On the Efficiency of Partial Information in Elections*

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Abstract

We study the relation between the electorate’s information about candidates’ policy platforms during an election, and the subsequent provision of inefficient local public goods (pork) by the elected government. We show that this relation is not monotonic: More information does not lead to better outcomes. In particular, we show that the efficient outcome in which no candidate proposes to provide any inefficient good is sustained in equilibrium only if voters are imperfectly informed. If the electorate is well informed, electoral competition leads candidates to provide inefficient pork in all equilibria. We show that this result is robust even if candidates care about efficiency.

Keywords: Elections, information, inefficiency, pork, campaigns.

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1 Introduction

During electoral campaigns, candidates running for office make policy proposals to woo voters. Voters pay only limited attention to electoral campaigns and as a result they do not become fully informed about the policies proposed by the candidates. We study the relation between the information acquired by the voters, and the policies that the candidates announce during the campaign and execute once in office. In particular, we explain the effect of voters’ information on the provision of socially inefficient particularistic public goods.

A particularistic or local public good provides a benefit only to the members of a single district or group. If the costs of provision are spread across society at large by general taxation, voters in each district want their own particularistic public good to be provided, while they prefer the public good in any other district not to be provided. Because voters enjoy the benefits of the public good provided to their own district fully, while they only pay a fraction of the cost of any given public good, they care more about the provision of their own good than about the non-provision of the public good in any one other given district. This leads politicians to promote inefficient policies that result in the over provision of particularistic public goods.
We find that if local public goods are very inefficient and their costs largely overwhelm benefits, societies with an informed electorate do not provide them, while if the electorate is poorly informed, the efficient equilibrium such that no candidate promises to provide inefficient public goods exists but it is one equilibrium among many. So if local public goods are very inefficient, more information is better for the electorate.

Such an optimistic view on voters’ information does not hold if public goods are inefficient but to a lesser extent. If the benefit/cost ratio of pork is between two thirds and one, the efficient outcome with no pork can be sustained in equilibrium only if voters are poorly informed; if voters are better informed, all equilibria are inefficient. The conclusion that an increase in voters’ information does not necessarily enhances social welfare is reinforced if candidates also care about social efficiency along with their desire to gain office: In this case there are many equilibria when voters are poorly informed; a unique efficient equilibrium arises for an intermediate level of voters’ information; and a unique inefficient equilibrium emerges if voters are well informed.

Examples of policies that distribute targeted goods of dubious efficiency abound: we highlight farm subsidies, military procurement, and some infrastructure investment in the United States and Europe. We believe that similarly inefficient allocations of funds to narrow constituencies occur as well at the regional and local level.

Farm subsidies distribute between $12 billion and $25 billion every year in the
United States, and around €55 billion per year in the European Union (almost one half of the European Union budget),\(^1\) distorting the market and creating an aggregate welfare loss.

Parochial interests also trump efficiency in contracts for military equipment. The US Air Force has seen a $40 billion contract to replace its refuelling tankers repeatedly delayed since 2002 amid controversies over waste and fraud; Congress spent $65 billion from 1991 to 2009 on the jet fighter F-22, which was designed to counter a Soviet threat, and which the US Air Force has found unsuitable for any combat mission in the post-Soviet world; controversy surrounds the plans to develop two parallel versions of the engine for the F-35 fighter, at an additional cost of $3 billion. President Obama declared purchases of the F-22 aircraft an “inexcusable waste” and the second engine for the F-35 an “unnecessary and extravagant expense.” These projects gain political support based on the funding and jobs they bring to specific districts, irrespective of their merit as a cost-efficient mean to satisfy the Air Force needs. Consequently, companies disperse production among multiple districts and states to maximize political support: Boeing promised to “create up to 50,000 jobs in 40 states” in its bid for the refueling tanker; building the F-22 that the Air Force has never used provided 25,000 jobs in 44 states, etc.\(^2\) In Europe, production of the European Typhoon jet


fighter was assigned to countries in proportion to their procurement orders and not based on any measure of efficiency.

Military procurement decisions thus degenerate into contests for pork that serve as local jobs programs. Younossi, Stem Lourell and Lussier [2005] show that the dispersion of production among many districts that makes a project politically viable, is precisely the cause of large cost overruns and delays. An audit on the European Typhoon conducted by the United Kingdom’s Ministry of Defense, which alone has spent over £20 billion on the project, agrees, identifying “the inefficient commercial and managerial arrangements on the project as the root cause of much of the cost escalation and schedule slippage on the project.”

With regard to investment in public infrastructure, an egregious example in the United States was the practice (halted in 2011) of approving earmarks, which allocate funds to local projects avoiding the scrutiny and debate of regular appropriations. The total cost of these projects ascended to $16 billion in 2010, and every state received some funds. Aside from earmarks, the cost effectiveness of other projects such as high speed rail lines, both existing ones in Europe (Ginés and Inglada [1997]) and

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3See the Report by the Comptroller and Auditor General, HC 755, session 2010–2011, 2 march 2011. These “inefficient commercial arrangements” were such that the left wings were built in Italy, the right ones in Spain, the front of the plane in the United Kingdom and the back in Germany.

4Taxpayers for Common Sense (www.taxpayer.org) defines earmarks as “legislative provisions that set aside funds within an account for a specific program, project, activity, institution, or location. These measures normally circumvent merit-based or competitive allocation processes.” See Taxpayers Against Earmarks at www.endingspending.com for a state by state breakdown.
future ones in the United States\textsuperscript{5} appears to be rather questionable.

We investigate whether this inefficient allocation of funds occurs because voters are not sufficiently informed about issues that do not concern them directly: Farm subsidies are important to farmers, and aircraft procurement to aviation industry workers. Most voters are not farmers or aviation industry workers, and they do not follow political campaigns closely enough to know each candidate’s funding plans for farming or aircraft purchases, or other projects that are not directly relevant to the voter. To the extent that voters remain unaware of policy details on issues that do not concern them directly, legislators have no incentives to pursue efficiency gains by eliminating programs that generate targeted benefits and diffused costs, even if the aggregate costs surpass the localized benefits.

This intuition, while true, is incomplete. First, a society with a perfectly informed electorate can also suffer from inefficient policies that favor organized lobbies and special interests (Grossman and Helpman [2002]). Furthermore, we show that the relation between the electorate’s information and the efficiency of policy outcomes depends on the motivation of candidates. We consider candidates that seek to win office by any means, and candidates that seek office but also care about policy outcomes, disliking inefficiency.

We find the equilibrium policy proposals in a Westminster democracy with two

parties who compete in multiple districts by proposing to implement local public good projects that are inefficient for society, but beneficial to the district in which they are developed. The party that wins in most districts implements its policy proposal. The model also fits presidential elections, where the two agents competing for votes are two individual politicians; we refer to the two agents contesting the elections as candidates to accommodate both cases.

Since voters care more about policies that directly affect their districts, if their attention span is limited, they naturally become better informed about these proposals than about projects in other districts that only affect them indirectly through general taxation. We solve the game in which voters observe the policy proposals for their own district with certainty, but observe the candidates’ spending plans in other districts only with some probability. If voters end up uninformed, they form beliefs about a candidate’s proposal based on what they observe in their district.

If the probability that voters are perfectly informed is very low, results depend on voters’ beliefs: many strategies can be supported in equilibrium. In particular, the efficient outcome with no pork is sustained in an equilibrium in which voters believe that a candidate who deviates to offer them their local public good must be a big spender who made similar inefficient offers in every other district. When information about what a candidate promises in the other districts increases, voters’ beliefs become less relevant. If public goods are not extremely inefficient, a minimal-winning majority of districts prefers to exploit the other districts by having the public good provided
only to the members of the coalition, and then with an informed electorate, the
competition among districts for being member of the winning coalition and between
candidates for attracting votes, induce an inefficient overprovision of public goods.

Therefore some degree of ignorance, together that the belief that a candidate who
deviates in one district must have offered to provide a lot of inefficient public goods
in other district, are necessary to sustain the efficient equilibrium.

There is an extensive literature on political science and economics on the redis-
tribution of inefficient local public goods. Seminal contributions by Weingast [1979],
Shepsle and Weingast [1981], Weingast, Shepsle and Johnsen [1981] and Niou and
Ordeshook [1985] analyze the provision of local inefficient public goods as the result
of a legislative bargaining game, and predict a "pork for everybody" outcome. Legis-
lators commit to a norm of universalism by which every district gets its own inefficient
project, rather than letting a minimal-winning majority distribute public goods only
to the districts in this majority. An objection to this seminal theory is that it cannot
explain why legislators do not embrace instead a Pareto-superior universalist norm
by which no inefficient local public goods are ever provided. In fact, if legislators do
not commit to any norm, Ferejohn, Fiorina and McElveen [1987] and Baron [1991]
show that only a minimal winning majority of districts benefit from the provision of
inefficient projects.

A stream of economic theories explain targeted redistribution as the equilibrium
outcome of a game in which candidates compete in elections (Lindbeck and Weibull
[1987]; Dixit and Londregan [1996]; Lizzeri and Persico [2001]; Chari, Jones and
Marimon [1997]; the survey by Persson and Tabellini [2000]; and the more recent
article by Roberson [2008]). These theories assume that citizens are fully informed
about the policy proposals made by the candidates. The assumption is unrealistic.
The empirical literature on voter behavior has conclusively established that in practice
voters have a sketchy idea of these policy proposals (Campbell, Converse, Miller and
Stokes [1980]; Bartels [1986] and Alvarez [1997]).

Our contribution narrows the gap between the assumptions in the theoretical
literature, and the accepted stylized facts of the empirical literature, by recognizing
that voters have only partial information about candidates’ policy proposals.

Ours is not the first theory of elections with voters that are not fully informed. At
the opposite extreme, Grosser and Palfrey [2010] assume that citizens do not know
anything about the candidates. McKelvey and Ordeshook [1985] assume that most,
but not all, voters are completely uninformed. Snyder and Ting [2002] argue that
party labels serve as cues to provide some information to voters who do not directly
know the ideological preferences of individual candidates. Glaeser, Ponzetto and
Shapiro [2005] assume that each voter becomes either fully informed or fully unin-
formed about the policy proposal of each of the two candidates, so that a given voter
may become perfectly informed about one candidate’s proposal but remain entirely
unaware of the other candidate’s proposal. All these models deal with ideological
preferences.\textsuperscript{6}

The closest references are models of pork distribution that assume that voters are not informed about candidates’ proposals. McKelvey and Riezman [1992] and Muthoo and Shepsle [2010] assume that citizens only know if the candidate is an incumbent or not. Baron [1994] assumes that some voters are fully informed, while others are fully uninformed.

We believe that it is more realistic to assume that voters have some but not all the information about each of the candidates’ policy proposals, and this intermediate approach is the one we pursue.

2 The Model

We consider a society partitioned into three subsets, with one representative voter \(i \in \{a, b, c\}\) in each subset. We refer to these subsets as districts, but they could also be population groups of similar size divided by ethnicity, age, profession or class.

Two candidates \(A\) and \(B\) compete for election. Let \(J \in \{A, B\}\) denote an arbitrary candidate and let \(-J\) denote the other candidate, so that \(\{J, -J\} \equiv \{A, B\}\).

The policy space consists on whether or not to provide a public good in each

\textsuperscript{6}Other models consider electorates that are not perfectly informed about the state of the world (Feddersen and Pesendorfer [1996] and [1999]) or about candidates’ competence (Krishna and Morgan [2010]), or allow candidates to deviate from their campaign proposals once in office (Banks [1990] and Callander and Wilkie [2007]). These models are more distantly related because they do not deal with distributive policies, and assume that voters are perfectly informed about candidates’ proposals.
district. A strategy for each candidate consists on proposing a policy in the policy space. Let $S^J = S^{-J} = \{0, 1\}^3$ be the strategy set of each candidate. Let $s^J = (s^J_a, s^J_b, s^J_c) \in \{0, 1\}^3$ be a strategy by candidate $J$, where $s^J_i = 1$ indicates that $J$ proposes to provide the public good in district $i$ and $s^J_i = 0$ indicates that $J$ proposes not to provide it. Let $s^J_{\bar{i}}$ denote the proposals for the other two districts, not including $i$.

Voters only observe some of the information contained in the policy proposals, as described below, and an election is held, where each voter chooses to vote either for $A$ or $B$, or abstain. The candidate with most votes wins, and in case of a tie, each candidate wins with equal probability. We assume that the winning candidate carries out her proposal once in office; or alternatively, we assume that voters vote as if this were to be the case.

Each public good brings a benefit only to voters of the district where it is provided but its cost is equally borne by all the voters. We assume that all public goods bring the same benefit and are equally costly. Voters’ preferences over candidates exclusively depend on a valuation of their policy proposals. Each voter votes for the candidate whose policy gives the largest expected utility. Let $\beta$ denote the ratio of the benefit/cost of each public good, or, alternatively, let $\beta$ be the benefit given a normalization of the cost to one. If candidate $J$ proposes to provide the public good in district $i$, then voter $i$’s expected utility of $J$’s proposal is equal to $\beta - \frac{1}{3}k$, where $k \in \{0, 1, 2, 3\}$ is the total number of public goods proposed by candidate $J$. 

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If candidate $J$ does not propose to provide the public good in district $i$, then voter $i$'s expected utility is equal to $-\frac{1}{3}k$. We assume that public goods are inefficient in the sense that no district would like to provide its own public good if it had to bear its full cost, but not too inefficient because, ceteris paribus, each voter prefers having the public good provided in her district. Formally, we assume that $\beta > \frac{1}{3}$.\footnote{If $\beta < 1/3$, the problem is trivial, since no district wants to implement any project.}

It is immediate to note that voter $i$'s ranking over policies depend on $\beta$. Namely, if $\beta \in \left(\frac{2}{3}, 1\right)$ then the preference order of voter $a$ over the eight possible policy outcomes is

\[
\{1, 0, 0\} \succ_a \{1, 1, 0\} \succ_a \{1, 0, 1\} \succ_a \{0, 0, 0\} \succ_a \{1, 1, 1\} \succ_a \{0, 0, 1\} \succ_a \{0, 1, 0\} \succ_a \{0, 1, 1\}.
\]

(1)

If $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$

\[
\{1, 0, 0\} \succ_a \{0, 0, 0\} \succ_a \{1, 1, 0\} \succ_a \{1, 0, 1\} \succ_a \{0, 0, 1\} \succ_a \{0, 1, 0\} \succ_a \{1, 1, 1\} \succ_a \{0, 1, 1\}.
\]

(2)

We assume that candidates and voters are fully strategic, rational agents. We consider two types of candidates: Purely office motivated, and efficiency concerned. Both types of candidates want to be elected. For the purely office motivated, this is their only concern: They obtain utility one if they win and zero otherwise. Efficiency
concerned candidates also care about the policies they implement; these candidates want to win, but they prefer to win proposing and implementing more efficient policies. Let $k$ the number of projects in the proposal of candidate $J$. Candidate $J$’s preferences are represented by the utility function:

$$U_J(k) = \begin{cases} \frac{1}{1 + \alpha(1 - \beta)k} & \text{if } J \text{ wins the election} \\ 0 & \text{otherwise.} \end{cases}$$

where $\alpha \in \{0, 1\}$. If candidate $J$ is purely office motivated $\alpha = 0$, if candidate $J$ is efficiency concerned, then $\alpha = 1$. The larger $(1 - \beta)k$, the greater the inefficiency of a policy that implements $k$ projects, and the lesser the utility that a efficiency concerned candidate experiences if she wins proposing such policy. We assume that candidates’ type is public information.

All candidates evaluate lotteries according to standard expected utilities. Each voter $i$ calculates an expected payoff for each candidate $J$, using all the information available to the voter about the policy proposal of the candidate. We refer to this calculated expected payoff as the expected payoff of candidate $J$ for voter $i$. When voter $i$ does not observe the full policy proposal $s^J$, the expected payoff depends on the beliefs of the voter about $s^J$, and if the beliefs are not correct, the expected payoff may not be the ex-post payoff that $i$ obtains if $J$ wins the election (note however that this can only occur outside of the equilibrium path). We assume that
voters vote for the candidate with the highest expected payoff, that is, voters are sequentially rational (Kreps and Wilson [1982]). We assume that voters do not use weakly dominated strategies. This rules out equilibria in which all voters vote for the same candidate even though some voters prefer the losing candidate’s proposal. If the expected payoffs of both candidates coincide, voters are indifferent given their beliefs. We assume that in this case they abstain, unless abstention has been eliminated as a weakly dominated strategy.

Our benchmark solution concept is Pure Perfect Bayesian Equilibria. If (and only if) there exists no equilibrium in pure strategies, we look at mixed strategy equilibria. When we study mixed strategy equilibria, we let $\sigma_k$ be the probability of playing strategy $s_k$ in a mixed strategy $\sigma$, and we let $p^J \in \{0, 1\}^3$ be the policy actually proposed (action taken) by $J$. Clearly, if $J$ chooses a pure strategy, $p^J = s^J$. Let $p = (p^A, p^B)$.

Unless otherwise specified, an “equilibrium” means a Perfect Bayesian Equilibrium in pure strategies in which no agent uses a weakly dominated strategy.

We are particularly interested in analyzing voters’ behavior when they have only have partial information on candidates’ policy proposals. Specifically we want to analyze how voters behavior modifies when the amount of information about candidates’ proposal changes. We assume that voters can easily be informed about what each candidate proposes regarding their own district, whether or not to provide the public good, but it is harder for them to know what each candidate proposes in the other
districts. We parameterize how informed is the electorate by a single parameter $\varepsilon$, which captures the probability that voters cast their votes possessing all the relevant information. Specifically, we assume that after candidates choose policy proposals, Nature determines whether these proposals become public (and common) knowledge or not. Policy proposals become common knowledge with probability $\varepsilon \in [0, 1]$. With probability $1 - \varepsilon$, each voter $i \in \{a, b, c\}$ only observes what candidates commit to do in her district, and she is completely unaware of what candidates promise in the other districts. Notice that the extreme case $\varepsilon = 1$ corresponds to the standard model with perfect information, and $\varepsilon = 0$ corresponds to an electorate in which voters only have local information about the proposals for their district, and never learns about the proposals to provide public goods in other districts.

For each candidate $J \in \{A, B\}$, the set of pure strategies $S^J$ consists of the following eight 3-dimensional vector.

- $s_1$ : Propose policy $(0, 0, 0)$;
- $s_2$ : Propose $(1, 0, 0)$;
- $s_3$ : Propose $(0, 1, 0)$;
- $s_4$ : Propose $(0, 0, 1)$;
- $s_5$ : Propose $(1, 1, 0)$;
- $s_6$ : Propose $(1, 0, 1)$;
- $s_7$ : Propose $(0, 1, 1)$ and
- $s_8$ : Propose $(1, 1, 1)$. 
If Nature makes proposals public knowledge, voters observe $p^A$ and $p^B$ and compare the two proposals and vote for the one they prefer, according to the preference order 1 or 2. If proposals do not become public knowledge, each voter $i$ remains unaware about what each candidate proposes in districts other than $i$. Note that voter $i$ has two information sets with respect to the strategy of candidate $J$: Voter $i$ observes either $p^J_i = 0$ or $p^J_i = 1$. Since there are two candidates, $(p^A_i, p^B_i) \in \{0, 1\} \times \{0, 1\}$, so that voter $i$ has four information sets in which to make a decision, and three possible actions (vote $A$, vote $B$ or abstain) in each of these sets.

