Optimal Investment under Credit Constraint

Mohamed Belhaj † Bertrand Djembissi ‡

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Abstract

This paper analyzes the interactions between debt financing and investment in a growth option. We find that, due to tax shields, a partially debt financed option is exercised earlier than an all-equity financed option. We show that in absence of credit constraints, the value of the project at the investment date is not affected by debt. On the contrary, when the entrepreneur is credit constrained, the value of the project at the investment date depends on the amount of debt available. Furthermore, the investment trigger is a non monotonic function of debt level. Indeed, As credit constraints relax, entrepreneurs with small debt capacity speed up investment to exploit tax shields whereas those with large debt capacity postpone investment to minimize default risk.

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JEL Classification: G31, G32, G33.

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†GREQAM, Ecole Centrale de Marseille. Email: mohamed.belhaj@ec-marseille.fr
‡GREMAQ, Université de Toulouse 1, 21 Allée de Brienne, 31000 Toulouse, France. Email: bdjembis@univ-tlse1.fr. Tel: +33 614772284 Fax: +33 561225563. Corresponding author.
1. Introduction

The literature on investment under uncertainty has mainly focused on the consequence of irreversibility on the investment decision. One important result highlighted is that when investment is irreversible, it is not optimal to invest the first time at which the net present value (NPV) of the project becomes positive. Instead, since waiting has a value, it is optimal to invest only when the operating value of the project reaches a critical threshold that is larger than the investment cost. Although the real option literature has analyzed deeply the optimal timing of investment, little concern has been devoted up to now to the financing decision. Our work contributes to the recent literature, presented in section 2, that tries to bridge the gap between investments decisions and financing decisions. Our main objective is to analyze the impact of debt financing on the exercise of a growth option. Since the availability of funding is an important determinant of the investment level, we focus mainly of the effect of a borrowing constraint on both the timing of investment and the value of the firm. We consider an entrepreneur that holds an option to invest in a risky project. The entrepreneur has access to credit markets but may be constrained by the amount of debt she can get from creditors. Debt is a perpetual coupon bond and is fairly priced at the investment date. Since tax shields represent additional cash flows at the investment date, the option partially financed by debt is exercised earlier than the all-equity financed one. We characterize the optimal investment decision, the financing and liquidation decisions of the entrepreneur.

First, we analyze the impact of optimal debt financing on the investment timing when the credit constraint is not binding. We characterize the optimal investment and financing policies. It is found that, the investment trigger depends on the tax shields whereas the value of the project at the investment date does not depend on the tax shields. It follows that the entrepreneur exercises the investment option the first time at which the value of the project reaches the same critical level where would invest an all-equity financed entrepreneur. This result explains that as long as credit constraints are not binding, the value of the projet at the investment date is not affected by tax shields whereas the investment date is affected by tax shields. For reasonable values of bankruptcy costs and tax rates, we find that the entrepreneur is willing to finance a large fraction of the investment cost by debt, which would result in a transfer of the venture risk to creditors. Moreover, it is shown that the coupon rate and the optimal amount of debt demanded by the entrepreneur are non monotonic
functions of the project risk, decreasing for low risk projects and increasing for high risk projects. Two effects are at work at the investment date. In line with the real option theory, the value of the venture increases with the project risk, whereas, consistent with the tradeoff theory, leverage decreases with the project risk.

Second, we examine the effect of a binding credit constraint on the investment trigger. Here, creditors agree to finance only up to a defined fraction of the investment cost. In this case, both the investment trigger and the value of the venture at the investment date depend on the amount of available debt. Hence, the value of the venture at the investment date will differ from that of an all-equity financed venture. Since the amount of debt available to the entrepreneur is implicitly given by the latter constraint, the only flexibility is the choice of the investment trigger or equivalently the coupon to be paid to creditors. We solve for the optimal investment and financing decisions. It is found that the optimal investment trigger and the optimal coupon are linear functions of the investment cost. Interestingly, the relationship between the investment trigger and the level of debt available is non monotonic. As the credit constraint relaxes, entrepreneurs with low debt speed up the investment decision whereas entrepreneurs with large debt rather postpone the investment decision. This result reflects the tradeoff, in presence of risky debt, between tax shields and the cost of default.

The paper is organized as follows. Section 2 reviews the literature. Section 3 describes the model. Section 4 analyzes two benchmarks. In the first one the project is financed only by equity and the option is exercised at an optimal date. In the second one the project is undertaken immediately, and is financed by an optimal mix of debt and equity. These two benchmarks directly follow from the seminal papers of McDonald and Siegel (1986) and Leland (1994). Section 5 characterizes the optimal investment and financing policies and contains the main results. We first analyze the investment and financing policies when there is no credit constraint. Then, we examine the case where the credit constraint is binding. Section 6 concludes. Proofs of propositions are given in Appendix.

