Dynamic Marriage Matching: An Empirical Framework*

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Abstract
This paper develops a model for empirically analyzing dynamic matching in the marriage market and then applies that model to recent changes in the U.S. marriage distribution. Its primary objective is to estimate gains by age from being married today (till death of at least one spouse) relative to remaining single for that same time period. An empirical methodology that relies on the model’s equilibrium outcomes identifies the marriage gains using a single cross section of observed aggregate matches. This behavioral dynamic model rationalizes a new marriage matching function. The model also solves the inverse problem of computing the vector of aggregate marriages, given a new distribution of available single individuals and estimated preferences. Finally, this paper develops a simple test of the model’s empirical validity. Using aggregate data of new marriages and available single men and women in the U.S. over two decades from 1970 to 1990, I investigate the changes in marriage gains over this period.

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1 Introduction

This paper develops a model for empirically analyzing dynamic matching in the marriage market and then applies that model to recent changes in the U.S. marriage distribution. Its primary objective is to estimate gains by age from being married today (till death of at least one spouse) relative to remaining single (for that same time span). An empirical methodology that relies on the model’s equilibrium outcomes identifies marriage gains using a single cross section of observed aggregate matches. These estimates are consistent with a dynamic equilibrium model of frictionless marriage matching with transferable utility. Marriage is inherently fluid with heterogeneous participants, who differ by age, race, income, etc., entering and leaving matches. Unattached individuals can also forgo matching today with the hope of finding a better match in the future, albeit as older, different (e.g., perhaps more educated and wealthier) individuals. The behavioral overlapping generations model developed in this paper emphasizes these dynamic features of the marriage market. It also rationalizes a new marriage matching function.

The model extends the first empirical implementation of the static Becker-Shapley-Shubik marriage matching model proposed in Choo and Siow(2006)(CS hereafter) by positing a distribution of male and female preferences over all possible spouse types (which, for expositional convenience has been restricted to age). These preferences are explicitly aggregated to derive a quasi-demand and quasi-supply for spouses by age, which in turn are combined with an equilibrium market clearing condition to generate the equilibrium number of marriages.

In particular, I assume that all individuals of the same age have identical mean preferences and an individual specific additively separable idiosyncratic preference parameter that is independent of the spouse’s identity. This is a strong assumption, but provides enough heterogeneity in preferences among observationally identical individuals.
to rationalize observed aggregate matches. In the model, a single unattached individual decides whether to marry or remain single, while taking into account how this decision affects future choices. The choice to marry incurs a one-time transfer of utility at the time of marriage that is dependent only on the ages (i.e., types) of the couple (and not on the identity of the individuals matching). Similarly, the mean joint marriage payoffs depend on the ages of the individuals matching and the expected value of being single in the future should the marriage dissolve. A decision to marry “locks” an individual into a stream of age-dependent, within-marital utilities, with an exogenous probability of divorce at each age. As an individual ages, the expected value of participating in the marriage market also changes.\(^1\)

Methodologically, the model uses the dynamic discrete choice framework of Rust (1987). It rationalizes a new marriage matching function in which, for each age pair, the flow of new marriages is a function of the number of unmatched agents by age and the intrinsic marital surplus parameters. These parameters are model primitives and invariant to shocks in the number of available single men and women. The marriage gains parameter measures the difference between the discounted within-match utility of being married today (till death of at least one spouse) and that of remaining single (for that same time span). The parameter is identified from the ratio of the product of the equilibrium probabilities of marriage for the couple’s age groups, over a weighted average of the equilibrium probabilities that these age groups remain single in the future. The ratio’s denominator captures the expected opportunity cost incurred from matching and forgoing future participation in the marriage market. Since marriage might end in divorce, the weights on the probabilities of remaining single reflect the differences in the opportunity costs between the choice to marry and the choice to remain single. The model also solves the inverse problem of computing the vector of aggregate marriage, given a new distribution of available single individuals and estimated preferences. This permits me to compute counterfactual marriage distributions when there are demographic changes, assuming that preferences remain fixed.

\(^1\)The framework can be easily extended to allow for other discrete characteristics, such as race and education.
Using aggregate data of new marriages and available single men and women in the U.S. over two decades from 1970 to 1990, I demonstrate the model’s application by examining the changes in the marriage gains over these decades and compare the results with those obtained from a static model. This paper also develops a simple test of the model’s empirical validity.

During the period from 1970 to 1990, there has been a well-documented fall in the marriage rates in the U.S.. Part of this decline can be explained by socio-political changes that affected the institution of marriage.\(^2\) I use the proposed empirical model to estimate marriage gains over this period and compare them with marriage gains estimates ignoring dynamic considerations. I show that the contribution of the dynamic component of the total marriage gains is large, especially since most marriages occur when individuals are young and face many future opportunities to participate in the marriage market as they age. The decision to marry early suggests that the implied present discounted relative returns from locking into marriage early is high. When analyzing the change in marriage gains over these two decades, I show that ignoring the dynamic component of marriage gains severely understates the decline in marriage gains among the young.

The model makes a number of simplifying assumptions that can be relaxed, discussed in Section 4. Throughout this paper, I assume that divorce occurs at a constant exogenous rate. This is clearly not supported in the data, as is well documented in the literature (Wolfers and Stevenson (2007) and Browning et al.(2013)). The model can be extended to allow for heterogeneity in divorce rates that are dependent on the couple’s characteristics and the tenure of the marriage. However, modeling how divorce occurs more formally is beyond this paper’s scope.\(^3\) I have also assumed that the vector of available men and women by age is exogenously given. This strong assumption permits the system of marriage matching functions to be recast in terms of the unmarried indi-

\(^2\)For instance, some argue that the national legalization of abortion following the U.S. Supreme Court ruling on Roe versus Wade (1973) has lowered the marriage gains.

\(^3\)This is an important area of research that has long interested social scientists. Modeling how match values evolves would however require more detailed individual-level within-marriage data that is not used in this paper. See Brien et al. (2006)
iduals (that is, the equilibrium number of individuals who choose to remain single). This has the advantage of reducing the model’s dimensionality. This assumption can also be relaxed, as outlined in Section 4. Here, I extend the model to allow the number of single individuals to be endogenously determined through a system of demographic accounting equations. Relaxing this assumption also means that we lose the dimension reducing simplification. As in CS, I assume that the additively separable idiosyncratic shock is i.i.d. with a Type I Extreme Value distribution. Section 4 also considers relaxing this assumption by allowing for correlation across idiosyncratic draws. I also propose a bootstrap procedure for deriving the standard errors of the estimated preferences.

The rest of the paper proceeds as follows: Section 2 discusses related papers. I present the model in Section 3. In Section 4, I discuss relaxing some of the model’s assumptions and examine the implications. Section 5 describes the empirical application and documents the results. Section 6 concludes.

2 Related Literature

The empirical framework presented in this paper owes much of its spirit to the empirical industrial organization literature. Many papers on demand and supply estimation have focused on identifying consumers’ preferences and firms’ cost parameters from market-level data. Seminal papers such as Bresnahan(1987), Berry(1994) and Berry, Levinsohn and Pakes(1995), among others, continue the tradition of the discrete choice literature by developing methodologies to estimate individuals’ demand preference parameters over attributes. As has become dominant standard in the demand and supply literature, I assume that agents in my model value the characteristics of the person with whom they are matching.

There is a growing body of structural empirical papers on matching applied to a variety of matching markets. For instance, Chen (2013) develops a new matching model of CEOs and firms. That paper investigates the inefficiencies generated from misallocat-
ing CEOs across firms due to moral hazard (induced by the misalignment of CEO and shareholder incentives). Using data on one-to-one matches and transfers (wage and firm profits), Chen (2013) proposes a framework that recovers the preferences (surplus) and unobserved types on both sides (CEOs and firms) of a two-sided matching market. The author exploits the structure of optimal compensation contracting to recover surplus on both sides.

Agrawal (2013) analyzes the placement of U.S. medical residents through the centralized clearinghouse known as the National Residency Matching Program. The author proposes a new empirical strategy using pairwise stability and a vertical preference restriction on one side to identify preferences for both sides (residents and hospitals) of the market.

Fox (2010) focuses on identifying preferences in transferable utility models in a many-to-many setting applied to firm-level car parts data. The author proposes a maximum-score estimator based on a pairwise stability requirement. Because the number of inequalities can get large, Fox (2010) shows that the estimator is consistent as long as the rank-order condition holds for the set of inequalities used in the estimation.

Browning et al. (2013) and Chiappori et al. (2011) provide a stable matching characterization for preference utility that maintains the additive separability structure introduced in CS. They show that this structure reduces the complexity of stable matching into a set of simple inequalities that is easily satisfied by the probabilistic discrete choice framework of CS.

A number of papers have proposed generalizations to the CS empirical framework. Galichon and Salanié (2012) extend the empirical framework to a larger class of unobserved heterogeneities. Focusing on the social surplus function as the basis for their empirical application, these authors propose a parametric approach allowing matching across many observable attributes. Likewise Chiappori et al. (2011), Graham (2011) and Graham et al. (2013) propose various extensions to the CS model. Chiappori et al. (2011) allows for heteroskedasticity in idiosyncratic taste to investigate the marital college premium. As opposed to the approach taken here, all these papers maintain a static characterization of marriage matching which the current paper generalizes.
Decker et al. (2013) provides a test for the CS model that exploits the symmetry restrictions on the cross-type marriage elasticity matrix. The symmetry restriction requires that the elasticity of type \( i \) single men with respect to the supply of type \( j \) women be equal to the elasticity of type \( j \) single women with respect to the supply of type \( i \) men. This restriction is reminiscent of the Independence of Irrelevant Alternatives (I.I.A.) property brought about by the i.i.d. additive utility error imposed by the discrete choice structure (see McFadden (1974) and Debreu (1960)). The exact cost of this restriction in the context of marriage matching models that maintain the CS structure remains to be seen.

Graham (2013) proposes a new equilibrium representation to derive comparative static results for the CS model. These results extend the qualitative results derived by Decker et al. (2013) and provide insight into the substitution patterns the CS model implies (i.e., how the match distribution changes in response to changes in the availability of different types of agents and other model parameters).

Few papers apply non-transferable utility models to individual-level matchings. Echenique et al. (2013) is the first paper to derive the empirical implications of stable two-sided matching. The authors develop a revealed preference theory for stable matching and propose a non-parametric test for stability. They show that transferable utility matching theory is empirically nested in non-transferable utility matching theory.

Hsieh (2013) separately identifies male and female marital preferences using a non-transferable utility model applied to aggregate marriage data. The author argues that the Gale-Shapley’s

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\(^5\) A large body of literature in Empirical Industrial Organization and the discrete choice is devoted to overcoming the IIA properties of the discrete choice models. Some papers include McFadden (1978), Berry, Levinsohn and Pakes (1995) and Petrin (2002).

\(^6\) There is also a growing number of empirical papers that investigate the nature of marriage matching preferences through field experiments. Dugar, Bhattacharya and Reiley (2012) conduct an experiment, using a reputable (real world) Bengali arranged marriage market, to analyze how single men and women in India are willing to trade social status (i.e., caste) in the presence of strong economic incentives. The authors placed newspaper advertisements of potential grooms that systematically vary their caste and income and focused on responses of higher-caste females to lower-caste males. Higher-caste females lose their caste-status if they marry a lower-caste male. The authors provide strong empirical evidence suggesting that despite strong caste-based discrimination, higher-status females are willing to trade caste-status for an increase in the advertised income of lower-status males.
deferred-acceptance algorithm represents a special demand and supply system that can be used to generate a joint distribution of matches implied by the model. A related paper using the non-transferable utility specification is Hitsch et al. (2010), which focuses on identifying preferences separately from the matching process. Employing a dataset from an online dating service, the authors estimate a rich specification of preference over spousal physical and socio-economic characteristics. Using the estimated preferences, they simulate the men- and women-optimal matchings using the deferred-acceptance algorithm. The paper goes on to compare these optimal matchings to the actual matches observed in the online dating dataset.\footnote{Using a major online dating website in South Korea, Lee and Niederle (2011) conducted an experiment to see how preference signaling in the form of a virtual rose can increase chances in online dating.}

Many researchers have recognized that the marriage market is an important determinant of redistribution between men and women. Many empirical studies also suggest there may be strong connections between features of the marriage market, such as sex ratios and divorce laws, and redistribution within marriage and household labor supply behavior. (See, for example, Chiappori et al. (2002), Amuedo-Dorantes and Grossbard-Schectman (2007), and Lundberg and Pollak (1996), among others). These studies motivate empirical investigation of the joint determination of marriage matching and intra-household allocations. Choo and Seitz (2013) develop the collective marriage matching model that integrates the collective framework of intra-household decision making of Chiappori (1988, 1992) and the marriage matching model of CS. The authors’ framework of endogenizing marriage decisions in the collective model generates two independent sets of sharing rule estimates: one from labor supplies and one from marriage decisions.\footnote{The collective marriage matching model in Choo and Seitz (2013) is based on the collective model of Chiappori, Fortin and Lacroix (2002). As in Chiappori, Fortin and Lacroix (2002), Choo and Seitz (2013) focuses on households in which all consumption and leisure is private. In such a setting, the households Pareto problem can be decentralized and the resource allocation in the household can be summarized by a lump sum transfer of income, the sharing rule. Choo and Seitz (2013) produces a sharing rule that is the equilibrium outcome of marriage matching. The authors show that partial derivatives of the equilibrium sharing rules are identified from marriage data.} Using U.S. Census data, Choo and Seitz (2013) tests whether the
sharing rule that clears the marriage market is consistent with the sharing rule that determines labor supplies in households where both spouses work. Choo et al. (2008) develops the general form of the collective marriage matching model, which allows for risk sharing and public goods in marriage.

The papers discussed so far all focused on static one-shot matching. The idea of modeling marriage decisions using a dynamic structural model is not new and several papers estimate the model primitives using longitudinal data. In particular, van der Klaauw (1996) builds a dynamic model of marriage and employment to investigate the interdependence of female labor force participation and women’s marital status. Seitz (2009) extends the partial equilibrium of van der Klaauw (1996) to a two-sided model of marriage and employment to investigate the racial differences in marriage and employment in the U.S.. Brien, Lillard and Stern (2006) extends the search and matching model of Jovanovic (1979) to examine the evolution of match values to better understand cohabitation, marriage and divorce decisions. More recently, Bruze, Svarer and Weiss (2013) extends the transferable utility frictionless CS matching model to study marriage and divorce in a longitudinal Danish dataset and uses entry rates of new marriage and divorce probabilities of married couples to identify and estimate the marital surplus and the surplus shares of husbands and wives.

An earlier paper, Choo and Siow (2007), shares a similar objective to the current paper in its attempt to model the bivariate marriage distribution by age using the dynamic discrete choice framework of Rust (1987). It focuses on developing a general equilibrium framework to test different theories of marital and home production. The framework puts restrictions on linearized per-period marriage gains. The linearization in Choo and Siow (2007) allows the authors to empirically approximate the benefit of delaying marriage for one period versus marrying today using the marriage growth rate. This approximation then becomes the basis on which the authors test different theories of marital and home production.

While the present paper shares many similar objectives with Choo and Siow (2007), the representation of the dynamic problem proposed in the present paper is new. This new representation permits me to derive a closed form marriage matching function which
is the dynamic analogue of the one proposed in CS. The estimation of marital surplus
developed in this paper using a single cross-section of aggregate matches is simple and
transparent.

Aggregate marriage matching data strongly suggest that matches occur between
individuals of the same age as well as between ages that are far apart. The marriage
matching functions developed in CS and this paper emphasize this heterogeneity in
matching. The model permits substitution across all age groups of spouse, though with
many restrictions. In this paper, the dynamics in marriage decision adds an implicit
interdependence across ages. A single individual faced with choices at age $i$ knows his or
her type’s marriage prospect in the future and internalizes that in his or her decisions.
In other words, a high expected value from being single in the future would raise the
opportunity cost of being locked in a match today, hence lowering marriage gains and
the relative attractiveness of the match.

There is much literature devoted to understanding how frictions in the decentralized
labor market affect employment through the use of an aggregate job matching function.
Labor markets are characterized by the simultaneous presence of job seekers and job
vacancies. Analogous to the marriage matching function, the aggregate job matching
function maps the stock of job vacancies and the stock of job seekers to the flow of
new jobs (Pissarides (2000)). The job matching function provides a convenient reduced-
form way to introduce frictions into conventional models without modeling the complex
process that generates job matches.