Under this informational structure, in which voters have imperfect information, an equilibrium must describe strategies for voters and candidates, and beliefs for voters. The beliefs of voter $i$, denoted $\delta_i$, is a vector $\delta_i = (\delta^A_i, \delta^B_i)$, where $\delta^J_i = (\delta^J_i(0), \delta^J_i(p^J_i))$ for each $J \in \{A, B\}$, and $\delta^J_i(p^J_i)$ is the probability distributions over the set of strategies played by candidate $J$ that voter $i$ holds as a belief after observing $p^J_i$. For $s^J_i \in \{0, 1\}$, let $\omega^i_{k,J}(s^J_i)$ be the sum of weights assigned by $\delta^J_i(s^J_i)$ to the set of strategies where $J$ proposes to carry out $k$ projects in districts other than $i$.

Beliefs along the equilibrium path must be correct. Beliefs off the equilibrium path must assign all the probabilities to undominated strategies if any such strategy is consistent with the information possessed by the agent.\textsuperscript{8}

\textsuperscript{8}If no undominated strategy is consistent with the information possessed by the agent, then we let the agent hold any beliefs over the entire strategy set.
The strategy pair \((s^A, s^B)\) determines the information \((s^A_i, s^B_i)\) received by each voter \(i\). This information, together with beliefs \(\delta_i\), determine the expected utility for \(i\) if \(A\) or \(B\) wins, which in turn determines agent \(i\)'s vote and therefore, aggregating over all three agents, it determines the electoral outcome and the payoffs to \(A\) and \(B\).

The set of equilibria depend on the efficiency of the public goods measured by \(\beta\), and on how informed is the electorate, measured by \(\varepsilon\).

3 Results

We say that a public good is very inefficient if any minimal majority coalition prefers that no public good be provided rather than that public goods be provided only for all the members of the coalition. Our first result concerns these goods: In equilibrium both candidates make the efficient proposal of not providing any public good if voters are sufficiently well informed. When voters are poorly informed there are multiple equilibria and the efficient equilibria is one of them. The following Proposition summarizes this result.

**Proposition 1** For any \((\alpha_i, \alpha_j) \in \{0, 1\}^2\), any \(\varepsilon \geq 0\), and any \(\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)\), an equilibrium in which both candidates propose to implement the efficient policy exists, and furthermore if \(\varepsilon > \frac{1}{2}\), it is the unique equilibrium.
This result is robust to either assumption on the motivation of candidates, whether they exclusively seek to win office, or whether they also have care about policy outcomes. Here is an intuition for this result: If voters are poorly informed ($\varepsilon < \frac{1}{2}$) the set of Pure Perfect Bayesian Equilibria is large, regardless of the efficiency of the public good, because since voters are unlikely to gain full information, their votes depend on their beliefs and different equilibria can be sustained for certain out of equilibrium beliefs.

If the probability that citizens are informed is high ($\varepsilon > \frac{1}{2}$), out of equilibrium beliefs are more often irrelevant since with high probability voters are able to fully observe a deviation. The policy $(0, 0, 0)$ is the second best policy for each voter when $\beta < \frac{2}{3}$: there is no other policy that can defeat it when voters have fully information about candidates’ policy and $(0, 0, 0)$ can defeat any other policy. It follows that proposing $(0, 0, 0)$ for both candidates is indeed the unique equilibrium. If both candidates proposes the efficient policy $(0, 0, 0)$, each candidate wins the election with probability $\frac{1}{2}$. No deviation is profitable when information is fully revealed. Therefore independently of voters’ out of equilibrium beliefs any deviation might be profitable only in case information is not fully revealed, but this occurs with probability $1 - \varepsilon < \frac{1}{2}$. Hence the strategy profile such that both candidates propose the efficient policy $(0, 0, 0)$ is an equilibrium. Moreover, suppose in equilibrium a candidate makes a different proposal. If both candidates make a proposal different than $(0, 0, 0)$, there exists at least one candidate who wins with probability not larger than
\[ \frac{1}{2} \]. This candidate can deviate proposing the efficient policy and win when information is fully revealed, that is with probability \( \varepsilon > \frac{1}{2} \), contradicting the existence of such equilibrium. If one candidate proposes the efficient policy and the other candidate a different policy, the former candidate wins with probability one. Again, this cannot be an equilibrium since the defeated candidate can mimic the winner and increase her probability of winning up to \( \frac{1}{2} \).

It is also worthy noticing that if both candidates are efficiency concerned, the efficient equilibrium such that both candidates promise the efficient policy \((0, 0, 0)\) is the unique equilibrium for a threshold \( \bar{\varepsilon} < \frac{1}{2} \). In fact, efficiency concerned candidates care about their policy and therefore prefer to make an efficient proposal to make an inefficient one, even if they slightly reduce their probability of winning by making an efficient proposal.

In summary, if local public goods are very inefficient, to increase voters’ degree of information about candidates’ policies is normatively desirable.

On the contrary, if the public goods are still inefficient but their benefit/cost ratio is closer to one, this optimistic finding regarding the relation between information and efficiency does not hold anymore. Our more striking result is that an increase in voters’ information about candidates’ policy may be detrimental because it can destroy the possibility of reaching an efficient outcome.

Theorem 2 below states our main result.
**Theorem 2** For any \((\alpha_i, \alpha_j) \in \{0, 1\}^2\) and any \(\beta \in (\frac{2}{3}, 1)\),

a) if \(\varepsilon \leq \varepsilon(\alpha_i, \alpha_j, \beta)\) an equilibrium in which both candidates propose to implement the efficient policy exists, and

b) if \(\varepsilon > \varepsilon(\alpha_i, \alpha_j, \beta)\), all equilibria are inefficient,

where \(\varepsilon(0, 0, \beta) = \varepsilon(0, 1, \beta) = \varepsilon(1, 0, \beta) = \frac{1}{2}\) and \(\varepsilon(1, 1, \beta) = \frac{3 - 2\beta}{2} > \frac{1}{2}\).

If citizens are poorly informed, \(\varepsilon \leq \bar{\varepsilon}\), there exist many equilibria and there is not a clear prediction on the level of inefficiency that may be observed in the provision of inefficient local public goods. With an uninformed electorate, many equilibria are supported by pessimistic beliefs about any observed deviation. For instance, there exist the efficient equilibrium in which both candidates propose zero projects as also the very inefficient equilibrium in which both candidates propose all projects. The intuition is straightforward. Suppose that voters’ beliefs are pessimistic in the sense that when a voter observes a deviating proposal in her district, the voter believes that the deviating candidate has proposed to provide the public goods in the other two districts. Given these beliefs, no voter votes for a deviating candidate unless information is fully revealed. It follows that if the probability that full information is revealed is low, deviating is never profitable. If we focus on the efficient equilibrium as a possible selection criterion, then we end up with a positive prediction when the level of voters’ information is low. However, when the level of information increases, such optimistic conclusion does not hold anymore. In fact the efficient equilibrium where each can-
candidate proposes the policy $(0, 0, 0)$ does not exist anymore. For $\varepsilon$ sufficiently large but small than $1$, the unique equilibrium which emerges is an equilibrium in mixed strategy where in expectation at least $9/7$ projects are implemented. It is easy to provide an intuition why no equilibrium in pure strategy exists when $\varepsilon$ is sufficiently large. Suppose that both candidates are office motivated and there exists a strategy profile of equilibrium in which both candidates play a pure strategy. There exists at least one candidate who wins the election with probability at most $\frac{1}{2}$. However, there always exists a pure strategy that this candidate can play and defeat the other candidate’s pure strategy when information is fully revealed. Since he is only interested in winning the competition, then such deviation is profitable because it allows the deviating candidate to win with probability $\varepsilon > \frac{1}{2}$.

The intuition holds for efficiency concerned candidates, too. In this case the threshold above which no pure strategy equilibrium exists is larger than $\frac{1}{2}$ since candidates may prefer to win with probability lower than $\frac{1}{2}$ and an efficient proposal, to win with slightly larger probability and an inefficient proposal. Hence the efficient equilibrium where both candidates makes the proposal $(0, 0, 0)$ still exists for $\varepsilon > \frac{1}{2}$ because both candidates prefer to win with probability $\frac{1}{2}$ and an efficient proposal to win with probability $\varepsilon$ and an inefficient proposal where two public goods are provided. However if $\varepsilon$ is sufficiently large, then the incentive to deviate, and win the competition when information is fully revealed, become strong enough to destroy the equilibrium.
It follows that when public goods are inefficient but not too much, some degree of voters’ ignorance is necessary to sustain an efficient outcome. The intuition behind this pessimistic result is straightforward. Candidates make a proposal to win the election. Even efficiency concerned candidates primarily care about winning the election. When public goods are not too inefficient, districts compete in order to belong to a minimal-winning majority. The efficient proposal cannot be part of an equilibrium because it is defeated by any proposal that provides the public good to two districts. No pure strategy can be sustained in equilibrium since there is always a different strategy that is able to attract a minimal winning majority of districts. If candidate A proposes to provide the public good in two districts, say $a$ and $b$, candidate $B$ can win the election when information is fully revealed by promising the public good in only one of those two districts, for instance $a$; candidate $B$ gets the vote of district $a$ and $c$. Similarly if candidate $A$ promises the public good only in district $a$, candidate $B$ can win the election when full information is revealed promising public goods in districts $b$ and $c$.

If full information is revealed with sufficiently high probability the unique equilibrium is in mixed strategy where voters the public good in each district is provided with positive probability.

The efficient equilibrium where both candidates promise the policy $(0,0,0)$ can only exist if candidates are not tempted to promise inefficient pork provision because voters cannot observe candidates’ policies and are sufficiently pessimistic to believe
that a candidate who offers them pork offers pork to everybody. In short: campaign promises to carve out a minimal winning coalition are not credible if the electorate is poorly informed.

Theorem 2 claims that an efficient equilibrium does not exist if voters are sufficiently informed. However a multiplicity of equilibria exists when voters are poorly informed. Standard refinement arguments\textsuperscript{9} do not sharpen the predictions if $\varepsilon < \bar{\varepsilon}$.

While the efficient equilibrium exists, we may wonder if it will be played, since different equilibria emerge depending on the voters’ out of equilibrium beliefs. Skeptical voters about candidates who promise pork and the inability for candidates to credibly announce that they favor a particular subset of districts instead of trying to get round all districts sustain the efficient equilibrium.

The observation that there is not a monotonic relation between voters information and efficiency is reinforced if we look at the particular case when candidates are efficiency concerned (and public goods are not too inefficient). In this case in fact we can provide a clear-cut prediction on the electoral outcome. Theorem 3 below describes the set of equilibria when both candidates are efficiency concerned.

\textbf{Proposition 3} \textit{Suppose that both candidates are efficiency concerned and public goods are not very inefficient $\beta \in (\frac{2}{3}, 1)$.}

\[ \text{If } \varepsilon \leq \frac{2-\beta}{\beta}, \text{ there exist multiple pure equilibria;} \]

\textsuperscript{9}Results showing that all equilibria are trembling hand perfect, sequential ([1982]) and rationalizable (Pearce [? ] and Battigali [?]), and satisfy the intuitive criterion and divinity (Cho and Kreps [1987]) and a forward induction refinement are available from the authors.
If \( \varepsilon \in \left(\frac{2-\beta}{6-4\beta}, \frac{3-2\beta}{2}\right) \), there is a unique pure strategy equilibrium in which both candidates propose the efficient policy; and

If \( \varepsilon > \frac{3-2\beta}{2} \) there is no pure strategy equilibrium, and there exists a mixed strategy equilibrium. The probability distribution over pure strategies varies with \( \varepsilon \) and \( \beta \); in expectation at least \(9/7\) projects are implemented.

An increase in information increases from low to intermediate is at least weakly beneficial because it makes the efficient equilibrium unique; however, a further increase of information is detrimental, as it leads to inefficient outcomes that reduce the ex-ante welfare of each voter. We illustrate these results in figure 1, and, as shown there, if candidates are concerned about efficiency and public goods are not too inefficient, some amount of voters’ ignorance is beneficial, as it is necessary to sustain the first best in equilibrium.

4 Discussion

We have developed a theory on the provision of particularistic goods that are socially inefficient. We study elections with a poorly informed electorate. We argue that citizens are better informed about government expenditures in their own district (which they favor) than about government expenditure in other districts (which they oppose). We analyze how this informational bias affects the provision of socially inefficient local public goods.
Figure 1: Equilibria and policy outcomes with efficiency-concerned candidates
If particularistic goods are very inefficient, returning less than 66cts of benefit per dollar of cost, we find that not providing any inefficient goods can always be supported in equilibrium, regardless of the information environment.

We find richer results if particularistic goods are inefficient but provide more than 67cts of benefit per dollar of cost. The equilibrium outcome depends on the information held by the electorate, as follows:

a) If voters are unlikely to be fully informed, votes depend more on beliefs than on the actual actions taken by the candidates, and, in consequence, many different strategies and levels of provision of inefficient goods can be sustained in equilibrium. Nevertheless, the efficient equilibrium exists and it is sustained by out of equilibrium beliefs of voters who assign sufficiently high probability that a candidate who offers them their public good is offering the same to other voters.

b) If the probability that voters are informed is sufficiently high, there is no pure strategy equilibrium because majority preferences experience a Condorcet cycle: a majority prefers to provide goods to no district rather than to all districts, and to provide to two districts over no district; a different majority prefers to provide the good to only one of those two districts instead of to both of them; and yet a different majority prefers to provide goods to all three districts instead of to just that one district, completing the cycle. Equilibria are in mixed strategies, and are ex-ante inefficient, since in expectation 43% of districts receive a unit of the inefficient public good.
These results hold whether candidates care only about winning the election, or also about the efficiency of the policies they propose and implement once in office.

The negative effect that an increase in information may induce on the social welfare is reinforced when we look at the case in which candidates care about efficiency. Undoubtedly in this case social welfare is not monotonically increasing in information: there is an intermediate range of information for which the unique pure strategy equilibrium is the efficient one in which no public good is provided. A higher level of information destroys this equilibrium, as each candidate is then always able to best respond to any pure strategy by the other candidate by crafting a proposal that is more beneficial to a simple majority of districts.

We therefore find that a more informed electorate can make every voter ex-ante worse off.

In a survey on the role of the media, Stromberg and Prat [2011] argue that an electorate that ignores what is the state of the world may become worse off if it gains information about candidates’ actions but not about outcomes, because it makes candidates’ pander by choosing actions that match the prior of the voter about the right action to take (Maskin and Tirole). In their framework, the electorate would always be better off learning about outcomes. We identify a novel channel by which information hurts the electorate: without any uncertainty about the state of the world, an informed electorate leads candidates to defeat the efficient policy by proposing inefficient policies designed to benefit a simple majority of districts, which ex-ante
makes every voter worse off.

These results have normative implications with regard to voter education: making the electorate fully informed does not suffice and can in fact harm the prospects of obtaining efficient policies from the electoral process. Citizens, or at least candidates, must care for efficiency in order for efficient outcomes to emerge in equilibria.

We considered several extensions to check the robustness of our results, available in detail from the authors: we study an election with candidates that are biased, favoring one districts so that they want this district’s project to be implemented. In this case, we find that the efficient outcome cannot be supported in equilibrium for any information level.

While the case with three districts that we have presented suffices to convey the intuition of the results in the clearest manner, results are robust to a society $N$ with an arbitrary odd number $n$ of districts, the only difference is that the relevant efficient cutoff is no longer 66.7cts of benefit per dollar of cost, but rather, $\frac{n+1}{2n}$ units of benefit per unit of cost, so that, for a large number of districts, we predict that projects that provide no more than 50cts of benefit on the dollar will not be provided, but on the other hand, projects that provide more than 51cts of benefit on the dollar will be provided at least in expectation, unless candidates are efficiency concerned and the electorate is not informed enough for candidates to craft policies that seek to benefit a majority at the expense of the electorate as a whole.

These sharp predictions can be easily tested in laboratory experiments, which we
will conduct in future research to assess the predictive success of our theory.

References


5 Appendix

It is useful to classify strategy pairs in classes of strategic equivalence, as follows:

Let \( S_1 = \{(s_1, s_1)\}; S_2 = \{(s_1, s_2), (s_1, s_3), (s_1, s_4), (s_2, s_1), (s_3, s_1), (s_4, s_1)\}; \)

\[ S_3 = \{(s_1, s_5), (s_1, s_6), (s_1, s_7), (s_5, s_1), (s_6, s_1), (s_7, s_1)\}; S_4 = \{(s_1, s_8), (s_8, s_1)\}; \]

\[ S_5 = \{(s_2, s_2), (s_3, s_3), (s_4, s_4)\}; S_6 = \{(s_2, s_3), (s_2, s_4), (s_3, s_4), (s_3, s_2), (s_4, s_2), (s_4, s_3)\}; \]

\[ S_7 = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), (s_5, s_2), (s_5, s_6), (s_5, s_3), (s_6, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}; \]

\[ S_8 = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), (s_7, s_2), (s_6, s_3), (s_5, s_4)\}; \]

\[ S_9 = \{(s_2, s_8), (s_3, s_8), (s_4, s_8), (s_8, s_2), (s_8, s_3), (s_8, s_4)\}; \]

\[ S_{10} = \{(s_5, s_5), (s_6, s_6), (s_7, s_7)\}; S_{11} = \{(s_5, s_6), (s_5, s_7), (s_6, s_5), (s_6, s_7), (s_5, s_7), (s_7, s_5)\}; \]

\[ S_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8), (s_8, s_5), (s_8, s_6), (s_8, s_7)\}; S_{13} = \{(s_8, s_8)\}. \]

Note that \( S = S^A \times S^B = \prod_{k=1}^{13} S_k \) is the set of all possible candidates’ pure strategy pairs. Within each class, it is without loss of generality to establish whether any one of the elements can or cannot be supported in equilibrium.
To simplify notation, and given that voters’ strategies are straightforward when full information is revealed, in all the analysis below we implicitly assume that if Nature fully reveals the policy proposals, voters vote according to their preferences and abstain when indifferent. This allows us to focus our analysis of voters on the branches of the game after Nature chooses not to reveal the full information so that voters face uncertainty.\footnote{We stress that this simplifies notation, but does not change our behavioral assumption that voters are fully strategic and rational. In the branches of the game with full information the voters’ decision problem can be solved by simple domination arguments, and we directly anticipate and impose the outcome that follows from the unique undominated solution.} For each voter $i \in \{a, b, c\}$, let $s^i : \{0, 1\} \times \{0, 1\} \rightarrow \{A, B, \emptyset\}$ be a behavioral strategy for voter $i$, which is a function that maps each information set of the voter when Nature does not reveal the policy proposals fully, into an action by the voter. A complete strategy for the voter specifies $s^i$, and the actions to be taken when information is fully revealed. We also express $s^i$ as a vector $s^i = (s^i_1, ..., s^i_4)$, where $s^i_k$ is the action chosen under the $k$-th information set according to the following order: $\{(0, 0), (0, 1), (1, 0), (1, 1)\}$.

It is also useful to define $v : S \rightarrow \{A, B, \emptyset\}^3$ as the list of votes by voters $\{a, b, c\}$ as a function of candidates’ strategies, given some beliefs. We stress that this function is only defined given beliefs, and we will only use it after specifying beliefs, or providing a strategy pair that the voters believe is being played.