2. Related literature review

As we mentioned before, there are two strands of literature with relevance for this work. The first strand relies on the real option theory. This theory analyzes optimal investment policy. It points out that, when investment is irreversible, waiting has a value. MacDonald and Siegel (1986) pioneered the use of option theory to analyze investment decisions. They
illustrate that it is optimal to postpone investment until the net present value of the project is larger than a positive threshold. Hence, the exercise of the investment option should take into account the opportunity cost of losing the option to wait. This work has been followed by an important development by Dixit and Pindyck (1994). However, these seminal papers considered frictionless credit market and therefore ignored the financing policy. Childs, Mauer and Ott (2005) and Mauer and Sarkar (2005) analyze the conflict between bondholders and equityholders on the timing of the exercise of a growth option. Childs, Mauer and Ott (2005) show that, by making the debt less sensitive to changes in firm value and by allowing for more frequent repricing of debt, short term debt can mitigate the underinvestment and overinvestment problems. Mauer and Sarkar (2005) assume that all debt terms are decided before the investment decision and find that an equity-maximizing firm exercises the option too early relative to a value-maximizing strategy. These works focus on agency problems and thus leave aside the problem of optimal financing of the growth option and possible credit constraints.

The second strand of literature analyzes the optimal capital structure. In a seminal paper, Leland (1994) proposes a continuous time contingent-claims analysis framework to derive qualitative and quantitative guidance for corporate finance decision making. He approaches the optimal capital structure balancing the tax benefits and the default costs associated to debt financing. This work was followed by a series of papers that addressed related questions in similar settings. This includes Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) or Décamps and Faure-Grimaud (2002). This literature also provides for instance, quantitative results on optimal amount and maturity of debt (Leland and Toft (1996)), debt restructuring (Goldstein, Ju and Leland (2001)), credit spreads when bondholders have only imperfect information on the firm’s cash flows (Duffie and Lando (2001)) or more recently, in a setting with dynamic volatility choice, the role of Warrant in mitigating the asset substitution problem (Henessy and Tserlukevich (2004)).

Our work is also in line with a more recent literature that analyzes the interaction between financing decisions and investment decisions. Most of these works analyze the sensitivity of investment to cash flows. Povel and Raith (2001) show that information asymmetries about firm profitability decrease the investment and optimal investment is a U-shaped function of the firm’s level of internal funds. Boyle and Guthrie (2003) examine the effect of a financing constraint on the timing of investment. In their work, the financial constraint takes the form
of a costly access to external funds, making the firm highly dependent on internal funds. They find that financial constraint accelerates the investment since the threat of future shortfalls in funding lowers the value of waiting. They conclude that more financially constrained firms are more aggressive in entering new markets than less financially constrained firms. We will obtain a more general result: more financially constrained firms may invest earlier or later, depending on the tightness of the financing constraint. Décamps and Villeneuve (2005) analyze the interaction between dividend policy and the exercise of a growth option for liquidity constrained firms, and find ambiguous effect of uncertainty and liquidity shocks on the investment decision. Lyandres (2004) distinguishes between two types of financial constraints: the firm does not have enough internal funds or it has access to a costly outside finance. He finds that investment increases with the level of liquid assets and can also be positively correlated to the cost of external financing for some firms. The firm may therefore increase current investment in order to obtain more funds for future investments. In Grenadier and Wang (2005), the owner of the option delegates its exercise to a manager, yielding asymmetries information problems due to the manager hidden action and hidden information on the cash flows. In our model there is no agency problem since cash flows are publicly observable, debt is fairly priced and no debt exists prior to the investment decision. Our model rather reflects the tradeoff tax advantages versus default costs of debt.

3. The model

Consider an entrepreneur with an option to invest in a facility that costs \( I \) and who can use for the purpose outside financing. We assume that the firm operating profits are taxed at rate \( \theta \). Following the tradeoff theory, because of the tax advantage of debt, it is optimal for the entrepreneur to raise debt, even if she has enough money to cover the investment cost. We assume that debt is issued exactly at the investment date. Creditors are perfectly informed about the investment cost and the project characteristics. They agree to lend any amount of debt lower than a fraction of the investment cost. Since our objective is not to justify the financing constraint, but to draw implications on investment of such a constraint, we also assume that the contribution of creditors to the investment cost is exogenously specified. This debt constraint could capture industry practices: to prevent moral hazard, creditors may require that entrepreneurs contribute a significant fraction of the investment cost. The constraint may also reflect creditors risk constraint in the sense that they are willing to lend
only to firms with a probability of failure lower than a prescribed level. Financing constraints on investment is not uncommon, and has also been analyzed in a context of liquidity risk by Boyle and Guthrie (2001), who interpret it as simple "credit and bank loans constraints, restricted access to bond markets or prohibitively expensive equity costs".

The facility allows for operating profits after the investment with dynamics:

\[
\frac{dX_t}{X_t} = \mu dt + \sigma dW_t,
\]

where \( W = \{W_t, \mathcal{F}_t, 0 \leq t < \infty\} \) is a standard Brownian motion on a probability space \((\Omega, \mathcal{F}, \mathbb{P})\) equipped with the filtration \( (\mathcal{F}_t)_{0 \leq t < \infty} \) which will be assumed to satisfy the usual conditions. The constants \( \mu \) and \( \sigma \) are respectively the drift and volatility of the operating profits. Investors are risk neutral and discount the future at a constant rate \( r > \mu \).

