (or crowding out) of private savings by pension savings. In his seminal article, Feldstein (1974) triggered a discussion of the effects of (state) pension systems on household savings. The author argues that the introduction of a system of social security need not have any effects on saving, especially if the retirement age is a choice variable. If individuals retire earlier, private savings may actually increase if there exists a social security system. Gale (1998) mentions several reasons why the displacement effect need not be equal to 100%: pensions are illiquid, tax-deferred annuities, households may save for other reasons than retirement or may lack a basic level of financial literacy. Moreover, Gale (1998) shows that a regression of non-pension wealth on pension wealth will bias downward the estimated displacement effect, if labor income and pension wealth enter the right-hand-side of the regression separately; the author stresses the importance of age effects resulting in different planning horizons. Whether or not household savings react to the accumulation of pension wealth thus remains an empirical issue. Many papers have made attempts to estimate the displacement effect. Gale (1998) estimates the displacement effect of pensions on non-pension wealth to be 82.3% (39.3%) using least absolute deviations (robust) regressions. Attanasio and Rohwedder (2003) and Attanasio and Brugiavini (2003) estimate savings equations derived from lifecycle models, using pension reforms in the United Kingdom and Italy respectively to identify the displacement effect. Attanasio and Brugiavini (2003) find that the displacement effect differs per age group, ranging from close to zero for young adults and nearly retired individuals to 200% for middle-aged individuals, although the coefficients differ per specification. Attanasio and Rohwedder (2003) find that the displacement effect is close to zero for the basic state pension, and ranges from 55% for middle aged to 75% for nearly retired individuals. Engelhardt and Kumar (2011) use data from the Health and Retirement Study in the US, and use an instrumental variables approach to account for measurement error in wealth and individual heterogeneity, such as tastes for saving. These authors find a displacement effect of 62%, although quantile regression estimates show that this estimate increases with the level of non-pension wealth. Kapteyn, Alessie, and Lusardi (2005) exploit productivity differences across cohorts and the introduction of social security in the Netherlands to find a small but statistically significant displacement effect of 11.5%.

The contribution of this paper is to relax two restrictive assumptions made in aforementioned papers studying the effect of pensions on household saving. First, although closely linked to a lifecycle model, the main regression equations in the papers above are based on either certainty or certainty equivalence. Yet, among others Skinner (1988), Zeldes (1989) and Caballero (1990, 1991) emphasize the importance of precautionary savings in aggregate savings. In this paper, we allow for a precautionary savings motive in the lifecycle model presented in Section 3, and argue that our data is suitable for estimating the effect of pension income uncertainty on private savings. Second, expectations are taken to be rational and static, meaning that the introduction of the social security system in Kapteyn, Alessie, and Lusardi (2005) or the reform of the pension system in Attana-
sio and Rohwedder (2003) or Attanasio and Brugiavini (2003) comes as a surprise, that households perfectly understand the consequences of the change in the pension system and that the change is considered to be permanent and, therefore, immediately incorporated into household consumption and saving programs over the lifecycle. Instead, we have available expectations of the pension income replacement rate for several time periods in a panel of households. We do not have to make restrictive assumptions on the expectation formation process, nor assume static expectations.

We use data from the DNB Household Survey (DHS), an annual survey collecting panel data from the Netherlands, and the Pension Barometer, an annual survey presented to a subset of respondents from the DHS, which elicits expectations of pension benefits. To be precise, the expectations of pension benefits are elicited from probabilistic survey questions of the type suggested by Dominitz and Manski (1997) and Manski (2004). These questions allow us to calculate the expected level of the retirement income replacement rate, as well as the variance of the replacement rate. We have subjective expectations data at our disposal for the period 2006-2009, and both the expected level of the replacement rate and the variance vary over time and over households.

Some other studies have also relaxed the assumption on static expectations by using subjective expectations data. Bottazzi, Jappelli, and Padula (2006) have panel data for Italian households at their disposal, and use a subjective measure of pension benefits to study displacement of private wealth by social security wealth; their IV estimate of the displacement effect equals 64.5% using Italian pension reforms to identify this effect. The survey questions these authors employed do not allow the calculation of a measure of uncertainty however, and thus excludes the precautionary savings motive. Guiso, Jappelli, and Padula (2009) use similar probabilistic survey questions as used in this paper to calculate individual-level expected replacement rates of pension income, as well as the standard deviation as measure of uncertainty. Using probit regressions on a cross-section of Italian investors, the authors find that the probability of investing in a pension fund decreases with the expected replacement rate, and increases with its standard deviation, in line with intuition. The same signs and significance are obtained for the probability of having health insurance. For life insurance and casualty insurance, only the expected replacement rate is significant, with the correct (negative) sign. This paper extends the analysis of Guiso, Jappelli, and Padula (2009) by using a savings equation derived from a lifecycle model with uncertain pension income, thus allowing for both precautionary saving motives and age effects, and by using panel data. Furthermore, we account for mortality risk, for which we use subjective survival expectations in the empirical section.

We show, in section 3, that the effects of mortality risk, pension risk and expected pension income can be heterogeneous in the population, due to the presence of liquidity constraints. Therefore, we use quantile regression techniques to estimate the savings equation, which allows for dependence of the marginal effects on the wealth and on the path for labour income. We find evidence of precautionary savings due to pension risk and uncertain lifespan for the higher income or wealth quantiles of our sample. This result is robust to different measures for savings or wealth. Only when we include correlated random effects, parameterized by household specific means of the independent variables in the spirit of Mundlak (1978), the significance disappears, but the household effects are not significant. We do not find evidence of a displacement effect; the expected
level of pension income is usually insignificant.

The paper is organized as follows. Section 2 briefly discusses the Dutch pension system. Section 3 presents the theoretical model, with derivations delegated to the appendix. Section 4 discusses the data and section 5 presents the results. Section 6 concludes.

2 Uncertainties in the Dutch pension system

The Dutch pension system consists of three pillars.\(^1\) The first pillar is the flat-rate state pension benefit, provided to all inhabitants aged 65 and above. In 2010, this amounted to €1057 for singles and €1470 for couples. The second pillar, the occupational pensions, are mandatory for most employees, and both employers and employees contribute to a (usually defined contribution) pension fund. Traditionally, the Dutch occupational pension system is one of the most developed in the world, with pension funds holding around 125% of Dutch GDP in wealth in 2008. Finally, the third pillar concerns private pension savings, such as annuities bought from banks or insurers or private saving accounts. The third pillar is much less developed in the Netherlands. This paper concerns pension benefit expectations from the first and second pillars together.

Bodie (1990) argues that employer pensions can serve as insurance against replacement rate inadequacy, deterioration of social security benefits, longevity risk, investment risk and inflation risk. However, this “insurance contract” is far from complete. The recent turmoil on financial markets after the subprime mortgage crisis in the US, followed by a global financial and economic crisis, and the aging of the population in many developed economies has led to revisions in the pension system for many countries. In the Netherlands, these include an increase in the statutory retirement age, from currently 65 to 66 in 2020 and another year in 2025, a reduction of nominal accrued pension rights, increasing the pension premium or not adjusting pension wealth to inflation. Especially for the nearly 50,000 employees that were hit by the reduction of nominal pension rights, it is clear that income after retirement may not be as certain as usually perceived. Pension income becomes more uncertain, and therefore could provoke changes in current savings. Hence, both social security and employer pension provisions are rather uncertain. Even apart from the risk of changes in the pension benefit formula or the welfare state, the level of post-retirement income is in itself hard to estimate for a currently working individual, as it depends amongst others on the future career.

Since our sample period (2006-2009) includes this period of turbulence, it is important that the expectations from the survey questions we use in this paper do reflect this. We have reason to believe that this is indeed the case. First, Van Santen, Alessie, and Kalwij (2011) reports that the (average) expected replacement rate, calculated from the same data as used in this study, has been decreasing over time. Likewise, the (average) variance of the replacement rate has increased over time. Second, Van der Wiel (2009) studies the effect of public debate on the expectations of the statutory retirement age in the Netherlands, using data from the monthly version of the Pension Barometer. The author finds that there is hardly any reaction from publicity on the expectations held by

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\(^1\)See Bovenberg and Gradus (2008) for an overview of the Dutch pension system and its reforms.
the high-educated or high-income groups, but a large effect on less-educated individuals and on those that stated not to read the newspapers. If, as is argued, the first group of individuals had initial expectations which are in accordance with the actual policy debate, that is, they already expected the retirement age to increase, while the second group had not, then the effect of publicity is to raise awareness for those that need it. 2

3 Model

The theory on income uncertainty we present here is not new in any respect. Our model is in the spirit of the two-period consumption model of Leland (1968), who analyzes precautionary savings if second-period (pension) income is unknown, but the individual does have a subjective distribution over future (retirement) income; savings are shown to increase with uncertainty. We consider a finite horizon discrete-time lifecycle model with uncertainty over pension income and mortality risk after retirement. The current period is denoted by period \( t \). The remaining lifetime is divided into two parts, the working stage and the retirement stage. The individual\(^3\) is assumed to retire in period \( K > t \), which is exogenously given. The per-period utility function is of the Constant Absolute Risk Aversion (CARA) type\(^4\) with coefficient of absolute risk aversion \( \alpha \). From today up to and including period \( K - 1 \) the individual receives (deterministic but possibly nonconstant) labor income \( y_\tau \) in period \( \tau \). Future consumption is discounted using the discount factor \( \beta = \frac{1}{1 + \rho} \) with \( \rho \) the rate of time preference. The interest factor is denoted by \( R = 1 + r \) with \( r \) the real interest rate. Previously accumulated assets are predetermined and denoted by \( A_{t-1} \). The instantaneous mortality rate in period \( i + 1 \) is \( m_{i+1} \). Hence, we can write the survival probability up to period \( \tau > K \) as \( a_\tau = \prod_{i=K}^{\tau} (1 - m_i) \). We assume that \( m_{L+1} = 1 \) (or \( a_{L+1} = 0 \)) such that \( L \) is the maximum attainable age. Furthermore, we assume survival up to period \( K \), so that \( m_{i-1} = 0 \) (or \( a_i = 1 \)) for \( i = t, ..., K \). During retirement, the individual receives a constant pension benefit \( y_K \), which is stochastic from today’s perspective. For analytical simplicity,\(^6\) we assume that \( y_K \) follows a normal distribution with expectation \( \mu \) and variance \( \sigma^2 \) to derive the expression for today’s consumption as a function of the expected value and variance of pension income.