An extensive empirical literature has largely taken a reduced-form approach and
focused on estimating the elasticities with respect to the number of job vacancies and the
number of job seekers. There is considerable empirical evidence to support a stable well-
behaved aggregate matching function of the Cobb-Douglas form with constant return
to scale in vacancies and unemployment (see Petrongolo and Pissarides (2001) for a
detailed survey). Other papers have identified heterogeneity in the form of individual
characteristics to be important in explaining individual hazard rates. This individual-
level heterogeneity in the matching function can be rationalized through the job seeker’s
choice of search intensity or through differences in their reservation wage.\(^9\)

As in the marriage market, many papers have recognized the importance of heterogeneity in workers and jobs in decentralized labor market. Frictions in job matching arise because of heterogeneities in job quality, worker skill, differences in job and worker locations and imperfect information about these and other relevant parameters (see Pissarides (2000)). Unlike the match heterogeneity seen in marriage markets (where couples of different types match), heterogeneity in workers and jobs generates unfilled jobs and unemployed workers.\(^10\) That is, heterogenous unemployed workers (skills, occupations, industries, locations) are seeking employment in sectors different from those where vacancies exist. The literature considering heterogeneity in vacancies and job seekers within a model of employment is extensive.\(^11\) Heterogeneity is usually cast in terms of worker or match productivity and job characteristics. (Lise and Robin (2013), Han and Yamaguchi (2013) and Hagedorn et.al.(2010) are some recent papers that allow for heterogeneity in productivity and job characteristics). Another unique feature of the marriage matching function that is absent in the macro and labor literature is the implicit interdependence between the number of matches for say age \(i,j\) couples, and the future marriage prospects of these age groups.

\(^9\)Merz and Yashiv (2007), Feve and Langot (1996) and Yashiv (2000) are part of the growing number of papers that take a structural approach to estimating the matching function.


3 A Dynamic Matching Model

3.1 Preview of Results

Consider a stationary environment in which men and women live for $Z$ periods. The equilibrium matching model delivers a new marriage matching function:

$$
\mu_{i,j} = \widehat{\Pi}_{i,j} \sqrt{m_i f_j \prod_{k=0}^{T_{i,j}} \left( \frac{\mu_{i+k,0} \mu_{0,j+k}}{m_{i+k} f_{j+k}} \right)^{\frac{1}{2}(\beta(1-\delta))}}.
$$

Equation (3.1) expresses the number of new marriages between age $i$ men and age $j$ women, $\mu_{i,j}$ as a function of the model primitives estimate, $\widehat{\Pi}_{i,j}$; the stationary number of available age $i$ men and available age $j$ women, $m_i$ and $f_j$ respectively; and a ‘scaled average’ of the proportion of single age $i$ men, $(\mu_{i,0}/m_i)$, and single age $j$ women, $(\mu_{0,j}/f_j)$. The term $T_{i,j} = Z - \max (i, j) \geq 0$ represents the maximum length of a match before one of the spouses in the match passes away at the terminal age $Z$. For convenience, I will refer to $T_{i,j}$ as the duration of the match. $(1 - \delta)$ denotes the probability a marriage survives in any period and $\beta$ is the per-period discount factor. $\mu_{i,0}$ and $\mu_{0,j}$ denote the number of type $i$ men and type $j$ women who chose to remain single at age $i$ and $j$ respectively. The parameter $\widehat{\Pi}_{i,j}$ represents an estimate of marriage gains from an $(i,j)$ marriage relative to remaining single for the duration of the match.

Like in CS, the matching function (3.1) is homogenous of degree zero in the vector of single individuals and marriages and allows for substitution effects across all spouse ages. That is, holding all else constant, a change in $m_i'$ or $f_j'$ will lead to a change in

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12 A marriage matching function, denoted by $\mu = G(m, f; \Pi)$, is a simple reduced form way of characterizing the entire distribution of aggregate marriages by age, denoted by a $(Z \times Z)$ matrix of marriages $\mu$ as a function of exogenous factors that include a $(Z \times 1)$ vector of available single men, $m$, a $(Z \times 1)$ vector of available single women, $f$ and a $(Z \times Z)$ matrix of parameters, $\Pi$. $\mu_{i,j}$ is the $(i,j)$th element of $\mu$, $m_i$ the $i$th element of $m$, and $f_j$ the $j$th element of $f$, and $\Pi_{i,j}$ denotes the $(i,j)$th element of the matrix $\Pi$.

13 For ease of exposition, the survival rate (which is one less the divorce rate) of a marriage is assumed to be constant across ages of matches and marriage tenure. Section 4 relaxes this assumption by allowing heterogeneity in the divorce rate.

14 The CS marriage matching function is $\mu_{i,j} = \pi_{i,j} \sqrt{\mu_{i,0} \mu_{0,j}}$. 


the number of equilibrium \((i, j)\) matches, \(\mu_{i,j}\), even if \(i' \neq i\) and \(j' \neq j\). Another unique feature of this function is the interdependence between the number of equilibrium \((i, j)\) matches and their respective future marriage prospects as captured by the equilibrium probabilities of remaining single in the future, \((\mu_{i+k,0}/m_{i+k})\) and \((\mu_{0,j+k}/f_{j+k})\). This interdependence arises because of the dynamic feature in the matching model.\(^{15}\)

Moving observed quantities to one side and model parameters to the other and taking logs, I obtain:

\[
2 \ln \hat{\Pi}_{i,j} = \left[ \ln \frac{\mu_{i,j}}{m_i} - \sum_{k=0}^{T_{i,j}} \ln \left( \frac{\mu_{i+k,0}}{m_{i+k}} \right)^{(\beta(1-\delta))k} \right] + \left[ \ln \frac{\mu_{i,j}}{f_j} - \sum_{k=0}^{T_{i,j}} \ln \left( \frac{\mu_{0,j+k}}{f_{j+k}} \right)^{(\beta(1-\delta))k} \right]. \quad (3.2)
\]

This marital surplus captured by \(2 \ln \Pi_{i,j}\) represents the couple’s discounted within-marriage utilities from being locked in an \((i, j)\) match today for the duration of the match relative to the present discounted sum of per-period payoffs for both individuals remaining single for the entire period. It comprises the male and female net gains from marriage relative to being single for the duration of the match as represented by \(n_{i,j}(\mu, m, f)\) and \(N_{i,j}(\mu, m, f)\) respectively. The tuple \((\mu, m, f)\) represents the matrix of observed marriages and the vector of available men and women, respectively.\(^{16}\)

The model assumptions (in particular, the Type 1 Extreme Value distributional assumption together with the transferable utility setup) allows me to express the net marital surplus as a dynamic log-odds ratio. This dynamic analogue to a ‘static’ log odds ratio, is comprised of an age \(i\) male’s equilibrium probability of an \((i, j)\) marriage

\(^{15}\)This marriage matching function also needs to satisfy a set of accounting constraints. The accounting constraints are

\[
\begin{align*}
\mu_{0,j} + \sum_{i=1}^{Z} \mu_{i,j} &= f_j \quad \forall j, \\
\mu_{i,0} + \sum_{j=1}^{Z} \mu_{i,j} &= m_i \quad \forall i, \\
\mu_{0,j}, \mu_{i,0}, \mu_{i,j} &\geq 0 \text{ for all } i \text{ and } j.
\end{align*}
\]

I will describe these constraints in more detail in Section 3.3.

\(^{16}\)Both \(n_{i,j}(\mu, m, f)\) and \(N_{i,j}(\mu, m, f)\) are endogenous quantities which depend on the relative scarcity of available men and women.
as estimated by \( \mu_{i,j} / m_{i,j} \) over a weighted average of the equilibrium probabilities that an age \( i \) male would remain single in the future as estimated by \( \prod_{k=0}^{T_{i,j}} \left( \mu_{i+k,0} / m_{i+k} \right)^{(\beta(1-\delta))^{k}} \).

The equilibrium probability of remaining single at a future age captures the expected value of being single and participating in the marriage market at that age. In the event that an \((i, j)\) marriage survives for \( k \) periods, the age \( i \) male’s expected cost of being unable to participate in the marriage market when he is age \( i + k \) is captured by \( \left( \mu_{i+k,0} / m_{i+k} \right) \). If the expected marriage rates at \( i + k \) are high, then the opportunity cost for men of being locked in a marriage at age \( i + k \), rather than being able to participate in the marriage market, is also high. This result implies that male net gains from the choice of an \((i, j)\) marriage relative to staying single are large.\(^{17}\)

The interpretation of \( N_{i,j}(\mu, m, f) \) is similar.

Equation (3.2) is also the estimating equation for the parameters \( 2 \ln \Pi_{i,j} \). These parameters are structural in that they are invariant to marriage market demand and supply changes and capture the preferences of individuals in the market. In empirical application, practitioners are often interested in the inverse problem. That is, given an estimated vector \( \hat{\Pi} \) consistent with a vector of aggregate matches \( (\mu, m, f) \), satisfying equation (3.1) and the accounting constraints, how do changes in the vector of available men and women affect the distribution of matches? I use this model to analyze the change in marriage gains in the U.S. between 1970 and 1990 and compare these results with the static matching model of CS.

\(^{17}\)In other words, when comparing male net marriage gains for pairs \((i, j)\) and \((i', j')\) that have the same duration of marriage and the same male probabilities of marriage (i.e. \( \mu_{i,j} / m_{i,j} = \mu_{i',j'} / m_{i',j'} \)), if the future marriage rates for \( i \) are greater than \( i' \), I expect the male net gains from an \((i, j)\) match to be larger than that of \((i', j')\). The reason is that in choosing to marry age \( j \) women, age \( i \) men on average are incurring a greater opportunity cost than age \( i' \) men who marry age \( j' \) women (i.e. \( n_{i,j}(\mu, m, f) > n_{i',j'}(\mu, m, f) \)). Since a decision by the couple to marry at ages \( i \) and \( j \) does not completely preclude the couple from participating again in the marriage market in the future, the weights in the ratio reflect the difference in the forgone future opportunity from marriage relative to being single.
3.2 The Model Environment

Consider a stationary society in discrete time populated by overlapping generations of adults. Each individual lives for $Z$ periods irrespective of gender.\textsuperscript{18} The youngest adult is of age one. The age of a male is indexed by $i$ and the age of a female is indexed by $j$. Agents are horizontally differentiated by age only.\textsuperscript{19} $m_i$ and $f_j$ denote the numbers of single males of age $i$ and females of age $j$ at the beginning of each period and I assume the vectors of single men and women $\mathbf{m} = (m_1, \ldots, m_Z)'$ and $\mathbf{f} = (f_1, \ldots, f_Z)'$ as exogenous and given. I relax this assumption in Section 4.

Any single age $i$ male $g$ (or age $j$ female $h$) in each period is characterized by two state variables:

- $i$ (or $j$) $\in \{1, \ldots, Z\}$ is his (or her) age when single, and
- $\epsilon_{i,g}$, (or $\epsilon_{j,h}$) is a $(Z+1)$ vector of i.i.d idiosyncratic payoffs or match specific errors specific to age $i$ male, $g$ (or age $j$ female, $h$) that are unobserved to the econometrician.

At each period, a single age $i$ male (or single age $j$ female) faces a random utility draw from each type of spouse available and from remaining single. He or she chooses the option maximizing his or her discounted expected utility. Male $g$ observes the $(Z+1) \times 1$ vector of idiosyncratic payoffs $\epsilon_{ig} = (\epsilon_{i,0,g}, \epsilon_{i,1,g}, \ldots, \epsilon_{i,Z,g})'$ at the beginning of each period before deciding on a utility-maximizing decision. $\epsilon_{i,0,g}$ is the idiosyncratic payoff that $g$ receives from remaining single, and $\epsilon_{i,j,g}$ is the idiosyncratic payoff that $g$ receives from matching with an age $j$ female. Similarly female $h$ observes a $[(Z + 1) \times 1]$ vector of idiosyncratic payoffs $\epsilon_{j,h} = (\epsilon_{0,j,h}, \epsilon_{1,j,h}, \ldots, \epsilon_{Z,j,h})'$.

These type-specific idiosyncratic draws do not depend on the identity of the spouse the single decision maker meets or matches with; they depend only on the the identity of

\textsuperscript{18}This assumption can be relaxed to allow for differential mortality by age and gender without changing the qualitative results of the model.

\textsuperscript{19}Sautmann (2011) extends the Shimer and Smith (2000) transferable utility model of search and matching to allow for types (defined by age) that change continuously over time. The author derives conditions for positive and negative assortative matching and differential age matching.
the decision maker male \( g \) (or female \( h \)). The crucial assumption first introduced in CS provides empirical tractability in explaining observed aggregate matches. In other words, the idiosyncratic contribution of two observationally equivalent age \( j \) female partners to male \( g \)'s utility are the same. I assume that the i.i.d. \( \epsilon \) are drawn from McFadden’s Type I extreme value distribution.\(^{20}\) In Section 4, I consider \( \epsilon \) drawn from the family of Generalized Extreme Value distributions to allow for persistence in the idiosyncratic preferences.

Only single adults can make utility maximizing decisions whether or not to marry. A decision to marry locks an individual into a stream of payoffs in the event the marriage does not dissolve due to divorce or death. Marriages dissolve at a constant exogenous per-period rate, \( \delta \), for all \((i,j)\) pairs.\(^{21}\) Divorce shocks are realized just before the end of a period after utility maximizing decisions are made. A single individual who chooses to marry during a period could be divorced before that period ends, in which case he (or she) re-enters the marriage market in the next period as a one-period-older individual. If divorce occurs in period \( k \) of a marriage, where \( 1 \leq k \leq (Z - \max(i,j) + 1) \), the individuals \( g \) and \( h \) re-enter the marriage market as single individuals of age \( i + k \) and \( j + k \) respectively. The duration of an \((i,j)\) match, \( T_{i,j} = (Z - \max(i,j) + 1) \), is the maximum length of a marriage that ends with the death of the oldest spouse.

Let \( \alpha_{i,j,k} \) (or \( \gamma_{i,j,k} \)) be the \( k^{th} \) period within marriage surplus accrued to an age \( i \) male (or \( j \) female) when married to an age \( j \) female (or age \( i \) male) where \( k \geq 1 \). \( a_{i,g} \) (or \( a_{j,h} \)) denotes the action of a single age \( i \) male \( g \) (or \( j \) female \( h \)) where \( a_{i,g} \) (or \( a_{j,h} \)) \( \in \mathcal{D} = \{0, 1, \ldots, Z\} \). If he (or she) chooses to remain single, \( a_{i,g} = 0 \) (or \( a_{j,h} = 0 \)). If he (or she) chooses to match with an aged \( k \) spouse, \( a_{i,g} = k \) (or \( a_{j,h} = k \)). The one period utility of male \( g \) with state vector \((i, \epsilon_{i,g})\) and action \( a_{i,g} \) is denoted by \( v(a_{i,g}, i, \epsilon_{i,g}) \).

\(^{20}\)The marginal density is given by \( f(\epsilon_{i,g} \mid i) = \prod_{a_{i,g}=0}^{Z} \exp[-\epsilon_{i,a,g} + c] \exp[-\exp(-\epsilon_{i,a,g} + c)] \), where \( c \) is the Euler constant.

\(^{21}\)This approximation while clearly restrictive, simplifies the equilibrium condition which I take to data and the estimation strategy used to estimate the model primitives. Understanding how within-match value evolves and endogenizing divorce is an important topic of research that is beyond the scope of the present paper. In Section 4.3, I formulate a model that allows for heterogeneity in the divorce rate that depends on the couple’s ages and the marriage tenure.
utility of female $h$ with state vector $(j, \epsilon_{j,h})$ and action $a_{j,h}$ is denoted by $w(a_{j,h}, j, \epsilon_{j,h})$.

\[ v(a_{i,g} = j, i, \epsilon_{i,g}) = \begin{cases} \alpha_i(j) - \tau_{i,j} + \epsilon_{i,j,g}, & \text{if } 1 \leq a_{i,g} \leq Z \\ \alpha_{i,0} + \epsilon_{i,0,g}, & \text{if } a_{i,g} = 0, \text{ and} \end{cases} \]  

(3.3)

\[ w(a_{j,h} = i, j, \epsilon_{j,h}) = \begin{cases} \gamma_j(i) + \tau_{i,j} + \epsilon_{i,j,h}, & \text{if } 1 \leq a_{j,h} \leq Z \\ \gamma_{0,j} + \epsilon_{0,j,h}, & \text{if } a_{j,h} = 0, \end{cases} \]  

(3.4)

where the present discounted gains from the match $\alpha_i(j)$ and $\gamma_j(i)$ take the form

\[ \alpha_i(j) = \sum_{k=1}^{T_{i,j}} (\beta(1-\delta))^{k-1} \alpha_{i,j,k}, \text{ and } \gamma_j(i) = \sum_{k=1}^{T_{i,j}} (\beta(1-\delta))^{k-1} \gamma_{i,j,k}. \]  

(3.5)

$\alpha_{i,0}$ and $\gamma_{0,j}$ are the per-period utilities from remaining single for $i$ age males and $j$ age females respectively. The time discount factor is denoted by $\beta \in (0, 1)$.