At the investment date \( \tau \), the entrepreneur finances a fraction of the investment cost by debt in order to take advantage of the tax shields offered for interest expenses. We consider a simple debt contract: the firm pays out to debtholders a constant coupon at rate \( c > 0 \). Tax benefits of debt are received at rate \( \theta c \) until default. We assume that default on debt service will trigger liquidation of the firm. As in Leland (1994) bankruptcy occurs when equityholders cease to inject cash in order to finance debt service. The firm is run as an all-equity concern after default and bankruptcy costs are a proportional fraction \( \gamma \) of the unlevered firm value. These costs may correspond to legal and administrative costs, loss of credibility and loss of customers and suppliers. We also assume limited liability for equityholders and absolute priority rule. The problem of the entrepreneur is to choose the optimal mix of debt and equity, the date of investment \( \tau \), and the date of operations abandonment \( \tau_L \) (the liquidation date), given that she can raise an amount of debt \( b \) up to a fraction \( \psi \) of the investment cost. It should be emphasized that in this environment, the only motive for issuing debt is the tax shields offered for interest expenses. Actually, absent tax exemption on coupon payments, it is not optimal to issue debt since it will decrease the value of the firm by the expected bankruptcy costs.

At date \( t=0 \), the entrepreneur chooses the coupon \( c \) or equivalently the amount of debt to be issued at the investment date, and the investment policy. When the entrepreneur makes these decisions, she anticipates that future equityholders will strategically abandon the project at date \( \tau_L \), the first time at which cash flows reach the liquidation threshold \( x_L \). The entrepreneur maximizes the expected value of equity net of the contribution of equityholders to the investment cost, subject to the constraint that the amount of debt \( b \) to
be issued is lower than the fraction $\psi I$ of the investment cost that creditors are willing to finance.

Let $E(X_\tau, X_{\tau_L}, c)$ and $D(X_\tau, X_{\tau_L}, c)$ denote respectively equity and debt value evaluated at the investment date $\tau$ when the firm is liquidated at date $\tau_L$ and pays a coupon rate $c$. Formally, the entrepreneur solves the following program:

$$\begin{align*}
\sup_{c, \tau < \infty} & \mathbb{E} \left[ e^{-r\tau} (E(X_\tau, X_{\tau_L}, c) - (I - b)) \right], \\
\text{s.t.} & b \leq \psi I.
\end{align*}$$ (2)

where $\tau_L = \inf \{ t \geq 0 \text{ s.t } X_t^x = x_L \}$ and $x_L$ is chosen to maximize the value of equity at the investment date. The financing policy and the investment policy are be jointly determined. The optimal level of debt will be the result of the tradeoff between tax advantages of debt and bankruptcy costs. The choice of the coupon will affect the timing of investment through its effects on the value of the firm at the investment date. Conversely, the timing of investment will also affect the financing policy since waiting decreases the exposure of the firm to default risk and therefore decreases the cost of debt.

We do not exclude that the entrepreneur be initially endowed with an amount of cash $A$. In this case, she will invest her initial outlay $A$ in a risk free asset. At an investment date $\tau$, the amount of capital available to the entrepreneur is equal to the sum of the accumulated capital $Ae^{r\tau}$ and the debt $b$ raised on the market. When the amount $Ae^{r\tau} + b$ is lower than the investment cost $I$, the question of financing the remaining fraction of the investment cost is raised. Since our objective is to analyze the effect of debt financing on the timing of investment, we abstract from the latter problem by assuming that the entrepreneur can raise outside equity whenever needed, that is the entrepreneur can dilute, at the investment date, a fraction $\omega(\tau)$ of equity in order to finance the amount needed:

$$Ae^{r\tau} + b + \omega(\tau)E(X_\tau, X_{\tau_L}, c) = I.$$ (3)

Note that in this case, the dilution $\omega(\tau)$ depends on the investment date and is therefore stochastic.

4. **Two Benchmarks**

In this section, we briefly survey two standard models that are essential to our work. The first one, drawn on the model by MacDonald and Siegel (1986), corresponds to the case where the entrepreneur has no access to debt, whereas the second benchmark, drawn on Leland...
(1994), corresponds to the case where the entrepreneur has access to unlimited amount of
debt and invests at date t=0.

4.1. The all-equity financed firm

This subsection reviews the model by MacDonald and Siegel (1986). The entrepreneur
finances all the investment cost by equity. Since operating profits are always positive, eq-
uityholders will never abandon operation. For a current value of the cash flows x, the net
expected profit of investing is given at the investment date by:

\[
\frac{(1 - \theta)X_t}{r - \mu} - I.
\]  

(4)

Therefore, equityholders solve the following optimal stopping problem:

\[
\sup_{\tau < \infty} \mathbb{E} \left[ e^{-r\tau} \left( \frac{(1 - \theta)X_t}{r - \mu} - I \right) \right] = \sup_{x^e_t} \left[ ((1 - \theta)\nu x^e_t - I) \left( \frac{x}{x^e_t} \right)^{\beta_2} \right].
\]  

(5)