The problem the currently young individual faces is to maximize lifecycle utility subject to the consolidated lifetime budget constraint. Formally, we write the problem as

\[^2\] A more critical note comes from Kotlikoff and Pakes (1988), who use the change in aggregate consumption to retrieve the change the expected value of lifetime income, and compare this to actual changes in aggregate income, to find that there is very low evidence for a consumption reaction to actual innovations in lifetime earnings. Yet, the authors themselves argue that the time series used are probably not the most suitable for analyzing this problem, but that panel data instead is needed for investigating this issue.

\[^3\] We abstract from intra-household decision making, and hence write "the individual" to mean the collective household. In the empirical application, we use data from the head of the household for estimating the model.

\[^4\] The choice of a CARA utility function is dominated by the possibility to obtain closed-form solutions, which is not possible with the class of Constant Relative Risk Aversion utility functions. This choice prevents buffer-stock saving behavior (Carroll, 1992), which needs decreasing absolute risk aversion.

\[^5\] If \( T \) is the time of death, then \( m_{i+1} = P(T = i + 1 | T > i) \).

\[^6\] Lam (1987) discusses more general distributions for income under CARA utility.
come is uncertain. The first term is the familiar expected present value of future income minus a precautionary savings term, where in our case uncertainty - risk aversion, in that the impatience effect is counterbalanced by increasing risk aversion: consumption decreases, a result that is also found under the well-documented 'certainty higher as the consumer becomes impatient. If the probability of survival increases (or doubling on net worth available at the beginning of retirement, \(A_{K-1}\). First, we solve the retirement stage and calculate the value of future utility streams, conditional on information available in period \(t\). We solve for today's consumption, \(c_t\), in three steps. First, we solve the retirement stage and calculate the value of future utility streams, conditional on net worth available at the beginning of retirement, \(A_{K-1}\). Second, we solve the problem for the working stage, and compute the value of utility conditional on leaving \(A_{K-1}\) available for future consumption. Finally, we choose \(A_{K-1}\) to maximize lifecycle utility. The Appendix shows the complete derivation of the model. Current consumption is given by

\[
\max_{c_t} \frac{1}{\alpha} \sum_{\tau=t}^{K-1} \beta^{\tau-t} \exp \left[ -\alpha c_t \right] - \frac{1}{\alpha} E_t \sum_{\tau=K}^{L} \beta^{\tau-t} a_\tau \exp \left[ -\alpha c_t \right] \tag{1a}
\]

s.t. \(\sum_{\tau=t}^{K-1} R^{\tau-t} c_\tau + \sum_{\tau=K}^{L} R^{\tau-t} c_\tau = RA_{t-1} + \sum_{\tau=t}^{K-1} R^{\tau-t} y_\tau + \sum_{\tau=K}^{L} R^{\tau-t} y_K \tag{1b}\)

where \(c_\tau\) is consumption in period \(\tau\) and \(E_t\) is the expectation operator conditional on information available in period \(t\). We solve for today's consumption, \(c_t\), in three steps. First, we solve the retirement stage and calculate the value of future utility streams, conditional on net worth available at the beginning of retirement, \(A_{K-1}\). Second, we solve the problem for the working stage, and compute the value of utility conditional on leaving \(A_{K-1}\) available for future consumption. Finally, we choose \(A_{K-1}\) to maximize lifecycle utility. The Appendix shows the complete derivation of the model. Current consumption is given by

\[
c_t = \frac{RA_{t-1} + \sum_{\tau=t}^{K-1} R^{\tau-t} y_\tau + \sum_{\tau=K}^{L} R^{\tau-t} y_K - \sum_{\tau=K}^{L} R^{\tau-t}\left( \frac{1}{\alpha} R^{1-\tau} \left( \frac{1}{2} \log(a_\tau) + \frac{1}{2} \alpha_\tau^2 \right) - \log(R\beta) \sum_{\tau=t}^{L} R^{\tau-t} (\tau-t) \right)}{\sum_{\tau=t}^{L} R^{\tau-t}} \tag{2}\)

This is the closed-form expression for today’s consumption when future pension income is uncertain. The first term is the familiar expected present value of future income streams (or permanent income). If \(r = \rho\) (or \(R = 1/\beta\)) we see that consumption equals permanent income minus a term related to two types of uncertainty: longevity and pension income uncertainty. Compared to the case without mortality risk, consumption is higher as the consumer becomes impatient. If the probability of survival increases (\(a_\tau \uparrow\)), consumption decreases, a result that is also found under the well-documented 'certainty equivalence' case\(^7\). Contrary to the quadratic utility function, we see an interaction with risk aversion, in that the impatience effect is counterbalanced by increasing risk aversion: a consumer with high mortality risk and high risk aversion would end up consuming as much as a consumer with low mortality risk and low risk aversion. The most interesting feature of the solution is the explicit relation of consumption to the variance in pension benefits, which is absent in the certainty equivalence case. Variability of pension income induces consumers to spend less, and hence to increase savings. Again we see the interaction with risk aversion, as a higher coefficient of risk aversion decreases the marginal effect of uncertainty. The consumption function we obtain is similar to the consumption function found in Caballero (1990, 1991): consumption equals permanent income minus a precautionary savings term, where in our case uncertainty

\[^7\] For the certainty equivalence case with \(r = \rho\), where \(U(c_t) = -\frac{1}{2} (c_t - c)^2\), we would get the solution \(c_t = \frac{RA_{t-1} + \sum_{\tau=t}^{K-1} R^{\tau-t} y_\tau + \sum_{\tau=K}^{L} R^{\tau-t} y_K - \sum_{\tau=K}^{L} R^{\tau-t}\left( 1 - \alpha_\tau \right) \frac{R^{1-\tau}}{\alpha_\tau}}{\sum_{\tau=t}^{L} R^{\tau-t}}\), with \(\frac{\partial c_t}{\partial c} < 0\).
stems from unknown lifespan and future pension income. Cantor (1985) has shown the same solution for current consumption when income is normally distributed.

We do not have consumption or expenditures data as our disposal, \(8\) but we do have data on savings. The closed-form solution for current savings, \(s_t = (R - 1) A_{t-1} + y_t - c_t\) can be written as

\[
s_t = -\frac{R^{t-L}}{\sum_{\tau=t}^L R^{t-\tau}} A_{t-1} + \frac{\sum_{\tau=t}^L R^{t-\tau} (y_t - \mu)}{\sum_{\tau=t}^L R^{t-\tau}} - \frac{\sum_{\tau=t}^{K-1} \Delta y_{\tau} \sum_{q=\tau}^{K-1} R^{t-q}}{\sum_{\tau=1}^L R^{t-\tau}}
\]

\[
+ \frac{\log (R\beta)}{\alpha} \frac{\sum_{\tau=t}^L R^{t-\tau} (\tau - t)}{\sum_{\tau=t}^L R^{t-\tau}} + \frac{\sum_{\tau=t}^K R^{t-\tau} \left( \frac{1}{\alpha} \log (a_{\tau}) + \frac{1}{2} \alpha \sigma^2 \right)}{\sum_{\tau=t}^L R^{t-\tau}} \tag{3}
\]

where \(\Delta\) is the backward difference operator. As consumption is smoothed (in fact, if \(r = \rho\) and expectations are static, the consumption path is flat), previously accumulated assets have a negative impact on savings, and if labor income increases with age (\(\Delta y_{\tau} > 0\)), savings will also be lower. Mortality decreases the saving rate, while uncertainty in pension benefits increases savings.

### 3.1 Model extensions

Although equations 2 and 3 are the subject of our empirical analysis in sections 4 and 5, we consider deviations from the assumptions implicitly or explicitly made so far. There are many deviations possible, such as \(9\) stochastic labor income (Caballero, 1991) or interest rates (Merton, 1973), endogenous labor supply and retirement (Feldstein, 1974), hyperbolic discounting (Laibson, 1998), habit formation (Angelini, 2009; Alessie and Teppa, 2010), home production (Aguiar and Hurst, 2005, 2007), bequest motives (Hurd, 1989), liquidity constraints (Mariger, 1987; Deaton, 1991) or differential mortality (Grossman, 1972; Attanasio and Hoynes, 2000; Knoef, Alessie, and Kalwij, 2009b), all of which will have an impact on current consumption. We consider liquidity constraints explicitly in the model presented above to guide our empirical approach.

We use the approach of Mariger (1987) to incorporate a liquidity constraint. We assume that \(R\beta = 1\) (or \(r = \rho\)) to ease computation. The approach of Mariger (1987) consists of the following steps:

1. Let today be denoted by time \(t\), and assume \(A_{t-1} > 0\). Assume that there exists a date \(v\) in the future, such that the liquidity constraint binds at this date for the first time, \(A_v = 0\).
2. Solve the unconstrained problem for periods \(t\) until \(v\).
3. The date \(v\) can be found by choosing the maximum date that minimizes period \(t\) consumption.

\(8\) Our data allows us to construct consumption as the residual part of (capital and labor) income minus savings, but we do not prefer to use this approach due to measurement error.

