Endogenous Housing Market Model:
A Deterministic Heterogeneous Agent Approach

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Abstract

Motivated by the complexity of price dynamics during the booms and busts of housing market, we examine two types of housing market crashes — sudden crashes and smooth crashes — using a deterministic heterogeneous agent model (HAM). By matching simulated price dynamics to historical data, we show that housing market crashes may have endogenous origins which can be explained by the interactions between heterogeneous agents in the market. Different types of crashes in housing market are results of strategy switches by different groups of agents. We also find that increasing transaction cost in buying and selling of properties can effectively moderate price volatility in housing market.
1 Introduction

The collapse of the U.S. housing market prior to the 2008 Global Financial Crisis has induced much post-Crisis analysis to study how the bubbles has been fueled up as well as the role it played in dragging the economy into a recession. Existing models on housing market prices are incapable to account for the magnitude and persistence of price decline during the recent crash. While intensive efforts have been devoted to analyze the impact of external shocks, policy shocks in particular, on housing market price movements, our study focuses the role of internal factors during housing market crises. We construct a deterministic heterogeneous agent model (HAM) with the belief that interactions of agents adopting different investment strategies can lead to the crash of housing market. Having identified the common characteristics of housing market and financial market crises, we incorporate the evolutionary mechanism, which specifies the switching between heterogeneous beliefs, in asset pricing model introduced by Brock and Hommes (1997, 1998) into our model. As our model is capable of generating price mechanisms in different types of housing market crises, it captures some, if not all, of the fundamentals of housing market crises.

In general, booms and busts in housing market exhibit themselves differently each time in terms of depth and length. Comparison studies on historical data and current situations have been carried out to illustrate how this time differs from past experience. A study by Agnello and Schuknecht (2011) on real estate price booms and busts in eighteen industrialized countries including the U.S. reveals that the recent housing booms have been amongst the longest in the past forty years. Strong correlation between the magnitude and persistence of booms and busts implies that the current bust in the U.S. will be severe and prolonged. Case (2008) performed a close analysis on the U.S. housing market, including both national and state-wise analyses, and proposed that the decline in housing market price might have not come to an end.

Existing literature on housing market crisis focuses more on the effects of external economic or policy shocks on housing markets. Analyses vary from the impacts of land restrictions (see Pollakowski and Wachter, 1990) to down-payment requirements (see Stein, 1995). Taylor (2007) tabulated the difference in macroeconomic policies between the occurrence of previous and current housing crises and suggested that the departure of interest rate from its ordinary path might be the cause of recent housing market boom and bust. On top of that, he proposed a series of suggestions regarding the implementation of monetary policies in stabilizing the housing market. Therefore, exogenous factors such as changes in interest rates, constraints on mortgages and down-payment requirements are noteworthy for explaining the price dynamics of the current housing bust.

However, changes in economic fundamentals or external factors fail to account for the magnitude and depth of the current bust. The most recent U.S. housing market boom starting from the mid 1990s demonstrates that despite that fact that interest rate has stayed low since 2003 and rental price has not increased much in most states around the U.S., housing price
continued to rise until it crashed in 2007. Indeed, the impact of policy shocks on housing prices is found to be small as compared to the magnitude of recent fluctuations in the U.S. (Negro and Otrok, 2007). Therefore, it is reasonable to hypothesize that the interaction of buyers and sellers contributed to the current housing crisis. That is, housing market crisis has an endogeneous origin.

Of the few literature which takes housing market endogeneity into account, Glaeser et al. (2008) incorporated endogenous housing supply into their model and discovered that places with higher elasticity of housing supply have fewer and shorter bubbles. Sommervoll et al. (2010) developed an endogeneous model and showed that the imposition of credit constraints would increase housing market volatility. However, these models focus on the effect of external shocks to price movement, and the price dynamics due to pure interactions among agents are not discussed.

Therefore, we intend to investigate the interactions among heterogeneous agents in the housing market to show that housing crises possibly have endogeneous origins. In our construction of the deterministic HAM, the fixed parameters capture the fundamentals of housing market. We are capable of simulating the price dynamics of different states in the U.S. with differences in the values of initial price only. Thus, the current U.S. housing market could be shown to have an endogeneous origin.

Despite the gap in housing market literature investigating the price mechanism based on interactions among heterogeneous agents, past works on financial markets provide us some insights on model set-up. A pioneer work by Day and Huang (1990) introduced heterogeneous beliefs in investment strategies to financial market. Follow up, Brock and Hommes (1998) developed a revolutionary framework for switches between different strategies. Boswijk et al. (2007) constructed a model based on such evolutionary selection mechanism to estimate the U.S. stock price, which provides strong supports to the existence of heterogeneous beliefs in investment demand. Built upon a similar fundation, the endogenous deterministic HAM constructed by Huang et al. (2010) is capable of generating three different types of financial crises. One defining characteristic of housing bubbles is the tendency to view housing as an investment (see Case and Shiller, 2003). This implies that housing markets share similar features with stock markets especially during the period of crises. With the presence of investment needs, agents who trade on houses act like investors in financial market. Thus, it is reasonable to assume that traders in housing market are forward-looking and update their trading strategies differently according to their price and profit expectations. The switches between different trading strategies drive price movements in the market.