The optimal policy is to invest in the new facility the first time at which operating profits
exceed the trigger \(x^e_t\). The constant \(\beta_2\) is the positive solution to the equation \(\frac{\sigma^2}{2} y(y - 1) + \mu y = r\) and \(\nu = \frac{1}{r - \mu}\). The stochastic discount factor \((\frac{x}{x^e_t})^{\beta_2}\) represents the value
at date \(t = 0\) of one unit of cash received at the investment date. When choosing the
investment trigger, the entrepreneur trades off low net present value and early exercise of
the option against large net present value but later exercise of the growth option. Standard
computations show that

\[
x^e_t = \frac{\beta_2}{\beta_2 - 1} \frac{I}{(1 - \theta)\nu}.
\]  

(6)

The value of the project at the investment date is \((1 - \theta)\nu x^e_t - I = \frac{I}{\beta_2 - 1}\). In consequence we
can write the investment rule in the following form:

\[
\tau^e_t \equiv \inf \left\{ t \geq 0 \ s.t \ X^e_t = x^e_t \right\} = \inf \left\{ t \geq 0 \ s.t \ (1 - \theta)\nu x^e_t - I = \frac{I}{\beta_2 - 1} \right\}.
\]  

(7)

The investment rule therefore states that the entrepreneur should invest as soon as the
net present value of the project reaches the level \(\frac{I}{\beta_2 - 1}\), or equivalently that it is optimal
to exercise the option whenever the ratio project operating value-to-the investment cost
reaches the critical threshold \((1 - \theta)\nu x^e_t = \frac{\beta_2}{\beta_2 - 1} > 1\). It directly follows from this equality that
the investment trigger and then the value of the option are increasing functions of the project
risk.
4.2. A model of optimal capital structure

This subsection reviews the standard model of Leland (1994). It corresponds in our model to the case where the entrepreneur invests immediately (at t=0) and has access to unlimited amount of debt. After the exercise of the option, any claim on the ongoing firm operating profits is obtained as the expected discounted value of future cash flows accruing to the specific stakeholders. In particular, once the investment has been made, the entrepreneur holds a claim in the form of equity, protected by limited liability. This yields to the following optimal stopping problem for the value of equity:

\[
E(x, c) \equiv \Sup_{t \geq 0} E \left[ \int_{t}^{\infty} e^{-r(t-s)} (X_s^x - c) \, ds \right]
\]  
(8)

or equivalently

\[
E(x, c) \equiv \Sup_{x \geq 0} \left( \nu x - \frac{c}{r} + \left( \frac{c}{r} - \nu x_L \right) \left( \frac{x}{x_L} \right)^{\beta_1} \right),
\]  
(9)

where the default date is \( \tau_L = \inf \{ t \geq 0 \, : \, X_t^x = x_L \} \) for \( x \geq x_L \). Absent any covenant restriction, equityholders will cease to inject cash needed to finance debt service at a trigger \( x_L \), which is chosen to maximize the value of equity. Standard computations yield to

\[
x_L = \frac{\beta_1 c}{\beta_1 - \frac{1}{r \nu}} \equiv \delta c.
\]  
(10)

The optimal default trigger \( x_L \) depends only on the coupon rate. In particular, it is independent of the initial value of operating profits \( x \).

The value of debt is equal to the value of a perpetual coupon bond net of the expected loss for bondholders due to equityholders default option. For \( x \geq x_L \), it is equal to

\[
D(x, c) = \frac{c}{r} - \left( \frac{c}{r} - (1 - \gamma)(1 - \theta) \nu x_L \right) \left( \frac{x}{x_L} \right)^{\beta_1}.
\]  
(11)

The value of the firm is equal to the net present value of after taxes operating profits plus the expected tax benefits of debt net of the expected bankruptcy costs. For \( x \geq x_L \), it is equal to

\[
V(x, c) = (1 - \theta) \nu x + \frac{\theta c}{r} \left[ 1 - \left( \frac{x}{x_L} \right)^{\beta_1} \right] - \gamma (1 - \theta) \nu x_L \left( \frac{x}{x_L} \right)^{\beta_1}.
\]  
(12)

The optimal coupon denoted \( c^* \) maximizes the firm value:

\[
c^* = \ArgMax_{x \geq 0} V(x, c).
\]  
(13)
Straightforward computations yield to
\[ c^* = \frac{\lambda}{\delta x} \] (14)

where,
\[ \lambda = \left[ \frac{\theta}{\theta - \beta_1 (\theta + \gamma (1 - \theta))} \right]^{-\frac{1}{\beta_1}}. \] (15)

The coupon rate is a non monotonic function of the project risk. Low risk and large risk firms will pay out larger coupons. However, firm value and leverage are decreasing functions of the project risk.

5. Optimal investment decisions

Now, we turn back to the main problem, namely we solve for the optimal investment and financing decisions as described by equation (2). Since the liquidation trigger depends only on the coupon rate \( c \), the decision taken by the owner of the option is to choose the pair \((\tau, c)\) to maximize the value of the option subject to the credit constraint.