Equation (3.3) states that if age $i$ male $g$ marries an age $j$ woman, he receives the mean utility from the match equal to $\alpha_i(j) - \tau_{i,j}$, plus an idiosyncratic shock $\epsilon_{i,j,g}$, with the mean utility from marriage depending only on the age of the men and women in the match.\(^{22}\) He also receives an idiosyncratic return specific to him, $\epsilon_{i,j,g}$, that does not depend on the precise identity of $g$’s spouse. The mean utility from marriage comprises of two terms. $\alpha_i(j) = \sum_{k=1}^{T_{i,j}} (\beta(1-\delta))^{k-1} \alpha_{i,j,k}$ captures the discounted stream of male within-marriage payoffs in the event that the marriage does not dissolve. If the divorce happens in the $k$’th period of marriage, the couple will re-enter the marriage market as an $i+k$ and a $j+k$ age single male and female (assuming that $T_{i,j} \geq k$). If $g$ chooses to remain single, his current-period mean utility (common to all men of his age) is $\alpha_{i,0}$. In choosing this match, $g$ commits to pay a one-off transfer, $\tau_{i,j}$, specific to the age pair $(i, j)$ of individuals matching.

Similarly, in equation (3.4), an age $j$ female $h$ who decides to marry an age $i$ man agrees to receive this equilibrium transfer. In accepting the match, she locks herself into a stream of marital payoffs, of which the present discounted value equals $\sum_{k=1}^{T_{i,j}} (\beta(1-\delta))^{k-1} \gamma_{i,j,k}$. Thus, if male $g$ wants to marry female $h$, he has to transfer $\tau_{i,j}$ of marital

\(^{22}\)This is, in part, due to the AS assumption, but transferable utility and frictionless matching are also necessary for this.
output to her. Similarly, if an age \( j \) woman \( h \) wants to marry an age \( i \) man \( g \), she has to be willing to accept \( \tau_{i,j} \) of marital output from him. Each individual takes \( \tau_{i,j} \) as exogenous. The marriage market clears when, given \( \tau_{i,j} \), for every \( i,j \), the number of age \( i \) men who want to marry age \( j \) women is equal to the number of age \( j \) women who want to marry age \( i \) men. This transfer can be positive or negative. In this full-commitment model, the one-time payment of \( \tau_{i,j} \) fully internalizes: the discounted stream of within-marriage utilities for this \((i,j)\) couple, the exogenous divorce probabilities and the relative scarcity of males and females in the system.

The full-commitment model adopts the standard Beckerian assumption that prospective spouses who marry make binding agreement on the stream of allocations within marriage and the one-off transfer that is specific to the age (or types) of the couples. This transferable utility assumption provides a simple way for each partner to be compensated for the desirability of his or her age (or type) using match specific marriage gains without affecting aggregate marriage surplus (see Weiss (1997)).

The specification of preferences over partners and the evolution of the state variables satisfy two assumptions: the Additive Separability (AS) and Conditional Independence (CI) assumptions. Both these assumptions were introduced by Rust (1987) in the context of a single agent dynamic discrete choice model. I assume that the following assumptions hold:

**Assumption AS Additive Separability:**

The utility functions \( v(a_{i,g}, i, \epsilon_{i,g}) \) and \( w(a_{j,h}, j, \epsilon_{j,h}) \) have additively separable decompositions of the form:

\[
\begin{align}
  v(a_{i,g}, i, \epsilon_{i,g}) &= v_a(i) + \epsilon_{i,a,g}, \\
  w(a_{j,h}, j, \epsilon_{j,h}) &= w_a(j) + \epsilon_{j,a,h},
\end{align}
\]

where \( \epsilon_{i,a,g} \) and \( \epsilon_{j,a,h} \) are the \( a \)th component of the vector \( \epsilon_{i,g} \) and \( \epsilon_{j,h} \) respectively. \( v_a(i) \) (or \( w_a(j) \)) is the mean utility accrued for all age \( i \) men (or \( j \) women) who

---

\(^{23}\)Given this is a full commitment model of marriage with exogenous divorce, transfers could equivalently be made as a stream over the course of the marriage. I use a one-time payment for analytical convenience and assume there is no resource or income constraint at the time of marriage.
chose action \( a \). From equations (3.3) and (3.4) above:

\[
v_a(i) = \begin{cases} 
\alpha_i(j) - \tau_{i,j}, & \text{if } 1 \leq a_{i,g} \leq Z \\
\alpha_{i,0}, & \text{if } a_{i,g} = 0, \text{ and}
\end{cases}
\]

\[
w_a(i) = \begin{cases} 
\gamma_j(i) + \tau_{i,j}, & \text{if } 1 \leq a_{j,h} \leq Z \\
\gamma_{0,j}, & \text{if } a_{j,h} = 0.
\end{cases}
\]

**Assumption CI Conditional Independence:**

The transition probability of the state variables for males and females respectively factorize as

\[
\mathbb{P}\{i', \epsilon'_{i,g} \mid i, \epsilon, a\} = \mathbb{f}(\epsilon' \mid i) \cdot \mathbb{F}_a(i' \mid i), \tag{3.8}
\]

\[
\mathbb{P}\{j', \epsilon'_{j,h} \mid j, \epsilon, a\} = \mathbb{f}(\epsilon' \mid i) \cdot \mathbb{R}_a(j' \mid j), \tag{3.9}
\]

where \( \mathbb{f}(\epsilon) \) is the multivariate pdf of the i.i.d \( \epsilon \), and \( \mathbb{F}_a(i' \mid i) \) (or \( \mathbb{R}_a(j' \mid j) \)) is the probability that the male (or female) individual will next be single again at \( i' \) (or \( j' \)) given action \( a \) and his (or her) current age \( i \) (or \( j \)).

The **AS** assumption is standard in the static and dynamic discrete choice literature. This structure ensures that the utility function depends separately on a function of observed state and action, \( v_a(i) \) and \( w_a(j) \), and the individual-specific idiosyncratic taste parameters, \( \epsilon_{i,a,g} \) and \( \epsilon_{a,j,h} \). This restriction on preferences preclude any interaction between the observed state and choice decision, and the idiosyncratic taste term. The **CI** assumption limits the dependence structure on the state variables. The author in Rust (1994), argues that the observed states \( i' \) (and \( j' \)) are sufficient statistics for the unobserved states \( \epsilon_{i',g} \) (and \( \epsilon_{j',h} \)). So any dependence between say \( \epsilon_{i,g} \) and \( \epsilon_{i+1,g} \) is only transmitted through the observed age, \( i + 1 \) and not through the unobserved \( \epsilon_{i,g} \). More importantly, the probability that any individual \( g \) is single again at age \( i' \) depends only on his action \( a \) and age \( i \) and not on the unobserved state \( \epsilon_{i,g} \). I use \( \mathbb{F}_a(i' \mid i) \) (or \( \mathbb{R}_a(j' \mid j) \)) to denote the transition probability that an age \( i \) male \( g \) (or \( j \) female \( h \)) will next find himself (or herself) single at age \( i' \) (or \( j' \)) given his (or her) action \( a \) at age \( i \) (or \( j \)). \( \epsilon \) are i.i.d. noise that are superimposed on this process. Appendix 7.1 provides details about the state transition probabilities, \( \mathbb{F}_a(i' \mid i) \) and \( \mathbb{R}_a(j' \mid j) \).
Figure 1: Timing Sequence of shocks and decisions

[Diagram showing the sequence of events: observe $\epsilon_{ig}$, choice made, divorce shock.]

Male $g$ of age $i$ is single at beginning of period

Period ends

Figure 1 above provides an overview of the sequence of events and decisions in a given period. A single agent observes the choice-specific idiosyncratic shocks $\epsilon$ at the beginning of the period. This is followed by expected utility maximizing decisions. If the agent chooses to marry, a divorce shock is revealed before the period ends.

Since the decision problem for a single individual is a finite-horizon problem, I solve it by backward induction. The model permits agents to make choices only when they are single. Consider a single male $g$ in his terminal age $Z$; his value function is given by

$$V_\alpha(Z, \epsilon_{Z,g}) = \max_{a} \left\{ v(0, Z, \epsilon_{Z,g}), v(1, Z, \epsilon_{Z,g}), \ldots, v(Z, Z, \epsilon_{Z,g}) \right\},$$

where the functional form of the utility from choosing action $j$ with state vector $(Z, \epsilon_{Z,g})$, $v(j, Z, \epsilon_{Z,g})$ is given by equation (3.3). Working backwards, for $i < Z$, we get the following Bellman equation for a single age $i$ individual $g$:

$$V_\alpha(i, \epsilon_{i,g}) = \max_{a} \left\{ \alpha_{i,0} + \beta \mathbb{E}[V_\alpha(i+1, \epsilon_{i+1,g}) | i, \epsilon_{i,g}, a_{i,g} = 0] + \epsilon_{i,0,g}, \right. $$

$$\left. \max_{a \in \{1, \ldots, Z\}} \left\{ \alpha_i(a) - \tau_{i,a} + \sum_{k=i+1}^{i+T_{i,a}} \beta^{k-i} \mathbb{E}[V_\alpha(k, \epsilon_{k,g}) | i, \epsilon_{i,g}, a_{i,g} + \epsilon_{i,j,g}] \right\} \right\}. \quad (3.10)$$

The CI assumption allows me to factorize the future expectation of the value function from being single at age $k$ (conditional on being single at age $i$, observing the vector of idiosyncratic payoffs $\epsilon_{i,g}$ and choosing decision $a_{i,g}$) as

$$\mathbb{E}[V_\alpha(k, \epsilon_{k,g}) | i, \epsilon_{i,g}, a_{i,g}] = \mathcal{F}_a(k \mid i) \int V_\alpha(k, \epsilon_{k,g}) f(d\epsilon_g).$$

The latter term, $\int V_\alpha(k, \epsilon_{k,g}) f(d\epsilon_g)$, is referred to as the *integrated value function*. The Type I Extreme Value distributional assumption on $\epsilon_g$ allows it to have a closed form.
representation. I will discuss this further below. The Bellman equation for a single-age
$j$ female (where $j < Z$) takes a similar form,

$$W_j(j, \epsilon_{j,h}) = \max \left\{ \gamma_0 + \beta \mathbb{E}[W_j(j+1, \epsilon_{j+1,h}) | j, \epsilon_{j,h}, a_{j,h} = 0] + \epsilon_{0,j,h}, \right.$$  

$$\max_{a \in \{1, \ldots, Z\}} \left\{ \gamma_j(a) + \tau_{a,j} + \sum_{k=j+1}^{j+T_{a,j}} \beta^{k-j} \mathbb{E}[W_j(k, \epsilon_{k,h}) | j, \epsilon_{j,h}, a_{j,h}] + \epsilon_{a,j,h} \right\}, \quad (3.11)$$

Consider decomposing the value functions $V_\alpha(i, \epsilon_{i,g})$ and $W_j(j, \epsilon_{j,h})$ in equations 
(3.10) and (3.11) into two parts: (1) a mean component that is dependent on the utility 
maximizing choice and the current age, and (2) an idiosyncratic component. The mean 
component is common to all individuals of the same age choosing the same action. 
$I(a \neq 0)$ and $I(a = 0)$ are indicator variables for the decisions to marry and to remain 
single respectively. I denote the mean component for age $i$ males and $j$ females by $\tilde{v}_{i,j}$ 
and $\tilde{w}_{i,j}$ respectively:

$$\tilde{w}_{i,j} = \begin{cases} 
\gamma_j(i) + \tau_{i,j} I(i \neq 0) + \gamma_0 I(i = 0) \\
+ \sum_{k=j+1}^{j+T_{i,j}} \beta^{k-j} \mathbb{E}[W_j(k, \epsilon_{k,h}) | j, \epsilon_{j,h}, a_{j,h} = i], & \text{if } j < Z, \\
\gamma_j(i) + \tau_{i,j} I(i \neq 0) + \gamma_0 I(i = 0), & \text{if } j = Z,
\end{cases} \quad (3.12)$$

$$\tilde{v}_{i,j} = \begin{cases} 
\alpha_i(j) - \tau_{i,j} I(j \neq 0) + \alpha_{i,0} I(j = 0) \\
+ \sum_{k=i+1}^{i+T_{i,j}} \beta^{k-i} \mathbb{E}[V_\alpha(k, \epsilon_{k,g}) | i, \epsilon_{i,g}, a_{i,g} = j], & \text{if } i < Z, \\
\alpha_i(j) - \tau_{i,j} I(j \neq 0) + \alpha_{i,0} I(j = 0), & \text{if } i = Z.
\end{cases} \quad (3.13)$$

Now, the individuals’ optimization problem can be represented as the familiar discrete 
choice problem:

$$V_\alpha(i, \epsilon_{i,g}) = \max_{a \in D} \{ \tilde{v}_{i,a} + \epsilon_{i,a,g} \}, \quad (3.14)$$

$$W_j(j, \epsilon_{j,h}) = \max_{a \in D} \{ \tilde{w}_{a,j} + \epsilon_{a,j,h} \}, \quad (3.15)$$

where $\tilde{v}_{i,a}$ and $\tilde{w}_{a,j}$ are commonly referred to in the literature as the choice specific value 
functions.
Let the conditional choice probability \( P_{i,j} \) denote the probability that choice \( j \) is the optimal choice for age \( i \), that is
\[
P_{i,j} = \int I\{j = \arg \max_{a \in \mathcal{D}} (\tilde{v}_{i,a} + \epsilon_{i,a,g})\} f(d\epsilon).
\]
Similarly, for females, \( Q_{i,j} \) is the probability that choice \( i \) is the optimal choice for age \( j \) females. That is,
\[
Q_{i,j} = \int I\{i = \arg \max_{a \in \mathcal{D}} (\tilde{w}_{a,j} + \epsilon_{a,j,h})\} f(d\epsilon).
\]

The conditional choice probability can be expressed as a function of the normalized choice value functions, \((\tilde{v}_{i,j} - \tilde{v}_{i,0})\). In this case, the probability that a type \( i \) male who matches with a type \( j \) female will have the familiar multinomial logit form is:
\[
P_{i,j} = \frac{\exp(\tilde{v}_{i,j} - \tilde{v}_{i,0})}{1 + \sum_{r=1}^{Z} \exp(\tilde{v}_{i,r} - \tilde{v}_{i,0})}.
\]
Similarly, for females, the conditional choice probability is:
\[
Q_{i,j} = \frac{\exp(\tilde{w}_{i,j} - \tilde{w}_{0,j})}{1 + \sum_{r=1}^{Z} \exp(\tilde{w}_{r,j} - \tilde{w}_{0,j})}.
\]

I will now delve deeper into the mean utilities \( \tilde{v}_{i,j} \) and \( \tilde{w}_{i,j} \) and derive a representation for the log-odds ratio of a match relative to remaining single. Consider the integrated value function as introduced by Rust (1987) where the unobserved state is integrated out of the Bellman equations. Let \( V_i \) and \( W_j \) be the integrated value function for a single age \( i \) male and age \( j \) female respectively. That is, \( V_i = \mathbb{E}\alpha_i(\epsilon_g) = \int V_{\alpha}(i, \epsilon_g) f(d\epsilon_g) \) and \( W_j = \mathbb{E}\gamma_j(\epsilon_h) = \int W_{\gamma}(j, \epsilon_h) f(d\epsilon_h) \). In this finite-horizon case, the Type I Extreme Value distributional assumption allows these integrated value functions to have a recursive structure,
\[
V_i = \begin{cases} 
\alpha_{i,0} + c - \ln P_{i,0} + \beta V_{i+1} & : i < Z, \\
\alpha_{i,0} + c - \ln P_{i,0} & : i = Z,
\end{cases}
\]
\[
W_j = \begin{cases} 
\gamma_{0,j} + c - \ln Q_{0,j} + \beta W_{j+1} & : j < Z, \\
\gamma_{0,j} + c - \ln Q_{0,j} & : i = Z,
\end{cases}
\]
where \( c \) is the Euler’s constant. Appendix 7.2 presents the derivations of equations (3.18) and (3.19).