\(9\) This list is certainly not exhaustive, as is the list of references given here. See Attanasio and Weber (2010) for a recent review.
See Mariger (1987) for details and a proof. In our model, we distinguish between the cases where $v$ falls in the working period, $t \leq v \leq K - 1$ or in the retirement period, $K \leq v < L$. In the corner solution, where $v \geq L$, the liquidity constraint never binds, and hence we end up in the model of section 3. We make the assumption that the liquidity constraint always binds after binding for the first time. For the first case, Appendix B shows that current consumption is equal to

$$c_t = \frac{RA_{t-1} + \sum_{\tau=t}^{v} R^{l-\tau} y_{\tau}}{\sum_{\tau=t}^{v} R^{l-\tau}}$$

with $v$ given implicitly by

$$\left( \log (R) y_v + \frac{\partial y_v}{\partial v} \right) \sum_{\tau=t}^{v} R^{l-\tau} - \log (R) \sum_{\tau=t}^{v} R^{l-\tau} y_{\tau} = \log (R) RA_{t-1}$$

Furthermore, it is shown that $\frac{dv}{dA_{K-1}} > 0$ for an increasing but concave time path of labor income. This case immediately gives some useful insights. First, for households reaching the zero-assets bound before retirement, consumption is independent of expected pension income or the variance of pension income, as $A_{K-1} = 0$. Second, the income path and the level of current wealth are important for determining whether or not the constraint is binding, and hence, the effects of expected pension income and its variance depend on these factors. For those households with positive current wealth and an upward sloping income profile, the liquidity constraint will not bind before retirement. In this case, Appendix B shows that current consumption is equal to

$$c_t = \frac{RA_{t-1} + \sum_{\tau=t}^{K-1} R^{l-\tau} y_{\tau} + \sum_{\tau=K}^{v} R^{l-\tau} \mu - \sum_{\tau=K}^{v} R^{l-\tau} \left( \frac{1}{a} \log (a_{\tau}) + a\sigma^2 \right)}{\sum_{\tau=t}^{v} R^{l-\tau}}$$

$$= \frac{\sum_{\tau=K}^{v} R^{l-\tau} \log \left( \frac{H \sum_{\tau=K}^{v} R^{l-\tau} + (\log (R))^2 R^{l-\tau}}{H \sum_{\tau=K}^{v} R^{l-\tau}} \right)}{a \sum_{\tau=t}^{v} R^{l-\tau}}$$

with $v$ given implicitly by

$$\log (R) \log (a_v) \sum_{\tau=K}^{v} R^{l-\tau} - \frac{\partial a_v}{\partial v} \sum_{\tau=K}^{v} R^{l-\tau} \log (a_{\tau})$$

and $H$ depending on the structure of the survival function. In particular,

$$\frac{dv}{dA_{K-1}} > 0 \iff H \equiv \frac{\partial^2 a_v}{\partial v^2} - \frac{\partial a_v}{\partial v} \log (R) a_v - \left( \frac{\partial a_v}{\partial v} \right)^2 > 0$$

As the household enters retirement with positive assets, current consumption will depend on the expected value and variance of pension income, but the effect (i.e. $v$) differs between households depending on their income path and the survival function.
Both of these consumption functions assume positive current wealth. For those with zero wealth, current consumption will be either 4 or 5 with \( A_{t-1} = 0 \) or equal to current income, \( c_t = y_t \).

It is clear from these consumption functions that the effect of pension income on current consumption heavily depends on whether or not the liquidity constraint binds, and if so, when it binds. We do not have information on who is (credit) constrained or not, but instead use quantile regressions where the coefficients depend on the quantile of savings, current wealth and labor income; more details follow in section 4.1.

4 Data and methodology

For the empirical analysis, we use two sources of survey data: the DNB Household Survey (DHS) and the Pension Barometer (PB). Both surveys are administered by CentERData, Tilburg, The Netherlands, and have unique identifiers allowing us to merge the two data sets at the individual level. The respondents represent the Dutch population aged 16 and above. Both surveys are administered via the internet, and internet access is provided to those that do not have access themselves. The DHS has been running since 1993, and the data from 2009 are the most recent available. The DHS collects information on many socio-economic characteristics of the household, including a detailed breakdown of household income and wealth holdings, which can be used to construct measures of total assets, financial assets and housing assets; see Nyhus (1996) and Alessie, Hochguertel, and van Soest (2002) for an extended description.

The Pension Barometer survey is administered to a subset of respondents from the DHS. The survey started in 2006, and 2009 is the most recent survey year at our disposal. Among other questions, the PB elicits expectations of pension benefits. More specifically, the PB contains probabilistic survey questions of the type suggested by Dominitz and Manski (1997) and Manski (2004) that elicit the subjective distribution of the pension income replacement rate. Using the responses to these questions allows us to construct individual-specific measures of expected pension benefits and subjective uncertainty of pension income, by calculating the first and second moment of the distribution.

The exact wording of these questions is as follows.

**Question 1** At which age do you think you can retire at the earliest, following your employer’s pension scheme?

The answer to this question, say age \( Y \), is used in the subsequent question:

**Question 2** If you would retire at age \( Y \), please think about your total net pension income including social security, compared to your current total net wage or salary. What do you think is the probability that the purchasing power of your total net pension income in the year following your retirement will be:

\[ \begin{align*}
    a) & \text{ more than 100\% of your current net wage? } ... \% \\
    b) & \text{ less than 100\% of your current net wage? } ... \% 
\end{align*} \]
A second set of questions is asked for the earliest retirement age, which we use as a robustness check. The probabilities answered by the respondent define the subjective cumulative density function of pension income, and we compute the expected pension income replacement rate as the first moment and the variance as a measure of replacement rate uncertainty. The determinants of the expected value and variance of the replacement rate have been investigated in Van Santen, Alessie, and Kalwij (2011), and show that the expected benefit is U-shaped in age with a minimum at 48, while uncertainty is inverted U-shaped with age with maximum at age 36. Educational attainment depresses the expectation, and increases uncertainty. The uncertainty was higher in 2007 and 2008, compared to 2005 and 2006, possibly due to the financial crisis. Similarly, the expected replacement rate was lower in these years.

Mortality is the second source of uncertainty in our model. For the empirical specification, we rely on questions from the DHS which ask respondents to provide subjective survival probabilities for certain target ages.

**Question 3** How likely is it that you will attain (at least) the age of 65 / 75 / 80 / 85 / 90 / 95 / 100? Please indicate your answer on a scale of 0 thru 10, where 0 means ‘no chance at all’ and 10 means ‘absolutely certain’.

We fit a two-parameter Gompertz distribution to the answers to estimate yearly survival probabilities for each respondent, in line with the theoretical model. The cumulative Gompertz distribution function reads (Willemse and Koppelaar, 2000)

\[
F(t) = P(T \leq t) = 1 - \exp \left[ -\frac{\lambda}{b} - \exp \left[ \frac{t - \lambda}{b} \right] \right]
\]

where \( T \) is the time of death and \( t \) is current age. We estimate the individual-specific parameters \( b \) and \( \lambda \) using a nonlinear least squares procedure. Furthermore, we compute remaining life expectancy as

\[
E(T|T \geq t) \approx \sum_{\tau=t}^{\infty} \tau [P(T \geq \tau|T \geq t) - P(T \geq \tau + 1|T \geq t)] = \sum_{\tau=t}^{\infty} P(T \geq \tau|T \geq t) = \sum_{\tau=t}^{\infty} \frac{\exp\left[-\exp\left[\frac{\tau - 1}{\tau}ight]\right]}{\exp\left[-\exp\left[\frac{\tau}{\tau}ight]\right]}
\]

Respondents answer at most three, but mostly two questions, depending on their actual age. Respondents younger than 55 provide survival probabilities up to age 65 and 75, while people aged 80-85 provide survival probabilities for age 95 and 100, with gradual transition for the in-between ages. This ensures that respondents do not have to answer survival probabilities up to ages lower than their actual age nor ages in the near (5 years) future.
and compute \( L = E(T|T \geq t) + t \). Since we lose many observations due to missing observations, we impute the missing survival probabilities and life expectancies by age and gender from Statistics Netherlands, CBS (2011b).

Our dependent variable is the level of savings. The measure we used is based on the following questions:

**Question 4** Did your household put any money aside in the past 12 months?

Respondents answer yes or no. For those that answer yes, the follow-up question reads

**Question 5** About how much money has your household put aside in the past 12 months?

1. less than €1,500
2. between €1,500 and €5,000
3. between €5,000 and €12,500
4. between €12,500 and €20,000
5. between €20,000 and €37,500
6. between €37,500 and €75,000
7. €75,000 or more
8. don’t know

We take class midpoints (for example €750 for answer 1) as our measure of saving, and impute €75,000 for answer 7. One problem associated with this question is that reported saving cannot be negative, while some individuals might actually dissave. We use additional information from the following question:

**Question 6** Over the past 12 months, would you say the expenditures of your household were higher than the income of the household, about equal to the income of the household, or lower than the income of the household?

For those that answer question 4 negatively, as well as indicate that expenditures and income are about equal, it is clear that saving is zero. For those that report to spend more than their income, we compute the change in net worth\(^{11}\) (if this is indeed negative) as savings measure. Finally, for those that report no savings, but claim that expenditures were lower than household income, we also compute the change in net worth if this change is positive. For robustness, we also consider an alternative savings measure by imputing the (cross-sectional) median change in net worth for each year separately, for those that report no saving and higher expenditures than income and for those that report no saving and lower expenditures than income. This alternative savings measure can take on 16 values at most (7 from question 5, 1 from the no-savers and 4 years \( \times \) 2 average

---

\(^{11}\) The DNB Household Survey asks detailed information on many assets and liabilities. We aggregate assets and liabilities for each household, and compute the difference between them as our measure of net worth. For robustness, we also consider a narrower measure of household wealth. For this narrower measure, we have picked the most liquid categories for assets (checking accounts, savings arrangements, linked to a Postbank account, deposit books, savings or deposit accounts, savings certificates) net of the most liquid categories of liabilities (private loans and extended lines of credit) and then taken first differences. For both measures, we have deleted extreme values in order to avoid including outliers in our imputations.
changes in net worth). We prefer these savings measures over the change in net worth for the whole sample, as the assets and liabilities reported by the respondents suffer from measurement error.

The DHS gives additional information on household characteristics, most notably the age of the household members, the size of the household and household income. Table 1 below shows the sample statistics of the variables used in this study.

Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savingsa</td>
<td>3612.944</td>
<td>10148.782</td>
<td>1033</td>
</tr>
<tr>
<td>Savingsb</td>
<td>3290.865</td>
<td>8769.614</td>
<td>1033</td>
</tr>
<tr>
<td>Savingsc</td>
<td>9774.662</td>
<td>91164.673</td>
<td>1033</td>
</tr>
<tr>
<td>Real income</td>
<td>33624.895</td>
<td>16980.239</td>
<td>1033</td>
</tr>
<tr>
<td>Net worth</td>
<td>168496.137</td>
<td>187854.504</td>
<td>1033</td>
</tr>
<tr>
<td>Liquid wealth</td>
<td>46656.22</td>
<td>82248.676</td>
<td>1033</td>
</tr>
<tr>
<td>Expected retirement age</td>
<td>66.525</td>
<td>2.334</td>
<td>1033</td>
</tr>
<tr>
<td>Expected replacement rate</td>
<td>0.815</td>
<td>0.175</td>
<td>1033</td>
</tr>
<tr>
<td>Variance replacement rate</td>
<td>0.362</td>
<td>0.244</td>
<td>1033</td>
</tr>
<tr>
<td>Life expectancy</td>
<td>82.125</td>
<td>7.28</td>
<td>1033</td>
</tr>
<tr>
<td>Age</td>
<td>45.833</td>
<td>9.332</td>
<td>1033</td>
</tr>
<tr>
<td># Children</td>
<td>0.933</td>
<td>1.149</td>
<td>1033</td>
</tr>
<tr>
<td>Home owner</td>
<td>0.761</td>
<td>0.427</td>
<td>1033</td>
</tr>
<tr>
<td>Education</td>
<td>0.494</td>
<td>0.5</td>
<td>1033</td>
</tr>
<tr>
<td>Female</td>
<td>0.196</td>
<td>0.397</td>
<td>1033</td>
</tr>
<tr>
<td>Work experience</td>
<td>24.845</td>
<td>11.814</td>
<td>964</td>
</tr>
</tbody>
</table>

a Savings measured from questions 4, 5 and 6.
b Savings measured by imputing cross-sectional median increase (decrease) in net worth if household reports not to save and to spend less (more) than their income.
c Savings measured from household-level change in net worth.

4.1 Estimation strategy

Since we have panel data, we need to index the variables correctly. We use \( t \) to index the age of the head of the household; obviously, \( t \) can also be read as the index for year. Households are indexed by \( i \). Savings of household \( i \) at age \( t \), writing out the summations where possible and setting \( r = \rho \), equals

\[
s_{it} = -\frac{(R - 1) R_{t-i} - L_i}{R - R_{t-i} - L_i} A_{it-1} + \frac{R_{t-i} - K_{it+1} - R_{t-i} - L_i}{R - R_{t-i} - L_i} y_{it} - \frac{\sum_{\tau=t-i+1}^{K_{it}-1} \Delta y_{i\tau} \left( R_{t-i} - \tau + 1 - R_{t-i} - K_{it} + 1 \right)}{R - R_{t-i} - L_i} \\
- \frac{R_{t-i} - K_{it+1} - R_{t-i} - L_i}{R - R_{t-i} - L_i} \mu_{it} + \frac{R_{t-i} - K_{it+1} - R_{t-i} - L_i}{2 (R - R_{t-i} - L_i)} a_{it}^2 + \frac{(R - 1) \sum_{\tau=K_{it}}^{L_i} R_{t-i} - \tau - 1}{R - R_{t-i} - L_i} \log (a_{it}) + u_{it}
\]

Note that all terms (except for \( \Delta y_{i\tau} \)) in equation 9 are known from the survey questions described above or background characteristics. In particular, \( t_i \) is the age of the head of
the household, $K_{it}$ is the (possibly time-varying) retirement age from question 1, $A_{it-1}$ are previous-period liquid assets, $\mu_{it}$ is the elicited expected replacement rate multiplied by income $y_{it}$ and $\sigma_{it}^2$ the variance of the replacement rate multiplied by income squared. The coefficient of risk aversion, $a$, is not available at the household level; we use $a = 5$ for computing the terms, although the value is not important in the regression analysis as these constants enter linearly. For $R$, we use $R = 1.03$ as the baseline value, but as this enters nonlinearly, we experiment with extreme values, $R = 1.001$ and $R = 1.15$ as well to check the robustness of our results.

As for the third term on the right hand side, we use a fixed effects model to predict future labor income, detailed in Appendix C. All amounts are measured in 2006 Euro’s, deflated using the CPI index provided by Statistics Netherlands (CBS, 2011a).

Before estimating, we need to take a possible multicollinearity problem into account. In particular, the present value of expected pension benefit receipts and the present value of future labor income, detailed in Appendix C. All amounts are measured in 2006 Euro’s, to check the robustness of our results.

Before estimating, we need to take a possible multicollinearity problem into account. In particular, the present value of expected pension benefit receipts and the present value of uncertainty in pension benefits are highly correlated with current income, by construction. Therefore, we divide all terms by $\frac{R_{pi}^{i-1} - R_{pl}^{i-1}}{R_{pi}^{i-1} - R_{pl}^{i-1}}y_{it}$.

Potentially more serious is measurement error in our income and wealth variables. Therefore, we take the first term on the right-hand-side, featuring predetermined assets $A_{it-1}$, and the third term, featuring $\Delta u_{it}$, to the left-hand-side. Furthermore, in order not to have income on both sides of the equation, we use the standard deviation of the replacement rate, $\sigma_{it}$ from question 2 as measure of uncertainty instead of the variance $\sigma_{it}^2$. Note that if we would take the variance, we would have the saving rate, $s/y$ on the left hand side, and $\sigma^2 \times y$ on the right hand side. If income $y$ is measured with error, the impact of this error is unclear a priori and could bias our results dramatically.

In the empirical application, we control for observable household characteristics, $X_{it}$, such as education and the size of the household, and time fixed effects. The final equation to be estimated reads

$$\begin{align*}
\frac{(R - R_{pi}^{i-1}) \frac{s_a}{y_{it}}}{R_{pi}^{i-1} - R_{pl}^{i-1}} + & \frac{(R - 1) R_{pi}^{i-1} - R_{pl}^{i-1}}{R_{pi}^{i-1} - R_{pl}^{i-1}} \frac{A_{it-1}}{y_{it}} + \frac{\sum_{\tau=K_{it}}^{K_{it}+1} \frac{\Delta u_{it}}{y_{it}} (R_{pi}^{\tau} - R_{pl}^{\tau} - R_{pi}^{i-1} - R_{pl}^{i-1})}{R_{pi}^{i-1} - R_{pl}^{i-1}} \\
= & \beta_{\mu} \frac{\mu_{it}}{y_{it}} + \beta_c \frac{\Delta c_{it}}{2y_{it}} + \beta_{\text{Survival}} \left( \frac{(R - 1) \sum_{\tau=K_{it}}^{K_{it}+1} R_{pi}^{\tau} - \frac{1}{a} \log (u_{it})}{R_{pi}^{K_{it}+1} - R_{pl}^{i-1}} \right) + X_{it} \gamma + u_{it}\end{align*}$$

We estimate equation 10 using quantile regressions (Koenker and Bassett, 1978; Koenker and Hallock, 2001), as the coefficients of interest, $\beta_{\mu}$, $\beta_c$ and $\beta_{\text{Survival}}$ depend on the level of wealth and labor income, as shown in section 3.1. Table 2 shows that all components of the dependent variable, the saving rate, the wealth-to-income ratio and the growth rate of labor income all increase over the quantiles of the dependent variable, such that the results are not driven by any of these components individually. Of course, quantile regressions have the additional robustness advantage to outliers, compared to linear regression as well.
Table 2: Who is in which quantile?

<table>
<thead>
<tr>
<th>Quartile</th>
<th>Statistic</th>
<th>Dep var</th>
<th>$s/y$</th>
<th>$A/y$</th>
<th>$\Delta y/y$</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1 ($N = 259$)</td>
<td>Mean</td>
<td>-0.74</td>
<td>-0.06</td>
<td>3.25</td>
<td>-0.16</td>
<td>44.69</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>1.54</td>
<td>0.49</td>
<td>2.90</td>
<td>0.17</td>
<td>8.77</td>
</tr>
<tr>
<td>Q2 ($N = 258$)</td>
<td>Mean</td>
<td>0.42</td>
<td>0.10</td>
<td>5.28</td>
<td>-0.02</td>
<td>50.20</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.16</td>
<td>0.10</td>
<td>4.34</td>
<td>0.14</td>
<td>8.50</td>
</tr>
<tr>
<td>Q3 ($N = 258$)</td>
<td>Mean</td>
<td>1.08</td>
<td>0.13</td>
<td>5.38</td>
<td>0.14</td>
<td>46.52</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.24</td>
<td>0.12</td>
<td>5.83</td>
<td>0.18</td>
<td>9.05</td>
</tr>
<tr>
<td>Q4 ($N = 258$)</td>
<td>Mean</td>
<td>3.96</td>
<td>0.26</td>
<td>6.14</td>
<td>0.38</td>
<td>41.92</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>4.01</td>
<td>0.43</td>
<td>9.09</td>
<td>0.39</td>
<td>9.05</td>
</tr>
<tr>
<td>Total ($N = 1033$)</td>
<td>Mean</td>
<td>1.18</td>
<td>0.11</td>
<td>5.01</td>
<td>0.09</td>
<td>45.83</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>2.76</td>
<td>0.35</td>
<td>6.08</td>
<td>0.31</td>
<td>9.33</td>
</tr>
</tbody>
</table>

5 Results

Table 3 shows the results of estimating equation 10 for the 10%, 25%, 50%, 75% and 90% quantiles. We use the abbreviations $\mu$, $\sigma$ and Survival to refer to the first, second and third right-hand-side variable, respectively. Clustered standard errors based on 499 bootstrap replications$^{12}$ are shown in parentheses. We also show the bias-corrected confidence intervals for each parameter estimate, which proves useful for checking whether or not the theoretical predictions show up in our data.