The value of the option is then given by the program below
\[
Sup_{c,\tau<\infty} \mathbb{E} \left[ e^{-r\tau} (E(X_{\tau}, c) - (I - b)) \right],
\]
\[
s.to \ b \leq \psi I. \] (16)

We start by showing that there is no conflict of interests between stakeholders on the timing of the investment when debt is fairly priced at the investment date. Recall that the investment cost is financed with a standard debt contract whereby debtholders are required a contribution \( b \leq \psi I \) in return of a perpetual coupon payment at rate \( c \). Rational expectations require that at the investment date \( \tau \), \( b \) fairly prices debt:
\[ D(X_{\tau}, c) = b. \] (17)

Finally, taking into account the fair pricing of debt in equation (17), the entrepreneur program is equivalent to
\[
F(x) = Sup_{x,\tau} \mathbb{E} \left[ e^{-r\tau} (V(X_{\tau}, c) - I) \right],
\]
\[
s.t \ D(X_{\tau}, c) \leq \psi I. \] (18)

The expression above indicates that the financing and investment policies are chosen by the entrepreneur to maximize the expected value of equity net of the contribution of equityholders to the investment cost (program (16)), and equivalently the expected firm value net of the
investment cost (program (18)). In others terms, an equity maximizing policy is equivalent to a firm maximizing policy. Consequently, there is no agency cost of debt financing. This result recognizes the interaction between the investment and the financing policies. Now we solve for the optimal investment and financing decisions. We start by analyzing the case where the credit constraint is not binding, then we consider the case when this constraint is binding.

5.1. Non binding credit constraint

When the entrepreneur has access to a sufficient amount of debt ($\psi$ should be large enough), the credit constraint is not binding. Technically, the constraint is irrelevant in program (18) and hence, the owner of the option solves:

$$F(x) = \text{Sup}_{\tau,c} E[e^{-r\tau}(V(X_\tau, c) - I)].$$  \hspace{1cm} (19)

This program is equivalent to

$$F(x) = \text{Sup}_{xI} E[e^{-r\tau} \text{Sup}_c ((V(xI, c) - I))],$$  \hspace{1cm} (20)

with $\tau = \inf\{t \geq 0 \ s.t \ X_t = xI\}$. We solve program (20) in two steps. First, given an investment trigger $xI$, we compute the associated optimal coupon $c^*(xI)$. This coupon maximizes the firm value at the investment date. Second, we solve for $xIF$ the optimal exercise trigger of the option and then we recover the optimal coupon $c^*(xIF)$.

For a given investment trigger $xI$, the corresponding optimal coupon $c^*(xI)$ is the coupon of Leland (1994) given by equation (14) and evaluated at $x = xI$:

$$c^*(xI) = \frac{\lambda}{\delta} xI.$$  \hspace{1cm} (21)

In the next step we take into account the above relation to compute the optimal investment trigger. At a given investment date, the value of the firm is given by equation (12) evaluated at $x = xI$ with $c^*(xI) = \frac{\lambda}{\delta} xI$:

$$V(xI) = V(xI, c^*(xI)) = [(1 - \theta) + \theta \lambda] \nu xI.$$  \hspace{1cm} (22)

The term $\theta \lambda$ captures the effect of tax advantage of debt net of bankruptcy costs. It then
follows from equation (20) that the optimal investment trigger \( x_I^f \) solves

\[
x_I^f = \text{ArgMax}_{x_I} \left[ \left( \frac{x}{x_I} \right)^{\beta_2} \left[ ((1 - \theta) + \theta \lambda) \nu x_I - I \right] \right].
\] (23)

The entrepreneur maximizes the value of the option to invest which is equal to the firm value at the investment date net of the investment cost, multiplied by the stochastic discount factor. Maximizing the value of the option with respect to the investment trigger yields:

\[
x_I^f = \frac{\beta_2}{\beta_2 - 1} \frac{I}{((1 - \theta) + \theta \lambda) \nu}.
\] (24)

Then the optimal coupon \( c_I^f \) is given by:

\[
c_I^f = c^*(x_I^f) = \frac{\lambda}{\delta} \frac{\beta_2}{\beta_2 - 1} \frac{I}{((1 - \theta) + \theta \lambda) \nu}.
\] (25)

The optimal level of debt \( D_I^f \) that the owner is willing to take is given by equation (11) evaluated at \( (x_I^f, c_I^f) \):

\[
D_I^f = D(x_I^f, c_I^f) = \varphi I
\] (26)

where,

\[
\varphi \equiv \frac{\beta_2}{\beta_2 - 1} \frac{\lambda}{((1 - \theta) + \theta \lambda) \nu \delta} \left[ \frac{1}{r} - \frac{\theta}{r} \frac{1 - (1 - \gamma)(1 - \theta) \nu r \delta}{\theta - \beta_1 (\theta + \gamma (1 - \theta))} \right].
\] (27)

It is interesting to observe that the optimal amount of debt is a linear function of the investment cost and the firm optimal leverage (defined as debt over firm value, at the investment date) is independent of the investment cost. It is also worth mentioning that the optimal level of debt \( D_I^f \) demanded by the entrepreneur may exceed the investment cost, especially if tax rate is high or costs of default are low. Since the owner has no interest in taking debt larger than \( D_I^f \), the credit constraint is not binding if and only if \( \varphi \leq \psi \). Then, the following results hold:

**Proposition 5.1** When the credit constraint is not binding \( (\varphi \leq \psi) \),

(i) A partially debt financed option is exercised earlier than an all-equity financed option:

\[
x_I^f < x_e^f.
\]

(ii) The net present value of the project at the investment date is independent of the tax shields.