Consider the integrated value function for a male at terminal age \( Z \). Equation (3.18) says that the expected value of participating in the marriage market at age \( Z \) is simply the familiar (McFadden’s) social surplus function (or expected maximum utility) given
by $\alpha_{Z,0} + c - \ln \mathcal{P}_{Z,0}$. It depends on the period utility from being single, $\alpha_{Z,0}$, and the equilibrium probability of a $Z$ type male remaining single, $\mathcal{P}_{Z,0}$. If the marriage rate for age $Z$ males is large, (that is, $\mathcal{P}_{Z,0}$ is small) then the expected value of participating in the marriage at age $Z$, $V_Z$ will also be large. This familiar functional form arises from the Type I Extreme Value additively separable idiosyncratic payoff. Moving age backward to $i < Z$, the integrated value function in equation (3.18) has an additional term represented by $\beta V_{i+1}$. This is part of the mean utility from choosing to remain single at age $i$, and reflects the continuation value of participating in the marriage market in the next period as an older ($i + 1$) individual. When the individual is at a terminal age $Z$, the continuation value in the next period is hence zero. Equation (3.19) has an analogous interpretation.

With repeated substitution of equation (3.19) into $\tilde{v}_{i,j}$ and $\tilde{v}_{i,0}$, and after some algebra, the mean utilities can be expressed as a function of the expected surplus from the per-period random utilities:

$$
\tilde{v}_{i,j} \quad = \quad \alpha_i(j) - \tau_{i,j} + \sum_{k=1}^{T_{i,j}-1} \beta^k (1 - (1 - \delta)^k) \left( \alpha_{i+k,0} + c - \ln \mathcal{P}_{i+k,0} \right) \\
+ \sum_{k=i+T_{i,j}}^Z \beta^{k-i} \left( \alpha_{k,0} + c - \ln \mathcal{P}_{k,0} \right) 
$$

(3.20)

$$
\tilde{v}_{i,0} \quad = \quad \alpha_{i,0} + \sum_{k=i+1}^Z \beta^{k-i} \left( \alpha_{k,0} + c - \ln \mathcal{P}_{k,0} \right) 
$$

(3.21)

Repeated substitution of equation (3.19) into the mean utilities from the female decision problem, $\tilde{w}_{i,j}$ and $\tilde{w}_{0,j}$, I get the following analogous expressions for the expected surplus to females:

$$
\tilde{w}_{i,j} \quad = \quad \gamma_j(i) + \tau_{i,j} + \sum_{k=1}^{T_{i,j}-1} \beta^k (1 - (1 - \delta)^k) \left( \gamma_{0,j+k} + c - \ln \mathcal{Q}_{0,j+k} \right) \\
+ \sum_{k=j+T_{i,j}}^Z \beta^{k-j} \left( \gamma_{0,k} + c - \ln \mathcal{Q}_{0,k} \right) 
$$

(3.22)

$$
\tilde{w}_{0,j} \quad = \quad \gamma_{0,j} + \sum_{k=j+1}^Z \beta^{k-j} \left( \gamma_{0,k} + c - \ln \mathcal{Q}_{0,k} \right). 
$$

(3.23)
The log-odds ratio of an \((i,j)\) match relative to \(i\) remaining single identifies the difference in mean utilities, \(\tilde{v}_{i,j} - \tilde{v}_{i,0}\). It describes the expected payoffs to an age \(i\) male from a match with an age \(j\) female relative to remaining single that period. This ratio has the following expression:

\[
\log \left\{ \frac{P_{i,j}}{P_{i,0}} \right\} = \begin{cases} 
\alpha_i(j) - \alpha_i,0(j) - \tau_{i,j} & \text{if } \max(i,j) < Z \\
- \sum_{k=0}^{T_{i,j}-1} (\beta(1-\delta))^k \left( c + \ln P_{i+k,0}^{-1} \right) & \text{if } \max(i,j) = Z
\end{cases}
\]

(3.24)

Appendix 7.3 presents the derivation of equations (3.24) and (3.26). The log-odds ratio comprises of three components: (1) the equilibrium transfer \(\tau_{i,j}\), (2) the parameters

\[
\alpha_i(j) - \alpha_i,0(j) = \sum_{k=0}^{T_{i,j}-1} (\beta(1-\delta))^{k-1}(\alpha_{i,j,k} - \alpha_{i+k,0}),
\]

(3.25)

representing the discounted stream of within marriage net utilities an age \(i\) male gets in the \(k\)th period of marriage relative to the per period utility from remaining single that period (in the event that the marriage does not dissolve) and (3) the term

\[
- \sum_{k=0}^{T_{i,j}-1} (\beta(1-\delta))^k \left( c + \ln P_{i+k,0}^{-1} \right),
\]

representing the discounted sum of future log male marriage rates.\(^{24}\)

Higher future marriage rates increases the opportunity cost of committing to an \((i,j)\) match today. Thus, lowering the mean utility of the match relative to remaining single.\(^{25}\)

Similarly for females, the log-odds ratio that a \(j\) type female marries an \(i\) type male relative to remaining single equals the difference in choice specific value functions \(\tilde{w}_{i,j} - \tilde{w}_{i,0}\). That is:

\[
\log \left\{ \frac{Q_{i,j}}{Q_{0,j}} \right\} = \begin{cases} 
\gamma_j(i) - \gamma_{0,j}(i) + \tau_{i,j} & \text{if } \max(i,j) < Z \\
- \sum_{k=1}^{T_{i,j}} (\beta(1-\delta))^k \left( c + \ln Q_{0,j+k}^{-1} \right) & \text{if } \max(i,j) = Z
\end{cases}
\]

(3.26)

---

\(^{24}\)When the marriage rates at \(i + k\) are high, so would \(\ln P_{i+k,0}^{-1}\).

\(^{25}\)Notice that the \(k\)th period discount weights in the log-odds ratio, \((\beta(1-\delta))^{k-1}\), reflects time discounting of future period utilities \((\beta^{k-1})\), as well as the probability that the marriage survives to that period given by \((1-\delta)^{k-1}\).
Equation (3.26) gives the difference in systematic expected payoffs for a $j$ type female marrying an $i$ type male relative to remaining single during that same period. The term

$$\gamma_j(i) - \gamma_{0,j}(i) = \sum_{k=0}^{T_{i,j}-1} (\beta(1-\delta))^{k-1}(\gamma_{i,j,k} - \gamma_{0,j+k})$$

represents the discounted stream of within-marriage net utilities accrued to an age $j$ female in the $k$th period of marriage relative to the per-period utility from remaining single during that same period. Contrary to the male’s equation, the transfer $\tau_{i,j}$ enters additively. Just as in the male case, the term $\sum_{k=0}^{T_{i,j}} (\beta(1-\delta))^{k} \left(c + \ln Q_{0,j+k}^{-1}\right)$ captures the opportunity cost incurred from choosing to marry at age $j$ instead of staying single and participating in the marriage market in the future.

### 3.3 Equilibrium and the Dynamic Marriage Matching Function

Rearranging the terms of equations (3.24) and (3.26) delivers a dynamic analogue of the log-odds ratio. The left-hand side of equation (3.28) has the natural log of the probability that an age $i$ male matches with an age $j$ female, $\mathcal{P}_{i,j}$, relative to the scaled products of the probabilities that an age $i$ male remains single during that period and for the duration of the proposed match, $\prod_{k=0}^{T_{i,j}} \mathcal{P}_{i+k,0}^{(\beta(1-\delta))^{k}}$. The denominator represents the opportunity cost of future participation in the marriage market that an $i$ type male incurs when he chooses to match with a $j$ type female. When the probability of remaining single in the future, $\mathcal{P}_{i+k,0}$ is large, then the forgone opportunity of being locked into marriage is small, and vice versa. The future probabilities of remaining single at age $i+k$ are scaled by the discount factor and the probabilities that the marriage survives for $k$ periods. I will refer to this quantity as the dynamic log-odds ratio for an age $i$ male marrying with an age $j$ female. The constant $\kappa$ is the geometric sum of Euler’s constants, $\kappa = c\beta(1-\delta)(1 - (\beta(1-\delta))^{T_{i,j}})/(1 - \beta(1-\delta))$.

$$\ln \mathcal{P}_{i,j} - \sum_{k=0}^{T_{i,j}} (\beta(1-\delta))^{k} \ln \mathcal{P}_{i+k,0} = \alpha_i(j) - \alpha_{i,0}(j) - \tau_{i,j} - \kappa$$

(3.28)

$$\ln \mathcal{Q}_{i,j} - \sum_{k=0}^{T_{i,j}} (\beta(1-\delta))^{k} \ln \mathcal{Q}_{0,j+k} = \gamma_{0,j}(i) - \gamma_j(0) + \tau_{i,j} - \kappa.$$

(3.29)
Equations (3.28) and (3.29) identify the discounted stream of within-marriage net utilities an age $i$ male gets relative to the per-period utility from remaining single, $\alpha_i(j) - \alpha_{i,0}(j)$, less the equilibrium transfer, $\tau_{i,j}$. The interpretation of the dynamic log-odds ratio for an age $j$ female marrying an age $i$ male in equation (3.29) is similar with the central difference is that the age $j$ female is the recipient of the transfer. I will denote the dynamic log-odds for an age $i$ male marrying an age $j$ female by $n_{i,j}(\mu, m, f) = \ln \left( \frac{P_{i,j}}{\prod_{k=0}^{T_{i,j}} P_{i+k,0}^{(\beta(1-\delta))k}} \right)$, and for the female counterpart, $N_{i,j}(\mu, m, f) = \ln \left( \frac{q_{i,j}}{\prod_{k=0}^{T_{i,j}} q_{0,j+k}^{(\beta S)k}} \right)$.

Since equations (3.28) and (3.29) are defined for every $(i, j)$ marriage pair, in aggregate they form a system of $(Z \times Z)$ quasi-demand and quasi-supply equations respectively.

**Definition 1:** A marriage market equilibrium consists of a vector of available males, $m$, and females, $f$, the vector of marriages $\mu$, and the vector of transfers, $\tau$, such that the number of age $i$ men who want to marry age $j$ spouses exactly equals the number of age $j$ women who agree to marry age $i$ men for all combinations of $(i, j)$. That is, for each of the $(Z \times Z)$ sub-markets,

\[ m_i P_{i,j} = f_j Q_{i,j} = \mu_{i,j}. \]

**Dynamic Marriage Matching Function:** Let $p_{i,j}$ and $q_{i,j}$ denote the maximum likelihood (ML) estimators of the probability that an age $i$ male matches with an age $j$ female, $P_{i,j}$, and an age $j$ female matches with an age $i$ male, $Q_{i,j}$ respectively. That is, $p_{i,j} = \mu_{i,j}/m_i$ and $q_{i,j} = \mu_{i,j}/f_j$. The above marriage market clearing conditions and the ML estimators for the choice probabilities are applied to the system of quasi-supply and demand equations, (3.28) and (3.29) respectively, to derive the Dynamic Marriage Equilibrium.

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26In the static framework of CS, we get an analogous representation of quasi-demand and quasi-supply of spouses corresponding to the case when $T_{i,j} = 0$. That is:

\[ \ln P_{i,j} - \ln P_{i,0} = \alpha_{i,j} - \alpha_{i,0} - \tau_{i,j} \]
\[ \ln Q_{i,j} - \ln Q_{0,j} = \gamma_{i,j} - \gamma_{0,j} + \tau_{i,j} \]
Matching Function for an \((i,j)\) marriage (when \(T_{i,j} > 0\))^27 given by equation (3.1):

\[
\mu_{i,j} = \hat{\Pi}_{i,j} \sqrt{m_i f_j} \prod_{k=0}^{T_{i,j}} \left( \frac{\mu_{i+k,0} \mu_{0,j+k}}{m_{i+k} f_{j+k}} \right)^{\frac{1}{2}(\beta(1-\delta)) k}
\]

where \(\ln \Pi_{i,j} = \frac{1}{2} \left( \alpha_i(j) + \gamma_j(i) - \alpha_{i,0}(j) - \gamma_{0,j}(i) \right) - \kappa.\)

The dynamic marriage matching function also needs to satisfy the accounting constraints given by equations (3.30), (3.31) and (3.32):

\[
\mu_{0,j} + \sum_{i=1}^{Z} \mu_{i,j} = f_j \forall j \tag{3.30}
\]

\[
\mu_{i,0} + \sum_{j=1}^{Z} \mu_{i,j} = m_i \forall i \tag{3.31}
\]

\[
\mu_{0,j}, \mu_{i,0}, \mu_{i,j} \geq 0 \forall i, j \tag{3.32}
\]

Equation (3.30) states that the total number of age \(j\) women who marry and the number of unmarried age \(j\) women must be equal to the number of available age \(j\) women for all \(j\). Similarly equation (3.31) states that the total number of women who marry age \(i\) men and the number of unmarried age \(i\) men must be equal to the number of available age \(i\) men for all \(i\). Equation (3.32) holds because the number of unmarrieds of any age and gender and the number of marriages between age \(i\) men and age \(j\) women must be non-negative.

Azevedo and Hatfield (2013) demonstrate that, in a quasilinear setting, a competitive equilibrium exists in two-sided markets with a continuum of agents with arbitrary preferences. In large two-sided markets with a finite number of agents of each type and quasilinear utility like that of this paper’s model, the authors show that equilibria exist that approximately clears the market. I refer readers to the results and proof in that paper.

Given the preference parameters of the system, \(\Pi\), practitioners are often interested in how variations in the supply population vectors, \(m\) and \(f\), affect the distribution

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27This condition ensures that neither spouse is at a terminal age. If \(T_{i,j} = 0\), the Dynamic Marriage Matching Function reduces to the static marriage matching function of CS, which is \(\mu_{i,j} = \hat{\Pi}_{i,j} \sqrt{\mu_{i,0} \mu_{0,j}}\).
of marriages as represented by \( \mu \). I’ll refer to this as the Dynamic Marriage Matching (DMM) Inverse Problem. A formal statement of this problem follows:

**Definition 2:** - *Dynamic Marriage Matching (DMM) Inverse Problem*

Given a matrix of preferences \( \Pi \), whose elements are non-negative and strictly positive population vectors, \( m \) and \( f \), does there exist a unique non-negative marital distribution \( \mu \) that is consistent with \( \Pi \), and that satisfies equations (3.30), (3.31), (3.32) and (3.26)?