We start with a simple version of equation 10, that does not control for background characteristics or survival probabilities, in panel A of table 3. We see that for the lower quantiles of the distribution of our dependent variable, which consists of households with low levels of wealth and low incomes, expected pension benefits nor uncertainty in benefits influence savings. From table 2, we know that these households are the most likely to be credit constrained, and hence may spend most of their income on (durable) consumption, although we cannot test for this. For the wealthier, higher-income quartiles, we see that the prediction of the lifecycle model, that more uncertainty in pension income increases savings, shows up in the data as well. The lower bound of the confidence interval is above zero; uncertainty induces additional savings. The expected level of pension benefits leads to higher saving as well, which is the opposite prediction from the lifecycle model. Although we cannot test this, we might be dealing with a ‘taste for saving’ effect, in that those with a preference for saving select themselves into jobs with higher occupational pensions or put more money aside themselves (Cagan, 1965). Alternatively, there could be a motive for leaving bequests (Hurd, 1989), although this is less likely given that the sample consists of currently young household heads. The same pattern emerges in Panel B, where we include the variable measuring survival probabilities and time fixed effects. The variance of pension income increases saving, as does the level of expected pension income. For the higher quantiles, except for the 90% quantile for which the parameter is imprecisely estimated, survival probabilities significantly increase savings, in line with the theoretical model. For the lower quantiles, survival probabilities have a negative or zero effect on savings, most likely due to the lower age of

$^{12}$We have experimented with different number of replications, but the results do not differ by changing the number of bootstraps.
these households. The time effects are generally insignificant, with the exception of the higher quantiles in 2008, in which savings were higher.

In panel C, we additionally control for work experience, in years, educational attainment, which is a binary variable equal to zero for low-educated and 1 for high-educated household heads, the number of children living in the household, home ownership, which is a binary variable equal to zero for tenants and 1 for home owners, and gender, equal to 1 for females. From these, we see that only the number of children is significantly negative for household savings. The results of our main variables of interest are largely unchanged compared to the results in Panel B, except that Survival is never significantly different from zero.

In Figure 1, we plot the quantile regression estimates for $\mu$, $\sigma$ and Survival from panel B with the bias-corrected confidence bounds and the OLS-slope coefficient for comparison. We immediately see that the quantile regression technique, in this case executed for all 19 (5%) quintiles, uncovers much more heterogeneity than the single-valued OLS coefficient. For all three variables, the slope coefficients are generally upward sloping over the quantiles, while the confidence intervals increase substantially in the lowest and highest quantiles, as these quantiles contain the persons with high debt and net worth, respectively.
Table 3: Results equation 10, quantile regressions

<table>
<thead>
<tr>
<th>Variables</th>
<th>Quantiles</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel A: No mortality risk, no time effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>-0.128 (0.479)</td>
<td>-0.310 (0.296)</td>
<td>0.0978 (0.313)</td>
<td>0.983 (0.556)</td>
<td>3.041 (1.255)</td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>-0.933 (1.107)</td>
<td>-0.818 (0.340)</td>
<td>-0.467 (0.735)</td>
<td>0.0663 (2.110)</td>
<td>-0.039 (5.422)</td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>-0.357 (0.166)</td>
<td>-0.161 (0.160)</td>
<td>0.407 (0.218)</td>
<td>1.619 (0.405)</td>
<td>3.932 (0.882)</td>
<td></td>
</tr>
<tr>
<td>Survival</td>
<td>-1.376 (0.104)</td>
<td>-0.434 (0.225)</td>
<td>0.0690 (0.997)</td>
<td>0.304 (2.425)</td>
<td>2.223 (5.633)</td>
<td></td>
</tr>
<tr>
<td>Dummy 2007</td>
<td>0.0669 (0.203)</td>
<td>0.0755 (0.104)</td>
<td>0.181 (0.114)</td>
<td>0.204 (0.168)</td>
<td>0.222 (0.384)</td>
<td></td>
</tr>
<tr>
<td>Dummy 2008</td>
<td>0.156 (0.212)</td>
<td>0.0799 (0.095)</td>
<td>0.0723 (0.095)</td>
<td>0.119 (0.207)</td>
<td>0.409 (0.523)</td>
<td></td>
</tr>
<tr>
<td>Dummy 2009</td>
<td>0.146 (0.191)</td>
<td>0.0818 (0.071)</td>
<td>0.0829 (0.096)</td>
<td>0.106 (0.176)</td>
<td>0.423 (0.516)</td>
<td></td>
</tr>
<tr>
<td>Panel B: Mortality risk, time effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>-0.146 (0.479)</td>
<td>-0.310 (0.296)</td>
<td>0.0978 (0.313)</td>
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<td>0.423 (0.516)</td>
<td></td>
</tr>
<tr>
<td>Panel C: Additional control variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
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<td>-0.310 (0.296)</td>
<td>0.0978 (0.313)</td>
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<td>0.0829 (0.096)</td>
<td>0.106 (0.176)</td>
<td>0.423 (0.516)</td>
<td></td>
</tr>
<tr>
<td>Experience</td>
<td>0.0143 (0.0067)</td>
<td>0.0000 (0.0067)</td>
<td>-0.0142 (0.00487)</td>
<td>-0.0283 (0.0079)</td>
<td>-0.0493 (0.0131)</td>
<td></td>
</tr>
<tr>
<td>High educated</td>
<td>0.0503 (0.134)</td>
<td>0.0140 (0.0083)</td>
<td>-0.0201 (0.0886)</td>
<td>-0.423 (0.136)</td>
<td>-0.918 (0.396)</td>
<td></td>
</tr>
<tr>
<td>Home owner</td>
<td>-0.241 (0.260)</td>
<td>-0.122 (0.183)</td>
<td>-0.361 (0.1068)</td>
<td>-0.684 (0.150)</td>
<td>-1.642 (0.130)</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>0.211 (0.185)</td>
<td>0.179 (0.116)</td>
<td>0.225 (0.127)</td>
<td>0.215 (0.228)</td>
<td>1.016 (1.081)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.316 (0.604)</td>
<td>-0.118 (0.369)</td>
<td>-0.0412 (0.455)</td>
<td>-0.168 (0.734)</td>
<td>-0.316 (3.585)</td>
<td></td>
</tr>
<tr>
<td>Cluster-bootstrapped standard errors in parentheses, 499 replications. Bias-corrected 95% confidence intervals plotted below estimate and standard error.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Quantile regression estimates

(a) $\mu$

(b) $\sigma$

(c) Survival
5.1 Sensitivity analysis

We consider a number of robustness checks for our results. In table 4, we use two alternative savings measures namely the cross-sectional median increase (decrease) in net worth for those households that do not save, but have expenditures lower (higher) than income in panel A. In panel B, we use the change in net worth as a direct measure of savings. In panel C, we use the narrower measure of wealth, in which we include only liquid assets and liabilities, as mentioned in section 4.

We see that the point estimates of $\mu$, $\sigma^2$ and Survival are similar in panel A compared to the main results in panel B of table 3, but the confidence intervals are slightly wider. In panel B, the point estimates are bigger in absolute sense, but again the same pattern emerges. The narrower measure of wealth also does not affect the main results.

Table 4: Results equation 10, alternative savings or wealth measures

<table>
<thead>
<tr>
<th>Variables</th>
<th>Quantiles</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: 1st alternative savings measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0737 (0.506)</td>
<td>-0.400 (0.338)</td>
<td>0.171 (0.324)</td>
<td>0.834 (0.517)</td>
<td>2.883 (1.217)</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.951 (0.443)</td>
<td>0.729 (0.392)</td>
<td>0.343 (0.216)</td>
<td>1.309 (0.391)</td>
<td>3.506 (0.928)</td>
<td></td>
</tr>
<tr>
<td>Survival</td>
<td>-2.280 (0.430)</td>
<td>0.588 (0.378)</td>
<td>1.182 (0.562)</td>
<td>1.750 (0.831)</td>
<td>3.468 (1.940)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.106 (0.186)</td>
<td>0.038 (0.321)</td>
<td>0.390 (0.305)</td>
<td>0.336 (0.513)</td>
<td>0.549 (1.123)</td>
<td></td>
</tr>
<tr>
<td>Panel B: 2nd alternative savings measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>-2.457 (3.346)</td>
<td>-0.489 (1.004)</td>
<td>0.341 (0.650)</td>
<td>3.489 (1.440)</td>
<td>4.275 (3.541)</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-4.122 (3.020)</td>
<td>0.729 (0.392)</td>
<td>0.343 (0.216)</td>
<td>1.309 (0.391)</td>
<td>3.506 (0.928)</td>
<td></td>
</tr>
<tr>
<td>Survival</td>
<td>-8.474 (1.079)</td>
<td>-3.435 (1.453)</td>
<td>0.064 (1.746)</td>
<td>0.388 (3.059)</td>
<td>6.943 (9.731)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.447 (3.852)</td>
<td>-0.204 (1.991)</td>
<td>0.406 (0.634)</td>
<td>0.162 (1.581)</td>
<td>7.402 (3.805)</td>
<td></td>
</tr>
<tr>
<td>Panel C: Narrower wealth measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>-0.434 (0.405)</td>
<td>-0.422 (0.235)</td>
<td>0.001 (0.305)</td>
<td>0.504 (0.516)</td>
<td>2.259 (1.205)</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.353 (0.284)</td>
<td>-0.128 (0.163)</td>
<td>0.511 (0.204)</td>
<td>1.201 (0.471)</td>
<td>3.497 (0.799)</td>
<td></td>
</tr>
<tr>
<td>Survival</td>
<td>-0.897 (0.198)</td>
<td>-0.448 (0.185)</td>
<td>0.080 (0.842)</td>
<td>0.374 (2.045)</td>
<td>1.698 (4.797)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.447 (3.852)</td>
<td>-0.204 (1.991)</td>
<td>0.406 (0.634)</td>
<td>0.162 (1.581)</td>
<td>7.402 (3.805)</td>
<td></td>
</tr>
</tbody>
</table>

Cluster-bootstrapped standard errors in parentheses, 499 replications.