The set of equilibria depend on the efficiency of the projects measured by $\beta$, and on how informed is the electorate, measured by $\varepsilon$. We focus in the following propositions
on the case in which \( \varepsilon \in (0, 1] \). All equilibria that exist for \( \varepsilon \in (0, \frac{1}{2}) \), also exist if \( \varepsilon = 0 \), but if \( \varepsilon = 0 \) we can sustain additional equilibria that do not seem very plausible, in which one candidate wins all votes with probability one. For instance, if \( \varepsilon = 0 \) there exists an equilibrium in which candidate A proposes the strategy \( s^A = (0, 0, 0) \) and candidate B proposes \( s^B = (1, 1, 1) \) and all voters vote for candidate A. This equilibrium is supported by voters’ beliefs that are pessimistic about candidate B’s proposals; namely if each voter \( i \) observes \( p^B_i = 0 \), \( i \) assigns probability one that \( p^B_k = 1 \) for both \( k \neq i \). If this is the case candidate B has no profitable deviation because even if B mimics A’s policy, B gets zero votes. These equilibria are fragile because they immediately disappears if \( \varepsilon > 0 \), because in this case by mimicking A and proposing \( \tilde{s}^B = (0, 0, 0) \) candidate B can at least tie the election with positive probability. This argument can be used to prove the following preliminary claim that holds both for office motivated and efficiency concerned candidates

**Lemma 4** If \( \varepsilon > 0 \), in any pure strategy equilibrium both candidates propose to carry out the same number of projects and the election is tied.

**Proof.** of Claim 4. Consider any strategy profile such that candidate J wins with probability less than \( \frac{1}{2} \). Since in equilibrium voters hold correct beliefs, the probability that J wins conditional on full information being revealed, or not revealed, is less than \( \frac{1}{2} \) in each case. In pure strategies, the probability of victory is in the set \( \{0, \frac{1}{2}, 1\} \) so if it is less than \( \frac{1}{2} \), it is zero. Deviating to \( s^J = s^{-J} \), candidate J ties the election if
full information is revealed, so the probability of winning is at least \( \frac{6}{7} \).

In the following Section we characterize the set of equilibria for all \( \beta \in \left[ \frac{1}{3}, 1 \right] \) and all pairs \((a_i, \alpha_j) \in \{0,1\}^2\). For sake of tractability we distinguish three cases: we characterize the set of equilibria first when both candidates are office motivated, second when both are also efficiency concerned and finally in the mixed case.

\section{Office Motivated Candidates}

\subsection{Pure strategy equilibria}

\textbf{Proposition 5} Assume \( \varepsilon \in \left( 0, \frac{1}{2} \right) \). For any \( \beta \in \left( \frac{2}{3}, 1 \right) \), an equilibrium in which candidates use the strategy pair \((s^A, s^B)\) exists if and only if \((s^A, s^B) \in S_k\) for some \(k \in \{1,11,13\}\). For any \( \beta \in \left( \frac{1}{3}, \frac{2}{3} \right) \), an equilibrium in which candidates use the strategy pair \((s^A, s^B)\) exists if and only if \((s^A, s^B) \in S_k\) for some \(k \in \{1,5,6,11,13\}\).

Assume \( \varepsilon \in \left( \frac{1}{2}, 1 \right) \). For any \( \beta \in \left( \frac{2}{3}, 1 \right) \) there is no equilibrium in pure strategies. For any \( \beta \in \left( \frac{1}{3}, \frac{2}{3} \right) \), there is a unique equilibrium such that candidates propose \(k = 0\).

\textbf{Proof.} We prove the high benefit case first.

\( S_1 \) : Voter strategy \( s^i = (\emptyset, A, B, \emptyset) \) for each voter \( i \) and beliefs such that \( \omega_{0}^{i,J}(0) = 1 \) and \( \omega_{2}^{i,J}(1) = 1 \) for any voter \( i \) and any candidate \( J \) make the election tied and if candidate \( J \) deviates to any \( s^J \neq s_1 \), then \( J \) loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters are correct along the equilibrium path, so
these strategies and beliefs are an equilibrium. If \( \varepsilon \leq \frac{1}{2} \), no gain when full information is revealed can compensate for this loss. Suppose now that \( \varepsilon > \frac{1}{2} \). If candidate A deviates and proposes \( s^A = s_5 \) she wins the election with probability \( \varepsilon \) and therefore the deviation is profitable.

\[ S_2 : \] Assume without loss of generality that \( (s^A, s^B) = (s_1, s_2) \). Given \( (s_1, s_2) \), \( v(s_1, s_2) = (B, A, A) \). By Lemma 4, this cannot occur in equilibrium.

\[ S_3 : \] Assume w.l.o.g. that \( (s^A, s^B) = (s_1, s_5) \). Given \( (s_1, s_5) \), \( v(s_1, s_5) = (B, B, A) \).

Ruled out by Lemma 4.

\[ S_4 : \] Assume w.l.o.g. that \( (s^A, s^B) = (s_1, s_8) \). Every voter votes for A. Ruled out by Lemma 4.

\[ S_5 : \] Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_2) \). Given \( (s_2, s_2) \), every voter abstains.

If candidate J deviates to \( s^J = s_8 \) wins the election both in case the information is revealed and in case it is not.

\[ S_6 : \] Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_3) \). Given \( (s_2, s_3) \), \( v(s_2, s_3) = (A, B, \emptyset) \).

If A deviates to \( s^A = s_6 \) voters \( a \) and \( c \) vote for A both in case full information is revealed and in case it is not revealed. Hence by deviating, A wins with for sure.

\[ S_7 : \] Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_5) \). Given \( (s_2, s_5) \), \( v(s_2, s_5) = (A, B, A) \).

Ruled out by Lemma 4.

\[ S_8 : \] Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_7) \). Given \( (s_2, s_7) \), \( v(s_2, s_7) = (A, B, B) \).

Ruled out by Lemma 4.

\[ S_9 : \] Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_8) \). Given \( (s_2, s_8) \), \( v(s_2, s_8) = (A, B, B) \).
Ruled out by Lemma 4.

$S_{10}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given $(s_5, s_5)$, all voters abstain.

Suppose first that $\varepsilon < \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_8$ and full information is not revealed, only voter $c$ observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate $A$ wins with probability at least $1 - \varepsilon > \frac{1}{2}$. If $\varepsilon = \frac{1}{2}$ then voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter $i$ and beliefs such that $\omega_i^J(0) = 1$ for $i = a, b$ and $J = A, B$ make the election tied and if candidate $J$ deviates to any $s^J \neq s_5$, then $J$ wins the election with probability at most $\frac{1}{2}$ and with complementary she looses the election. Hence no deviation is profitable. If $\varepsilon > \frac{1}{2}$ then candidate $A$ can win the election with probability $\varepsilon$ deviating to $s^A = s_2$.

$S_{11}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given $(s^A, s^B) = (s_5, s_6)$, beliefs such that $\omega_i^J(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ support an equilibrium in which $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. Suppose $\varepsilon \leq \frac{1}{2}$. It suffices to check that $A$ has no incentives to deviate. If $A$ deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, $A$ loses the election. If $A$ deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, $A$ loses the election. In any case, after a deviation $A$ wins the election with probability less than $\frac{1}{2}$. If $\varepsilon > \frac{1}{2}$ candidate $A$ can win the election with probability $\varepsilon$ deviating to $s^A = s_2$.

$S_{12}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_5, s_8)$, $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.
\(S_{13}:\) Voter strategy \(s^i = (\emptyset, B, A, \emptyset)\) for each voter \(i\) and beliefs such that \(\omega_i^{i,J}(0) = \omega_i^{i,J}(1) = 1\) for any voter \(i\) and any candidate \(J\) make the election tied. Suppose \(\varepsilon \leq \frac{1}{2}\); if candidate \(J\) deviates to any strategy \(s^J \neq s_8\) and full information is not revealed, \(J\) loses the election. Suppose \(\varepsilon > \frac{1}{2}\); candidate \(J\) wins the election probability \(\varepsilon\) deviating to \(s^J = s_1\).

Next we prove the low benefit case. To sustain equilibria, assume that off-equilibrium path beliefs in cases \(S_1, S_5, S_6, S_{11}\) and \(S_{13}\) are such that given the equilibrium proposal \(s^i, \omega_i^{i,J}(1 - s_i^J) = 1\) for each \(i \in \{a, b, c\}\) and \(J \in \{A, B\}\). That is, a voter who observe a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

\(S_1:\) Suppose \(\varepsilon \leq \frac{1}{2}\). Voter strategy \(s^i = (\emptyset, A, B, \emptyset)\) for each voter \(i\) and beliefs such that \(\omega_0^{i,J}(0) = 1\) and \(\omega_2^{i,J}(1) = 1\) for any voter \(i\) and any candidate \(J\) make the election tied and if candidate \(J\) deviates to any \(s^J \neq s_1\), then \(J\) loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If full information is revealed there is not a different proposal that can defeat \(s_1\). Hence there are no profitable deviations for all \(\varepsilon \in [0, 1]\).

\(S_2:\) Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_2)\). Given \((s_1, s_2)\), \(v(s_1, s_2) = (B, A, A)\). Ruled out by Lemma 4.

\(S_3:\) Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_5)\). Given \((s_1, s_5)\), \(v(s_1, s_5) = (A, A, A)\).
Ruled out by Lemma 4.

\( S_4 \): Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_8)\). Given \((s_1, s_5)\), \(v(s_1, s_5) = (A, A, A)\).

Ruled out by Lemma 4.

\( S_5 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_2)\). Given \((s_2, s_2)\), every voter abstains. If \(J\) deviates and full information is not revealed, any voter who observes the deviation votes for \(-J\) and \(J\) loses the election. If \(\varepsilon > \frac{1}{2}\) then if candidate \(J\) deviates and proposes \(s^J = s_1\) then she wins with probability \(\varepsilon\) and therefore the deviation is profitable.

\( S_6 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_3)\). Given \((s_2, s_3)\), \(v(s_2, s_3) = (A, B, \emptyset)\).

If \(J\) deviates and full information is not revealed, any voter who observes the deviation votes for \(-J\) and \(J\) loses the election. If \(\varepsilon > \frac{1}{2}\) then candidate \(A\) wins with probability \(\varepsilon\) deviating to \(s^A = s_1\)

\( S_7 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_5)\). Given \((s_2, s_5)\), \(v(s_2, s_5) = (A, B, A)\).

Ruled out by Lemma 4.

\( S_8 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_7)\). Given \((s_2, s_7)\), \(v(s_2, s_7) = (A, B, B)\).

Ruled out by Lemma 4.

\( S_9 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_8)\). All three voters vote for \(A\). Ruled out by Lemma 4.

\( S_{10} \): Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_5)\). Given \((s_5, s_5)\), all voters abstain. Suppose \(\varepsilon < \frac{1}{2}\). If candidate \(A\) deviates to \(s^A = s_8\) and full information is not revealed, only voter \(c\) observes the deviation and \(v(s_8, s_5) = (\emptyset, \emptyset, A)\). Hence by
deviating candidate A wins with probability at least $1 - \varepsilon > \frac{1}{2}$. Suppose $\varepsilon = \frac{1}{2}$, for any deviation played by candidate J, she wins with probability $\frac{1}{2}$ and looses with the same probability. Hence there is not a profitable deviation. If $\varepsilon > \frac{1}{2}$, candidate J wins when full information is revealed deviating to $s^J = s_1$.

$S_{11}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given $(s_5, s_6), v(s_5, s_6) = (\emptyset, A, B)$. Suppose $\varepsilon \leq \frac{1}{2}$. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Suppose $\varepsilon > \frac{1}{2}$ if candidate J deviates to $s^J = s_1$ she wins the election with probability $\varepsilon$.

$S_{12}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_5, s_8), v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

$S_{13}$: All voters abstain and the election is tied. Suppose $\varepsilon \leq \frac{1}{2}$. If candidate J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. Suppose $\varepsilon > \frac{1}{2}$; if candidate J deviates to $s^J = s_1$ she wins the election with probability $\varepsilon$.

This proposition shows that there is not a monotonic relationship between information and welfare if at least one candidate is efficiency concerned. If we start from a situation where voters are totally uninformed, $\varepsilon = 0$, the social welfare is increasing in $\varepsilon$ because the set of equilibria shrinks to equilibria that support a lower number of
projects the larger is ε, until reaching for ε large enough but lower than $\frac{1}{2}$ the efficient outcome as the unique equilibrium outcome. However, if the amount of information increases further then the unique equilibrium is in mixed strategy with a positive expected number of projects implemented.

5.1.2 Mixed strategy equilibria

We look at the set of mixed equilibria in case there are no equilibria in pure strategies. By the previous proposition this happens when $\varepsilon > \frac{1}{2}$ and $\beta \in (\frac{2}{3}, 1)$. We know from (Eguia and Nicolo 2011) that if $\varepsilon = 1$, the equilibrium strategy pair is $\sigma^A = \sigma^B = (1/7, ..., 1/7, 0)$. Let $s_L = (0, 1/3, 1/3, 1/3, 0, 0, 0, 0)$ and $s_H = (0, 0, 0, 0, 1/3, 1/3, 1/3, 0)$ be the special mixed strategies that consist, respectively, on proposing exactly one project and randomizing which one, and proposing exactly two projects and randomizing which two. The first proposition almost characterizes the set of mixed equilibria for $\varepsilon \in (\frac{1}{2}, \frac{3}{4})$

**Lemma 6** If $\beta \in (\frac{2}{3}, 1)$ and $\varepsilon \in (\frac{1}{2}, \frac{3}{4})$, candidate strategies $(s_H, s_H)$ are supported in equilibrium. Conversely, in any symmetric equilibrium, $\sigma^I_5 + \sigma^I_6 + \sigma^I_7 = 1$ for $J \in \{A, B\}$.

**Proof.** First we prove that $(s_H, s_H)$ can be supported in equilibrium. Suppose $(s_H, s_H)$ is played, for any candidate $J$ and voter $i$, if $i$ does not observe the full proposals, beliefs are such that $\omega^{i,J}_1(1) = \omega^{i,J}_2(0) = 1$ and hence $s^i(1, 0) = A, s^i(0, 1) = B$.
and \( s^i(p^A_i, p^B_i) = \emptyset \) if \( p^A_i = p^B_i \). Therefore, given that \((s_H, s_H)\) is played as expected by voters, either all voters abstain if \( p^A = p^B \) or one voter votes for \( A \), one for \( B \) and one abstains if \( p^A \neq p^B \); in either case the election is tied and the probability that each candidate is elected is \( \frac{1}{2} \), so the expected payoff for each candidate is \( \frac{1}{2} \).

Suppose \( A \) deviates to \( s_1 \), \( A \) loses \( 0 - 2 \) if Nature does not reveal \( p \), and \( A \) loses \( 1 - 2 \) if Nature reveals \( p \). Suppose \( A \) deviates to \( s_k \in \{ s_2, s_3, s_4 \} \). If Nature reveals \( p \), with probability \( \frac{2}{3} \) \( A \) wins and with probability \( \frac{1}{3} \) \( A \) loses; while if \( p \) is not revealed, \( A \) loses for sure. Hence, \( A \) wins with probability \( \frac{2}{3} \), the expected utility deviating is \( \frac{2}{3} \epsilon \) and the deviation is profitable if and only if \( \frac{2}{3} \epsilon > \frac{1}{2} \), that is, \( \epsilon > \frac{3}{4} \). Suppose \( A \) deviates to \( s_k \in \{ s_5, s_6, s_7 \} \). Then \( A \) achieves the same expected electoral outcomes and utilities as not deviating. Suppose \( A \) deviates to \( s_8 \). If Nature reveals \( p \), \( A \) loses \( 1 - 2 \). If Nature does not reveal \( p \), then \( A \) wins \( 2 - 1 \). But Nature reveals \( p \) with probability \( \epsilon \), hence the expected utility for \( A \) is \( 1 - \epsilon \) which is less than \( \frac{1}{2} \) for any \( \epsilon > \frac{1}{2} \). Hence, there is no profitable deviation.

Second, we prove that in any symmetric equilibrium both candidates propose two projects.

Suppose \( (\sigma^J, \sigma^J) \) is part of a symmetric equilibrium. Since the equilibrium is symmetric, for any \( i \in \{ a, b, c \} \), if \( i \) does not observe \( p \), then \( s^i(p^A_i, p^B_i) = \emptyset \) if \( p^A_i = p^B_i \) and furthermore, for \( k \in \{ 0, 1 \} \), \( s^i(k, 1 - k) = A \) if and only if \( s^i(1 - k, k) = B \). Suppose \( s^i(1, 0) \neq A \), so \( s^i(0, 1) \neq B \). Then \( s_8 \) is not a best response and it is not played in equilibrium. But if \( s_8 \) is not played, the expected payoff for \( i \) if \( A \) wins is strictly higher.
than if $B$ wins given $(p_i^A, p_i^B) = (1, 0)$, so by assumption $s'(1, 0) = A$, a contradiction. Thus, it must be $s'(1, 0) = A$ and $s'(0, 1) = B$. Then, given that $\varepsilon \in (\frac{1}{2}, \frac{3}{4})$, for any strategy $\sigma^{-J}$, a best response by $J$ must propose two projects. Thus no strategy that proposes any other number of projects can be used in a symmetric equilibrium. \[\blacksquare\]

**Proposition 7** If $\beta \in (\frac{2}{3}, 1)$ and $\varepsilon \in [\frac{3}{4}, \frac{11+\sqrt{61}}{20}]$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (0, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{4\varepsilon-3}{10\varepsilon-3})$. The expected number of projects is weakly larger than $\frac{5}{3}$ and converges to $\frac{5}{3}$ as $\varepsilon \to \frac{3}{4}$.

If $\beta \in (\frac{2}{3}, 1)$ and $\varepsilon \in [\frac{11+\sqrt{61}}{20}, 1)$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (\frac{4\varepsilon-3}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, \frac{2\varepsilon-1}{10\varepsilon-3}, 0)$. The expected number of projects is weakly larger than $\frac{9}{7}$ and converges to $\frac{9}{7}$ as $\varepsilon \to 1$.

**Proof.** Let $(\sigma^A, \sigma^B)$ be a symmetric candidate strategy profile so $\sigma^A = \sigma^B$. Since the candidates’ strategies are symmetric, voters’ strategies must be such that $s^i(k, k) = \emptyset$ for $k \in \{0, 1\}$, and $\{s^i(1, 0) = A \text{ and } s^i(0, 1) = B\}$ or $\{s^i(1, 0) = B \text{ and } s^i(0, 1) = A\}$ or $\{s^i(1, 0) = \emptyset \text{ and } s^i(0, 1) = \emptyset\}$ for every voter $i$. Suppose not $\{s^i(1, 0) = A \text{ and } s^i(0, 1) = B\}$. Then, given any strategy $\sigma^{-J}$, candidate $J$ obtains a greater expected payoff playing $s_1$ than playing $s_8$, and a strictly greater payoff if $\sigma^{-J}_8 > 0$. Therefore, in a symmetric mixed equilibrium, $\sigma^J_8 = 0$. Then, it follows that for any voter $i$ who observes $p^J_i = 1$ and $p_i^{-J} = 0$, the expected payoff for voter $i$ is greater if $J$ wins, thus by assumption, $i$ votes $J$. Therefore, $s^i(1, 0) = A$ and $s^i(0, 1) = B$. 