Taking \( \Pi_{i,j} \), \( m \) and \( f \) as exogenously given, equation (3.1) defines a \( Z \times Z \) system of polynomials with the \( Z \times Z \) elements of \( \mu \) as unknowns. The exogeneity of the number of single individuals, \( m \) and \( f \), allows us to reformulate the model as a \( 2Z \) system of polynomials with \( 2Z \) number of unmarrieds by age, \( \mu_{i,0} \) and \( \mu_{0,j} \), as unknowns. I derive this “reduced” system as defined by equations (3.33) and (3.34) below by summing equation (3.1) over all \( i \)’s and equation (3.1) over all \( j \)’s respectively. The \( 2Z \) system represented by equations (3.33) and (3.34) has a significant computational advantage when solving the inverse problem of calculating \( \mu_{i,0} \) and \( \mu_{0,j} \), consistent with an estimate of \( \Pi \) and a vector of single individuals, \( m \) and \( f \). Equation (3.1) fully determines the distribution of marriage \( \mu \) after solving for \( \mu_{i,0} \) and \( \mu_{0,j} \).

\[
m_i - \mu_{i,0} = \sum_{i=1}^{I} \tilde{\Pi}_{i,j} \sqrt{m_i f_j} \prod_{k=0}^{T_{i,j}} \left( \frac{\mu_{i+k,0} \mu_{0,j+k}}{m_{i+k} f_{j+k}} \right)^{\frac{1}{2}} (\beta(1-\delta))^k \quad (3.33)
\]

\[
f_j - \mu_{0,j} = \sum_{j=1}^{J} \tilde{\Pi}_{i,j} \sqrt{m_i f_j} \prod_{k=0}^{T_{i,j}} \left( \frac{\mu_{i+k,0} \mu_{0,j+k}}{m_{i+k} f_{j+k}} \right)^{\frac{1}{2}} (\beta(1-\delta))^k. \quad (3.34)
\]

### 3.4 Identification

#### 3.4.1 Dynamic Marriage Gains

CS introduced a statistic for the *Total Gains* to an \((i, j)\) marriage relative to remaining single. It is the ratio of the number of \((i, j)\) to the geometric mean of the number of unmarrieds of each type, which:

\[
\ln \frac{\mu_{i,j}^2}{\mu_{i,0} \mu_{0,j}} = \ln \frac{p_{i,j} q_{i,j}}{p_{i,0} q_{0,j}} = (\alpha_{i,j} + \gamma_{i,j}) - (\alpha_{i,0} + \gamma_{0,j}) = 2\ln \pi_{i,j}. \quad (3.35)
\]
The dynamic analogue of this statistic derived from equation (3.1) takes the following form:

$$\ln \prod_{k=0}^{T_{i,j}} (p_{i+k,0}q_{0,j+k})^{\beta(1-\delta)} = \alpha_i(j) + \gamma_j(i) - \alpha_i(0) - \gamma_j(0) - 2\kappa = 2\ln \Pi_{i,j}. \quad (3.36)$$

The interpretation of the statistic is similar to the static case. $2\ln \Pi_{i,j}$ gives the couple’s present discounted utility from being locked in an $(i,j)$ match today for the duration of the match relative to the present discounted sum of the per-period payoff from being single for that same time span. This statistic is point identified from a single cross-section of data on aggregate marriages. The right hand side of equation (3.36) is comprised of the primitives of the model only, and these are invariant to changes in the vectors of unmarried men, $m$, and women, $f$. This statistic becomes the basis of the empirical application in Section 5. I will refer to the statistic $2\ln \pi_{i,j}$ as defined by equation (3.35) as Static Gains and to $2\ln \Pi_{i,j}$ from equation (3.36) as Dynamic Gains.

### 3.4.2 Bootstrap Standard Errors

Any inference using the estimates of marriage gains $\Pi$ would necessitate the computation of its standard error. Since the estimator in equation (3.36) is a function of multinomial probabilities, one approach would be to derive the limiting distribution of $\hat{\Pi}$ using the delta method applied to multinomial random vectors. The heuristics of that approach go as follows: Consider a universe where the unit of observations are households. There are $(Z^2 + 2Z) \times 1$ possible types of households. A household could be comprised of a married couple of age pair $(i,j)$ or an unmarried male or female of age $i$ and $j$ respectively. Let the $(Z^2 + 2Z) \times 1$ vector $\theta_0$ denote the true probability that we observe a particular type of household.\(^{28}\) Given a random sample of $N$ households (from which $\mu, m, f$ are constructed), the ML estimator for $\theta_{i,j}$ is $\hat{\theta}_{i,j} = \mu_{i,j}/N$ where $N = \sum_{i=1}^{Z} \sum_{j=1}^{Z} \mu_{i,j} + \sum_{i=1}^{Z} \mu_{i,0} + \sum_{j=1}^{Z} \mu_{0,j}$. $\hat{\theta}_N$ denotes the $(Z^2 + 2Z) \times 1$ random vector of ML estimators. Applying the Central Limit Theorem for multinomial random

\(^{28}\)For example, $\theta_{i,j}$ denotes the probability that we observe a household of $(i,j)$ married couple, $\theta_{i,0}$ denotes the probability that we observe a household of a single $i$ age male, and so on.
vectors, the limiting distribution is
\[ \sqrt{N} (\hat{\theta}_N - \theta_0) \rightsquigarrow N(0, \Omega), \]  
(3.37)
where \( \Omega = \text{diag}(\theta) - \theta \theta' \). Suppose the vector valued function \( \chi(\cdot) \) is continuous and once differentiable in \( \theta \). The \( Z^2 \times 1 \) parameter vector of interest \( \Pi = \chi(\theta) \), and its corresponding estimator \( \hat{\Pi} = \chi(\hat{\theta}) \). By the continuous mapping theorem, consistency in \( \hat{\theta}_N \) implies that \( \hat{\Pi} \) is a consistent estimator of \( \Pi \). Applying the delta methods (see Theorem 3.1 of van der Vaart (2000)), the limiting distribution is as follows:
\[ \sqrt{N} \left( \chi(\hat{\theta}_N) - \chi(\theta_0) \right) \rightsquigarrow N \left( 0, \nabla_{\theta} \chi(\theta_0) \Omega \nabla_{\theta} \chi(\theta_0)' \right), \]  
(3.38)
where \( \nabla_{\theta} \chi(\theta_0) \) is the \( Z^2 \times (Z^2 + 2Z) \) matrix of first derivatives with respect to \( \theta \).

The structure of equation (3.36) suggests that computing the closed form of \( \nabla_{\theta} \chi(\theta_0) \) is algebraically tedious and complicated.

The empirical application instead uses the bootstrap method to compute the standard errors of the marriage gains. I propose the following bootstrapping algorithm to compute the variance of the limiting distribution. Consider a sample of \( N \) observations comprised of matches and single individuals by age. The quantities \( \mu_{i,j}, \mu_{i,0}, \) and \( \mu_{0,j} \) are simply count frequencies of \((i,j)\) matches, single age \( i \) men and age \( j \) women observed in the sample of \( N \). Let this empirical distribution be denoted by \( \hat{F} \). The vector of probabilities \( \theta_0 \) can be estimated by its ML estimate \( \hat{\theta} = (\mu_{1,1}/N, \ldots, \mu_{1,0}/N, \ldots \mu_{0,Z}/N) \). Consider re-sampling with replacement \( B \) times, drawing \( N \) observations each time from the distribution \( \hat{F} \). Let \( b \) index the \( b' \)th bootstrap sample and \( \mu_b^* = (\mu_{1,1,b}^*, \mu_{1,2,b}^*, \ldots, \mu_{Z,Z,b}^*) \) denote the vector of count frequencies in the \( b \)th bootstrap sample. Accordingly, we can compute \( \hat{\theta}_b^* = (\mu_{1,1,b}^*/N, \ldots, \mu_{1,0,b}^*/N, \ldots \mu_{0,Z,b}^*/N) \) and the corresponding structural parameters \( \hat{\Pi}_b^* = \chi(\hat{\theta}_b) \). The variance of the \((i,j)\)th marriage gain \( \hat{\Pi}_{i,j} \) can computed by
\[ \text{Var}(\hat{\Pi}_{i,j}) = \frac{1}{B-1} \sum_{b=1}^{B} \left( \hat{\Pi}_{i,j}^* - \hat{\Pi}_{i,j} \right)^2. \]
3.4.3 A Test of the Model

Equations (3.28) and (3.29) can be expressed in terms of the ML estimators $p_{i,j}$ and $q_{i,j}$. That is

\[
\ln \left( \frac{p_{i,j}}{\prod_{k=0}^{T_{i,j}} p_{i+k,0}^{(\beta(1-\delta))k}} \right) = \alpha_i(j) - \alpha_i(0) - \tau_{i,j} - \kappa, \tag{3.39}
\]

\[
\ln \left( \frac{q_{i,j}}{\prod_{k=0}^{T_{i,j}} q_{0,j+k}^{(\beta(1-\delta))k}} \right) = \gamma_j(i) - \gamma_j(0) + \tau_{i,j} - \kappa. \tag{3.40}
\]

Let

\[ n_{i,j}(\mu, m, f) = \ln \left( \frac{p_{i,j}}{\prod_{k=0}^{T_{i,j}} p_{i+k,0}^{(\beta(1-\delta))k}} \right) \quad \text{and} \]

\[ N_{i,j}(\mu, m, f) = \ln \left( \frac{q_{i,j}}{\prod_{k=0}^{T_{i,j}} q_{0,j+k}^{(\beta(1-\delta))k}} \right). \]

Proposition 1 below provides a simple test for our model:

**Proposition 1** Holding $\alpha_{i,j,k}$, $\gamma_{i,j,k}$, and $\delta$ fixed for all $(i, j, k)$, any changes in available men $m_i$ or women $f_j$ leading to a non-decreasing change in $n_{i,j}(\mu, m, f)$ would also lead to a non-increasing change in $N_{i,j}(\mu, m, f)$, and vice versa.

In other words, any changes in the relative scarcity of men and women that affect the market clearing division of surplus $\tau_{i,j}$ would make $n_{i,j}$ and $N_{i,j}$ move in opposite directions. If our model is true, a simple regression of estimates of $\hat{n}_{i,j}(\mu, m, f)$ against $\hat{N}_{i,j}(\mu, m, f)$ should yield a slope coefficient of -1.\(^{29}\)

4 Supplies, Commitment, Divorce and Persistence,

4.1 Endogenizing Supplies

The model so far assumes that the vector of single available individuals at the beginning of each period is given by $m$ and $f$. This assumption is convenient for several reasons.

\(^{29}\)Changes in $m_i$ or $f_j$ that leaves $\tau_{i,j}$ unchanged would also not affect $n_{i,j}(\mu, m, f)$ and $N_{i,j}(\mu, m, f)$. 

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For one, when faced with data on the number of single individuals by age, it allows me to abstract from the economic and demographic factors that relate \((m_i', f_j')\) to \((m_i, f_j)\) where \(i \neq i'\) and \(j \neq j'\) and ignore mortality and morbidity factors that become important as an individual ages. Data also present prominent inflow and outflow migrations at younger ages. Computationally, the assumption also allows me to reduce the model’s dimensionality by expressing marriage matching functions in terms of the number of unmarried individuals, as described in the previous section.

A more realistic assumption would be to allow the number of single individuals by age to be endogenously determined by marriage decisions. Consider a similar environment to that described in Section 3 where only \(m_1\) and \(f_1\) individuals are exogenous. That is, these are the number of year 1 individuals born into the marriage market. With a little algebra, I can derive these equations of motion for the number of single men and women:

\[
\begin{align*}
  m_{k+1} &= \begin{cases} 
    m_k - \sum_{j=1}^{Z-1} (1 - \delta) \mu_{k,j} & \text{if } k = 1 \\
    m_k - \sum_{j=1}^{Z-1} (1 - \delta) \mu_{k,j} + \sum_{i=1}^{k-1} \sum_{l=1}^{Z-1} (1 - \delta)^{k-i} \delta \mu_{i,l} & \text{if } k > 1 
  \end{cases} \\
  f_{k+1} &= \begin{cases} 
    f_k - \sum_{i=1}^{Z-1} (1 - \delta) \mu_{i,k} & \text{if } k = 1 \\
    f_k - \sum_{i=1}^{Z-1} (1 - \delta) \mu_{i,k} + \sum_{j=1}^{k-1} \sum_{l=1}^{Z-1} (1 - \delta)^{k-j} \delta \mu_{j,l} & \text{if } k > 1 
  \end{cases}
\end{align*}
\]

These equations are mirror images of one another. The second term accounts for the number of individuals who got married in the previous period and are still married. These individuals leave the pool of single individuals participating in the marriage market during this period at age \(k + 1\). The third term accounts for individuals who return to the marriage market as aged \(k + 1\) individuals after the dissolution of earlier marriages. In this case, where supplies are endogenous, the marriage matching function in equation (3.1) needs to be augmented with equations (4.1) and (4.2) to complete the model. The model now comprises of a system of \(Z \times Z\) polynomial equations in \(\mu\). The estimation of \(\Pi_{i,j}\) would require that I first compute the vector of available men and women by age, \(m\) and \(f\). Hence, given data on \((\mu, m_1, f_1)\), I would first need to compute the vector of \(m\) and \(f\), as implied by these demographic equations of motion, before computing the
4.2 Allowing for Persistence in Preferences - Generalized Extreme Value Distribution

The i.i.d. assumption on the unobserved state vectors $\epsilon_i$ (and $\epsilon_j$) is not innocuous. It is well known that static discrete choice model with independent Type 1 Extreme Value distribution exhibits the I.I.A. property, which imposes proportional substitution across alternatives. McFadden (1978) proposed the Generalized Extreme Value class of distributions that maintains the convenient closed form representations, such as the Type I Extreme Value, while permitting a variety of less restrictive substitution patterns. Following McFadden (1978), the distribution function for this class of models takes the form of:

$$ F(\epsilon_1, \epsilon_2, \cdots, \epsilon_K) = \exp(-H(e^{-\epsilon_1}, e^{-\epsilon_2}, \cdots, e^{-\epsilon_K})), \quad (4.4) $$

where $H(\cdot)$ is a member of the class of functions from $\mathbb{R}_+^K \rightarrow \mathbb{R}_+^+$, with properties outlined in Appendix 7.2.

A widely used GEV distribution in the discrete choice literature is the nested logit model. Consider a marriage market where an individual’s type is characterized by race and age, where race is either white or black. Let $r,i$ be the index for an aged $i$ male of

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30I can also allow for the differential mortality rate to affect the number of males and females in the system. Consider a system where only the number of men and women, $m_1$ and $f_1$, born each period is exogenous. Let the mortality rate for an age $i$ male be $(1 - \varrho_i)$; that is, the probability that an age $i$ male is alive at the beginning of the next period is $\varrho_i$. For an age $j$ female, I denote the mortality rate as $(1 - \rho_j)$. For a couple who marry at age $(i,j)$, let $\delta_{i,j,d}$ be the exogenous divorce hazard in the $d^{th}$ year of the marriage where $1 \leq d \leq T_{i,j}$. I can show that the number of single men for $2 \leq k \leq Z$ at each period is given by:

$$ m_{k+1} = \varrho_k \left[ m_k - \sum_{j=1}^{Z-1} \rho_j (1 - \delta_{k,j,1}) \mu_{k,j} \right. $$

$$ + \sum_{i=1}^{k-1} \sum_{t=1}^{Z-1} \left\{ \prod_{r=0}^{k-1-t} \varrho_{t+r} \rho_{t+r}(1 - \delta_{i,t,r+1})(1 - \rho_{t+k-i}(1 - \delta_{i,t,k+i+1})) \mu_{i,d} \right\} \right]. \quad (4.3) $$

The female equation would be a mirror image of the male equation.
race \( r \in \{w, b\} \) and \( s, j \) be the index for an aged \( j \) female of race \( s \in \{W, B\} \). Suppose choices within each race group are correlated. For a white male choosing a white female, let the statistic \( \lambda_{w,W} \) be a measure of correlation between spouses who are white and \( \lambda_{w,B} \) be a measure of correlation between spouses who are black. A value of \( \lambda_{w,W} = 1 \) indicates that the group of spouse choices who are white are also independent.\(^{31}\)

The choice to remain single is contained in its own (single-hood) nest and is uncorrelated with the choice to match with either race. Suppressing the subscript for the \((w, i)\) type men, the function \( H(\cdot) \) becomes:

\[
H(Y_0, Y_{W,1}, \ldots, Y_{W,Z}, \ldots, Y_{B,1}, \ldots, Y_{B,Z}) = Y_0 + \left( \sum_{j=1}^{Z} Y_{W,j}^{\delta_{w,W}} \right)^{1/\delta_{w,W}} + \left( \sum_{j=1}^{Z} Y_{B,j}^{\delta_{w,B}} \right)^{1/\delta_{w,B}} \tag{4.5}
\]

I can derive an analogous system of quasi-demand and quasi-supply equations for matches that are consistent with the nested logit error structure. For expositional convenience, I will suppress the race subscript where possible. For \((w, i, W, j)\) matches,

\[
\ln \frac{P_{i,j}^{1/\lambda_{w,w}}}{P_{i,j}} - \sum_{k=0}^{T_{i,j}} (\beta (1 - \delta))^k \ln P_{i+k,0} = \alpha_i(j) - \alpha_i(0)(j) - \tau_{i,j} - \kappa \tag{4.6}
\]

\[
\ln \frac{Q_{i,j}^{1/\lambda_{w,w}}}{Q_{i,j}} - \sum_{k=0}^{T_{i,j}} (\beta (1 - \delta))^k \ln Q_{0,j+k} = \gamma_j(i) - \gamma_j(0)(i) + \tau_{i,j} - \kappa.
\]

Equations (4.6) and (4.7) maintain a very similar structure to their i.i.d. counterpart in equations (3.28) and (3.29). The derivation of these two equations is shown in Appendix 7.4. The terms \( P_i = \sum_{j=1}^{Z} P_{w,i,W,j} \) and \( Q_j = \sum_{i=1}^{Z} Q_{w,i,W,j} \) refers to the probability that a white aged \( i \) man marries a white spouse, and a white aged \( j \) woman marries a white spouse respectively. There are a corresponding pair of equations for the race pairs \((w, B), (b, B)\) and \((b, W)\). The correlations of marital choices within a race pair are captured by parameters \( \lambda_{w,B}, \lambda_{b,B}, \) and \( \lambda_{b,W} \). Unlike the i.i.d. counterpart, this dynamic model with the nested-logit error (without additional structure) is unidentified from a single cross-section of aggregate matching data. The constant \( \kappa \) is the geometric sum of Euler’s constants, \( \kappa = c \beta S (1 - (\beta S)^T_{i,j})/(1 - \beta S) \).