1. Panel A: 1st alternative savings measures savings as imputing the annual, cross-sectional median increase in net worth for those that report not to save and to spend less (more) than their income; see text just below question 6.
2. Panel B: 2nd alternative savings measures savings as the change in net worth, $s_{it} = \Delta A_{it}$.
3. Panel C: Narrower assets measure includes only liquid assets and liabilities to construct $A_{it-1}$; see footnote 11.

Table 5 shows the results of changing the interest factor $R$, as this enters nonlinearly in equation 10. Panel A assumes an interest factor of 1.001, or an interest rate of $r = 0.1\%$, while panel B assumes that interest rates are extremely high at 15%. The estimates of precautionary savings remain unchanged in panel A, but are much larger in absolute value in panel B. Still, qualitatively the same results hold: for the higher quantiles, uncertainty increases savings. The survival variable is significantly positive at the highest two
Finally, we experiment with ‘random effects’ quantile regressions, where the random household effect $\alpha_i$ is parameterized as the mean of the independent variables, following Mundlak (1978). Table 6 shows the results of estimating equation 10 with random effects (panel A) and with additional control variables (panel B). The regressors Education, Home owner and Female are constant over time for each household, and are therefore omitted from the random effect specification to prevent perfect collinearity. In panel A, the point estimates for $\mu$ are generally lower, but are never significantly different from zero. The point estimates for $\sigma^2$ are supporting the lifecycle model, but the parameters are imprecisely estimated, and hence we cannot claim precautionary savings to influence savings. The random effects are insignificant. In panel B, we see that a similar pattern arises; the point estimates are comparable to our main results in table 3, but the estimates are less precise. The expected pension benefits, the uncertainty of pension income and the risk of longevity are no longer significant. We should be cautious with interpreting these results however, as the parameterized random effects are correlated with the independent variables, inflating the standard errors. Moreover, the bottom of table 6 shows that the random effects are not significant in either specification.
Table 6: Results equation 10, random effects quantile regression

<table>
<thead>
<tr>
<th>Variables</th>
<th>Quantiles</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>-0.533 (0.758)</td>
<td>-0.229 (0.441)</td>
<td>-0.184 (0.466)</td>
<td>0.207 (0.771)</td>
<td>0.110 (1.955)</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.216 (0.514)</td>
<td>-0.260 (0.301)</td>
<td>0.0291 (0.311)</td>
<td>0.537 (0.606)</td>
<td>1.102 (1.639)</td>
<td></td>
</tr>
<tr>
<td>Survival</td>
<td>-0.617 (1.559)</td>
<td>-0.432 (0.918)</td>
<td>1.978 (0.949)</td>
<td>2.124 (1.364)</td>
<td>5.641 (4.427)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.189 (0.814)</td>
<td>-0.278 (0.418)</td>
<td>-0.564 (0.419)</td>
<td>-0.655 (0.673)</td>
<td>-2.551 (1.352)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1033</td>
<td>1033</td>
<td>1033</td>
<td>1033</td>
<td>1033</td>
<td></td>
</tr>
<tr>
<td>Households</td>
<td>560</td>
<td>560</td>
<td>560</td>
<td>560</td>
<td>560</td>
<td></td>
</tr>
<tr>
<td>Individual effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Same model as Panel B of Table 3

| $\mu$     | 0.291 (0.808) | -0.261 (0.400) | -0.505 (1.047) | 0.681 (0.761) | 0.046 (1.810) |
| $\sigma$  | -1.364 (1.910) | -1.169 (0.427) | -0.271 (1.427) | -1.118 (1.956) | -3.130 (3.559) |
| Survival  | 0.321 (1.395) | 0.707 (0.954) | 1.930 (1.539) | 2.139 (1.771) | 9.080 (5.669) |
| Constant  | -1.559 (4.579) | -0.756 (3.062) | 0.609 (4.281) | -0.657 (5.297) | 0.802 (6.190) |
| Observations | 964 | 964 | 964 | 964 | 964 |
| Households | 529 | 529 | 529 | 529 | 529 |
| Individual effects | Yes | Yes | Yes | Yes | Yes |
| Time effects | Yes | Yes | Yes | Yes | Yes |

Panel B: Same model as Panel C of Table 3

1 Random effects are parameterized by household-specific averages of all independent variables (Mundlak, 1978).

$p$-value of $H_0: \alpha_i = 0 \forall i = 0.392$ (Panel A), 0.118 (Panel B).

Cluster-bootstrapped standard errors in parentheses, 499 replications.

Bias-corrected 95% confidence intervals plotted below estimate and standard error.
6 Conclusion

This paper shows evidence of precautionary retirement savings, as the level of pension income is uncertain for the currently young population. To guide our empirical approach, we construct and estimate a lifecycle consumption model in which pension benefits and lifespan are uncertain. Our theoretical model features rational individuals that work and consume or save before retirement. After retirement, they receive pension income, the level of which is uncertain from today’s perspective. This model shows that savings decrease with the expected value of pension income, and increase with its variance; longer lifespan should increase savings. We also present the solution to the model if liquidity constraints are presents; the same predictions follow, but the important difference is that the effect of uncertain pension income or mortality risk on savings now depend on the level of wealth and labor income.

We estimate the savings equation implied by the model using panel data for Dutch households. We use the answers from subjective probabilistic survey questions to compute expected pension income and the variance of pension income as a measure for uncertainty, as well as mortality risk. Our quantile regression results show that, for the higher wealth and income quantiles, savings increase with the uncertainty in pension income and increase with longevity risk, as predicted by the theory. The results are robust to different measures for savings, household wealth and including control variables.

For policy purposes, the paper shows that Dutch employees do prepare for retirement, as indicated by the expected sign for pension uncertainty and mortality risk. However, there is no evidence of a displacement effect, in the sense that households do not save more if the expected pension benefit decreases. Given the current developments in the private and public pension provisions in the Netherlands, this could lead to undersaving for retirement. Furthermore, the risk of undersaving is already present for the lower wealth and income groups, as these households have a negative estimated displacement effect, and do not increase savings with increased pension risk or longevity risk. These groups especially might be targeted by fiscal policies regarding retirement savings. In future research, we hope to allow for endogenous labor supply and retirement, the importance of which is stressed by among others Feldstein (1974). To implement this approach empirically, we should consider either cohorts near or in retirement, for which the entire work career can be elicited, or use younger households career planning prospects to estimate the career path based on individual expectations.
A Derivation of equation 2

The problem the individual faces is to maximize lifecycle utility subject to the consolidated lifetime budget constraint. Formally, we write the problem as

$$\max_{c_t} \sum_{\tau=t}^{K-1} \beta^{\tau-l} U(c_{\tau}) + E_t \sum_{\tau=K}^{L} \beta^{\tau-l} a_{\tau} U(c_{\tau})$$

s.t. \( \sum_{\tau=t}^{K-1} R^{l-\tau} c_{\tau} + \sum_{\tau=K}^{L} R^{l-\tau} c_{\tau} = RA_{t-1} + \sum_{\tau=K}^{K-1} R^{l-\tau} y_{\tau} + \sum_{\tau=K}^{L} R^{l-\tau} y_{K} \) \hspace{1cm} (11b)

where \( c_{\tau} \) is consumption in period \( \tau \) and \( E_t \) is the expectation operator conditional on information available in period \( t \). We solve for today’s consumption, \( c_t \), in three steps. First, we solve the retirement stage and calculate the value of future utility streams, conditional on net worth available at the beginning of retirement, \( A_{K-1} \). Second, we solve the problem for the working stage, and compute the value of utility conditional on leaving \( A_{K-1} \) available for future consumption. Finally, we choose \( A_{K-1} \) to maximize lifecycle utility.

A.1 Consumption during retirement

During retirement, when the uncertainty over pension income has resolved, the problem can be written as

$$\max_{c_{\tau}} \sum_{\tau=k}^{L} \beta^{\tau-k} a_{\tau} U(c_{\tau})$$

s.t. \( \sum_{\tau=k}^{L} R^{\tau-k} c_{\tau} = RA_{K-1} + \sum_{\tau=k}^{L} R^{\tau-k} y_{K} \) \hspace{1cm} (12b)

\( A_{\tau} = RA_{\tau-1} + y_{\tau} - c_{\tau} \) \hspace{1cm} (12c)

\( A_{L} = 0 \) \hspace{1cm} (12d)

For \( (K \leq \tau \leq L) \), the solutions are given by\(^{13}\)

$$c_{\tau} = (\tau - K) \frac{\log (R \beta)}{\alpha} + \frac{\log (a_{\tau})}{\alpha} + c_K$$

\[ c_K = \frac{\log (R \beta)}{\alpha \sum_{\tau=K}^{L} R^{\tau-k}} \sum_{\tau=K}^{L} R^{\tau-k} (\tau - K) - \frac{1}{\alpha \sum_{\tau=K}^{L} R^{\tau-k}} \sum_{\tau=K}^{L} R^{\tau-k} \log (a_{\tau}) \]

\[ + \frac{R^{L-K+1}}{\sum_{\tau=K}^{L} R^{\tau-k}} A_{K-1} + y_{K} \]  

\(^{13}\)We use the first-order condition, the consolidated retirement budget constraint (26) and the terminal condition (28).
For convenience we define
\[
G = -\frac{\log (R\beta)}{\alpha} \sum_{\tau = K}^{L} R^{L-\tau} (\tau - K)
\]
\[
\Pi = -\frac{1}{\alpha} \sum_{\tau = K}^{L} R^{L-\tau} \log (a_{\tau})
\]

So that we can write for \( K \leq q \leq L \)
\[
c_{q} = (q - K) \frac{\log (R\beta)}{\alpha} + G + \Pi + \frac{R^{L-K+1}}{\sum_{\tau = K}^{L} R^{L-\tau}} A_{K-1} + y_{K}
\]

The value of utility after retirement can then be written as
\[
\sum_{q = K}^{L} \beta^{q-t} a_{q} U (c_{q}) = -\frac{1}{\alpha} \exp \left[ -\alpha \left( -K \frac{\log (R\beta)}{\alpha} + G + \Pi + \frac{R^{L-K+1}}{\sum_{\tau = K}^{L} R^{L-\tau}} A_{K-1} \right) \right] 
\cdot \exp [-a y_{K}] \sum_{q = K}^{L} \beta^{q-t} a_{q} \exp \left[ -\alpha \left( q \frac{\log (R\beta)}{\alpha} + \frac{\log (a_{q})}{\alpha} \right) \right] 
\]
\[
= -\frac{1}{\alpha} \exp \left[ -\alpha \left( -K \frac{\log (R\beta)}{\alpha} + G + \Pi + \frac{R^{L-K+1}}{\sum_{\tau = K}^{L} R^{L-\tau}} A_{K-1} \right) \right] 
\cdot \exp [-a y_{K}] \cdot \sum_{q = K}^{L} \frac{R^{q-K}}{\beta}
\]

which is a function of the state variable \( A_{K-1} \).