Proposition (5.1) follows directly from, on the one hand, the comparison of the triggers $x^e_I = \frac{\beta_2}{\beta_2-1} \frac{I}{(1-\theta)\nu}$ and $x^f_I = \frac{\beta_2}{\beta_2-1} \frac{I}{((1-\theta)+\theta\lambda)\nu}$, and on the other hand from the expression $V(x^f_I) - I = \frac{I}{\beta_2-1}$ which is the same whether the option is financed only by equity or not. It is important to emphasize that this result crucially depends on both the fact that the amount of debt is chosen optimally, and the fact that credit constraints are not binding. The proposition suggests some comments.

First, financing a fraction of the investment cost by debt leads to earlier exercise of the option. The effect of the tax shields is to speed up the investment decision. Intuitively, financing a fraction of the investment cost by debt will increase the value of the firm at the investment date since the owner of the option will enjoy the tax shields. This allows the entrepreneur to invest in the project at a lower trigger compared to the all-equity financed case. As we will see in the next subsection, this result will still hold when the entrepreneur is limited by the credit constraint, since she can still partially enjoy tax shields.

Second, reminiscent of the Modigliani Miller theorem, the value of the project at the investment date is the same as in the case of an all-equity financed entrepreneur, $V(x^f_I) - I = ((1-\theta)+\theta\lambda)\nu x^f_I - I = (1-\theta)\nu x^e_I - I = \frac{I}{\beta_2-1}$. The investment rule is consequently identical to the investment rule in absence of debt financing, as specified by equation (7). Precisely, the investment date is

$$\tau^f_I \equiv \inf \left\{ t \geq 0 \text{ s.t. } X^x_t = x^f_I \right\} = \inf \left\{ t \geq 0 \text{ s.t. } V(X^x_t) - I = \frac{I}{\beta_2-1} \right\}. \quad (28)$$

The second part of proposition (5.1) has the connotation of the Modigliani Miller theorem. It is interesting to see that the fact that the value of the firm is not affected by the tax shields does not imply that the investment trigger is not affected neither. In fact, the effect of tax shield is completely captured by the investment trigger, as shown by the first part of our proposition.

**Corollary 5.1** The investment trigger and the value of the option increase with the volatility of the operating profits.

Since $\lambda$, defined in equation (15), decreases with $\sigma$, the investment trigger $x^f_I$ is increasing with $\sigma$. The value of the option is equal to the difference between the firm value and the investment cost, multiplied by a stochastic discount factor. Equation (22) shows that, at a given investment trigger, an increase in the project risk decreases the value of the firm.
because of the default risk, but increases the investment trigger due to the value of waiting. The optimal firm value at the investment date \( V(x_I^f) = (1 - \theta) \nu x_I^f \) increases with the project risk and therefore, the value of the option increases with the project risk. Finally, since tax rates and bankruptcy costs affect negatively the firm value, proposition 5.1 suggests also that the entrepreneur will postpone the investment decision as tax rates or bankruptcy costs increase.

5.2. Numerical illustration

From numerical simulations (Table 1), we observe several features of the optimal investment/debt financing decisions.

(a) Equation (14) shows that an increase of the project risk has two different effects on the optimal coupon. First, for a given investment trigger, as in the tradeoff theory (see Leland (1994)), the coupon rate is a U-shaped function of the project risk. Second, as in the real option theory, the investment trigger increases as the project risk increases. Our numerical result shows that the first effect dominates and consequently the optimal coupon rate remains U-shaped.

(b) Although leverage decreases with the operating profits volatility, the optimal debt level \( \varphi I \) is a U-shaped function of the project risk. Low risk entrepreneurs as well as large risk ones are willing to take large amount of debt.

(c) A large fraction of the investment cost is financed by debt, resulting in a transfer of an important part of the project risk to debtholders. This can justify ceilings on the amount of debt creditors are willing to invest in the project and suggests also that credit constraints will prevent entrepreneurs from extracting the full tax benefits of debt.

5.3. Binding credit constraint

It is not uncommon that creditors require from entrepreneurs to contribute to the investment cost. Since numerical analysis in the previous subsection show that the entrepreneur is willing to finance a large fraction of the investment cost by debt, the credit constraint \( D(x_I, c) \leq \psi I \) is likely to be binding. In this subsection, we consider the case where the credit constraint is binding, that is \( \psi < \varphi^1 \), so that the entrepreneur cannot get all the amount she is willing to

\[1\psi I \] is the maximum amount of debt given by creditors, whereas \( \varphi I \) is the optimal level of debt needed by the entrepreneur to fully exploit tax advantages of debt.
borrow from creditors. Since the value of the project is an increasing function of debt level, the entrepreneur will contract on the maximum amount of debt \(\psi I\) allowed by creditors. That is \(D(x_I, c) = \psi I\). Let first describe the optimization problem faced by the entrepreneur, given that she is constrained on the amount she can get from creditors. The expression \(D(x_I, c)\) is the fair price of debt given in equation (11). Solving the equation \(D(x_I, c) = \psi I\) implies that the investment trigger and the coupon rate will satisfy the following relation:

\[
x_I \equiv x_I(c) = \left[\frac{\xi - \psi I}{\xi - (1 - \gamma)(1 - \theta)\nu dc} \right]^{\frac{1}{\beta_2}} \delta c.
\] (29)