\(^{31}\)More precisely, McFadden (1978) refers to \( (1 - 1/\lambda_k) \) as an index of similarity for choices in nest \( k \). McFadden (1978) also pointed out that the actual correlation coefficient is more complicated. See Chapter 4 of Train (1993) for further discussion.
There are other approaches to allowing for heterogeneity and persistence in choices. Galichon and Salanié (2012) generalized the Type I Extreme Value distribution in the static CS model to allow for a wide class of unobserved heterogeneity distributions (of which the Generalized Extreme Value is a subset).

4.3 Commitment and Divorce - Generalizing the Divorce Hazard

The model assumes full commitment where couples commit to an allocation of marital surplus that is predetermined by their respective ages at marriage. Divorce is an exogenous shock that dissolves otherwise fully committed stable couples. This simplification is integral to identifying the nett gains and the total marriage gains from a single cross section of data on aggregate marriage behaviors.

The literature has been long recognized that marital dissolution and the level of marital commitment are clearly interconnected. Life-cycle decisions on labor supply, savings and human-capital accumulations affect individuals’ outside options and accordingly their relative marital bargaining power. The full commitment assumption allows me to side-step these important though complicating considerations. A cost of this approach is that the model has nothing to say about the division of within-marriage surplus over the life-cycle, or how that is affected by changes in the labor market or the marriage market. The identification strategy is predicated on the observation that the aggregate number of new marriages, suitably scaled by the proportion of individuals who stays single at their current and future ages, identifies the marriage surplus parameter. Any changes in the marriage market that affects the outside option does so through the number of available individuals in the marriage market. A more complete equilibrium model that allows for limited commitment within marriage would of course require more detailed labor supply data for different couple types. The model here is therefore a building-block toward such an analysis.32

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32 A number of papers allow for intra-household allocation and the level of commitment to be jointly determined. Using data from the Panel Study of Income Dynamics, Mazzocco et al (2007) estimates a dynamic model to examine how labor supply and wealth accumulation decisions change with transition
So far, the model has assumed that divorce is exogenous and occurs at a constant rate of $\delta$. The heterogeneity in divorce patterns in the US has been well studied. For example, using data from the National Longitudinal Study of the High School Class of 1972, Willis and Weiss (1997) identify a number of interesting patterns. The authors find that the divorce hazard initially increases with the duration of a match before decreasing. Couples with similar education levels, religion and ethnicity at the time of marriage are less likely to divorce. While allowing for the endogeneity of divorce is beyond this paper’s scope, I can allow the divorce hazard to reflect some of the heterogeneity observed in data. One way would be to allow the divorce hazard to depend on the ages of the couple and duration of the match.

For a couple who marry at age $(i,j)$, let $\delta_{i,j,d}$ be the exogenous divorce hazard in the $d^{th}$ year of the marriage where $1 \leq d \leq T_{i,j}$. Hence, this marriage will survive until year $d$ with an unconditional probability given by $\prod_{l=1}^{d}(1 - \delta_{i,j,l})$. Divorce couples reenter the marriage market without penalty.

Working through the algebra of the model, the corresponding log-odds that allow for heterogeneity in the divorce hazard (analogous to equations (3.24) and (3.26)) are

$$
\log \left\{ \frac{P_{i,j}}{P_{i,0}} \right\} = \begin{cases} 
\alpha_i(j) - \alpha_{i,0}(j) - \tau_{i,j} & \text{if } \max(i,j) < Z, \\
- \sum_{k=1}^{T_{i,j}} \beta_k \prod_{l=1}^{k}(1 - \delta_{i,j,k}) \left( c + \ln P_{i+k,0}^{-1} \right) & \\
\alpha_i(j) - \alpha_{i,0}(j) - \tau_{i,j} & \text{if } \max(i,j) = Z
\end{cases}
$$

(4.7)

for an aged $i$ male marrying an aged $j$ female relative to remaining single. And for an in and out of marriage. In this model, married couples cooperate in their decision making without committing to future allocations of marital surplus. Divorce happens when no within-marriage reallocation can make the married couple better off than when single. Iyigun (2009) proposes a theoretical model of intra-household allocation where couples decide on the level of commitment and whether to marry or cohabitate. The paper investigates how individuals sort into marriage and cohabitation with different commitment levels as the cost of commitment and marital preferences vary across individuals.

This simplification can be relaxed. Assuming it is possible to differentiate the “previously married” from the “never married” individuals, the state vector can be extended to allow for this status. Accordingly, I can also allow for the parameters to be indexed by this additional state.
aged \( j \) female marrying an aged \( i \) male relative to remaining single, the log-odds are

\[
\begin{align*}
\log \left\{ \frac{Q_{i,j}}{Q_{0,i,j}} \right\} &= \begin{cases} 
\gamma_j(i) - \gamma_{0,j}(i) + \tau_{i,j} & \text{if } \max(i, j) < Z, \\
- \sum_{k=1}^{T_{i,j}} \beta^k \prod_{l=1}^{k} (1 - \delta_{i,jk}) \left( c + \ln Q_{0,j+k}^{-1} \right) & \\
\gamma_j(i) - \gamma_{0,j}(i) + \tau_{i,j} & \text{if } \max(i, j) = Z.
\end{cases}
\end{align*}
\] (4.8)

Solving for the equilibrium number of marriages, I obtain an alternate marriage matching function in equation (4.9). If I continue to assume \( m \) and \( f \) are exogenous, I can still employ the dimensionality reducing transformation where the system of equations is expressed in terms of the equilibrium number of unmarried individuals such as in equations (3.33) and (3.34).

\[
\mu_{i,j} = \prod_{i,j} \sqrt{m_i f_j} \prod_{k=0}^{T_{i,j}} \left( \frac{\mu_{i+k,0} \mu_{0,j+k}}{m_{i+k} f_{j+k}} \right)^{\frac{1}{2}} \beta^k \prod_{l=1}^{(1 - \delta_{i,jk})}
\]

where \( \ln \Pi_{i,j} = \frac{1}{2} (\alpha_i(j) + \gamma_j(i) - \alpha_{i,0}(j) - \gamma_{0,j}(i)) - \kappa \).34

5 Empirical Application

5.1 Changes in the U.S. Marriage Distribution, 1970-1990

The model is used to analyze the changes in the U.S. marriage distribution over two decades from 1970 to 1990. From a demographic viewpoint, this period saw significant

---

34The composite terms representing the present discounted utility within marriage are,

\[
\begin{align*}
\alpha_i(j) &= \alpha_{i,j1} + \sum_{k=1}^{T_{i,j}-1} \beta^k \prod_{l=1}^{k} (1 - \delta_{i,jk}) \alpha_{i,jk+1} \\
\alpha_{i,0}(j) &= \alpha_{i,0} + \sum_{k=1}^{T_{i,j}} \beta^k \prod_{l=1}^{k} (1 - \delta_{i,jk}) \alpha_{i+k,0} \\
\gamma_j(i) &= \gamma_{i,j1} + \sum_{k=1}^{T_{i,j}-1} \beta^k \prod_{l=1}^{k} (1 - \delta_{i,jk}) \gamma_{i,jk+1} \\
\gamma_{0,j}(i) &= \gamma_{0,j} + \sum_{k=1}^{T_{i,j}} \beta^k \prod_{l=1}^{k} (1 - \delta_{i,jk}) \gamma_{0,j+k}.
\end{align*}
\]
changes in the number of single men and women. In particular, the baby boomers entered a marriageable age in the 1980s and 1990s. This was also a period of major socio-political changes that affected marriage as an institution. Many have argued that federal legislative changes like the legalization of abortion and no-fault divorce have changed marriage gains. I construct the distribution of new marriages and single available men and women aged 16-75 over the two decades for the U.S.. To minimize sparseness in the marriage distribution, a two-year distribution of new marriages is used (instead of a one-year). The marriage distributions by age $\hat{\mu}_{i,j}$ for 1971/72, 1981/82 and 1991/92 is constructed using the Vital Statistics data taken from the N.B.E.R. collection of the National Center for Health Statistics. These files contain a sample of the new marriage records from 42 reporting states across the three periods. The number of single available men and women by age, $\hat{\mu}_{i,0}$ and $\hat{\mu}_{0,j}$, come from the 1970, 1980 and 1990 U.S. Census. To be consistent, only unmarried individuals from matching reporting states are included. The individuals are aged between 16 and 75. An individual is considered unmarried if his or her marital status is (i) not equal to married spouse present, or (ii) married spouse absent. Census weights are used to get an estimate of the total unmarried counts.

In this empirical application of the model, the number of available men and women are assumed to be exogenous and the divorce rate is held constant for all couples age and duration of the marriage. The divorce rate, $\delta$ is set at 3.6 per 1000 population for the 1971/72 period, 5.2 per 1000 population for the 1981/82 period, and 4.74 per 1000 population for the 1991/92 period. The time discount factor, $\beta$ is set at 0.95.

---

35 This data is collected by the US Department of Health and Human Services.
36 Weights in the Vital Statistics files are used to get an estimate of the total number of each type of marriage in reporting states.
Figures 1 a) and 1 b) plot the distribution of single men and women according to the three U.S. Census. The plots show some familiar patterns. There are fewer single individuals at older ages than at younger ages. Moving from 1970 to 1980 and 1990, there is a dramatic change in the number of available single men and women as the baby boomers enter marriageable age. Gender differences in later ages arise due to higher mortality rates among older men and lower remarriage rates among divorced women. Figures 1 c) and d) graph the marginal distribution of two-year new marriages over the period. There is a clear shift to the right in the distribution of new marriages across genders due to more delayed marriage. The modal age of marriage across gender also increases. For males, the modal age of marriage goes from approximately 21 in 1971/72 to 23 in 1981/82 and to 25 in 1991/92. The modal age for females, which is slightly younger compared to males, also increases from approximately 18 in 1971/72 to 20 in 1981/82 and to 22 in 1991/92.
Table 1 provides summary statistics of the data. According to the 1970 Census, there were 16.0 million and 19.6 million single men and women respectively between the ages of 16 and 75. By 1980, the number of available men and women had increased by 46.2% and 39% respectively to 23.4 million men and 27.2 million women. The change in population between 1980 and 1990 was more modest. In 1990, there were 28.4 million men and 31.6 million women, an increase of 21.4% and 15.9%, respectively, from 1980. In the data sample constructed from the Vital Statistics, there were 3.24 million new marriages recorded in the two-years 1971/72, while in 1981/82 there were 3.45 million new marriages. This is an increase of 6.5% compared to the approximately 40% increase in the number of single men and women. In 1991/92, the number of new marriages fell to 3.22 million, a drop of 7.1% from the level in 1981/82.
### Table 1: Data Summary

#### A: US Census Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Available Males, (mill.)</td>
<td>16.018</td>
<td>23.412</td>
<td>28.417</td>
</tr>
<tr>
<td>Percentage Change</td>
<td>46.2</td>
<td>21.4</td>
<td></td>
</tr>
<tr>
<td>Number of Available Females, (mill.)</td>
<td>19.592</td>
<td>27.225</td>
<td>31.563</td>
</tr>
<tr>
<td>Percentage Change</td>
<td>39.0</td>
<td>15.9</td>
<td></td>
</tr>
<tr>
<td>Average Age of Available Males</td>
<td>30.4</td>
<td>29.6</td>
<td>31.7</td>
</tr>
<tr>
<td>Average Age of Available Females</td>
<td>39.1</td>
<td>37.1</td>
<td>37.9</td>
</tr>
</tbody>
</table>

#### B: Vital Statistics Data

<table>
<thead>
<tr>
<th></th>
<th>1971/72</th>
<th>1981/82</th>
<th>1991/92</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Number of Marriages (mill.)</td>
<td>3.236</td>
<td>3.449</td>
<td>3.220</td>
</tr>
<tr>
<td>Percentage Change</td>
<td>6.6</td>
<td>-7.11</td>
<td></td>
</tr>
<tr>
<td>Average Age of Married Males</td>
<td>27.1</td>
<td>29.2</td>
<td>31.2</td>
</tr>
<tr>
<td>Average Age of Married Females</td>
<td>24.5</td>
<td>26.4</td>
<td>28.9</td>
</tr>
<tr>
<td>Average Couple Age Difference</td>
<td>2.6</td>
<td>2.7</td>
<td>2.3</td>
</tr>
</tbody>
</table>
5.2 Estimating Marriage Gains

Figure 2

Figure 2 a) graphs the distribution of new marriages by age in 1971/72, while Figures 2 b) and 2 c) graph the estimates of the Dynamic Gains and Static Gains from marriage implied by this distribution of new marriages.\textsuperscript{38} The Static Gains plot in Figure 2 c) shows strong assortative matching by age with the gains being highest for couples closest in age along the diagonal.\textsuperscript{39} A peak occurs at an early age when young couples matched with each other. The plot of the Dynamic Gains in Figure 2 b) maintains much of the qualitative features of Figure 2 c). The Dynamic Gains plot shows a strong assortative matching pattern by age with the peak rising a lot higher and occurring at an even earlier age. Aside from the difference in the peak height, a significant portion of the Dynamic Gains plot for young age couples are now positive compared to Figure 2 c) where the Static Gains are all negative.

The Static Gains for an \((i, j)\) pair is computed by taking the natural log of the number of current \((i, j)\) matches divided by the geometric averages of those age \(i\) men and

\textsuperscript{38}The plots in Figures 2 b) and 2 c) uses a non-parametric estimate to predict the gains for those age pairs where no marriages are observed. This typically happens for matches with large age differentials; that is, when a young individual is matched with a much older individual.

\textsuperscript{39}CS provides a smoothed version of the Static Gains plot in Figure 2 c).
age \( j \) women that chose to remain single. This statistic ignores dynamic considerations in terms of forgone future opportunities of participating in the marriage market if individuals remain single. The Dynamic Gains statistic compensates for this shortcoming by internalizing the forgone future marriage market opportunities in the gains calculation. It approximates the future value of participating in the marriage market by using the probabilities of remaining single in the future given by \( \prod_{k=0}^{T_{i,j}} (p_{i+k,0}q_{0,j+k})^{(\beta S)^k} \). Young individuals have the greatest opportunity to participate in the marriage market, albeit as increasingly older individuals as they age. Given that most marriages occur when individuals are young, the implied Dynamic Marriage Gains, accounting for the forgone future marriage market opportunities are much larger than the Static Gains. The Static Gains statistic in effect assumes that there is only one opportunity to match and that, in the future, agents will remain single with certainty. In other words, future probabilities of remaining single, \( p_{i+k,0} \) and \( q_{i+k,0} \), equal 1.

Figure 3 graphs various cross-sections of the 1971/72 Static and Dynamic Gains against the age of individuals’ spouses on the horizontal axis. Figures 3 a) and 3 b) plot the marriage gains for females aged 18, 25 and 34 years old, and Figure 3 c) and 3 d) plot the gains for males of the same ages. The graphs also plot the bootstrap 95% confidence interval computed using the procedure described in Section 3.4.2. These set of graphs provide a more detailed picture of the differences between the Static Gains and Dynamic Gains from marriage. In terms of magnitude, it is clear that the difference between these two statistics is biggest when at least one of the spouses is young. The Dynamic Gains for an 18 and a 25 year old far exceed their corresponding Static Gains and is positive when the spouse is young. The tight bootstrap confidence interval also suggests that the gains estimates are most precisely estimated for young individuals where most of the data lie.