### A.2 Consumption during working life

For the working period \((t \leq \tau \leq K - 1)\) the problem reads
\[
\max_{c_{\tau}} \sum_{t = t}^{K-1} \beta^{\tau-t} U (c_{\tau})
\]
\[
\text{s.t. } \sum_{t = t}^{K-1} R^{\tau-t} c_{\tau} = RA_{t-1} + \sum_{t = t}^{K-1} R^{\tau-t} y_{\tau} - R^{t-K+1} A_{K-1}
\]

The solution for the consumption path during working life is given by
\[
c_{\tau} = \frac{(\tau - t) \log (R\beta)}{\alpha} + c_{t}
\]
\[
c_{t} = \frac{RA_{t-1} + \sum_{\tau = t}^{K-1} R^{\tau-t} y_{\tau} - R^{t-K+1} A_{K-1} - \sum_{t = t}^{K-1} R^{\tau-t} \left( \frac{(\tau-t) \log (R\beta)}{\alpha} \right)}{\sum_{\tau = t}^{K-1} R^{\tau-t}}
\]

\footnote{Note that we assumed that \( a_{K} = 1 \), so that the solution for \( c_{K} \) is still as in (13b).}
The value of utility during the working stage can be written as

\[ \sum_{\tau=t}^{K-1} \beta^{t-\tau} U(c_\tau) = -\frac{1}{\alpha} \sum_{\tau=t}^{K-1} \beta^{t-\tau} \exp[-\alpha c_\tau] = -\frac{\exp[-\alpha c_t]}{\alpha} \sum_{\tau=t}^{K-1} R^{t-\tau} \] (18)

which is also a function of the wealth stock to be used during retirement, \( A_{K-1} \).

### A.3 Lifecycle consumption

Finally, we choose \( A_{K-1} \) to optimize the lifecycle utility function (11a) using (18) and (15) with the constraint (17b).

The first-order condition for \( A_{K-1} \) gives

\[ A_{K-1} = \frac{\sum_{t=K}^{L} R^{L-t} \gamma}{R^{L-K+1}} \left( c_t + \frac{(K-t) \log(R\beta)}{\alpha} - G - \Pi + \frac{\log(E_t(\exp[-\alpha y_K]))}{\alpha} \right) \]

Use the constraint (17b) and plug in \( G \) and \( \Pi \) to obtain

\[
c_t = \frac{RA_{t-1} + \sum_{\tau=t}^{K-1} R^{t-\tau} y_\tau}{\sum_{\tau=t}^{L} R^{t-\tau}} - \frac{\log(R\beta)}{\alpha} \frac{\sum_{\tau=t}^{K-1} R^{t-\tau} (\tau - t)}{\sum_{\tau=t}^{L} R^{t-\tau}} - \frac{\log(R\beta)}{\alpha} \frac{\sum_{\tau=t}^{L} R^{L-\tau} (\tau - K)}{\sum_{\tau=t}^{L} R^{L-\tau}} - 1 \frac{\sum_{\tau=t}^{L} R^{L-\tau} \log(a_\tau)}{\sum_{\tau=t}^{L} R^{L-\tau}} - \frac{\sum_{\tau=t}^{L} R^{L-\tau}}{\sum_{\tau=t}^{L} R^{L-\tau}} \left( \frac{(K-t) \log(R\beta)}{\alpha} + \frac{\log(E_t(\exp[-\alpha y_K]))}{\alpha} \right)
\]

The final task is to obtain an expression for \( \log(E_t(\exp[-\alpha y_K])) \). For any random variable \( x \), we know that

\[ M(\gamma) = E(\exp[\gamma x]) \]

represents the moment-generating function. For the case that \( y_K \sim N(\mu, \sigma) \), we have that

\[ M(\gamma) = \exp[\mu \gamma] \exp[\sigma^2 \gamma^2 / 2] \]

so that we can write

\[ \log(E_t(\exp[-\alpha y_K])) = -\alpha \mu + \frac{1}{2} \sigma^2 \alpha^2 \] (19)

Using this result and simplifying the remaining terms, we end up with equation (2):

\[
c_t = \frac{RA_{t-1} + \sum_{\tau=t}^{K-1} R^{t-\tau} y_\tau + \sum_{\tau=K}^{L} R^{t-\tau} \mu}{\sum_{\tau=t}^{L} R^{t-\tau}} - \frac{\sum_{\tau=t}^{L} R^{t-\tau} \left( \frac{1}{2} \log(a_\tau) + \frac{1}{2} \alpha \sigma^2 \right)}{\sum_{\tau=t}^{L} R^{t-\tau}} \]

(20)

This is equation 2.
B Derivation of equations 4 and 5

Suppose that at some date \( v < K - 1 \) the constraint binds, and that \( A_{t-1} > 0 \). Then the problem before retirement is

\[
\max_{c_t} \quad \frac{1}{R} \sum_{\tau = t}^{v} \beta^{\tau-t} \exp \left[ -\alpha c_{\tau} \right] - \frac{1}{R} \sum_{\tau = v+1}^{K-1} \beta^{\tau-t} \exp \left[ -\alpha y_{\tau} \right]
\]

s.t. \( \sum_{\tau = t}^{v} R^{\tau-t} c_{\tau} = R A_{t-1} + \sum_{\tau = t}^{v} R^{\tau-t} y_{\tau} \) \hspace{1cm} (21)

We know from equation 17b in Appendix A that the solution for current consumption, imposing \( R \beta = 1 \), must equal

\[
c_t = \frac{R A_{t-1} + \sum_{\tau = t}^{v} R^{\tau-t} y_{\tau}}{\sum_{\tau = t}^{v} R^{\tau-t}}
\]

which is equation 4.

In general, there exists no closed-form solution for the date \( v \) at which the liquidity constraint becomes binding. Furthermore, in a discrete-time setting, minimizing consumption with respect to time \( \left( \frac{dc_t}{dv} \right) \) is not a valid approach, since \( dv \) never approaches zero. Still, we proceed in this manner, as the intuition carries over directly to the case of discrete differences, but is computationally more attractive for characterizing the complete solution. We admit that we make an approximation error of order \( O(1) \).

Given these limitations, we minimize period \( t \) consumption to find the date \( v \):

\[
\frac{\partial c_t}{\partial v} = \frac{\left( -R^{1-v} \log (R) y_v + R^{1-v} \frac{\partial y_v}{\partial v} \right) \sum_{\tau = t}^{v} R^{\tau-t}}{(\sum_{\tau = t}^{v} R^{\tau-t})^2} \sum_{\tau = t}^{v} R^{\tau-t} + \frac{R^{1-v} \log (R) \left( R A_{t-1} + \sum_{\tau = t}^{v} R^{\tau-t} y_{\tau} \right)}{(\sum_{\tau = t}^{v} R^{\tau-t})^2} = 0
\]

Simplifying gives

\[
\left( \log (R) y_v + \frac{\partial y_v}{\partial v} \right) \sum_{\tau = t}^{v} R^{\tau-t} - \log (R) \sum_{\tau = t}^{v} R^{\tau-t} y_{\tau} = \log (R) R A_{t-1}
\]

There is no closed form solution for the exact date \( v \), but note that for a concave time path of income, with \( \partial y_v / \partial v > 0 \) and \( \partial^2 y_v / \partial v^2 < 0 \),

\[
\frac{d v}{d A_{t-1}} = \frac{R \log (R)}{\left( \log (R) \frac{\partial y_v}{\partial v} - \frac{\partial^2 y_v}{\partial v^2} \right) \sum_{\tau = t}^{v} R^{\tau-t}} > 0 \hspace{1cm} (24)
\]

Under the assumption that the liquidity constraint is always binding\(^{15}\) this suffices

\(^{15}\)This is not unreasonable under the same concavity assumption for income, as \( v \) is already chosen as far away in the future as possible depending on the level of \( A_{t-1} \), see (24).
for current consumption. Concluding, \(c_t\) is independent of \(A_{K-1}\) and hence of \(\mu, \sigma^2\) and survival probabilities.