This relation is appealing, and points out that when the entrepreneur faces a financing constraint, she is not free to choose independently the investment policy and the financing policy, as it would have been the case if she was not facing the credit constraint. The entrepreneur will actually solve the following program:

\[
F(x) = \text{Sup}_{\tau, c} E \left[ e^{-r\tau} (V(X_{\tau}, c) - I) \right] \text{ subject to } (29) \tag{30}
\]

Or equivalently

\[
F(x) = \text{Sup}_c \left[ \left( \frac{x}{x_I(c)} \right)^{\beta_2} [V(x_I(c), c) - I] \right], \tag{31}
\]

where \(X_{\tau} \equiv x_I(c)\) and \(V(x, c)\) given by the expression of equation (12). It is then easy to solve program (31) in \(c\) and to recover the investment trigger \(x_I(c)\). We obtain the following result:

**Proposition 5.2** When the credit constraint is binding \((\varphi > \psi)\), the net present value of the project at the investment date depends on the level of debtholders contribution to the investment cost.

Proposition 5.2 states that the value of the firm at the investment date depends on the available credit. Hence, the value of the project at the investment date may differ from that of an all-equity financed entrepreneur. Consequently the investment rule of equation (7) does not hold anymore. Precisely, we have:

\[
\tau_i^* = \inf \{ t \geq 0 \text{ s.t } X_i^x = x_i^s \} \neq \inf \left\{ t \geq 0 \text{ s.t } V(X_i^x) - I = \frac{I}{\beta_2 - 1} \right\}. \tag{32}
\]
investment cost. The parallel with the Modigliani Miller theorem pointed out in the previous subsection does not hold here because of the friction on the credit market. The value of the option will increase as the credit constraint relaxes. The optimal coupon and investment trigger are linear functions of the investment cost. The optimal capital structure will differ from that of Leland (1994) because the entrepreneur cannot fully exploit the tax shields.

5.4. Numerical illustration

In this subsection, we illustrate how the credit constraint affects the investment trigger and the capital structure. For a set of parameters, Table 2 illustrates the impact of a restriction in the credit constraint on the project.

(a) The effect of an increase of debt on the timing of investment results from two conflicting incentives: the willingness of the entrepreneur to reduce the cost of debt by investing later, and her desire to exploit tax advantages of debt by investing earlier. It is found that the optimal investment trigger is a U-shaped function of debt. It is a decreasing for low levels of debt and increasing thereafter. For small levels of debt, the marginal tax benefit of debt is larger than the marginal default cost of debt. Then, the effect of tax shields dominates and the entrepreneur speeds up investment as debt increases. But, for large amount of available debt, the marginal default cost of debt is larger than the marginal tax gain associated, and thus the entrepreneur postpones the investment decision as debt increases. This has some implications.

The U-shaped form of the investment trigger implies that, on the one hand two entrepreneurs with different borrowing capacities may invest at the same date and, on the other hand, consistent with the finding of Boyle and Guthrie (2003), an entrepreneur with limited access to debt may invest earlier than another entrepreneur that faces no credit constraint. Now, consider for instance two entrepreneurs with different levels of debt but who invest at the same date. Since the firm value strictly increases with the level of debt, then at the investment date, the net present values of the two projects are necessarily different. Consequently, the credit constraint clearly affects the investment trigger, but also the value of the project at the investment date, as shown in proposition 5.2. To summarize, a binding credit constraint does not necessary lead the entrepreneur to invest later than she would do with full access to debt financing. Note also that if we interpret the entrepreneur contribution to the investment cost as firm’s internal funds, our result confirms an earlier result of Povel and Raith
(2003), who found that the investment trigger is a U-shaped function of internal funds. The result of Povel and Raith (2003) however relies on asymmetric information between the firm and outside investors. Again, there is no asymmetric information in our model and all our results rely on the standard tradeoff theory.

(b) The value of the venture at the investment date, \( V(x^*_I) \), is a U-shaped function of debtholders contribution to the investment cost. This is a conjunction of two effects: the investment trigger is a U-shaped function of debtholders contribution when for a given investment trigger, the value of the project is an increasing function of debt level. However, a stronger credit constraint reduces the value of the option.

(c) At the investment date, the expected probability of default is an increasing function of the fraction \( \psi \) of the investment cost financed by debt. Hence creditors willingness to keep a low probability of default at the date of investment/debt issuance gives a rationale for the credit constraint.

6. Conclusion

In this paper, we analyze the impact of debt financing on the timing of exercise of a growth option. Debt is fairly priced at the investment date and leads to an interaction between the investment and financing decisions. Because of the tax shields, the entrepreneur partially financed by debt invests at a lower trigger compared to the case where the investment is all-equity financed. We then analyze the impact of an exogenous constraint on the amount of debt that the owner of the option can get from creditors. When the constraint is not binding, the net present value of the project at the optimal investment date is not affected by the presence of debt, and the entrepreneur chooses the capital structure of Leland (1994). When the entrepreneur faces a binding credit constraint, a non monotonic relationship between debt level and the optimal investment trigger is obtained. Less constrained and extremely constrained entrepreneurs will invest later compared to intermediate constrained ones. Entrepreneurs use the flexibility in the investment timing to trade off the tax advantages of debt with the default risk associated to debt financing. In this environment, the net present value of the project at the optimal investment date depends on the amount of debt obtained from debtholders.
References


7. Appendix

7.1. Proof of Propositions

Proof of Proposition 5.1. Let \( x^e_I \) (resp. \( x^f_I \)) be the investment trigger when the option is financed by equity only (resp. by optimal mix of equity and debt).