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Next we prove that in any symmetric mixed strategy equilibrium, \( \sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} > 0 \). Suppose not. Notice that given \( \varepsilon > \frac{3}{4} \), \( s^i(1, 0) = A \) and \( s^i(0, 1) = B \), if \(-J\) proposes one project and \(J\) proposes two projects, in expectation \(J\) wins the election more often, whereas if \(J\) proposes two and \(-J\) proposes zero or three, \(J\) wins more often. So if \(-J\) never proposes one project, proposing two projects in expectation defeats any other proposal with probability more than one half. Then any best response by \(J\) to \(\sigma^{-J}\) with \(\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} = 0\) must be such that \(\sigma_5^J + \sigma_6^J + \sigma_7^J = 1\), which in turns means that any best response by \(J\) must be such that \(\sigma_2^J + \sigma_3^J + \sigma_4^J = 1\), a contradiction.

Similarly, in any symmetric mixed strategy equilibrium, \(\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} > 0\). Suppose not. Any best response by \(J\) must be such that \(\sigma_2^J + \sigma_3^J + \sigma_4^J = 0\), but then the best response by \(-J\) must be \(\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} = 1\), a contradiction.

Therefore, in any symmetric mixed strategy equilibrium, both candidates propose one project, and two projects, with positive probability. But then, it must be that \(\sigma_2^J = \sigma_3^J + \sigma_4^J\) and \(\sigma_5^J = \sigma_6^J = \sigma_7^J\). Given that the randomization between districts, subject to choosing a number of projects, assigns equal weight to all districts, we can reduce the strategic problem to that of assigning weights to strategies \(s_1, s_8, s_L, s_H\). The payoff matrix is as follows:
\[
\begin{pmatrix}
    s_1 & s_L & s_H & s_8 \\
    s_1 & \frac{1}{2}, \frac{1}{2} & \varepsilon, 1 - \varepsilon & 0, 1 & \varepsilon, 1 - \varepsilon \\
    s_L & 1 - \varepsilon, \varepsilon & \frac{1}{2}, \frac{1}{2} & \frac{2}{3} \varepsilon, 1 - \frac{2}{3} \varepsilon & 0, 1 \\
    s_H & 1, 0 & 1 - \frac{2}{3} \varepsilon, \frac{2}{3} \varepsilon & \frac{1}{2}, \frac{1}{2} & \varepsilon, 1 - \varepsilon \\
    s_8 & 1 - \varepsilon, \varepsilon & 1, 0 & 1 - \varepsilon, \varepsilon & \frac{1}{2}, \frac{1}{2}
\end{pmatrix}
\]

An equilibrium with \( \sigma_1^I > 0, \sigma_L^I > 0, \sigma_H^I > 0 \) and \( \sigma_8^I = 0 \) must satisfy

\[i) \ \frac{1}{2} \sigma_1^I + \varepsilon \sigma_L^I = (1 - \varepsilon) \sigma_1^I + \frac{1}{2} \sigma_L^I + \frac{2}{3} \varepsilon (1 - \sigma_1^I - \sigma_L^I)\]

\[ii) \ \frac{1}{2} \sigma_1^I + \varepsilon \sigma_L^I = \sigma_1^I + (1 - \frac{2}{3} \varepsilon) \sigma_L^I + \frac{1}{3} (1 - \sigma_1^I - \sigma_L^I)\]

\[iii) \ \frac{1}{2} \sigma_1^I + \varepsilon \sigma_L^I \geq (1 - \varepsilon) \sigma_1^I + \sigma_L^I + (1 - \varepsilon) (1 - \sigma_1^I - \sigma_L^I).\]

Dropping the superindex and simplifying \( i) \) we obtain

\[0 = \frac{2}{3} \varepsilon + \frac{1}{2} \sigma_1 + \frac{1}{2} \sigma_L - \frac{5}{3} \varepsilon \sigma_1 - \frac{5}{3} \varepsilon \sigma_L,\]

\[(10 \varepsilon - 3) \sigma_1 = 4 \varepsilon + (3 - 10 \varepsilon) \sigma_L\]

\[\sigma_1 = \frac{4 \varepsilon}{10 \varepsilon - 3} - \sigma_L.\]

Dropping the superindex and simplifying \( ii) \) we obtain \( \varepsilon \sigma_L = \frac{1}{2} \sigma_L - \frac{2}{3} \varepsilon \sigma_L + \frac{1}{2} \) and with simple computation

\[\sigma_L = \frac{3}{10 \varepsilon - 3}.\]

Thus

\[\sigma_1 = \frac{4 \varepsilon - 3}{10 \varepsilon - 3}\]
Then simplifying inequality iii):

\[
\frac{1}{2}\sigma_1 + \varepsilon \sigma_L \geq \sigma_1 - \varepsilon \sigma_1 + \sigma_L + 1 - \sigma_1 - \sigma_L - \varepsilon + \varepsilon \sigma_1 + \varepsilon \sigma_L
\]

\[
\frac{1}{2} \sigma_1 \geq 1 - \varepsilon
\]

and substituting the value of \( \sigma_1 \):

\[
\frac{4\varepsilon - 3}{20\varepsilon - 6} \geq 1 - \varepsilon
\]

The previous expression holds as an equality for \( \varepsilon = \frac{11 + \sqrt{61}}{20} \) (for \( \varepsilon \in (\frac{2}{3}, 1) \)). So for \( \varepsilon > \frac{11 + \sqrt{61}}{20} \), the above mix is an equilibrium, with expected number of projects

\[
\frac{3}{10\varepsilon - 3} + 2(1 - \frac{3}{10\varepsilon - 3} - \frac{4\varepsilon - 3}{10\varepsilon - 3}) = \frac{12\varepsilon - 3}{10\varepsilon - 3}.
\]

The probability of proposing two projects is

\[
\frac{10\varepsilon - 3 - 4\varepsilon}{10\varepsilon - 3} = \frac{6\varepsilon - 3}{10\varepsilon - 3}
\]

which converges to \( \frac{9}{7} \) as \( \varepsilon \) converges to 1. The initial assumption that voters vote \( s^i(1, 0) = A \) and \( s^i(0, 1) = B \) is supported because \( \sigma_8 = 0 \) and \( \beta > 2/3 \).

If instead \( \varepsilon < \frac{11 + \sqrt{61}}{20} \), this equilibrium does not exist. We look then for a fully mixed equilibrium, which must satisfy:

\( i \)

\[
\frac{1}{2}\sigma_1 + \varepsilon \sigma_L + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H) = (1 - \varepsilon)\sigma_1 + \frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon \sigma_H
\]

\( ii \)

\[
\frac{1}{2}\sigma_1 + \varepsilon \sigma_L + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H) = \sigma_1 + (1 - \frac{2}{3})\sigma_L + \frac{1}{2}\sigma_H + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H)
\]

\( iii \)

\[
\frac{1}{2}\sigma_1 + \varepsilon \sigma_L + \varepsilon(1 - \sigma_1 - \sigma_L - \sigma_H) = (1 - \varepsilon)\sigma_1 + \sigma_L + (1 - \varepsilon)\sigma_H + \frac{1}{2}(1 - \sigma_1 - \sigma_L - \sigma_H)
\]

Simplifying the first equation
\[ \varepsilon = \frac{1}{2}\sigma_1 + \frac{1}{2}\sigma_L + \frac{5}{3}\varepsilon\sigma_H \]

i) \[ 3\sigma_1 + 3\sigma_L + 10\varepsilon\sigma_H = 6\varepsilon \]

Simplifying the second equation

\[ \varepsilon\sigma_L = \frac{1}{2}\sigma_1 + (1 - \frac{2\varepsilon}{3})\sigma_L + \frac{1}{2}\sigma_H \]

\[ 0 = \frac{1}{2}\sigma_1 + (1 - \frac{5\varepsilon}{3})\sigma_L + \frac{1}{2}\sigma_H \]

ii) \[ 3\sigma_1 + (6 - 10\varepsilon)\sigma_L + 3\sigma_H = 0 \]

Simplifying the third equation

\[ \varepsilon = \frac{1}{2}\sigma_L + \frac{1}{2}\sigma_H + \frac{1}{2} \]

iii) \[ \sigma_L + \sigma_H = 2\varepsilon - 1 \]

From \( iii) \)

\[ \sigma_H = 2\varepsilon - 1 - \sigma_L. \]

From \( i) - ii) \)

\[ 3\sigma_L + 10\varepsilon\sigma_H - (6 - 10\varepsilon)\sigma_L - 3\sigma_H = 6\varepsilon \]

\[ (10\varepsilon - 3)(\sigma_H + \sigma_L) = 6\varepsilon \]

\[ \sigma_H = \frac{6\varepsilon}{10\varepsilon - 3} - \sigma_L. \]
The two equalities together imply that

\[
\frac{6\varepsilon}{10\varepsilon - 3} = 2\varepsilon - 1 \\
\varepsilon = \frac{1}{20}\sqrt{61} + \frac{11}{20}.
\]

Which means that the fully mixed equilibrium is non-generic. Consider equilibria such that \(\sigma_1 = 0\), so that candidates mix between proposing one, two and three projects. An equilibrium with these characteristics requires:

\[
\begin{bmatrix}
\frac{1}{2}, \frac{1}{2} & \varepsilon, 1 - \varepsilon & 0, 1 & \varepsilon, 1 - \varepsilon \\
1 - \varepsilon, \varepsilon & \frac{1}{2}, \frac{1}{2} & \frac{2}{3}\varepsilon, 1 - \frac{2\varepsilon}{3} & 0, 1 \\
1, 0 & 1 - \frac{2\varepsilon}{3}, \frac{2\varepsilon}{3} & \frac{1}{2}, \frac{1}{2} & \varepsilon, 1 - \varepsilon \\
1 - \varepsilon, \varepsilon & 1, 0 & 1 - \varepsilon, \varepsilon & \frac{1}{2}, \frac{1}{2}
\end{bmatrix}
\]

i) \(\varepsilon\sigma_L + \varepsilon(1 - \sigma_L - \sigma_H) \leq \frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon\sigma_H\)

ii) \(\frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon\sigma_H = (1 - \frac{2\varepsilon}{3})\sigma_L + \frac{1}{2}\sigma_H + \varepsilon(1 - \sigma_L - \sigma_H)\)

iii) \(\frac{1}{2}\sigma_L + \frac{2}{3}\varepsilon\sigma_H = \sigma_L + (1 - \varepsilon)\sigma_H + \frac{1}{2}(1 - \sigma_L - \sigma_H)\).

Simplifying the three expressions:

i) \(\varepsilon \leq \frac{1}{2}\sigma_L + \frac{5}{3}\varepsilon\sigma_H\)

ii) \((\frac{5\varepsilon}{3} - \frac{1}{2})(\sigma_L + \sigma_H) = \varepsilon\)

iii) \((\frac{5}{3}\varepsilon - \frac{1}{2})\sigma_H = \frac{1}{2} \).

From iii) and ii)

\((\frac{5\varepsilon}{3} - \frac{1}{2})\sigma_L = \varepsilon - \frac{1}{2}\), so \(\sigma_L = \frac{6\varepsilon - 3}{10\varepsilon - 3}\) and \(\sigma_H = \frac{3}{10\varepsilon - 3}\) and thus \(\sigma_8 = \frac{4\varepsilon - 3}{10\varepsilon - 3}\).
We then check that the first inequality is satisfied:

\[
\varepsilon \leq \frac{1}{2} \frac{6\varepsilon - 3}{10\varepsilon - 3} + \frac{5\varepsilon}{3} \frac{3}{10\varepsilon - 3}
\]

\[
2\varepsilon (10\varepsilon - 3) \leq 6\varepsilon - 3 + 10\varepsilon
\]

\[
20\varepsilon^2 - 22\varepsilon + 3 \leq 0
\]

which is true for \(\varepsilon \in \left(\frac{3}{4}, \frac{111 + \sqrt{61}}{20}\right)\). The expected number of projects in this equilibrium is \(\frac{18\varepsilon - 6}{10\varepsilon - 3}\) which converges to \(\frac{5}{3}\) as \(\varepsilon \to \frac{3}{4}\) and then increases but stays below \(\frac{7}{4}\). Finally, there cannot be a symmetric mixed strategy equilibrium such that \(\sigma_1 = \sigma_8 = 0\), because if \(\sum_{k=2}^{7} \sigma_k^J = 1\), any best response by \(-J\) must be such that \(\sigma_5^J + \sigma_6^J + \sigma_7^J = 1\), which, as shown above, cannot hold in a symmetric mixed strategy equilibrium with \(\varepsilon > \frac{3}{4}\). ■

### 5.2 Efficiency Concerned Candidates

#### 5.2.1 Pure strategy equilibria

**Proof.** As before, to sustain equilibria, we assume off-equilibrium path beliefs such that given the equilibrium proposal \(s_i^J, \omega_i^J(1 - s_i^J) = 1\) for each \(i \in \{a, b, c\}\) and \(J \in \{A, B\}\). That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts. We prove the high benefit case first.
$S_1$: Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter $i$ and beliefs such that $\omega_{i,J}^0(0) = 1$ and $\omega_{i,J}^2(1) = 1$ for any voter $i$ and any candidate $J$ make the election tied and if candidate $J$ deviates to any $s^J \neq s_1$, then $J$ loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Hence when $\varepsilon < \frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss. Since candidates are efficiency concerned and any deviation implies a welfare loss, they have not incentive to deviate. Suppose that $\varepsilon \geq \frac{1}{2}$. If candidate $J$ deviates to $s^J \in \{s_2, s_3, s_4, s_8\}$ loses the election and makes a more inefficient proposal. If candidate $J$ deviates to $s^J = s_5$ (or to $s^J = s_6$, or $s^J = s_7$) wins the election with probability $\varepsilon$ (when full information is revealed) and therefore the deviation is profitable if and only if

$$
\frac{\varepsilon}{1 + 2(1 - \beta)} > \frac{1}{2}, \text{ or } \\
\varepsilon > \frac{3 - 2\beta}{2}.
$$

$S_2$: Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given $(s_1, s_2)$, $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

$S_3$: Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given $(s_1, s_5)$, $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.
\[ S_4: \text{Assume w.l.o.g. that } (s^A, s^B) = (s_1, s_8). \text{ Given beliefs such that } \omega^i_{2B}(0) = 1 \text{ for all } i \in \{a, b, c\}, \text{ every voter votes for } A. \text{ Ruled out by Lemma 4.} \]

\[ S_5: \text{Assume w.l.o.g. that } (s^A, s^B) = (s_2, s_2). \text{ Given } (s_2, s_2), \text{ every voter } i \text{ abstains. If candidate } A \text{ deviates to } s^A = s_8, \text{ then candidate } A \text{ wins the election. The deviation is profitable if only if} \]

\[
\frac{1}{1 + 3(1 - \beta)} > \frac{1}{2} \frac{1}{1 + (1 - \beta)}
\]

which holds for all \( \beta \in (0, 1) \).

\[ S_6: \text{Assume w.l.o.g. that } (s^A, s^B) = (s_2, s_3). \text{ If candidate } A \text{ deviates to } s^A = s_6, \text{ then candidate } A \text{ wins the election. The deviation is profitable since for all } \beta, \text{ we have that} \]

\[
\frac{1}{1 + 2(1 - \beta)} > \frac{1}{2} \frac{1}{1 + (1 - \beta)}
\]

\[ S_7: \text{Assume w.l.o.g. that } (s^A, s^B) = (s_2, s_5). \text{ Given } (s_2, s_5), v(s_2, s_5) = (A, B, A). \text{ Ruled out by Lemma 4.} \]

\[ S_8: \text{Assume w.l.o.g. that } (s^A, s^B) = (s_2, s_7). \text{ Given } (s_2, s_7), v(s_2, s_7) = (A, B, B). \text{ Ruled out by Lemma 4.} \]

\[ S_9: \text{Assume w.l.o.g. that } (s^A, s^B) = (s_2, s_8). \text{ Given } (s_2, s_8), v(s_2, s_8) = (A, B, B). \text{ Ruled out by Lemma 4.} \]

\[ S_{10}: \text{Assume w.l.o.g. that } (s^A, s^B) = (s_5, s_5). \text{ Consider the deviation } s^J = s_8. \text{ If information is not fully revealed candidate } J \text{ wins the election because voter } c \text{ votes for } J. \text{ If information is fully revealed candidate } J \text{ loses the election. Hence, candidate} \]
J prefers to deviate if and only if

\[(1 - \varepsilon) \frac{1}{1 + 3(1 - \beta)} > \frac{1}{2} \frac{1}{1 + 2(1 - \beta)}\]

\[\varepsilon < \frac{2 - \beta}{6 - 4\beta}\]

Consider the deviation \(s^J = s_2\). If information is not fully revealed candidate J loses the election because voter \(b\) votes for \(J\). If information is fully revealed candidate \(J\) wins the election. Hence, candidate \(J\) prefers to deviate if and only if

\[\varepsilon \frac{1}{1 + (1 - \beta)} > \frac{1}{2} \frac{1}{1 + 2(1 - \beta)}, \text{ or} \]

\[\varepsilon > \frac{2 - \beta}{6 - 4\beta}\]

Hence there is always profitable deviation for all \(\varepsilon \neq \frac{2 - \beta}{6 - 4\beta}\).

\(S_{11}\) : Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_6)\). If candidate \(J\) deviates to \(s^J \in \{s_1, s_2, s_3, s_4, s_6, s_7\}\) and full information is not revealed, \(J\) loses the election. If \(J\) deviates to \(s^J = s_8\) and full information is not revealed, the election is tied, but if full information is revealed, \(J\) loses the election. It follows that the best deviation for an efficiency concerned candidate is \(s_2\) since candidate \(J\) wins the election when information is fully revealed and minimizes the number of proposed projects (if \(J\) proposes \(s_1\) she loses the election). Candidate \(J\) prefers to deviate to \(s^J = s_2\) if only
if

$$\varepsilon \frac{1}{1 + (1 - \beta)} > \frac{1}{2} \frac{1}{1 + 2(1 - \beta)}; \text{ or}$$

$$\varepsilon > \frac{2 - \beta}{6 - 4\beta}. \tag{4}$$

$S_{12}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_5, s_8)$, $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

$S_{13}$: If candidate $J$ deviates to any strategy $s^J \neq s_8$ and full information is not revealed, $J$ loses the election. Hence the best deviation for an efficiency concerned candidate is $s_1$ because $J$ wins the election when information is fully revealed and the proposal is efficient. Candidate $J$ prefers to deviate to $s_1$ if only if

$$\varepsilon > \frac{1}{8 - 6\beta}. \tag{5}$$

Next we prove the low benefit case.

$S_1$: Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter $i$ and beliefs such that $\omega^i_{0,J}(0) = 1$ and $\omega^i_{2,J}(1) = 1$ for any voter $i$ and any candidate $J$ make the election tied and if candidate $J$ deviates to any $s^J \neq s_1$, then $J$ loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If full informa-
tion is revealed there is not a different proposal that can defeat $s_1$. Hence there are no profitable deviation for all $\varepsilon \in [0, 1]$.

$S_2$: Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given $(s_1, s_2)$, $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

$S_3$: Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given $(s_1, s_5)$, $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

$S_4$: Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given $(s_1, s_8)$, $v(s_1, s_8) = (A, A, A)$. Ruled out by Lemma 4.