40For age pairs where no matches were observed, a non-parametric conditional mean was used to predict the gains. In those cases, no standard error nor confidence interval is computed accounting for the gaps in the plots of the 95% confidence intervals.
Figures 4, 5, and 6 attempt to document the inter-temporal changes in marriage gains over the period from 1970 to 1990. In Figure 4, I construct a simple difference in the Static Gains and Dynamic Gains of marriage from 1970 to 1980. Figure 4 plots various cross sections of this difference against the age of individuals’ spouses on the horizontal axis. Figures 4 a) and 4 b) plot the differences for males and Figures 4 c) and 4 d) plot the differences for females. Generally, all these plots suggest a fall in marriage gains over this decade. After accounting for the forgone future marriage opportunities using the Dynamic Gains, Figures 4 a) and 4 c) suggest that the drop in marriage gains is even larger than initially suggested by the Static Gains calculation. Comparing Figures 4 a) and b), the drop in marriage gains for 18 year-old males is no longer confined to matches with spouses age 18 to 22 but experienced over the entire distribution of the spouse’s age. While differing in magnitude, the qualitative features of the plots for 25- and 34-year-old males are very similar.  

Figure 3

Figures 4, 5, and 6 attempt to document the inter-temporal changes in marriage gains over the period from 1970 to 1990. In Figure 4, I construct a simple difference in the Static Gains and Dynamic Gains of marriage from 1970 to 1980. Figure 4 plots various cross sections of this difference against the age of individuals’ spouses on the horizontal axis. Figures 4 a) and 4 b) plot the differences for males and Figures 4 c) and 4 d) plot the differences for females. Generally, all these plots suggest a fall in marriage gains over this decade. After accounting for the forgone future marriage opportunities using the Dynamic Gains, Figures 4 a) and 4 c) suggest that the drop in marriage gains is even larger than initially suggested by the Static Gains calculation. Comparing Figures 4 a) and b), the drop in marriage gains for 18 year-old males is no longer confined to matches with spouses age 18 to 22 but experienced over the entire distribution of the spouse’s age. While differing in magnitude, the qualitative features of the plots for 25- and 34-year-old males are very similar.  

The plot of this marriage gains difference from 1980 to 1990 is qualitatively similar to Figure 4 and
Figures 5 a), b) and c) present contour plots of the Dynamic Gains for 1971/72, 1981/82 and 1991/92 respectively. These plots trace out iso-level contour curves or locus of points that have equal level of Dynamic Gains. The curves in these plots are labeled with their corresponding levels of net utility. In general, the shape of these contour curves is in line with the qualitative features of the three-dimensional plot of the Dynamic Gains shown in Figure 2 b). The elliptical shape of the contours confirms the strong positive assortative matching pattern found in earlier plots. Couples that are closest in age realize the highest gains with the peak in Dynamic Gains occurring among couples that marry early.

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has been omitted.
Figure 5

Comparing the contour plots over the two decades, one also notices the changes in the range of contour levels over these periods. In Figure 5 a), the contour levels for
the 1970 Dynamic Gains range in values from +5 to -15. This range of contour levels decrease to 0 to -15, in the contour plot for 1981/82 in Figure 5 b), and decrease further to -3 to -15 in the 1991/92 contour plot in Figure 5 c). As the higher level contours disappear in the plots for 1981/82 and 1991/92, the lower level contours (e.g. the -5, -7, -10 level curves,) also become more centered towards the lower left corner. All these are further confirmation of the drop in marriage gains highlighted in Figure 4, with much of the decrease in Dynamic Gains concentrated among young couples below the age of around 30 years old. Comparing the shape of the curves in Figure 5 a), b) and c), the curves have become more elliptical especially when comparing the 1971/72 and the 1981/82 contour plots. This is strongly suggestive of an increase in assortative matching by age over this periods.

Figure 6 investigates the marriage gains maximizing spousal age for men and women.
These plots are meant to highlight the increase in assortative matching by age over the two decades from 1970 to 1990. Figure 6 a) plots the marriage gains maximizing spousal age for each female age over the two decades and Figure 6 c) repeats this for males. Figure 6 b) plots the difference between the marriage gains maximizing spousal age and the corresponding female age, i.e. it plots \( (i_j^* - j) \) on the vertical axis and \( j \) on the horizontal axis where \( i_j^* \) is the marriage gains maximizing spousal age for age \( j \) females. Similarly Figure 6 d) repeats this for males, i.e. it plots \( (j_i^* - i) \) against \( i \). Focusing first on the plots for females, Figure 6 a) suggests that the marriage gains maximizing spousal age for females is very close to the corresponding female’s age. Looking at Figure 6 b), women below the age of 25 years old maximize their Dynamic Gains when matched with slightly younger men, while women between the age of 25 and 45 years old maximize their marriage gains when matched with slightly older men. Over the two decades, Figures 6 a) and b) suggest that this age difference has decreased where the marriage gains maximizing spousal age is closer to the corresponding female’s age. The plots for males in Figures 6 c) and d) tell a slightly different story. They suggest that men’s Dynamic Gains is maximized when matched with considerably younger women. For example, the estimates in 1971/72 suggest that a 25 year-old men (on average) maximize his marriage gains when matched with 21 year-old women, a 30 year-old men with 21 year-old women, a 35 year-old men with 24 year-old women, and so on. This age difference appears to increase with older men. The 1971/72 estimates suggest a big decrease in age difference after approximately 47 years old, where a 50 year-old men (on average) maximize his marriage gains when matched with 46 year-old women.\(^{42}\) Comparing the 1971/72 estimates with that of 1981/82 and 1991/92, there is a clear decrease in the age difference between the marriage gains maximizing spousal age and the corresponding male’s age. For example, by 1991/92 the estimates suggest that a 25 year-old men (on average) maximize his marriage gains when matched with 23 year-old women, a 30 year-old men with 27 year-old women, a 35 year-old men with 29 year-old women, and so on.

\(^{42}\)Given the sparseness of new marriages after the age of 45, these estimates should be interpreted with caution.
6 Conclusion

I propose and estimate a dynamic model of marriage matching. It generalizes the contribution of CS into a dynamic setting while maintaining the empirical tractability and convenience of the static model. Applying the model to US marriage data, I show that ignoring the dynamic returns from marriage severely understates the marriage gains, especially among the young. The model is sufficiently flexible to allow for matching along other attributes.

References


7 Appendix

7.1 State Transition Matrices, $F_a(i' \mid i)$ and $R_a(j' \mid j)$:

$F_a(i' \mid i)$ (or $R_a(j' \mid j)$) denotes the transition probability that an age $i$ male $g$ (or $j$ female $h$) will next find himself (or herself) single at age $i'$ (or $j'$) given his (or her) action $a$ at age $i$ (or $j$). Clearly, $r > i$ and all $a$.\textsuperscript{43} If $g$ chooses to be single, $a = 0$, then $F_0(i + 1 \mid i) = 1$ and $F_0(r \mid i) = 0$ where $r \neq i + 1$. That is, if $g$ forgoes the opportunity to match at age $i$, he will be single with certainty in the next period at age $i + 1$. Similarly if an age $i < Z$ male matches with a female in her terminal age $Z$, he will return to the marriage market in the next period at age $i + 1$ with certainty. That is, if $i < Z$ and $a = Z$, then $F_Z(i + 1 \mid i) = 1$. Consider now if $g$ (of age $i << Z$) chooses to match with an older spouse of age $j$ (where $i < j << Z$). The marriage might dissolve in the first period and $g$ returns to the marriage market at age $i + 1$. This occurs with probability $F_j(i + 1 \mid i) = \delta$. The marriage might survive the first period but dissolve in the second period, in which case he finds himself single again at age $i + 2$. This occurs with probability $F_j(i + 2 \mid i) = \delta(1 - \delta)$. The marriage could survive till the death of the older spouse and the younger $g$ reenters the marriage market at age $i + T_{i,j} + 1$.

\textsuperscript{43}That is, whatever the actions an age $i$ male chooses, he can never return as a single age $i$ or younger self in the future.
This occurs with probability \( F_j(i + T_{i,j} + 1 \mid i) = (1 - \delta)^{T_{i,j}} \). The transition probability \( F_a(r \mid i) \) for \( a \neq 0 \) takes the form:

For \( a = j \leq i \), (that is, \( g \) marries someone younger),

\[
F_a(r \mid i) = \begin{cases} 
\delta(1 - \delta)^{r - (i + 1)}, & \text{if } i + 1 \leq r \leq Z \\
0, & \text{elsewhere,}
\end{cases}
\]

for \( a = j > i \), (\( g \) marries someone older),

\[
F_a(r \mid i) = \begin{cases} 
\delta(1 - \delta)^{r - (i + 1)}, & \text{if } i + 1 \leq r \leq i + T_{i,j} \\
(1 - \delta)^{T_{i,j}}, & \text{if } r = i + T_{i,j} + 1, \\
0, & \text{elsewhere.}
\end{cases}
\]

There is a similar analogous structure for the female transition matrix. \( R_a(r \mid j) = 0 \) for all \( r \leq j \) and all \( a \). If \( h \) chooses to remain single at age \( j \), she returns to the marriage market in the next period as an older female aged \( j + 1 \) with certainty. That is if \( a = 0 \), \( R_a(j + 1 \mid j) = 1 \) and zero elsewhere. The structure of \( R_a(r \mid j) \) for \( a \neq 0 \) is as follows:

For \( a = i \leq j \), (\( h \) marries a younger men)

\[
R_a(r \mid j) = \begin{cases} 
\delta(1 - \delta)^{r - (j + 1)}, & \text{if } j + 1 \leq r \leq Z \\
0, & \text{elsewhere,}
\end{cases}
\]

for \( a = i > j \), (\( h \) marries someone older),

\[
R_a(r \mid i) = \begin{cases} 
\delta(1 - \delta)^{r - (j + 1)}, & \text{if } j + 1 \leq r \leq j + T_{i,j} \\
(1 - \delta)^{T_{i,j}}, & \text{if } r = j + T_{i,j} + 1, \\
0, & \text{elsewhere.}
\end{cases}
\]

### 7.2 Derivation of Equation (3.18) and (3.19):

McFadden(1977) introduced the family of choice models based on the Generalized Extreme Value Distribution, with distribution function

\[
\tilde{F}(\epsilon_1, \epsilon_2, \ldots, \epsilon_K) = \exp(-\mathcal{H}(e^{-\epsilon_1}, e^{-\epsilon_2}, \ldots, e^{-\epsilon_K})).
\]

\( \mathcal{H}(\cdot) \) is a member of the class of functions from \( \mathbb{R}^K_+ \to \mathbb{R}_+ \) with the following properties:
1. homogeneous of degree one,

2. \( \lim_{r_j \to \infty} \mathcal{H}(r_1, \ldots, r_j \ldots, r_K) = \infty \)

3. the first partials of \( \mathcal{H} \) are positive, and all the distinct cross-partialss of order \( k \) (for example \( \partial^k \mathcal{H} / \partial r_i \ldots \partial r_l \) for \( i \ldots l \) are all distinct) are non-positive if \( k \) is even, and non-negative if \( k \) is odd.

\[ \text{For the Type I Extreme Value distribution, } \mathcal{H}(Y_{i,0}, Y_{i,1}, \cdots, Y_{iZ}) = \sum_{j=0}^{Z} Y_{i,j}. \text{ Theorem 1 of McFadden(1977) showed that for utilities of the form } u_{ia} = \tilde{v}_{ia} + \epsilon_{iag}, \text{ where the vector } \epsilon_i = (\epsilon_{i,0}, \epsilon_{i,1}, \cdots, \epsilon_{iZ}) \text{ is distributed } F, \text{ then the probability that } j \text{ is selected satisfies, } \mathcal{P}_{i,j} = e^{\tilde{v}_{i,j}} \mathcal{H}_j(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \cdots, e^{\tilde{v}_{iZ}}) / \mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \cdots, e^{\tilde{v}_{iZ}}). \]

I reproduce the arguments of the proof here:

\[
\mathcal{P}_{i,j} = \int \mathbb{I}\{j = \arg \max_{a \in D} (\tilde{v}_{ia} + \epsilon_{iag})\} f(\epsilon) d\epsilon \\
= \int_{\epsilon_{i,j}=-\infty}^{\epsilon_{i,j}=\infty} \mathcal{F}_j(\tilde{v}_{i,j} - \tilde{v}_{i,0} + \epsilon_{i,j}, \tilde{v}_{i,j} - \tilde{v}_{i,1} + \epsilon_{i,j}, \ldots, \tilde{v}_{i,j} - \tilde{v}_{iZ} + \epsilon_{i,j}) d\epsilon_{i,j} \\
= \int_{\epsilon_{i,j}=-\infty}^{\epsilon_{i,j}=\infty} e^{-\epsilon_{i,j}} \mathcal{H}_j(e^{\tilde{v}_{i,0} - \epsilon_{i,j} - \tilde{v}_{i,j}}, e^{\tilde{v}_{i,1} - \epsilon_{i,j} - \tilde{v}_{i,j}}, \ldots, e^{-\epsilon_{i,j}}) d\epsilon_{i,j} \\
= \int_{\epsilon_{i,j}=-\infty}^{\epsilon_{i,j}=\infty} e^{-\epsilon_{i,j}} \mathcal{H}_j(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \cdots, e^{\tilde{v}_{iZ}}) \exp\left[-e^{-\epsilon_{i,j}} \mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \cdots, e^{\tilde{v}_{iZ}})\right] d\epsilon_{i,j} \\
= e^{\tilde{v}_{i,j}} \mathcal{H}_j(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \cdots, e^{\tilde{v}_{iZ}}) / \mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \cdots, e^{\tilde{v}_{iZ}}),
\]

where \( \mathcal{H}_j(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \cdots, e^{\tilde{v}_{iZ}}) = \partial \mathcal{H}(\cdot) / \partial e^{\tilde{v}_{i,j}}. \) The fourth equality uses the homogeneity of degree one of \( \mathcal{H} \) and consequent homogeneity of degree zero of \( \mathcal{H}_j. \)

\[ ^{44} \text{Taking partial derivative } \mathcal{F}_1 \text{ and using property (1) above and the fact that } \mathcal{H}_1 \text{ is homogeneous of degree } 0, \text{ we can show that } \\
p_1 = e^{v_1} \mathcal{H}(e^{v_1}, e^{v_2}, \ldots, e^{v_K}) / \mathcal{H}(e^{v_1}, e^{v_2}, \ldots, e^{v_K}).\]
The following is a special case of the proof of Lemma 1 in Arcidiacono and Miller (2008). Consider the integrated value function

\[ V_i = \mathbb{E} V_\alpha(i, \epsilon_g) = \int V_\alpha(i, \epsilon_g) \, d\tilde{\mathcal{F}}(\epsilon_g) = \int_{\epsilon = -\infty}^{\infty} \max_{a \in D} (\tilde{v}_{ia} + \epsilon_{iag}) \, f(\epsilon) \, d\epsilon \]

\[ = \sum_{j=0}^{Z} P_{i,j}(\tilde{v}_{i,j} + \epsilon_{i,j}) \]

\[ = \sum_{j=0}^{Z} P_{i,j} \tilde{v}_{i,j} + \sum_{j=0}^{Z} \int \{ j = \text{arg max}_{a \in D} (\tilde{v}_{ia} + \epsilon_{ia}) \} \epsilon_{i,j} f(\epsilon) \, d\epsilon. \]

Subtracting \( \tilde{v}_{i,0} \) on both sides, I get

\[ V_i - \tilde{v}_{i,0} = \sum_{j=0}^{Z} P_{i,j}(\tilde{v}_{i,j} - \tilde{v}_{i,0}) + \sum_{j=0}^{Z} \int \{ j = \text{arg max}_{a \in D} (\tilde{v}_{ia} + \epsilon_{ia}) \} \epsilon_{i,j} f(\epsilon) \, d\epsilon. \] (7.3)