If the level of current wealth is large, and/or the income path is increasing, the constraint will not bind before retirement. If the constraint binds after retirement, the problem after retirement can be stated as follows:

\[
\max_{c_t} \sum_{\tau=K}^\nu \beta^{\tau-K} (a_{\tau}) U(c_{\tau}) \quad (25)
\]

subject to:

\[
\sum_{\tau=K}^\nu R^{K-\tau} c_{\tau} = R A_{K-1} + \sum_{\tau=K}^\nu R^{K-\tau} y_{\tau} \quad (26)
\]

\[
A_{\tau} = R A_{\tau-1} + y_{\tau} - c_{\tau} \quad (27)
\]

\[
A_{v} = 0 \quad (28)
\]

We know from equation 13b that the solution to this problem in period \(K\), imposing \(R\beta = 1\), equals

\[
c_K = -\frac{\sum_{\tau=K}^\nu R^{1-\tau} \log (a_{\tau})}{\alpha \sum_{\tau=K}^\nu R^{1-\tau}} + \frac{R^{1-K+1}}{\sum_{\tau=K}^\nu R^{1-\tau}} A_{K-1} + y_K \quad (29)
\]

We minimize period \(K\) consumption to find the date \(v\):

\[
\frac{\partial c_K}{\partial v} = \frac{R^{1-v} \log (R) \log (a_v) - \frac{\partial a_v}{\partial v} R^{1-v}}{\alpha \sum_{\tau=K}^\nu R^{1-\tau}} - \frac{\frac{\partial a_v}{\partial v} \sum_{\tau=K}^\nu R^{-\tau} - \log (R) \sum_{\tau=K}^\nu R^{-\tau} \log (a_{\tau})}{\alpha (\sum_{\tau=K}^\nu R^{1-\tau})^2} = 0
\]

Simplifying this FOC the implicit solution for \(v\):

\[
\log (R) \log (a_v) \sum_{\tau=K}^\nu R^{-\tau} - \frac{\frac{\partial a_v}{\partial v} \sum_{\tau=K}^\nu R^{-\tau} - \log (R) \sum_{\tau=K}^\nu R^{-\tau} \log (a_{\tau})}{\alpha} = -\alpha \log (R) R^{1-K} A_{K-1} \quad (30)
\]

There is no closed form solution for the date \(v\), but we can gain more insight by taking the total differential of the expression above, and solving for the effect of \(A_{K-1}\) on \(v\):

\[
\frac{dv}{dA_{K-1}} = \frac{\alpha \log (R) R^{1-K}}{\left(\frac{\partial^2 a_v}{\partial v^2} - \frac{\partial a_v}{\partial v} \log (R) a_v - \left(\frac{\partial a_v}{\partial v}\right)^2\right) \sum_{\tau=K}^\nu R^{-\tau}}
\]

The sign of this expression depends on the structure on the survival function:

\[
\frac{dv}{dA_{K-1}} > 0 \iff H = \frac{\partial^2 a_v}{\partial v^2} - \frac{\partial a_v}{\partial v} \log (R) a_v - \left(\frac{\partial a_v}{\partial v}\right)^2 > 0 \quad (31)
\]
After period \( v \), the optimal consumption pattern simply equals the (realized) value of pension income, as the survival probabilities make the agent impatient. The value of consumption after retirement can now be written as

\[
- \frac{E_t \exp \left[ -\alpha c_k \right]}{\alpha} \sum_{\tau = K}^{\nu} R^{l-\tau} - \frac{E_t \exp \left[ -\alpha y_k \right]}{\alpha} \sum_{\tau = \nu+1}^{L} R^{l-\tau} (a_{\tau})
\]

with \( c_k \) given by (29) and \( v \) implicitly by (30).

The working stage has not changed, so we immediately infer that

\[
c_t = \frac{RA_{l-1} + \sum_{\tau = l}^{K-1} R^{l-\tau} y_{\tau} - R^{l-K+1} A_{K-1}}{\sum_{\tau = l}^{K-1} R^{l-\tau}}
\]

and value of pre-retirement consumption equal to

\[
- \frac{1}{\alpha} \sum_{\tau = l}^{K-1} \beta^{\tau-l} \exp \left[ -\alpha c_{\tau} \right] = - \frac{\exp \left[ -\alpha c_l \right]}{\alpha} \sum_{\tau = l}^{K-1} R^{l-\tau}
\]

The lifecycle value function equals

\[
\exp \left[ \frac{\sum_{\tau = K}^{\nu} R^{l-\tau} \log \left( R \right)}{\sum_{\tau = K}^{\nu} R^{l-\tau}} \right] \exp \left[ \frac{-\alpha R^{l-K+1} A_{K-1}}{\sum_{\tau = K}^{\nu} R^{l-\tau}} \right] E_t \exp \left[ -\alpha y_k \right] \quad \text{(32)}
\]

The value function depends on \( A_{K-1} \), both directly, via \( c_t \) and \( c_K \), and indirectly, due to its effect on \( v \). We choose wealth for retirement by maximizing the value function, as in the unconstrained case in Appendix A. We use the fact that \( v = \arg \min_{c_K} \) to infer that \( \partial c_K / \partial A_{K-1} \) equals the direct effect of \( A_{K-1} \) on period \( K \) consumption, that is, \( \partial c_K / \partial A_{K-1} = R^{l-K+1} / \sum_{\tau = K}^{\nu} R^{l-\tau} \).

The FOC for \( A_{K-1} \) equals

\[
E_t \exp \left[ -\alpha c_k \right] R^{l-K+1} + \frac{\partial v}{\partial A_{K-1}} R^{-v} \log \left( R \right) \frac{E_t \exp \left[ -\alpha c_k \right]}{\alpha} = \exp \left[ -\alpha c_l \right] R^{l-K+1}
\]

Plugging in \( c_K \) gives

\[
\exp \left[ \frac{\sum_{\tau = K}^{\nu} R^{l-\tau} \log \left( a_{\tau} \right)}{\sum_{\tau = K}^{\nu} R^{l-\tau}} \right] \exp \left[ \frac{-\alpha R^{l-K+1} A_{K-1}}{\sum_{\tau = K}^{\nu} R^{l-\tau}} \right] E_t \exp \left[ -\alpha y_k \right]
\]

\[
 \cdot \left( R^{l-K+1} + \frac{\partial v}{\partial A_{K-1}} R^{-v} \log \left( R \right) \frac{1}{\alpha} \right) = \exp \left[ -\alpha c_l \right] R^{l-K+1}
\]

The solution for \( A_{K-1} \) equals

\[
A_{K-1} = \frac{\sum_{\tau = K}^{\nu} R^{l-\tau} \log \left( a_{\tau} \right)}{\alpha R^{l-K+1}} - \frac{\sum_{\tau = K}^{\nu} R^{l-\tau} \left( \mu - \alpha \sigma^2 \right)}{R^{l-K+1}} + \frac{\sum_{\tau = K}^{\nu} R^{l-\tau} c_l}{R^{l-K+1}}
\]

\[
+ \frac{\sum_{\tau = K}^{\nu} R^{l-\tau} \log \left( R^{l-K+1} + \frac{\partial v}{\partial A_{K-1}} R^{-v} \log \left( R \right) \frac{1}{\alpha} \right)}{\alpha R^{l-K+1}} - \frac{\sum_{\tau = K}^{\nu} R^{l-\tau} \log \left( R^{l-K+1} \right)}{\alpha R^{l-K+1}}
\]

(32)
And the solution for current consumption is equal to

\[
c_t = RA_{t-1} + \sum_{\tau=t}^{K-1} R^{1-\tau} y_{\tau} + \sum_{\tau=K}^{\nu} R^{1-\tau} \mu - \frac{\sum_{\tau=K}^{\nu} R^{1-\tau}}{\sum_{\tau=1}^{\nu} R^{1-\tau}} \left( \frac{1}{r} \log(a_{\tau}) + a_{\tau}^2 \right) - \frac{\sum_{\tau=K}^{\nu} R^{1-\tau} \log \left( \frac{H \sum_{\tau=K}^{\nu} R^{1-\tau} + (\log(R))^2 R^{1-\tau}}{H \sum_{\tau=K}^{\nu} R^{1-\tau}} \right)}{a \sum_{\tau=1}^{\nu} R^{1-\tau}}
\]

with \( H \) defined in (31). This is equation 5.
C Forecasting household income

As savings are decreasing with future (labor) income, we need to estimate household income until the retirement age. For this purpose, we employ the waves of 1996 until 2009 of the DHS to estimate a model for household income. We employ a parsimonious model with age as the only explanatory variable. More specifically, we estimate

$$\log(y_{it}) = m(\text{age}_{it}) + u_i + \epsilon_{it}$$

where $m(\text{age}_{it})$ is a linear spline function with nodes at 30, 35, 40, 45, 50, 55 and 60 years of age, $u_i$ is a household fixed effect and $\epsilon_{it}$ a random error term. The sample is restricted to household heads aged 25 until 70. We experimented with different parameter estimates by education group and inserting Deaton and Paxson (1994)-type of orthogonalized time effects, but this does not affect the results. The results are shown in table 7.

Table 7: Fixed effects model for log income

<table>
<thead>
<tr>
<th>Age</th>
<th>Coefficient</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>26-30</td>
<td>0.0619***</td>
<td>(0.00880)</td>
</tr>
<tr>
<td>31-35</td>
<td>0.0304***</td>
<td>(0.00708)</td>
</tr>
<tr>
<td>36-40</td>
<td>0.0317***</td>
<td>(0.00578)</td>
</tr>
<tr>
<td>41-45</td>
<td>0.0329***</td>
<td>(0.00586)</td>
</tr>
<tr>
<td>46-50</td>
<td>0.0195***</td>
<td>(0.00517)</td>
</tr>
<tr>
<td>51-55</td>
<td>0.0125**</td>
<td>(0.00553)</td>
</tr>
<tr>
<td>56-60</td>
<td>0.000628</td>
<td>(0.00588)</td>
</tr>
<tr>
<td>61+</td>
<td>0.0142**</td>
<td>(0.00660)</td>
</tr>
<tr>
<td>Constant</td>
<td>9.460***</td>
<td>(0.0527)</td>
</tr>
</tbody>
</table>

Observations: 20390
Number of households: 5731
$R^2$: 0.023
$\sigma_u$: 0.546
$\sigma_\epsilon$: 0.385
$p$-value Age effects: 0.000

Clustered standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Household income is increasing rapidly for young household heads ($\pm 6\%$ per year), and income growth is decreasing with age. Using these parameter estimates, the forecast of household income at age $t+s$ is calculated as

$$\hat{y}_{it+s} = \exp \left[ m(\hat{\text{age}}_{it+s}) + \hat{u}_i + z \right]$$

where $z$ is a random draw from the normal distribution with mean zero and standard deviation $0.5 \times \sigma_\epsilon$, in line with Knoef, Alessie, and Kalwij (2009a), who estimate a similar model using administrative data from Statistics Netherlands, and obtain an estimate of $\sigma_\epsilon$ of 0.205. Their estimate of the variance of the error term is smaller due to the inclusion of household characteristics and the tax-records data not suffering from measurement error as our survey data. Note that we use forecasted income as a left-hand-side variable in our main estimations to prevent generated regressor bias.
References


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