$S_5$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. If candidate $J$ deviates, she loses the election when information is not fully revealed. Hence the most profitable deviation is $s^J = s_1$ because $J$ wins the election when information is fully revealed and the proposal is efficient. Candidate $A$ prefers to deviate if and only if

$$\varepsilon > \frac{1}{4 - 2\beta}.$$  

$S_6$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given voters’ beliefs, $v(s_2, s_3) = (A, B, \emptyset)$. If $J$ deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and $J$ loses the election. For the same argument as above, the best deviation for an efficiency concerned candidate $J$ is $s^J = s_1$. Candidate $J$ prefers to deviate if and only if

$$\varepsilon > \frac{1}{4 - 2\beta}.$$
$S_7$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given $(s_2, s_5)$, $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

$S_8$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given $(s_2, s_7)$, $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

$S_9$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for $A$. Ruled out by Lemma 4.

$S_{10}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given $(s_5, s_5)$, all voters abstain.

If candidate $J$ deviates to $s^J = s_8$ and full information is not revealed, only voter $c$ observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate $J$ wins with probability $1 - \varepsilon$. Candidate $J$ prefers to deviate if and only if

\[
(1 - \varepsilon) \frac{1}{1 + 3(1 - \beta)} > \frac{1}{2} \frac{1}{1 + 2(1 - \beta)}
\]

\[
\varepsilon < \frac{2 + \beta}{6 - 2\beta}
\]

Consider the deviation $s^J = s_1$. Candidate $J$ only wins when information is fully revealed, and therefore the deviation is profitable if and only if

\[
\varepsilon > \frac{1}{2} \frac{1}{1 + 2(1 - \beta)}, \text{ or} \quad \varepsilon > \frac{1}{6 - 4\beta}
\]

Since $\frac{1}{6 - 4\beta} < \frac{2 + \beta}{6 - 2\beta}$ there is always a profitable deviation.
$S_{11}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given $(s_5, s_6), v(s_5, s_6) = (\emptyset, A, B)$. If $J$ deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, $J$ loses the election. If $J$ deviates to $s^J = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, $J$ loses the election. It follows that the most profitable deviation is $s^J = s_1$ since candidate $J$ wins the election when information is fully revealed and the proposal is efficient. Candidate $J$ prefers to deviate to $s_1$ if and only if

$$\varepsilon > \frac{1}{6 - 4\beta}.$$ $S_{12}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_5, s_8), v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

$S_{13}$: All voters abstain and the election is tied. If candidate $J$ deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and $J$ loses the election. If candidate $J$ deviates $s^J = s_1$, she wins the election when information is fully revealed, and this proposal is efficient. Therefore candidate $J$ prefers to deviate to $s_1$ if and only if

$$\varepsilon > \frac{1}{8 - 6\beta}.$$ (6)
5.2.2 Mixed strategy Equilibria

We look at the set of mixed equilibria when pure strategy equilibria do not exist. We know that if \( \beta > \frac{2}{3} \) and \( \varepsilon > \frac{3}{2} - \beta \) there exists no pure strategy equilibrium, and otherwise there exists at least one. Let \( s_L = (0, 1/3, 1/3, 1/3, 0, 0, 0) \) and \( s_H = (0, 0, 0, 1/3, 1/3, 1/3, 0) \) be the special mixed strategies that consist, respectively, on proposing exactly one project and randomizing which one, and proposing exactly two projects and randomizing which two. To characterize the set of mixed equilibria when candidates are efficiency concerned, is far from being trivial. The following two propositions provide a sufficiently detailed picture of the set of mixed equilibria in the area of parameters value \((\varepsilon, \beta)\) where a pure strategy equilibrium does not exist.

**Proposition 8** If \( \varepsilon \in \left( \frac{3}{2} - \beta, \frac{3}{2} - \frac{2-\beta}{1-2\beta} \right) \), candidate strategies \((s_H, s_H)\) are supported in equilibrium. Conversely, in any symmetric equilibrium, \( \sigma^J_5 + \sigma^J_6 + \sigma^J_7 = 1 \) for \( J \in \{A, B\} \).

**Proof.** First we prove that \((s_H, s_H)\) can be supported in equilibrium. Given \((s_H, s_H)\) is played, for any candidate \( J \) and voter \( i \), if \( i \) does not observe the full proposals, beliefs are such that \( \omega^i_{1,J}(1) = \omega^i_{2,J}(0) = 1 \) and hence \( s^i(1,0) = A \), \( s^i(0,1) = B \) and \( s^i(p^A_i, p^B_i) = \emptyset \) if \( p^A_i = p^B_i \). Therefore, given that \((s_H, s_H)\) is played as expected by voters, either all voters abstain if \( p^A = p^B \) or one voter votes for \( A \), one for \( B \) and one abstains if \( p^A \neq p^B \); in either case the election is tied and the probability that each candidate is elected is \( \frac{1}{2} \), so the expected payoff for each candidate is \( \frac{1}{2} \frac{1}{3} \frac{1}{2} \).
Suppose \( A \) deviates to \( s_1 \), \( A \) loses 0\(-2\) if Nature does not reveal \( p \), and \( A \) loses 1\(-2\) if Nature reveals \( p \). Suppose \( A \) deviates to \( s_k \in \{ s_2, s_3, s_4 \} \). If Nature reveals \( p \), with probability \( \frac{2}{3} \) \( A \) wins and with probability \( \frac{1}{3} \) \( A \) loses; while if \( p \) is not revealed, \( A \) loses for sure. Hence, \( A \) wins with probability \( \frac{2}{3} \), the expected utility deviating is \( \frac{2}{3} \); and the deviation is profitable if and only if \( \frac{2}{3} > \frac{1}{3} \), that is, \( \varepsilon > \frac{3}{4} \).

Suppose \( A \) deviates to \( s_k \in \{ s_5, s_6, s_7 \} \). Then \( A \) achieves the same expected electoral outcomes and utilities as not deviating. Suppose \( A \) deviates to \( s_8 \). If Nature reveals \( p \), \( A \) loses 1\(-2\). If Nature does not reveal \( p \), then \( A \) wins 2\(-1\). But Nature reveals \( p \) with probability \( \varepsilon \), hence the expected utility for \( A \) is \( (1 - \varepsilon) \frac{1}{4} \) which is less than \( \frac{1}{2} \) for any \( \varepsilon > \frac{1}{2} \). Hence, there is no profitable deviation.

Second, we prove that in any symmetric equilibrium both candidates propose two projects.

Suppose \((\sigma^J, \sigma^J)\) is part of a symmetric equilibrium. Since the equilibrium is symmetric, for any \( i \in \{ a, b, c \} \), if \( i \) does not observe \( p \), then \( s^i(p^A_i, p^B_i) = \emptyset \) if \( p^A_i = p^B_i \) and furthermore, for \( k \in \{ 0, 1 \} \), \( s^i(k, 1 - k) = A \) if and only if \( s^i(1 - k, k) = B \).

Suppose \( s^i(1, 0) \neq A \), so \( s^i(0, 1) \neq B \). Then \( s_8 \) is not a best response and it is not played in equilibrium. But if \( s_8 \) is not played, the expected payoff for \( i \) if \( A \) wins is strictly higher than if \( B \) wins given \((p^A_i, p^B_i) = (1, 0)\), so by assumption \( s^i(1, 0) = A \), a contradiction. Thus, it must be \( s^i(1, 0) = A \) and \( s^i(0, 1) = B \). Then, given that \( \varepsilon \in (\frac{3}{2} - \beta, \frac{3}{2} \), for any strategy \( \sigma^{-J} \), a best response by \( J \) must propose two projects. Thus no strategy that proposes any other number of projects can be used.
in a symmetric equilibrium. ■

**Claim 9** There exists an increasing function \( \varepsilon(\beta) \) such that

if \( \varepsilon \in \left( \max\left\{ \frac{3}{2} - \beta, \frac{3}{4} \frac{2 - \beta}{3 \beta} \right\}, \varepsilon(\beta) \right) \), there is a unique symmetric mixed strategy equilibrium, in which \( \sigma^J = (\alpha_1, \alpha_1, \alpha_1, \alpha_2, \alpha_2, 1 - 3\alpha_1 - 3\alpha_2) \) and the expected number of projects is \( \frac{18\varepsilon - 6}{10\varepsilon - 3} \); and

if \( \varepsilon \in (\varepsilon(\beta), 1) \), there is a unique symmetric mixed strategy equilibrium, in which \( \sigma^J = (0, \beta_1, \beta_1, \beta_1, \beta_2, \beta_2, 1 - 3\beta_1 - 3\beta_2) \) and the expected number of projects is \( \frac{12\varepsilon - 3}{10\varepsilon - 3} \geq \frac{9}{7} \) and which converges to \( \frac{9}{7} \) as \( \varepsilon \to 1 \).

We find the exact functional form of \( \varepsilon(\beta) \) and the weights of the mixed strategies as part of the proof.

**Proof.** Let \((\sigma^A, \sigma^B)\) be a symmetric candidate strategy profile so \( \sigma^A = \sigma^B \). Since the candidates’ strategies are symmetric, voters’ strategies must be such that \( s^i(k, k) = \emptyset \) for \( k \in \{0, 1\} \), and \{\( s^i(1, 0) = A \) and \( s^i(0, 1) = B \)\} or \{\( s^i(1, 0) = B \) and \( s^i(0, 1) = A \)\} or \{\( s^i(1, 0) = \emptyset \) and \( s^i(0, 1) = \emptyset \)\} for every voter \( i \). Suppose not \{\( s^i(1, 0) = A \) and \( s^i(0, 1) = B \)\}. Then given any strategy \( \sigma^{-J} \), candidate \( J \) obtains a greater expected payoff playing \( s_1 \) than playing \( s_8 \), and a strictly greater payoff if \( \sigma^{-J}_8 > 0 \). Therefore, in a symmetric mixed equilibrium, \( \sigma^J_8 = 0 \). Then, it follows that for any voter \( i \) who observes \( p^J_i = 1 \) and \( p^{-J}_i = 0 \), the expected payoff for voter \( i \) is greater if \( J \) wins, thus by assumption, \( i \) votes \( J \). Therefore, \( s^i(1, 0) = A \) and \( s^i(0, 1) = B \).
Next we prove that in any symmetric mixed strategy equilibrium, $\sigma_2^J + \sigma_3^J + \sigma_4^J > 0$. Suppose not. Notice that given $\varepsilon > \frac{3}{4}$ and $s^i(1,0) = A$ and $s^i(0,1) = B$, if candidate $-J$ proposes one project and candidate $J$ proposes two, in expectation $J$ wins the election more often, whereas if $J$ proposes two and $-J$ proposes zero or three, $J$ wins more often. So if candidate $-J$ never proposes one project, proposing two projects in expectation defeats any other proposal with probability more than one half. Then, any best response by candidate $J$ to $\sigma^{-J}$ with $\sigma_2^J + \sigma_3^J + \sigma_4^J = 0$ must be such that $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$, which in turns means that any best response by $J$ implies $\sigma_2^J + \sigma_3^J + \sigma_4^J = 1$, a contradiction.

Similarly, in any symmetric mixed strategy equilibrium, $\sigma_5^J + \sigma_6^J + \sigma_7^J > 0$. Suppose not. Any best response by $J$ must be such that $\sigma_2^J + \sigma_3^J + \sigma_4^J = 0$, in which in turns implies that the best response by $-J$ is $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$.

Therefore, in any symmetric mixed strategy equilibrium, both candidates propose one project, and two projects, with positive probability. But then, it must be that $\sigma_2^J = \sigma_3^J + \sigma_4^J$ and $\sigma_5^J = \sigma_6^J = \sigma_7^J$. Given that the randomization among districts (subject to choosing a number of projects) assigns equal weight to all districts, we can reduce the strategic problem to that of assigning weights to strategies $s_1, s_8, s_L, s_H$. The payoff matrix is as follows:
\[
\begin{pmatrix}
  s_1 & s_L & s_H & s_8 \\
  s_1 & \frac{1}{2}, \frac{1}{2} & \varepsilon, 1 - \varepsilon & 0, \frac{1}{3 - 2\beta} & \varepsilon, \frac{1 - \varepsilon}{4 - 3\beta} \\
  s_L & \frac{1 - \varepsilon}{2 - \beta}, \varepsilon & \frac{1}{2(2 - \beta)}, \frac{1}{2(2 - \beta)} & \frac{2\varepsilon}{3(2 - \beta)}, \frac{3 - 2\varepsilon}{3(3 - 2\beta)} & 0, \frac{1}{4 - 3\beta} \\
  s_H & \frac{1}{3 - 2\beta}, 0 & \frac{3 - 2\varepsilon}{3(3 - 2\beta)}, \frac{2\varepsilon}{3(2 - \beta)} & \frac{1}{2(3 - 2\beta)}, \frac{1}{2(3 - 2\beta)} & \varepsilon, \frac{1 - \varepsilon}{3 - 2\beta}, \frac{1 - \varepsilon}{4 - 3\beta} \\
  s_8 & \frac{1 - \varepsilon}{4 - 3\beta}, \varepsilon & \frac{1}{4 - 3\beta}, 0 & \frac{1 - \varepsilon}{3 - 2\beta}, \frac{\varepsilon}{4 - 3\beta} & \frac{1}{2(4 - 3\beta)}, \frac{1}{2(4 - 3\beta)}
\end{pmatrix}
\]

Dropping the superindex (so as to use the Maple feature of Scientific Workplace to solve the equations), a symmetric equilibrium strategy with \(\sigma_1^J > 0, \sigma_L^J > 0, \sigma_H^J > 0\) and \(\sigma_8^J = 0\) must satisfy

\(^i\) \(\frac{1}{2}\sigma_1 + \varepsilon\sigma_L = \frac{1 - \varepsilon}{2 - \beta}\sigma_1 + \frac{1}{2(2 - \beta)}\sigma_L + \frac{2\varepsilon}{3(2 - \beta)}(1 - \sigma_1 - \sigma_L)\)

\(^{ii}\) \(\frac{1}{2}\sigma_1 + \varepsilon\sigma_L = \frac{1}{3 - 2\beta}\sigma_1 + \frac{3 - 2\varepsilon}{3(3 - 2\beta)}\sigma_L + \frac{1}{2(3 - 2\beta)}(1 - \sigma_1 - \sigma_L)\)

\(^{iii}\) \(\frac{1}{2}\sigma_1 + \varepsilon\sigma_L \geq \frac{1 - \varepsilon}{4 - 3\beta}\sigma_1 + \frac{1}{4 - 3\beta}\sigma_L + \frac{1 - \varepsilon}{4 - 3\beta}(1 - \sigma_1 - \sigma_L)\).

Solving we obtain

\[
\sigma_1 = \frac{4\varepsilon + (3 - 16\varepsilon + 6\beta\varepsilon)\sigma_L}{10\varepsilon - 3\beta}
\]

and

\[
\sigma_L = \frac{3 - 6(1 - \beta)\sigma_1}{22\varepsilon - 12\beta\varepsilon - 3} = \frac{3 - 6(1 - \beta)^{4\varepsilon + (3 - 16\varepsilon + 6\beta\varepsilon)\sigma_L}}{10\varepsilon - 3\beta} \frac{10\varepsilon - 3\beta}{22\varepsilon - 12\beta\varepsilon - 3} = \frac{9\beta + 6\varepsilon + 24\beta\varepsilon}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta\varepsilon + 120\beta\varepsilon^2 - 18}
\]

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which as $\beta \to 1$ converges to $\sigma_L = \frac{-9+6\varepsilon+24\varepsilon}{60\varepsilon-100\varepsilon^2-9} = \frac{3(10\varepsilon-3)}{(10\varepsilon-3)^2} = \frac{3}{10\varepsilon-3}$ as it ought to.

Also,

$$
\sigma_1 = \frac{4\varepsilon - (3 - 16\varepsilon + 6\beta \varepsilon) \frac{-9\beta+6\varepsilon+24\beta \varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta \varepsilon+120\beta \varepsilon^2-18}}{10\varepsilon - 3\beta}
$$

$$
\sigma_1 = -\frac{88\varepsilon^2 - 60\varepsilon + 18\beta \varepsilon - 48\beta \varepsilon^2 + 9}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta \varepsilon + 120\beta \varepsilon^2 - 18}
$$

which as $\beta \to 1$ converges to $\sigma_1 = \frac{4\varepsilon-3}{10\varepsilon-\beta}$ as it ought to.

Then simplifying inequality $iii)$,

$$
\frac{1}{2} \sigma_1 + \varepsilon \sigma_L \geq \frac{1 - \varepsilon}{4 - 3\beta} \sigma_1 + \frac{1}{4 - 3\beta} \sigma_L + \frac{1 - \varepsilon}{4 - 3\beta} (1 - \sigma_1 - \sigma_L)
$$

$$
\sigma_1 \geq \frac{6\varepsilon(1 - \beta)\sigma_L - 2(1 - \varepsilon)}{3\beta - 4}
$$

$$
-\frac{88\varepsilon^2 - 60\varepsilon + 18\beta \varepsilon - 48\beta \varepsilon^2 + 9}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta \varepsilon + 120\beta \varepsilon^2 - 18} \geq \frac{-6\varepsilon(1 - \beta)\frac{-9\beta+6\varepsilon+24\beta \varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta \varepsilon+120\beta \varepsilon^2-18} - 2(1 - \varepsilon)}{3\beta - 4}
$$

$$
\frac{6\varepsilon(1 - \beta) \frac{-9\beta+6\varepsilon+24\beta \varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta \varepsilon+120\beta \varepsilon^2-18} + 2(1 - \varepsilon)}{3\beta - 4} \geq \frac{88\varepsilon^2 - 60\varepsilon + 18\beta \varepsilon - 48\beta \varepsilon^2 + 9}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta \varepsilon + 120\beta \varepsilon^2 - 18} = 0
$$

$$
48\varepsilon - 9\beta - 304\varepsilon^2 + 440\varepsilon^3 + 48\beta \varepsilon + 24\beta \varepsilon^2 - 240\beta \varepsilon^3 \frac{3\beta - 22\varepsilon - 12\beta \varepsilon - 6}{(3\beta - 4)(10\varepsilon - 3)(3\beta + 22\varepsilon - 12\beta \varepsilon - 6)} = 0
$$

$$
48\varepsilon - 9\beta - 304\varepsilon^2 + 440\varepsilon^3 + 48\beta \varepsilon + 24\beta \varepsilon^2 - 240\beta \varepsilon^3 = 0
$$

The solution, solved by Mathematica, is a cumbersome expression that simplifies to the desired $\varepsilon > \frac{11 + \sqrt{61}}{20}$ for $\beta = 1$. 

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The probability of proposing two projects is 

\[
\frac{3(44\varepsilon^2 - 24\varepsilon + 8\beta \varepsilon - 24\beta \varepsilon^2 + 3)}{(10\varepsilon - 3)(3\beta + 22\varepsilon - 12\beta \varepsilon - 6)}
\]

and the expected number of projects is 

\[
\frac{6(44\varepsilon^2 - 24\varepsilon + 8\beta \varepsilon - 24\beta \varepsilon^2 + 3)}{(10\varepsilon - 3)(3\beta + 22\varepsilon - 12\beta \varepsilon - 6)} - \frac{\frac{-9\beta + 6\varepsilon + 24\beta \varepsilon + 88\varepsilon^2 - 60\varepsilon + 18\beta \varepsilon - 48\beta \varepsilon^2 + 9}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta \varepsilon + 120\beta \varepsilon^2 - 18}}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta \varepsilon + 120\beta \varepsilon^2 - 18}
\]

\[
= \frac{1}{10\varepsilon - 3} (12\varepsilon - 3) = \frac{12\varepsilon - 3}{10\varepsilon - 3},
\]

which converges to \( \frac{9}{7} \) as \( \varepsilon \) converges to 1. The initial assumption that voters vote \( s^i(1,0) = A \) and \( s^i(0,1) = B \) is supported because \( \sigma_8 = 0 \) and \( \beta > 2/3 \).