(i) We have
\[
x^e_I = \frac{\beta_2}{\beta_2 - 1} \frac{I}{(1 - \theta) \nu} - \frac{1}{\beta_2 - 1} I (1 - \theta)
\]
and
\[
x^f_I = \frac{\beta_2}{\beta_2 - 1} \frac{I}{(1 - \theta) \nu + \theta \lambda} - \frac{1}{\beta_2 - 1} I (1 - \theta) + \theta \lambda.
\]
From \( \theta \lambda > 0 \) it follows that \( x^f_I < x^e_I \).

(ii) The NPV of the project at the investment date is
\[
V(x^f_I) - I = I \frac{\beta_2}{\beta_2 - 1} \frac{I}{(1 - \theta) \nu} - \frac{1}{\beta_2 - 1} I (1 - \theta)
\]
for an optimally debt financed project and
\[
(1 - \theta) \nu x^e_I - I = I \frac{\beta_2}{\beta_2 - 1} \frac{I}{(1 - \theta) \nu} - \frac{1}{\beta_2 - 1} I (1 - \theta)
\]
for an all-equity financed project. □

Proof of Proposition 5.2. Consider the following notation \( \xi \equiv \frac{c}{I} \). From equation (29) and for a coupon rate \( c \), the investment trigger can be written
\[
x_I(c) \equiv f(\xi) I,
\]
and the firm value at the investment date
\[
V(x_I(c), c) = g(\xi) I,
\]
where the function \( f(.) \) and \( g(.) \), independent of \( I \), are given by:
\[
f(\xi) = \left[ \frac{1 - \psi^\xi}{1 - (1 - \gamma)(1 - \theta) \nu \delta r} \right]^{\frac{1}{\beta_1}} \delta \xi
\]
and
\[
g(\xi) = (1 - \theta) \nu f(\xi) + \frac{\theta \xi}{r} - \frac{\gamma (1 - \theta) \nu \delta + \frac{\rho}{r}}{1 - (1 - \gamma)(1 - \theta) \nu \delta r} (\xi - \psi r).
\]
The optimization problem (31), faced by the owner of the option can be rewritten
\[
F(x) = I^{1 - \beta_2} \sup_{\xi} \left( \frac{x}{f(\xi)} \right)^{\beta_2} [g(\xi) - 1].
\]
Let
\[
\xi^* = \arg \max_{\xi} \left( \frac{g(\xi) - 1}{f(\xi)^{\beta_2}} \right),
\]
the optimal coupon \( c^* \) and the optimal investment trigger \( x^*_I \) are then given by:
\[
c^* = \xi^* I and x^*_I = f(\xi^*) I.
\]
The value of the firm at the investment date is:
\[
V(x^*_I, c^*) = g(\xi^*) I.
\]
Since \( \xi^* \) depends on \( \psi \), the coupon rate \( c^* = \xi^* I \), the investment trigger \( x^*_I = f(\xi^*) I \) and the value of the firm at the investment date \( V(x^*_I, c^*) = g(\xi^*) I \) will depend on the available debt \( \psi I \). It is also the case for the value of the option □
7.2. Tables

Table 1. Optimal debt and investment policies when the credit constraint is not binding as function of the instantaneous volatility of the firm operating profits after the exercise of the growth option and for an expected growth of $\mu = 0.1\%$. In this table, $\varphi$ is the optimal debt as percentage of the investment cost and $F$ is the value of the option for an initial value of the state variable normalized to $x = 5$. $x_f^I$ and $c^f$ are respectively the optimal investment trigger and coupon rate. The parameters values are: the default costs $\gamma = 0.4$, the tax rate $\theta = 0.30$, the fixed market interest rate $r = 0.06$ and the investment cost $I = 100$.

Table 1.

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<th>6</th>
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Table 2. Optimal debt and investment policies when the credit constraint is binding as function of the fraction of the investment cost received from debtholders, for an expected growth rate of the cash flows $\mu = 0.1\%$ and its volatility $\sigma = 5\%$. In this table, $\psi$ is the fraction of the investment cost financed by debt. $x^*_I$ and $c^*$ are respectively the optimal investment trigger and coupon rate. $V$ is the value of the firm at the investment date, $F$ is the value of the option for an initial value of the state variable normalized to $x = 5$ and $p$ the probability of default on debt at the investment date. The second column $\psi = \varphi$ corresponds to the benchmark case where the credit constraint is not binding. The parameters values are: the default costs $\gamma = 0.4$, the tax rate $\theta = 0.30$, the fixed market interest rate $r = 0.06$ and the investment cost $I = 100$.

Table 2.

<table>
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