Solving the second term,

\[ \int \{ j = \text{arg max}_{a \in D} (\tilde{v}_{ia} + \epsilon_{ia}) \} \epsilon_{i,j} f(\epsilon) \, d\epsilon \]

\[ = \int_{\epsilon_{i,j} = -\infty}^{\infty} \epsilon_{i,j} \delta_{j}(\tilde{v}_{i,j} - \tilde{v}_{i,0} + \epsilon_{i,j}, \tilde{v}_{i,j} - \tilde{v}_{i,1} + \epsilon_{i,j}, \ldots, \tilde{v}_{i,j} - \tilde{v}_{i,Z} + \epsilon_{i,j}) \, d\epsilon_{i,j} \]

\[ = \mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \ldots, e^{\tilde{v}_{i,Z}}) \int_{\epsilon_{i,j} = -\infty}^{\infty} \epsilon_{i,j} e^{-\epsilon_{i,j}} \exp \left[ -e^{-\epsilon_{i,j}} - \tilde{v}_{i,j} \mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \ldots, e^{\tilde{v}_{i,Z}}) \right] \, d\epsilon_{i,j} \]

\[ = \mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \ldots, e^{\tilde{v}_{i,Z}}) \frac{e^{\tilde{v}_{i,j}} (\ln \mathcal{H}(\cdot) - \tilde{v}_{i,j} + c)}{\mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \ldots, e^{\tilde{v}_{i,Z}})} \]

\[ = P_{i,j}(\ln \mathcal{H}(\cdot) - \tilde{v}_{i,j} + c). \]

Using the property of the mean of a Type I Extreme Value distribution where \( \int_{x=-\infty}^{\infty} \frac{x}{\phi} e^{-(x-\xi)/\phi} \exp(-e^{-(x-\xi)/\phi}) \, dx = \xi + c \phi \), where \( c \approx 0.57722 \). Hence,

\[ V_i - \tilde{v}_{i,0} = \sum_{j=0}^{Z} P_{i,j}(\tilde{v}_{i,j} - \tilde{v}_{i,0}) + \sum_{j=0}^{Z} P_{i,j}(\ln \mathcal{H}(\cdot) - \tilde{v}_{i,j} + c) \]

\[ \Rightarrow V_i = \ln \mathcal{H}(\cdot) + c. \] (7.4)

In the case where \( \epsilon_i \) is i.i.d., Type I Extreme Value,

\[ P_{i,0} = \frac{e^{\tilde{v}_{i,0}}}{\mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_{i,1}}, \ldots, e^{\tilde{v}_{i,Z}})} \]
Taking logs on both sides,

$$\ln \mathcal{H}(e^{\tilde{v}_{i,0}}, e^{\tilde{v}_1}, \ldots, e^{\tilde{v}_Z}) = \tilde{v}_{i,0} - \ln \mathcal{P}_{i,0}$$

and substituting this into equation (7.4), I get

$$V_i = \tilde{v}_{i,0} + c - \ln \mathcal{P}_{i,0}$$

Consider a single individual in his terminal age $Z$, from equation (3.13) $\tilde{v}_{i,0} = \alpha_{i,0}$. This gives me equation (3.18) when $i = Z$,

$$V_i = \alpha_{i,0} + c - \ln \mathcal{P}_{i,0}.$$  

Consider next when $i < Z$, accordingly the mean utility from being single as given by equation (3.13):

$$\tilde{v}_{i,0} = \alpha_{i,0} + \sum_{k=i+1}^{Z} \beta^{k-i} \mathbb{E}[V_{\alpha}(k, \epsilon_{kg}) \mid i, \epsilon_{ig}, a_{ig} = 0].$$

The CI assumption allows us to factorize the expectation $\mathbb{E}[V_{\alpha}(k, \epsilon_{kg}) \mid i, \epsilon_{ig}, a_{ig}] = \mathcal{F}_a(k \mid i) \int [V_{\alpha}(k, \epsilon_{kg})]dF_{\epsilon_{kg}}$. Since $\mathcal{F}_0(i + 1 \mid i) = 1$ and 0 elsewhere, $\mathbb{E}[V_{\alpha}(k, \epsilon_{kg}) \mid i, \epsilon_{ig}, a_{ig}] = \int [V_{\alpha}(i+1, \epsilon_{i+1,g})]dF_{\epsilon_{i+1,g}} = V_{i+1}$. Hence $\tilde{v}_{i,0} = \alpha_{i,0} + \beta V_{i+1}$. Substituting this into equation (7.4), I get equation (3.18) when $i < Z$,

$$V_i = \alpha_{i,0} + c + \beta V_{i+1} - \ln \mathcal{P}_{i,0}.$$  

7.3 Derivation of Equations (3.24) and (3.26):

Since the two equations are mirror images of one another with just different parameters and signs on transfer, I will focus only on the male equation (3.24) only. Consider the log odd-ratio of a marriage for a male in his terminal age $Z$ with a age $j$ female relative to his remaining single. Then,

$$\ln \frac{\mathcal{P}_{Z,j}}{\mathcal{P}_{Z,0}} = \tilde{v}_{Z,j} - \tilde{v}_{Z,0} = \alpha_Z(j) - \tau_{Z,j} - \alpha_{Z,0}(j)$$  

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where $\alpha_{Z,0}(j) = \sum_{k=0}^{T_{Z,j}} \alpha_{Z+k,0} = \alpha_{Z,0}$ since $T_{Z,j} = Z - \max(Z, j) = 0$. Recall that $T_{Z,j}$ refers to the maximum length of the match before one of the spouse passes away. The first equality arises from equation (3.16), and the second equality arises from the definition in equation (3.13). The structure of this equation applies to any match involving a man in his terminal age.

Doing the same for the log-odds ratio for a $(Z - 1, j)$ marriage relative to remaining single (where the spouse age $j \leq Z - 1$),

\[
\ln \frac{\mathcal{P}_{Z-1,j}}{\mathcal{P}_{Z-10}} = \tilde{v}_{Z-1,j} - \tilde{v}_{Z-1,0} = \alpha_{Z-1}(j) - \tau_{Z-1,j} + \beta \delta V_Z - (\alpha_{Z-1,0} + \beta V_Z)
\]

\[
= \alpha_{Z-1}(j) - \tau_{Z-1,j} - \alpha_{Z-1,0} - \beta(1 - \delta)(\alpha_{Z,0} + c - \ln \mathcal{P}_{Z,0})
\]

\[
= \alpha_{Z-1}(j) - \tau_{Z-1,j} - \alpha_{Z-1,0}(j) - \beta(1 - \delta)(c + \ln \mathcal{P}_{Z,0}^{-1})
\]

where $\alpha_{Z-1,0}(j) = \alpha_{Z-10} + \beta(1 - \delta)\alpha_{Z0} = \sum_{k=0}^{T_{Z-1,j}} (\beta(1 - \delta))^k \alpha_{Z-1+k,0}$. The term $T_{Z-1,j} = Z - \max(Z - 1, j) = 1$ if $j \leq Z - 1$. The third equality uses equation (3.18) derived above. In the case where the spouse is older, that is $j = Z$, the log-odds ratio is given by:

\[
\ln \frac{\mathcal{P}_{Z-1,Z}}{\mathcal{P}_{Z-10}} = \alpha_{Z-1}(Z) - \tau_{Z-1,Z} + \beta V_Z - (\alpha_{Z-1,0} + \beta V_Z)
\]

\[
= \alpha_{Z-1}(Z) - \tau_{Z-1,Z} - \alpha_{Z-1,0}(Z).
\]

Again $\alpha_{Z-1,0}(Z) = \sum_{k=0}^{T_{Z-1,Z}} (\beta(1 - \delta))^k \alpha_{Z-1+k,0} = \alpha_{Z-1,0}$.

Repeating this exercise for $Z - 2$ aged male, the log-odds for a $(Z - 2, j)$ marriage where $j \leq Z - 2$ is:

\[
\ln \frac{\mathcal{P}_{Z-2,j}}{\mathcal{P}_{Z-20}} = \alpha_{Z-2}(j) - \tau_{Z-2,j} + \beta \delta V_{Z-1} + \beta^2(1 - \delta)\delta V_Z - (\alpha_{Z-2,0} + \beta V_{Z-1})
\]

\[
= \alpha_{Z-2}(j) - \tau_{Z-2,j} - \alpha_{Z-2,0} - \beta(1 - \delta)(\alpha_{Z-1,0} + c - \ln \mathcal{P}_{Z-1,0} + \beta V_Z)
\]

\[
+ \beta^2(1 - \delta)\delta V_Z
\]

\[
= \alpha_{Z-2}(j) - \tau_{Z-2,j} - \alpha_{Z-2,0} - \beta(1 - \delta)(\alpha_{Z-1,0} + c - \ln \mathcal{P}_{Z-1,0})
\]

\[
- \beta^2(1 - \delta)^2(\alpha_{Z,0} + c - \ln \mathcal{P}_{Z,0})
\]

\[
= \alpha_{Z-2}(j) - \tau_{Z-2,j} - \alpha_{Z-2,0}(j) - \sum_{k=1}^{T_{Z-2,j}} \beta^k(1 - \delta)^k(c + \ln \mathcal{P}_{Z-2+k,0}^{-1}).
\]
The terms $\alpha_{Z-2,0}(j) = \sum_{k=0}^{T_{Z-2,j}} \alpha_{Z-2+k,0}$. In the case where the spouse is of age $j = Z - 1$, the log-odds ratio looks like this:

$$\ln \frac{P_{Z-2,Z-1}}{P_{Z-2,0}} = \alpha_{Z-2}(Z-1) - \tau_{Z-2,Z-1} + \beta \delta V_{Z-1} - (\alpha_{Z-2,0} + \beta V_{Z-1})$$

$$= \alpha_{Z-2}(Z-1) - \tau_{Z-2,Z-1} - \alpha_{Z-2,0}(Z-1) - \beta(1 - \delta)(c + \ln P_{Z-1,0}).$$

Generalizing further, we get Equation (3.24).

### 7.4 Derivation of Equations (4.6) and (4.7):

I first derive the choice probabilities for an $(ri)$ type male. Using equation (7.2) and for $\mathcal{H}(\cdot)$ given by equation (4.5),

$$P_{r,i,0} = \frac{e^{\tilde{v}_{r,i,0}}}{e^{\tilde{v}_{r,i,0}} + \left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,W,j})^{\delta_{w,W}}\right)^{1/\delta_{w,W}}} + \left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,B,j})^{\delta_{w,B}}\right)^{1/\delta_{w,B}} \tag{7.5}$$

$$P_{r,i,W,k} = \frac{e^{\tilde{v}_{r,i,W,k}} \left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,W,j})^{\delta_{w,W}}\right)^{1/\delta_{w,W} - 1}}{e^{\tilde{v}_{r,i,0}} + \left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,W,j})^{\delta_{w,W}}\right)^{1/\delta_{w,W}} + \left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,B,j})^{\delta_{w,B}}\right)^{1/\delta_{w,B}}} \tag{7.6}$$

Summing $P_{r,i,W,k},$

$$P_{r,i,W,\bullet} = \sum_{k=1}^{Z} P_{r,i,W,k} \tag{7.7}$$

$$= \frac{\left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,W,j})^{\delta_{w,W}}\right)^{1/\delta_{w,W}}}{e^{\tilde{v}_{r,i,0}} + \left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,W,j})^{\delta_{w,W}}\right)^{1/\delta_{w,W}} + \left(\sum_{j=1}^{Z} \exp(\tilde{v}_{r,i,B,j})^{\delta_{w,B}}\right)^{1/\delta_{w,B}}} \tag{7.8}$$

Substituting equation (7.8) into equation (7.6) and taking log-odds ratios,

$$\ln \left(\frac{P_{r,i,W,k}^{1/\delta_{w,W}}}{P_{r,i,0}^{1-1/\delta_{w,W}}}\right) = \tilde{v}_{r,i,W,k} - \tilde{v}_{r,i,0}.$$  

An analogous log-odds ratio equation can be derived for the female side of the market:

$$\ln \left(\frac{Q_{w,k,r,j}^{1/\delta_{w,W}}}{Q_{r,j,0}^{1-1/\delta_{w,W}}}\right) = \tilde{w}_{w,k,r,j} - \tilde{w}_{r,j,0}.$$  

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Consider a white male at his terminal age, i.e. $i = Z$, 

$$
\ln \left( \frac{\mathcal{P}_{r,Z,W,k}^{1/\delta_{w,w}} \mathcal{P}_{r,Z,W,k}^{1-1/\delta_{w,w}}}{\mathcal{P}_{r,i,0}} \right) = \tilde{v}_{r,Z,W,k} - \tilde{v}_{r,Z,0} = \alpha_{r,Z,W,k} - \tau_{r,Z,W,k} - \alpha_{r,Z,0}.
$$

The equation for a white male at age $i = Z - 1$, who marries a white female of age $k \leq Z - 1$ has

$$
\ln \left( \frac{\mathcal{P}_{r,Z-1,W,k}^{1/\delta_{w,w}} \mathcal{P}_{r,Z-1,W,k}^{1-1/\delta_{w,w}}}{\mathcal{P}_{r,Z-1,0}} \right) = \tilde{v}_{r,Z-1,W,k} - \tilde{v}_{r,Z-1,0} = \alpha_{r,Z-1}(Wk) - \tau_{r,Z-1,W,k} + \beta \delta \mathcal{V}_{r,Z} - \alpha_{r,Z-1,0} - \beta \mathcal{V}_{r,Z} = \alpha_{r,Z-1}(Wk) - \tau_{r,Z-1,W,k} - \alpha_{r,Z-1,0} - \beta(1 - \delta)(\alpha_{r,Z,0} - \ln \mathcal{P}_{r,Z,0} + c)
$$

where $\mathcal{V}_{r,Z} = \alpha_{r,Z,0} - \ln \mathcal{P}_{r,Z,0} + c$ and $\alpha_{r,Z-1,0}(Wk) = \alpha_{r,Z-1,0} + \beta(1 - \delta)\alpha_{r,Z,0}$. After some algebra, I get

$$
\ln \left( \frac{\mathcal{P}_{r,Z-1,W,k}^{1/\delta_{w,w}} \mathcal{P}_{r,Z-1,W,k}^{1-1/\delta_{w,w}}}{\mathcal{P}_{r,Z-1,0}} \mathcal{P}_{r,Z-1,0}^{\beta(1-\delta)} \right) = \alpha_{r,Z-1}(Wk) - \tau_{r,Z-1,W,k} - \alpha_{r,Z-1,0}(Wk) + \beta(1 - \delta)c.
$$

The equation for a white male at age $i = Z - 2$, who marries a white female of age $k \leq Z - 2$ has

$$
\ln \left( \frac{\mathcal{P}_{r,Z-2,W,k}^{1/\delta_{w,w}} \mathcal{P}_{r,Z-2,W,k}^{1-1/\delta_{w,w}}}{\mathcal{P}_{r,Z-2,0}} \right) = \tilde{v}_{r,Z-2,W,k} - \tilde{v}_{r,Z-2,0} = \alpha_{r,Z-2}(Wk) - \tau_{r,Z-2,W,k} - \beta(1 - \delta)\mathcal{V}_{r,Z-1} + \beta^2(1 - \delta)\delta \mathcal{V}_{r,Z} - \alpha_{r,Z-2,0} + \beta(1 - \delta)\mathcal{V}_{r,Z} - \alpha_{r,Z-2,0} + \beta^2(1 - \delta)\delta \mathcal{V}_{r,Z} - \alpha_{r,Z-2,0} = \alpha_{r,Z-2}(Wk) - \tau_{r,Z-2,W,k} - \beta(1 - \delta)(\alpha_{1,Z-1,0} + \beta \mathcal{V}_{r,Z} - \ln \mathcal{P}_{1,Z-1,0} + c) - \beta^2(1 - \delta)^2(\alpha_{r,Z,0} - \ln \mathcal{P}_{r,Z,0} + c) - \alpha_{r,Z-2,0}
$$

$$
\ln \left( \frac{\mathcal{P}_{r,Z-2,W,k}^{1/\delta_{w,w}} \mathcal{P}_{r,Z-2,W,k}^{1-1/\delta_{w,w}}}{\mathcal{P}_{r,Z-2,0}} \right) - \sum_{l=0}^{2} (\beta(1 - \delta))^l \ln \mathcal{P}_{r,Z-2+l,0} = \alpha_{r,Z-2}(Wk) - \tau_{r,Z-2,W,k} - \alpha_{r,Z-2,0}(Wk) + \beta(1 - \delta)c.
$$