If instead \( \varepsilon \) is below the cutoffs, candidates mix between proposing one, two and three projects. An equilibrium with these characteristics requires:

\[
\begin{align*}
\sigma_L & = \frac{1}{2(2-\beta)} \varepsilon + \frac{1}{2(2-\beta)} \sigma_H + \frac{2\varepsilon}{3(2-\beta)} \sigma_H
\end{align*}
\]

\[
\sigma_H = \frac{1}{2(2-\beta)} \varepsilon + \frac{2\varepsilon}{3(2-\beta)} \sigma_H + \frac{1}{2(3-2\beta)} \sigma_H + \frac{\varepsilon}{3-2\beta} (1 - \sigma_L - \sigma_H)
\]

Simplifying the first inequality expressions, we get 

\[
\varepsilon \leq \frac{1}{2(2-\beta)} \sigma_L + \frac{2\varepsilon}{3(2-\beta)} \sigma_H
\]

From ii)

\[
\sigma_L = \frac{12\varepsilon - 6\beta \varepsilon + (14\beta \varepsilon + 6 - 3\beta - 24\varepsilon) \sigma_H}{20\varepsilon - 10\beta \varepsilon - 3}
\]

From iii)

\[
\sigma_H = \frac{(3\beta + 6\sigma_L - 6\beta \sigma_L - 6)}{3\beta + 28\varepsilon - 18\beta \varepsilon - 6}
\]
So,

\[
\sigma_L = \frac{12\varepsilon - 6\beta\varepsilon - (14\beta\varepsilon + 6 - 3\beta - 24\varepsilon) \cdot \frac{3\beta + 6\sigma_L - 6\beta\varepsilon - 6}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6}}{20\varepsilon - 10\beta\varepsilon - 3}
\]

\[
\sigma_L = \frac{9\beta + 108\varepsilon - 168\varepsilon^2 - 60\beta\varepsilon + 108\beta\varepsilon^2 - 18}{18\beta + 174\varepsilon - 280\varepsilon^2 - 114\beta\varepsilon + 180\beta\varepsilon^2 - 27}
\]

which simplifies to

\[
\sigma_L = \frac{9 + 108\varepsilon - 168\varepsilon^2 - 60\varepsilon + 108\varepsilon^2 - 18}{18 + 174\varepsilon - 280\varepsilon^2 - 114\varepsilon + 180\varepsilon^2 - 27} = \frac{6\varepsilon - 3}{10\varepsilon - 3}
\]

when \(\beta = 1\) as desired. Also,

\[
\sigma_H = -\frac{3\beta - 6 + 6(1 - \beta)\sigma_L}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6} = -\frac{3\beta - 6 + 6(1 - \beta) \cdot \frac{9\beta + 108\varepsilon - 168\varepsilon^2 - 60\beta\varepsilon + 108\beta\varepsilon^2 - 18}{18\beta + 174\varepsilon - 280\varepsilon^2 - 114\beta\varepsilon + 180\beta\varepsilon^2 - 27}}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6}
\]

\[
\sigma_H = \frac{1}{10\varepsilon - 36\beta + 28\varepsilon - 18\beta\varepsilon - 9}
\]

which simplifies to

\[
\frac{1}{10\varepsilon - 36\beta + 28\varepsilon - 18\beta\varepsilon - 9} = \frac{3}{10\varepsilon - 3}
\]

and, the expected number of projects in this equilibrium is:

\[
2 \cdot \frac{1}{10\varepsilon - 36\beta + 28\varepsilon - 18\beta\varepsilon - 9} + \frac{9\beta + 108\varepsilon - 168\varepsilon^2 - 60\beta\varepsilon + 108\beta\varepsilon^2 - 18}{18\beta + 174\varepsilon - 280\varepsilon^2 - 114\beta\varepsilon + 180\beta\varepsilon^2 - 27} + 3(1 - \frac{24\varepsilon + 6\beta\varepsilon - 9 - (9\beta + 108\varepsilon - 168\varepsilon^2 - 60\beta\varepsilon + 108\beta\varepsilon^2 - 18)}{(10\varepsilon - 3)(6\beta + 28\varepsilon - 18\beta\varepsilon - 9)})
\]

\[
= \frac{18\varepsilon - 6}{10\varepsilon - 3}.
\]

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5.3 One Office Motivated and one Efficiency Concerned candidate

We characterize the set of pure equilibria when one candidate is office motivated and the other one is efficiency concerned.

**Proposition 10** If projects are not very inefficient $\beta \in (\frac{2}{3}, 1)$, there exist a cutoff function $\varepsilon_1(\beta)$ such $0 < \varepsilon_1(\beta) < \frac{1}{2}$ and:

If $\varepsilon \in [0, \varepsilon_1(\beta)]$, there exist multiple pure equilibria;

If $\varepsilon \in (\varepsilon_1(\beta), \frac{1}{2})$, there is a unique pure strategy equilibrium in which both candidates propose the efficient policy; and

If $\varepsilon > \frac{1}{2}$ there is no pure strategy equilibrium.

If projects are very inefficient $\beta \in (\frac{1}{3}, \frac{2}{3})$, there exists a cutoff function $\varepsilon_1(\beta)$ such $0 < \varepsilon_1(\beta) < \frac{1}{2}$ and:

If $\varepsilon \in [0, \varepsilon_1(\beta)]$, there exist multiple pure equilibria;

If $\varepsilon \in (\varepsilon_1(\beta), 1]$ in the unique pure equilibrium both candidates propose the efficient policy.

**Proof.** Without loss of generality, let assume that candidate A is office motivated. We prove the high benefit case first.
$S_1$ : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter $i$ and beliefs such that $\omega^i_0(0) = 1$ and $\omega^i_0(1) = 1$ for any voter $i$ and any candidate $J$ make the election tied and if candidate $J$ deviates to any $s^J \neq s_1$, then $J$ loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If $\varepsilon \leq \frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss. If a candidate is efficiency concerned, deviations give her a lower payoff so there is not incentive to deviate. Suppose now that $\varepsilon > \frac{1}{2}$. By assumption candidate $A$ is office motivated. If $A$ deviates to $s^A = s_5$ she wins the election with probability $\varepsilon$ (when full information is revealed) and therefore such deviation is profitable for the office motivated candidate.

$S_2$ : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given $(s_1, s_2)$, $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

$S_3$ : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given $(s_1, s_5)$, $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

$S_4$ : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs such that $\omega^i_2(s) = 1$ for all $i \in \{a, b, c\}$, every voter votes for $A$. Ruled out by Lemma 4.

$S_5$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given $(s_2, s_2)$, every voter $i$ abstains. If candidate $A$ deviates to $s^J = s_8$ wins the election both in case the information is revealed and in case it is not.
$S_6$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given $(s_2, s_3), v(s_2, s_3) = (A, B, \emptyset)$.

If $A$ deviates to $s^A = s_6$ both in case full information is revealed and in case it not revealed, $A$ wins the election. Hence the deviation is profitable for all $\varepsilon \geq 0$.

$S_7$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given $(s_2, s_5), v(s_2, s_5) = (A, B, A)$.

Ruled out by Lemma 4.

$S_8$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given $(s_2, s_7), v(s_2, s_7) = (A, B, B)$.

Ruled out by Lemma 4.

$S_9$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given $(s_2, s_8), v(s_2, s_8) = (A, B, B)$.

Ruled out by Lemma 4.

$S_{10}$ : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given $(s_5, s_5), \text{all voters abstain}$.

Suppose first that $\varepsilon < \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_8$ and full information is not revealed, only voter $c$ observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate $A$ wins with probability at least $1 - \varepsilon > \frac{1}{2}$. Suppose that $\varepsilon > \frac{1}{2}$.

If $A$ deviates to $s^A = s_2$ then $A$ wins the election when information is fully revealed.

Hence the deviation is profitable.

$S_{11}$ : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given $(s^A, s^B) = (s_5, s_6), \text{beliefs such that } \omega^i_{J}(1 - s_i^f) = 1 \text{ for each } i \in \{a, b, c\} \text{ and } J \in \{A, B\} \text{ support an equilibrium in which } v(s_5, s_6) = (\emptyset, A, B) \text{ and each candidate wins with equal probability. Consider first the office motivated candidate } A. \text{ It suffices to check that } A \text{ has no incentives to deviate. If } A \text{ deviates to } s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\} \text{ and full information is not revealed, } A \text{ loses the election. If } A \text{ deviates to } s^A = s_8$ and
full information is not revealed, the election is tied, but if full information is revealed, 
A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Consider now candidate B who, by assumption, is efficiency concerned.

Deviating to $s^B = s^8$ is clearly unprofitable. Playing any other deviation candidate
B looses the election when information is not fully revealed. So the best deviation is $s^B = s^2$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed.(playing $s^B = s_4$ gives the same payoff as $s_2$ when information is fully revealed, but if B plays $s_4$ she looses 3-0 when information is not fully revealed while if B plays $s_1$ she looses 2 - 1) Candidate B prefers to deviate to $s_2$ if and only if

$$\frac{\varepsilon}{1 + (1 - \beta)} > \frac{1}{2} \frac{1}{1 + 2(1 - \beta)}, \text{ or}$$

$$\varepsilon > \frac{2 - \beta}{6 - 4\beta} \quad (7)$$

Hence there is a profitable deviation for candidate B if the previous condition holds.

Suppose now that $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_2$, then A wins the election when full information is revealed. Hence the deviation is profitable.

$S_{12}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_5, s_8), v(s_5, s_6) = (A, A, B)$.

Ruled out by Lemma 4.

$S_{13}$: Let voter strategy $s^i = (\emptyset, B, A, \emptyset)$ for each voter $i$ and beliefs such that

$$\omega^{i,j}_2(0) = \omega^{i,j}_2(1) = 1 \text{ for any voter } i.$$ Suppose first that $\varepsilon \leq \frac{1}{2}$. In equilibrium the
election is tied; consider the office motivated candidate A. If candidate A deviates to any strategy \( s^A \neq s_8 \) and full information is not revealed, A loses the election. Consider the efficiency concerned candidate B. If information is not fully revealed, candidate B loses the election if she deviates. If information is fully revealed, the unique profitable deviation is \( s^B = s_1 \). Candidate B prefers to deviate if and only if

\[
\varepsilon > \frac{1}{8 - 6\beta} \tag{9}
\]

Hence there is a profitable deviation for B if the previous condition holds. Suppose that \( \varepsilon > \frac{1}{2} \). If candidate A deviates to \( s^A = s_0 \), then A wins the election when full information is revealed. Hence the deviation is profitable.

Next we prove the low benefit case. To sustain equilibria, assume that off-equilibrium path beliefs such that given the equilibrium proposal \( s^J_i, \omega^i_{J}(1 - s^J_i) = 1 \) for each \( i \in \{a, b, c\} \) and \( J \in \{A, B\} \). That is, a voter who observe a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

\( S_1 : \) Voter strategy \( s^i = (\emptyset, A, B, \emptyset) \) for each voter \( i \) and beliefs such that \( \omega^i_{J}(0) = 1 \) and \( \omega^i_{J}(1) = 1 \) for any voter \( i \) and any candidate \( J \) make the election tied. There is no a different proposal that can defeat \( s_1 \) both in case the information is revealed and in case it is not. The efficiency concerned candidate has lower incentive to deviate because \( s_1 \) is the efficient proposal. It is also straightforward to check that the voting
strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Hence no candidate has profitable deviations for all \( \varepsilon \in [0, 1] \).

\( S_2 \): Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_2)\). Given \((s_1, s_2)\), \(v(s_1, s_2) = (B, A, A)\).
Ruled out by Lemma 4.

\( S_3 \): Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_5)\). Given \((s_1, s_5)\), \(v(s_1, s_5) = (A, A, A)\).
Ruled out by Lemma 4.

\( S_4 \): Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_8)\). Given \((s_1, s_8)\), \(v(s_1, s_8) = (A, A, A)\).
Ruled out by Lemma 4.

\( S_5 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_2)\). Suppose first that \( \varepsilon \leq \frac{1}{2} \). Given \((s_2, s_2)\), every voter abstains. If the office motivated candidate \( A \) deviates and full information is not revealed, any voter who observes the deviation votes for \( B \) and \( A \) loses the election. Consider candidate \( B \). If candidate \( B \) deviates, \( B \) looses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate \( B \) is \( s^B = s_1 \), since \( B \) wins the election and the proposal is efficient. The deviation \( s^B = s_1 \) is profitable if and only if

\[
\varepsilon > \frac{1}{4 - 2\beta}.
\]
Suppose now $\varepsilon > \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_1$, $A$ wins the election when full information is revealed. Hence the deviation is profitable.

$S_6$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Suppose $\varepsilon \leq \frac{1}{2}$. Given $(s_2, s_3)$, $v(s_2, s_3) = (A, B, \emptyset)$. If the office motivated candidate $A$ deviates and full information is not revealed, any voter who observes the deviation votes for $B$ and $A$ loses the election. Any deviation makes the candidate loses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate $B$ is $s^B = s_1$, since she wins the election and the proposal is efficient. The deviation $s^B = s_1$ is profitable if and only if

$$\varepsilon > \frac{1}{4 - 2\beta}.$$ 

Suppose now $\varepsilon > \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_1$, $A$ wins the election when full information is revealed. Hence the deviation is profitable.

$S_7$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given $(s_2, s_5)$, $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

$S_8$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given $(s_2, s_7)$, $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

$S_9$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for $A$. Ruled out by Lemma 4.
$S_{10}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given $(s_5, s_5)$, all voters abstain. If the office motivated candidate $A$ deviates to $s^A = s_8$ and full information is not revealed, only voter $c$ observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate $A$ wins with probability at least $1 - \varepsilon > \frac{1}{2}$.

Suppose now $\varepsilon > \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_1$, $A$ wins the election when full information is revealed. Hence the deviation is profitable.

$S_{11}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given $(s_5, s_6)$, $v(s_5, s_6) = (\emptyset, A, B)$. The office motivated candidate $A$ has no incentives to deviate. If $A$ deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, $A$ loses the election. If $A$ deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, $A$ loses the election. In any case, after a deviation $A$ wins the election with probability less than $\frac{1}{2}$. Consider candidate $B$ who is efficiency concerned. Deviating to $s^B = s^8$ is clearly unprofitable. By deviating candidate $B$ loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate $B$ wins the election when information is fully revealed. Candidate $B$ prefers to deviate to $s_1$ if and only if

$$\varepsilon > \frac{1}{6 - 4\beta}$$

Suppose now $\varepsilon > \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_1$, $A$ wins the election when full information is revealed. Hence the deviation is profitable.
$S_{12}$ : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_8, s_8), v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

$S_{13}$ : All voters abstain and the election is tied. Suppose $\varepsilon \leq \frac{1}{2}$. If the office motivated candidate $A$ deviates and full information is not revealed, any voter who observes the deviation votes for $B$ and $A$ loses the election. Consider candidate $B$ who is efficiency concerned. By deviating candidate $B$ loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate $B$ wins the election when information is fully revealed. Candidate $B$ prefers to deviate to $s^B = s_1$ if and only if

$$\varepsilon > \frac{1}{8 - 6\beta} \quad (10)$$

Hence there is a profitable deviation for candidate $B$ if the previous condition holds. Suppose now $\varepsilon > \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_1$, $A$ wins the election when full information is revealed. Hence the deviation is profitable. ■

The proofs of Proposition 3 and of Theorem 2 follow by the previous result. To conclude the proof of Proposition 1 we show in the following lemma that there is no equilibrium in mixed strategy when $\beta \in (\frac{1}{3}, \frac{2}{3})$ and $\varepsilon > \frac{1}{2}$.

**Lemma 11** For any $(\alpha_i, \alpha_j) \in \{0, 1\}^2$ and any $\beta \in (\frac{1}{3}, \frac{2}{3})$, if $\varepsilon > \frac{1}{2}$ there is no equilibrium in which a candidate proposes to provide a public good with positive probability.
Proof. Suppose candidate $J$ plays with positive probability any strategy different than $s_1$. If candidate $-J$ plays $s_1$, then candidate $J$ looses the election when full information is revealed. If candidate $-J$ is playing any strategy $s_k^{-J} \neq s_1^{-J}$ then candidate $J$ wins with probability one by playing $s_1$ when full information is revealed. Therefore if candidate $J$ replace in the mixed strategy any $s_k^J \neq s_1^J$ with strategy $s^J$ increases her proability of winning the election. ■

6 Appendix for Referees

6.1 Equilibria with $\varepsilon = 0$

In this subsection we characterize the set of Bayesian equilibria when $\varepsilon = 0$ (and voters do not play weakly dominated strategies).

Lemma 12 Assume $\varepsilon = 0$. Every strategy is undominated.

Proof. No candidate strategy is weakly dominated, because the payoffs to candidates depend on the strategies of the voters.

For the voters, consider the generic information set $(p_i^A, p_i^B) = (x, y)$ with $x, y \in \{0, 1\}$. If $s^b(x, y) = s^c(x, y) = \emptyset$, $s^J$ consists of proposing $(x, 0, 0)$, and $s^{-J}$ consists of proposing $(y, 1, 1)$, then $a$ is strictly better off voting for $J$, while if $s^J$ consists of $(x, 1, 1)$ and $s^{-J}$ consists of $(y, 0, 0)$, then $a$ is strictly better off voting for $-J$. Thus, any strategy is undominated. ■
Proposition 13 Assume $\epsilon = 0$ and $\beta \in (\frac{2}{3}, 1)$. An equilibrium in which candidates use the strategy pair $(s^A, s^B)$ exists if and only if $(s^A, s^B) \in S_k$ for some $k \in \{1, 2, 3, 4, 8, 11, 13\}$.

Proof. For each strategy pair class, we find whether an element of the class can be sustained in equilibrium:

$S_1$ : Voter strategy $s^i = (0, A, B, \emptyset)$ for each voter $i$ and beliefs such that $\omega_{0}^{i,j}(0) = 1$ and $\omega_{2}^{i,j}(1) = 1$ for any voter $i$ and any candidate $J$ make the election tied and if candidate $J$ deviates to any $s^J \neq s_1$, then $J$ loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

$S_2$ : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (B, A, A)$. Let $\omega_2^{a,B}(0) = 1$ and $\omega_2^{b,A}(0) = \omega_2^{c,A}(0) = 1$. Then candidate $A$ cannot win the election by deviating and candidate $B$ cannot increase her vote margin by deviating.

$S_3$ : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, $v(s_1, s_5) = (B, B, A)$. Suppose $\delta$ is such that $\omega_2^{a,A}(1) = \omega_2^{b,A}(1) = \omega_2^{c,B}(1) = 1$. Then neither candidate can improve her electoral outcome by deviating.

$S_4$ : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs such that $\omega_2^{i,B}(0) = 1$ for all $i \in \{a, b, c\}$, in equilibrium every voter votes for $A$ and continues to vote for
A after any deviation by B.

\( S_5 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_2)\). Given \((s_2, s_2)\), every voter \( i \) abstains.

If \( A \) deviates to \( s^A = s_6 \), \( v(s_6, s_2) = (\emptyset, \emptyset, A) \).

\( S_6 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_3)\). Given \((s_2, s_3)\), \( v(s_2, s_3) = (A, B, \emptyset) \).

If \( A \) deviates to \( s^A = s_6 \), only voter \( c \) observes the deviation, so \( v(s_6, s_2) = (A, B, A) \).

\( S_7 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_5)\). Given \((s_2, s_5)\), \( v(s_2, s_5) = (A, B, A) \).

If \( B \) deviates to \( s^B = s_7 \), only voter \( c \) observes the deviation, so \( v(s_2, s_7) = (A, B, B) \).

\( S_8 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_7)\). Beliefs such that \( \omega_2^{a,B}(1) = \omega_2^{b,A}(1) = \omega_2^{c,A}(1) = 1 \) support an equilibrium in which \( v(s_2, s_7) = (A, B, B) \). It is easy to check that no candidate can improve her electoral outcome with any deviation.

\( S_9 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_8)\). Given \((s_2, s_8)\), \( v(s_2, s_8) = (A, B, B) \).

If candidate \( A \) deviates to \( s_8 \), voters \( b \) and \( c \) either vote for candidate \( A \) or abstains, depending upon their beliefs, and therefore candidate \( A \) wins the election (voter \( a \)'s beliefs do not change).

\( S_{10} \): Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_5)\). Given \((s_5, s_5)\), all voters abstain.

If candidate \( A \) deviates and proposes \( s_8 \), beliefs of voters \( a \) and \( b \) are unaffected, while voter \( c \) votes for candidate \( A \) for all possible beliefs over candidate \( A \)'s strategy.

Therefore candidate \( A \) wins the election.

\( S_{11} \): Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_6)\). Beliefs such that \( \omega_2^{a,A}(0) = \omega_2^{a,B}(0) = \omega_2^{b,B}(1) = \omega_2^{c,A}(1) = 1 \) support an equilibrium in which \( v(s_5, s_6) = (\emptyset, A, B) \) and it is again easy to check that no candidate can gain any vote by deviating.
$S_{12}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_8, s_8)$, $v(s_5, s_6) = (A, A, B)$.

If $A$ deviates to $s_8$, voter $c$ either votes for $A$ as well, or abstains, hence $A$ is better off deviating.

$S_{13}$: Voter strategy $s^i = (\emptyset, B, A, \emptyset)$ for each voter $i$ and beliefs such that $\omega_2^{i,J}(0) = \omega_2^{i,J}(1) = 1$ for any voter $i$ and any candidate $J$ make the election tied, and if candidate $J$ deviates to any strategy $s^J \neq s_8$, $J$ loses the election. ■

**Proposition 14** Assume $\varepsilon = 0$ and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair $(s^A, s^B)$ exists if and only if $(s^A, s^B) \notin S_{10} \cup S_{12}$.

**Proof.** First note that $(s^A, s^B) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given $(s_5, s_5)$, all voters abstain. If candidate $J$ deviates and proposes $s_8$ voters $a$ and $b$ beliefs are unaffected, while voter $c$ votes for candidate $J$ for all possible beliefs over candidate $J$ strategy. Therefore candidate $J$ wins the election.

Similarly, $(s^A, s^B) \in S_{12}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given $(s_5, s_8)$, voters $a$ and $b$ vote $A$, while voter $c$ votes $B$. Suppose $A$ deviates to $s_8$. Voters $a$ and $b$ do not observe the deviation, and continue to vote $A$, while voter $c$ now abstains or votes for $A$ depending on her beliefs. Hence now $A$ wins the election by a greater margin.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs’ over equilibrium strategies are correct. Out-of-equilibrium beliefs are such
that given the equilibrium proposal \( s_i' \), then \( \omega_i J(1 - s_i') = 1 \) for both \( J \in \{A, B\} \) and for \( i \in \{a, b, c\} \). These are most pessimistic beliefs that a voter can have regarding candidates’ strategy when she observes a deviation. We list the equilibrium electoral outcome for each class of candidates’ strategies. Voters’ strategies follow straightforwardly from their beliefs. Note that given the pessimistic beliefs a candidate who deviates cannot increase the number of votes she gets in any of the cases below.

\( S_1 \): All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

\( S_2 \): Assume without loss of generality that \( (s^A, s^B) = (s_1, s_2) \). In equilibrium, \( v(s_1, s_2) = (A, B, B) \), and any voter who observes a deviation votes against the candidate who deviates.

\( S_3 \): Assume w.l.o.g. that \( (s^A, s^B) = (s_1, s_5) \). In equilibrium, every voter votes for \( A \) and continues to vote for \( A \) after any deviation by \( B \).

\( S_4 \): Assume w.l.o.g. that \( (s^A, s^B) = (s_1, s_8) \). In equilibrium every voter votes for \( A \) and continues to vote for \( A \) after any deviation by \( B \).

\( S_5 \): Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_2) \). Given \( (s_2, s_2) \), every voter abstains, and votes against any candidate who deviates.

\( S_6 \): Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_3) \). Given \( (s_2, s_3) \), \( v(s_2, s_3) = (A, B, \emptyset) \). Every voter votes against any deviating candidate.

\( S_7 \): Assume w.l.o.g. that \( (s^A, s^B) = (s_2, s_5) \). Given \( (s_2, s_5) \), \( v(s_2, s_5) = (A, B, A) \). Voters \( a \) and \( c \) do not vote for \( B \) after any deviation by \( B \), and voter \( b \) does not vote.
for $A$ after any deviation.

$S_8$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given $(s_2, s_7)$, $v(s_2, s_7) = (A, B, B)$, and given any deviation by $J$, no voter changes her vote from voting for $-J$ to abstention or voting for $J$.

$S_9$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for $A$ and continue to do so after any deviation by $B$.

$S_{10}$: Not an equilibrium as shown above.

$S_{11}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given $(s_5, s_6)$, $v(s_5, s_6) = (\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.

$S_{12}$: Not an equilibrium as shown above.

$S_{13}$: All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates. ■

6.2 Equilibria in the $n$-district case.

Let $N$ be a society with $n$ districts, where each district has one voter and one project associated to it. Let $i \in N$ be an arbitrary voter or district. Let there be two candidates $A$ and $B$ who are not one of the $n$ voters and are not associated to any district.

Let $S = \{0, 1\}^n$ be the set of feasible pure strategies for each candidate. Let $(s^A, s^B) \in S \times S$ be an arbitrary candidates’ strategy pair. Let $\Sigma$ be a probability distribution over $S$. Let $\{1\}^n$ denote the strategy of proposing to carry out all projects,
and let \( \{0\}^n \) be the strategy of proposing to carry out no projects. For any \( k \in \{0, 1, \ldots, n\} \), let \( S_k = \{s^J : \sum_{i=1}^{n} s_i^J = k\} \), that is, the set of pure strategies that propose to carry out \( k \) projects. Candidates can be either both office motivated or both efficiency concerned. Before considering these two cases we provide some useful preliminary results that are useful in both cases.

**Lemma 15** In any pure strategy equilibrium, both candidates win with equal probability.

**Proof.** Consider any candidates’ strategy pair \((s^A, s^B)\) such that candidate \( J \) wins with probability less than \( \frac{1}{2} \). Since in equilibrium voters hold correct beliefs, the probability that \( J \) wins conditional on full information being revealed, or not revealed, is less than \( \frac{1}{2} \) in each case. In pure strategies, the probability of victory is in the set \( \{0, \frac{1}{2}, 1\} \) so if it is less than \( \frac{1}{2} \), it is zero. Deviating to \( s^J = s^{-J} \), candidate \( J \) ties the election if full information is revealed, so the probability of winning is at least \( \frac{\xi}{2} \).

Since for both types, office motivated and efficiency concerned, to win with positive probability gives a positive payoff, then there is no equilibrium where a candidate wins with zero probability. ■

**Corollary 16** Assume \( \beta \neq \frac{k}{n} \) for any integer \( k \). Then both candidates propose to carry out the same number of projects.

**Proof.** By lemma 15, both candidates win with equal probability. If \( \beta \neq \frac{k}{n} \) and candidates propose to implement a different number of projects, then no voter is
indifferent between the two candidates, and hence no voter abstains, so that either A or B win for sure, which is a contradiction.

Lemma 17 Assume $\beta \neq \frac{k}{n}$ for any integer $k$. In equilibrium, $s^i(0, 0) = s^i(1, 1) = \emptyset$, $s^i(1, 0) = A$ and $s^i(0, 1) = B$ for any voter $i \in N$.

Proof. $s^i(0, 0) = s^i(1, 1) = \emptyset$ follows directly from lemma 15. Note that if voter $i$ observes $(s^A_i, s^B_i) = (1, 0)$, by lemma 15 it means that $B$ proposes to carry out one more project than $A$ in districts other than $i$. So $i$ prefers $A$. A symmetric argument holds to show that $s^i(0, 1) = B$.

Consider the following out-of-equilibrium beliefs.

B1: Given an equilibrium strategy pair $(s^A, s^B)$, let the out of equilibrium beliefs of any voter $i \in N$ be such that $\delta^J_i (1 - s^J_i)$ assigns probability 1 to strategy $(1 - s^J_i, s^J_{-i})$ where $s^J_{-i} = \{1\}^{n-1}$, that is, voter $i$ believes candidate $J$ proposes to carry out a project in every other district.

Lemma 18 If a candidate strategy pair $(s^A, s^B)$ cannot be supported in an equilibrium with out of equilibrium beliefs B1, then it cannot be supported in equilibrium.

Proof. Suppose that $(s^A, s^B)$ and beliefs B1 are not an equilibrium but there exists a system of beliefs that sustains $(s^A, s^B)$ as an equilibrium strategy profile. If $(s^A, s^B)$ is an equilibrium strategy pair, it follows that there are no profitable deviation for any candidate. Consider, without loss of generality, any deviating proposal $\tilde{s}^A \neq s^A$
by candidate $A$. For each voter $i \in N$ who observes a deviation the utility attached at strategy $\tilde{s}^A$ under beliefs $B_1$ is equal or lower than the utility attached to strategy $s^A$ under the original beliefs. Therefore if there were no profitable deviation under the original beliefs there are no profitable deviation when each voter $i$ have beliefs $B_1$.

6.2.1 Purely Office Motivated Candidates

**Proposition 19** Assume $\varepsilon \in (0, \frac{1}{2})$ and $\beta \neq \frac{k}{n}$ for any integer $k$. A candidates’ strategy pair $(s^A, s^B)$ can be supported in equilibrium if and only if $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = x$ and one of the two following conditions holds:

i) $x < (1 - \beta)n$; or

ii) $x > \frac{n+1}{2}$ and $s_i^A + s_i^B \geq 1$ for any $i \in N$.

**Proof.** By corollary 16, in equilibrium $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B$. Let $\sum_{i=1}^{n} s_i^A = \sum_{i=1}^{n} s_i^B = x$ and assume $x < (1 - \beta)n$. Consider four subsets of voters: $N_{x,y} = \{i \in N : (s_i^A, s_i^B) = (x,y)\}$ for $x, y \in \{0,1\}$. In equilibrium, $i \in N_{00} \cup N_{11}$ abstain, $i \in N_{10}$ vote $A$ and $i \in N_{01}$ vote $B$. Without loss of generality, consider deviations by $A$. Suppose $A$ deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{00}$. With probability $1 - \varepsilon > \frac{1}{2}$, full information is not revealed, and given out of equilibrium beliefs $B_1$, voter $i$ expects utility $\beta - 1$ if $A$ wins, and utility $-\frac{x}{n}$ if $B$ wins. Note that $x < (1 - \beta)$ implies $\frac{x}{n} > 1 - \beta$ so voter $i$ votes for $B$ and thus candidate $A$ loses the election.

Suppose $A$ deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{01}$; whether or not full
information is revealed, voter \( i \) prefers \( B \) and continues to vote for \( B \), while other voters, if they observe the deviation, reduce their expected utility from a victory by \( A \), so \( A \) gains nothing by deviating.

Suppose \( A \) deviates to offer \( \tilde{s}_i^A = 1 - s_i^A \) to a voter \( i \in N_{10} \cup N_{11} \). With probability \( 1 - \varepsilon > \frac{1}{2} \), full information is not revealed, and given out of equilibrium beliefs \( B1 \), voter \( i \) votes for \( B \) and \( A \) loses the election.

Assume \( x \in ((1 - \beta)n, \frac{n+1}{2}) \). Suppose \( A \) deviates to offer \( \tilde{s}_i^A = 1 - s_i^A \) to a voter \( i \in N_{00} \). With probability \( 1 - \varepsilon > \frac{1}{2} \) full information is not revealed, and under out of equilibrium beliefs \( B1 \), voter \( i \) expects utility \( \beta - 1 \) if \( A \) wins, and utility \( -\frac{\varepsilon}{n} \) if \( B \) wins; since \( \beta - 1 > -\frac{\varepsilon}{n} \), it follows \( i \) prefers \( A \) and votes for \( A \) and \( A \) wins the election.

Hence there is no equilibrium with out of equilibrium beliefs \( B1 \), but then by lemma 18, there is no equilibrium.

Assume \( x > \frac{n+1}{2} \) but \( N_{00} \neq \emptyset \). Suppose \( A \) deviates to offer \( \tilde{s}_i^A = 1 - s_i^A \) to a voter \( i \in N_{00} \). With probability \( 1 - \varepsilon > \frac{1}{2} \) full information is not revealed, and under out of equilibrium beliefs \( B1 \), voter \( i \) expects utility \( \beta - 1 \) if \( A \) wins, and utility \( -\frac{\varepsilon}{n} \) if \( B \) wins; since \( \beta - 1 > -\frac{\varepsilon}{n} \), it follows \( i \) prefers \( A \) and votes for \( A \) and \( A \) wins the election.

Hence there is no equilibrium with out of equilibrium beliefs \( B1 \), but then by lemma 18, there is no equilibrium.

Assume \( x > \frac{n+1}{2} \) and \( N_{00} = \emptyset \). Suppose \( A \) deviates to offer \( \tilde{s}_i^A = 1 - s_i^A \) to a voter \( i \in N_{01} \); whether or not full information is revealed, voter \( i \) prefers \( B \) and continues to vote for \( B \) and other voters either do not observe the deviation or if they do, they
now have a lower expected utility from \( A \) winning, so \( A \) gains nothing by deviating. Suppose \( A \) deviates to offer \( s_i^A = 1 - s_i^A \) to a voter \( i \in N_{10} \cup N_{11} \). With probability \( 1 - \epsilon > \frac{1}{2} \), full information is not revealed, and given out of equilibrium beliefs \( B_1 \), voter \( i \) votes for \( B \) and \( A \) loses the election. \( \blacksquare \)

**Proposition 20** Assume \( \epsilon \in (\frac{1}{2}, 1] \). If \( \beta > \frac{n+1}{2n} \), then there is no equilibrium in pure strategy; if \( \beta < \frac{n+1}{2n} \), then there is a unique equilibrium where both candidates propose zero projects.

**Proof.** Consider first the case \( \beta > \frac{n+1}{2n} \) and suppose there is an equilibrium in pure strategy. By corollary 16 let \( k \) be the number of projects proposed in equilibrium by each candidate. Suppose without loss of generality that candidate \( A \) proposes in districts \( 1, 2, \ldots, k \). Consider first that \( k < \frac{n+1}{2} \). Candidate \( B \) can win the election when full information is revealed, that is with probability \( \epsilon > \frac{1}{2} \), proposing a project in districts \( k + 1, k + 2, \ldots, n \) and no project in district \( i \leq k \). Consider now the case with \( k \geq \frac{n+1}{2} \). Then candidate \( B \) can propose a project in districts \( 1, 2, \ldots, k-1 \) and no project in the other districts. All voters \( 1, 2, \ldots, k-1 \) and voters with \( i \geq k+1 \) (if any) vote for candidate \( B \) when full information is revealed and therefore candidate \( B \) wins the election with probability \( \epsilon > \frac{1}{2} \). Consider now the case \( \beta < \frac{n+1}{2n} \). If \( B \) deviates and proposes zero projects, then \( B \) wins the election when full information is revealed. In fact, if \( 1 \leq k < \frac{n+1}{2} \) and full information is revealed all voters to which candidate \( A \) does not propose a project vote for \( B \). If \( k \geq \frac{n+1}{2} \) each voter \( i \leq k \) votes for \( B \),
because if $A$ is elected voter $i$ gets $\beta - \frac{k}{n}$, while voter $i$ gets 0 if $B$ is elected; hence each voter $i \leq k$ votes for $B$ if $\beta < \frac{k}{n}$ which holds by assumption. Finally suppose both candidates propose no projects. If candidate $J$ deviates and proposes some projects loses the election both when full information is not revealed (by assumption B1) and when information is revealed, because, if $\beta < \frac{n+1}{2n}$, any minimum majority coalition formed by $\frac{n+1}{2}$ districts prefers zero projects than $k = \frac{n+1}{2}$ implemented. 

6.2.2 Efficiency concerned candidates

**Proposition 21** Assume $\varepsilon \in (0, \frac{1}{2})$ and $\beta \neq \frac{k}{n}$ for any integer $k$. A candidates’ strategy pair $(s^A, s^B)$ can be supported in equilibrium if and only if $\sum_{i=1}^{n} s^A_i = \sum_{i=1}^{n} s^B_i = x$ and one of the two following conditions holds:

i) $x < (1 - \beta)n$ and $\varepsilon \leq \frac{1}{2 (1-\beta)x+1}$; or

ii) $x > \frac{n+1}{2}$, $s^A_i + s^B_i \geq 1$ for any $i \in N$ and iia) $\varepsilon < 1 - \frac{(n-1)(1-\beta)}{(1-\beta)x+1}$ if $\beta > \frac{n+1}{2n}$; iib) $\varepsilon < \frac{1}{2 (1-\beta)x+1}$ if $\beta < \frac{n+1}{2n}$.

**Proof.** By corollary 16, in equilibrium $\sum_{i=1}^{n} s^A_i = \sum_{i=1}^{n} s^B_i$. Let $\sum_{i=1}^{n} s^A_i = \sum_{i=1}^{n} s^B_i = x$ and assume $x < (1 - \beta)n$. Consider four subsets of voters: $N_{x,y} = \{i \in N : (s^A_i, s^B_i) = (x, y)\}$ for $x, y \in \{0, 1\}$. In equilibrium, $i \in N_{00} \cup N_{11}$ abstains, $i \in N_{10}$ votes $A$ and $i \in N_{01}$ votes $B$. Without loss of generality, consider deviations by $A$. In the proof of proposition 19 we already showed that there do not exist profitable deviations for a office motivated candidate such that more projects are
proposed in the deviation. Therefore, these are not profitable deviations in case the candidate is efficiency concerned, too. Consider any deviation such that \( x' < x \) projects are proposed. Given, voters’ beliefs, this deviation can be profitable only if full information is revealed. It follows that the most profitable deviation for an efficiency concerned candidate is to propose zero project. This deviation is profitable if and only if

\[
\varepsilon > \frac{1}{2} \frac{1}{(1 - \beta)x + 1}.
\]

Assume \( x \in \left((1 - \beta)n, \frac{n+1}{2}\right) \) or \( x > \frac{n+1}{2} \) but \( N_00 \neq \emptyset \). Suppose \( A \) deviates to offer \( \tilde{s}_i^A = 1 - s_i^A \) to a voter \( i \in N_00 \). Since \( x > (1 - \beta)n \) then voter \( i \) votes for \( A \). This deviation is profitable if and only if

\[
(1 - \varepsilon) \frac{1}{(1 - \beta)(x + 1) + 1} > \frac{1}{2} \frac{1}{(1 - \beta)x + 1}, \quad \text{or}
\]

\[
\varepsilon \frac{1}{(1 - \beta)(x + 1) + 1} < \frac{1}{(1 - \beta)(x + 1) + 1} - \frac{1}{2} \frac{1}{(1 - \beta)x + 1}.
\]

Suppose \( A \) deviates to offer zero projects. This deviation is profitable if and only if

\[
\varepsilon > \frac{1}{2} \frac{1}{(1 - \beta)x + 1} \equiv \varepsilon''.
\]
For all $x$, $\epsilon' \geq \epsilon''$ and there is always a profitable deviation. Hence there is no equilibrium with out of equilibrium beliefs $B1$, but then by lemma 18, there is no equilibrium.

Assume $x > \frac{n+1}{2}$ and $N_{00} = \emptyset$. Suppose $A$ deviates to offer $\tilde{s}_i^A = 1 - s_i^A$ to a voter $i \in N_{01}$; whether or not full information is revealed, voter $i$ prefers $B$ and continues to vote for $B$ and other voters either do not observe the deviation or if they do, they now have a lower expected utility from $A$ winning, so $A$ loses by deviating since she is proposing more projects. When full information is not revealed, and given out of equilibrium beliefs $B1$, each voter $i$ who observes a deviation from candidate $A$ votes for $B$. So candidate $A$ can only find a deviation that is profitable when full information is revealed. Consider first the case with $\beta > \frac{n+1}{2n}$. The most profitable deviation is such that candidate $A$ proposes $x'$ projects such that $\tilde{s}_i^A = 1$ only if $\tilde{s}_i^B = 1$ and $(n - x) + x' = \frac{n+1}{2}$ that is $x' = \frac{2x+1-n}{2}$. This deviation is profitable if and only if

$$
\epsilon \left(1 - \beta\right) x' + 1 \geq \frac{1}{2} \left(1 - \beta\right) x + 1 ; \text{ or } \epsilon > 1 - \frac{(n-1)(1-\beta)}{(1-\beta)x + 1}.
$$

If $\beta < \frac{n+1}{2n}$, then the most profitable deviation is to propose zero project.
deviation is profitable if and only if

\[ \varepsilon > \frac{1}{2} \left( \frac{1}{1 - \beta} \right) x + 1. \]

\[ \Box \]

**Proposition 22** Assume \( \varepsilon \in (\frac{1}{2}, 1] \) and \( \beta \neq \frac{k}{n} \) for any integer \( k \). If \( \beta < \frac{n+1}{2n} \) then the candidates’ strategy pair \((s^A, s^B)\) is supported in equilibrium if and only if

\[ \sum_{i=1}^{n} s^A_i = \sum_{i=1}^{n} s^B_i = 0. \]

If \( \beta > \frac{n+1}{2n} \) then the candidates’ strategy pair \((s^A, s^B)\) is supported in equilibrium if and only if

\[ \sum_{i=1}^{n} s^A_i = \sum_{i=1}^{n} s^B_i = 0 \quad \text{and} \quad \varepsilon < \frac{(1-\beta)(n+1)+2}{4}. \]

If \( \beta > \frac{n+1}{2n} \) and \( \varepsilon \) satisfy

\[ \varepsilon > \frac{(1-\beta)(n+1)+2}{4} \]

then there is no equilibrium in pure strategies.

**Proof.** First of all it is easy to check that there is no equilibrium in pure strategy with \( x > 0 \) if \( \varepsilon > \frac{1}{2} \); in fact there is always a profitable deviation when full information is revealed such that less projects are implemented. Suppose then that both candidates proposes zero projects. If \( \beta < \frac{n+1}{2n} \), there is no profitable deviation because the minimum winning coalition prefers zero projects to have all their projects implemented (and no other project implemented). If \( \beta > \frac{n+1}{2n} \) then candidate \( J \) who proposes \( \frac{n+1}{2} \) projects wins the election when full information is revealed. This deviation is
profitable if and only if

\[ \varepsilon \left( \frac{1}{1 - \beta} \frac{n+1}{2} + 1 \right) > \frac{1}{2}, \quad \text{or} \quad \varepsilon > \frac{(1 - \beta)(n + 1) + 2}{4}. \]

\[ \square \]

6.3 Beta larger than one: The provision of local efficient public goods

We consider here the case of local public goods that are efficient, that is \( \beta > 1 \). In this case the unique efficient outcome is the provision of the efficient public good in each district. However, a public good in another district does not provide any benefit to a voter, but it only is a cost for her; voters’ preferences over the set of outcomes are the following:

\[ \{1, 0, 0\} \succ_i \{1, 1, 0\} \sim_i \{1, 0, 1\} \succ_i \{1, 1, 1\} \succ_i \{0, 0, 0\} \succ_i \{0, 0, 1\} \sim_i \{0, 1, 0\} \succ_i \{0, 1, 1\}. \]

As before, purely office motivated candidates want to win the election and they do not care about the policy. Efficiency concerned candidates want to win, but they prefer to win proposing and implementing more efficient policies, that is their utility is increasing in the number of the efficient public good provided. Let \( k \) the number
of projects in the proposal of candidate $J$. If candidate $J$ is efficiency concerned, her preferences are represented by the utility function:

$$U_J(k) = 1 + \beta k \text{ if } J \text{ wins the election}$$
$$U_J(k) = 0 \text{ otherwise.}$$

It is immediate to check that Lemma 4 still holds when $\beta > 1$.

### 6.3.1 Purely Office Motivated Candidates

**Proposition 23** Assume $\varepsilon \in (0, \frac{1}{2})$ and $\beta \neq \frac{1}{2}$. A candidates’ strategy pair $(s^A, s^B)$ can be supported in equilibrium if and only if $(s^A, s^B) \in S_k$ for some $k \in \{11, 13\}$. Assume $\varepsilon \in \left(\frac{1}{2}, 1\right]$, then there is no equilibrium in pure strategies.

**Proof.** $S_1$: If candidate $J$ deviates and proposes $s^J = s_5$ then voter $a$ and $b$ votes for $J$ for all $\varepsilon \geq 0$.

$S_2$: Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given $(s_1, s_2)$, $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

$S_3$: Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given $(s_1, s_5)$, $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

$S_4$: Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Every voter votes for $B$. Ruled out by Lemma 4.

$S_5$: Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given $(s_2, s_2)$, every voter $i$ abstains.
If candidate $A$ deviates proposing $s^A = s_7$ both when information is fully revealed and when it is not.

$S_6$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given $(s_2, s_3), v(s_2, s_3) = (A, B, \emptyset)$.

If $A$ deviates to $s^A = s_6$ voters $a$ and $c$ vote for $A$ both in case full information is revealed and in case it is not revealed. Hence by deviating, $A$ wins with for sure.

$S_7$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given $(s_2, s_5), v(s_2, s_5) = (A, B, A)$.

Ruled out by Lemma 4.

$S_8$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given $(s_2, s_7), v(s_2, s_7) = (A, B, B)$.

Ruled out by Lemma 4.

$S_9$ : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given $(s_2, s_8), v(s_2, s_8) = (A, B, B)$.

Ruled out by Lemma 4.

$S_{10}$ : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given $(s_5, s_5)$, all voters abstain.

Suppose first that $\varepsilon < \frac{1}{2}$. If candidate $A$ deviates to $s^A = s_8$ and full information is not revealed, only voter $c$ observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate $A$ wins with probability at least $1 - \varepsilon > \frac{1}{2}$. If $\varepsilon > \frac{1}{2}$ then candidate $A$ can win the election with probability $\varepsilon$ deviating to $s^A = s_2$.

$S_{11}$ : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given $(s_5, s_6)$, beliefs such that $\omega_i^J (1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ support an equilibrium in which $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. Suppose $\varepsilon \leq \frac{1}{2}$. It suffices to check that $A$ has no incentives to deviate. If $A$ deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, $A$ loses the election. If
A deviates to \( s^A = s_8 \) and full information is not revealed, the election is tied, but if full information is revealed, \( A \) loses the election. In any case, after a deviation \( A \) wins the election with probability less than \( \frac{1}{2} \). If \( \varepsilon > \frac{1}{2} \) candidate \( A \) can win the election with probability \( \varepsilon \) deviating to \( s^A = s_2 \).

\( S_{12} \): Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_8)\). Given \((s_5, s_8), v(s_5, s_6) = (A, A, B)\). Ruled out by Lemma ??.

\( S_{13} \): Voter strategy \( s^i = (\emptyset, B, A, \emptyset) \) for each voter \( i \) and beliefs such that \( \omega_2^i, J(0) = \omega_2^i, J(1) = 1 \) for any voter \( i \) and any candidate \( J \) make the election tied. Suppose \( \varepsilon \leq \frac{1}{2} \); if candidate \( J \) deviates to any strategy \( s^J \neq s_8 \) and full information is not revealed, \( J \) loses the election. Suppose \( \varepsilon > \frac{1}{2} \); candidate \( J \) wins the election probability \( \varepsilon \) deviating to \( s^J = s_5 \).

So if \( \varepsilon > \frac{1}{2} \) there is no equilibrium in pure strategy, if \( \varepsilon < \frac{1}{2} \) the class of equilibria are \( S_{11} \) and \( S_{13} \). ■

6.3.2 Efficiency Concerned Candidates

**Proposition 24** If \( \varepsilon < \frac{\beta}{1+3\beta} \) the unique equilibrium is the efficient one; if \( \varepsilon \in \left( \frac{\beta}{1+3\beta}, \frac{1+3\beta}{2+4\beta} \right) \) we have a multiplicity of equilibria and either two or three projects are implemented; if \( \varepsilon \in \left( \frac{1+3\beta}{2+4\beta}, \frac{1+2\beta}{2+2\beta} \right) \) the unique equilibrium is such that only two projects are implemented. For \( \varepsilon > \frac{1+2\beta}{2+2\beta} \) no pure strategy equilibria exist.

**Proof.** \( S_1 \): If candidate \( J \) deviates and proposes \( s^J = s_5 \) then voter \( a \) and \( b \) votes for \( J \) for all \( \varepsilon \geq 0 \). Since the deviation is more efficient than the strategy \( s_1 \) an efficiency
concerned candidate finds profitable to deviate.

\( S_2 \): Assume without loss of generality that \((s^A, s^B) = (s_1, s_2)\). Given \((s_1, s_2)\), \(v(s_1, s_2) = (B, A, A)\). By Lemma ??, this cannot occur in equilibrium.

\( S_3 \): Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_3)\). Given \((s_1, s_3)\), \(v(s_1, s_3) = (B, B, A)\). Ruled out by Lemma ??.

\( S_4 \): Assume w.l.o.g. that \((s^A, s^B) = (s_1, s_5)\). Given beliefs such that \(\omega_2^{s^B}(0) = 1\) for all \(i \in \{a, b, c\}\), every voter votes for \(A\). Ruled out by Lemma ??.

\( S_5 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_2)\). Given \((s_2, s_2)\), if candidate \(J\) deviates to \(s^J = s_6\) voters \(a\) and \(c\) vote for \(J\) both in case full information is revealed and in case it is not revealed. Hence by deviating, \(J\) wins with for sure and makes a more efficient proposal

\( S_6 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_3)\). The same argument as in case \(S_5\) applies.

\( S_7 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_5)\). Given \((s_2, s_5)\), \(v(s_2, s_5) = (A, B, A)\). Ruled out by Lemma ??.

\( S_8 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_7)\). Given \((s_2, s_7)\), \(v(s_2, s_7) = (A, B, B)\). Ruled out by Lemma ??.

\( S_9 \): Assume w.l.o.g. that \((s^A, s^B) = (s_2, s_8)\). Given \((s_2, s_8)\), \(v(s_2, s_8) = (A, B, B)\). Ruled out by Lemma ??.

\( S_{10} \): Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_5)\). Suppose first that \(\varepsilon < \frac{1}{2}\). If candidate \(J\) deviates to \(s^J = s_8\), only voter \(c\) observes the deviation and candidate \(J\) wins with
probability $1 - \varepsilon$ proposing more projects. Suppose now that $\varepsilon > \frac{1}{2}$. If candidate $J$ deviates to $s_2$ wins the election when full information is revealed and looses when it is not revealed, but she proposes less projects. So this deviation is profitable if and only if

$$\varepsilon(1 + \beta) > \frac{1}{2}(1 + 2\beta), \text{ or}$$

$$\varepsilon > \frac{1 + 2\beta}{2 + 2\beta}$$

Consider the deviation $s'_J = s_8$. Now candidate $J$ wins only with probability $1 - \varepsilon$, that is when full information is not revealed. This deviation is profitable only if

$$(1 - \varepsilon)(1 + 3\beta) > \frac{1}{2}(1 + 2\beta), \text{ or}$$

$$\varepsilon < \frac{(1 + 4\beta)}{(2 + 6\beta)}$$

Any other deviation does not increase the efficiency of the proposal and decreases the probability of winning so it is not profitable. It follows that the equilibrium $S_{10}$ exists for $\varepsilon \in \left(\frac{(1+4\beta)}{(2+6\beta)}, \frac{1+2\beta}{2+2\beta}\right)$

$S_{11}$: Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given $(s^A, s^B) = (s_5, s_6)$, beliefs such that $\omega^A_J(1 - s'_i) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ and $v(s_5, s_6) = (\emptyset, A, B)$. If $A$ deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and
full information is not revealed, A loses the election. If A deviates to \( s^A = s_8 \) and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. Hence an efficiency concerned candidate deviates only if

\[
(1 - \varepsilon)\frac{1}{2}(1 + 3\beta) > \frac{1}{2}(1 + 2\beta), \text{ or } \\
\varepsilon < \frac{\beta}{(1 + 3\beta)}
\]

If candidate J deviates to \( s_2 \) wins the election when full information is revealed and looses when it is not revealed, but she proposes less projects. So this deviation is profitable if

\[
\varepsilon(1 + \beta) > \frac{1}{2}(1 + 2\beta), \text{ or } \\
\varepsilon > \frac{1 + 2\beta}{2 + 2\beta}
\]

Any other deviation does not increase the efficiency of the proposal and decreases the probability of winning so it is not profitable. It follows that the equilibrium \( S_{11} \) exists for \( \varepsilon \in \left( \frac{\beta}{(1 + 3\beta)}, \frac{1 + 2\beta}{2 + 2\beta} \right) \); notice that \( \frac{1 + 2\beta}{2 + 2\beta} > \frac{1}{2} \).

\( S_{12} \): Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_8)\). Given \((s_5, s_8)\), \( v(s_5, s_6) = (A, A, B) \).

Ruled out by Lemma ??.

\( S_{13} \): Suppose first that \( \varepsilon \leq \frac{1}{2} \). Voter strategy \( s^i = (\emptyset, B, A, \emptyset) \) for each voter \( i \) and
beliefs such that \( \omega_{2i}^J(0) = \omega_{2i}^J(1) = 1 \) for any voter \( i \) and any candidate \( J \) make the election tied, and if candidate \( J \) deviates to any strategy \( s' \neq s_8 \) and full information is not revealed, \( J \) loses the election. Suppose \( \varepsilon > \frac{1}{2} \). Then candidate \( J \) can win the election by proposing \( s' = s_5 \). The deviation is profitable if and only if

\[
\varepsilon (1 + 2\beta) > \frac{1}{2} (1 + 3\beta), \text{ or }
\varepsilon > \frac{1 + 3\beta}{2 + 4\beta}.
\]

It is worthy to notice that when both candidates are efficiency concerned, if \( \varepsilon < \frac{\beta}{(1 + 3\beta)} \) the unique equilibrium is the efficient one; if \( \varepsilon \in \left( \frac{\beta}{(1 + 3\beta)}, \frac{1 + 3\beta}{2 + 4\beta} \right) \) we have multiplicity of equilibria and either two or three projects are implemented, while if \( \varepsilon \in \left( \frac{1 + 3\beta}{2 + 4\beta}, \frac{1 + 2\beta}{2 + 2\beta} \right) \) the unique equilibrium is such that only two projects are implemented. For \( \varepsilon > \frac{1 + 2\beta}{2 + 2\beta} \) no pure strategy equilibria.

### 6.3.3 One Candidate Office Motivated and One Candidate Efficiency Concerned

**Proposition 25** If \( \varepsilon < \frac{\beta}{(1 + 3\beta)} \) in the unique equilibrium both candidates propose all the projects, if \( \varepsilon \in \left( \frac{\beta}{(1 + 3\beta)}, \frac{1}{2} \right) \) there are a multiplicity of equilibria (and at least two projects are proposed in equilibrium) and if \( \varepsilon > \frac{1}{2} \) there are no equilibria in pure strategies.

**Proof.** First of all note that there cannot be any equilibrium of this case that is
not an equilibrium in the case when both candidates are office motivated, because the office motivated candidate has the same incentives to deviate than any candidate of the previous case. So we have only to check whether the efficiency concerned candidate has incentive to deviate in the following two cases:

\( S_{11} \) : Assume w.l.o.g. that \((s^A, s^B) = (s_5, s_6)\) and \(\varepsilon < \frac{1}{2}\). Given \((s^A, s^B) = (s_5, s_6)\), beliefs such that \(\omega_{s_i}^J(1 - s_i^J) = 1\) for each \(i \in \{a, b, c\}\) and \(J \in \{A, B\}\) support an equilibrium in which \(v(s_5, s_6) = (\emptyset, A, B)\) and each candidate wins with equal probability. It suffices to check that the efficiency concerned candidate has no incentives to deviate. Suppose \(A\) is the efficiency concerned candidate. If she deviates to \(s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}\) and full information is not revealed, \(A\) loses the election. Since the number of projects is not larger than two, \(A\) has not incentive to deviate. If \(A\) deviates to \(s^A = s_8\) and full information is not revealed, the election is tied, but if full information is revealed, \(A\) loses the election. Hence \(A\) has incentive to deviate only if

\[
(1 - \varepsilon)\frac{1}{2}(1 + 3\beta) > \frac{1}{2}(1 + 2\beta), \quad \text{or} \\
\varepsilon < \frac{\beta}{(1 + 3\beta)}
\]

It follows that \(S_{11}\) is an equilibrium if \(\varepsilon \in \left(\frac{\beta}{(1 + 3\beta)}, \frac{1}{2}\right)\)

\( S_{13} \) and \(\varepsilon < \frac{1}{2}\). Voter strategy \(s^i = (\emptyset, B, A, \emptyset)\) for each voter \(i\) and beliefs such that \(\omega_{s_i}^J(0) = \omega_{s_i}^J(1) = 1\) for any voter \(i\) and any candidate \(J\) make the election tied.
Suppose $\varepsilon \leq \frac{1}{2}$; if a candidate deviates to any strategy $s' \neq s_8$ and full information is not revealed, $J$ loses the election. Moreover since any deviation implies a lower number of proposed projects the efficiency concerned candidate has no incentive to deviate. ■