Several important aspects of housing market distinguish it from financial market. One prominent feature is the presence of utility demand driven by the need of buying houses for living purposes, which is incorporated into the decision-making of all traders. Therefore, in our model, each agent simultaneously possesses two types of demand, investment demand and utility demand. Due to differences in market evaluation and expectation, investment demand,
which is formed due to expectation of future price appreciation, differs among different groups of agents. Utility demand comes from agents’ own occupancy of houses as well as the dividends received from renting out extra houses. Consequently, utility demand is sensitive to market rental price.

The existence of search costs and transaction costs makes the housing market much less liquid than stock market. Such frictions in the market reduce agents’ selling or buying incentives. Consequently, less volatility in price movements is observed in housing market. Rosser (2000) grouped financial crises into three different types (namely sudden, disturbing and smooth crisis) according to their depth and length. Adopting a similar strategy, we classify housing market crises into two groups, namely the sudden crisis and the smooth crisis.\textsuperscript{1} In the sudden crisis, price falls from the peak down to the bottom within six quarters. In contrast, the price falls smoothly from the peak to the bottom with a moderate but persistent decline during the smooth crisis, which may last longer than six years. No visible crash occurs in this type of crises. Our model demonstrates both types of crises by capturing the essential properties of housing market.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses how external shocks may affect the magnitude of price fluctuations. Section 4 simulates the different types of crises and provides possible economic interpretations. Section 5 concludes.

\section{Deterministic Dynamic Model}

In housing market, buyers search for houses in the market according to their highly differentiated preferences. Because of high friction, demand almost never matches supply. The excess demand for housing is defined to be the amount of the housing units demanded less that of the housing units supplied in the market. The market would adjust the housing market price according to the amount of excess demand in the direction so as to clear the market in the long run. In our set-up, the excess demands are measured in sizes of houses to account for differentiated price impacts from housing in various sizes. This section is devoted to deriving a price impact function that links excess market demand to market price. In Section 2.1, a general individual demand function is derived. In Section 2.2, we specify two belief types for individual agents when they buy houses for capital gains. How agents allocate their income between utility and investment for housing, as well as how agents choose investment strategies would be discussed in Section 2.3. Finally in Section 2.4, we present the price impact function.

\subsection{Individual Demand Function}

In housing market, agents demand houses both for utility purposes (including occupancy to the house and rents received) as well as for future capital gains. Moreover, each agent’s

\textsuperscript{1}The price dynamics of housing market exhibits fewer patterns due to less volatility.
demand for housing would be affected by the transaction and search cost in housing market. The individual demand function is developed based upon the above three factors.

2.1.1 Utility Demand

Suppose the market consists of $N$ agents, each agent allocates some income to housing purchases for utility. Agents consider their income level, the mortgage rate, and the current economic conditions, when deciding on how many units of housing to demand. Moreover, the rental rate would be taken into account as occupying or renting out the houses are two alternatives of obtaining utility from houses. Therefore, given the housing fundamentals, there would exist a market price high enough that agents would be indifferent between consuming houses or not. We define such price threshold as the reservation price and denote it as $p^*$. Note that $p^*$ is positively correlated with rental price since the higher the rents, the more an individual can receive from renting out extra houses.

Mathematically, utility demand for each agent is expressed as,

$$D_{U,i,t} = s(p^* - p_t)h \kappa_{1,i,t}$$

where $s, h > 0$ are scaling factors and $\kappa_{1,i,t}$ is an indicator variable such that

$$\kappa_{1,i,t} = \begin{cases} 1 & p^* > p_t \\ 0 & \text{otherwise} \end{cases}$$

The indicator variable $\kappa_{1,i,t}$ ensures that when price is above $p^*$, utility demand is eliminated.

2.1.2 Investment Demand

Apart from demanding houses for utility, agents also buy houses in the hope of future capital gains. More specifically, agents care about excess gains of investing in housing market with respect to investment alternatives. Suppose the rate of return of risk-free investment alternatives is $r$, the investment demand for each agent could be expressed as,

$$D_{I,i,t} = p_{e} - (1 + r)p_t$$

where $p_{e}$ is the expected future housing price.

2.1.3 Weighted Individual Demand

Each agent in the market is constrained by limited resources when faced with the decision of house purchase. Under resource constraints, agents inclined to investment in housing are restrained to demand less for utility. Therefore, the combination of utility and investment
demand on housing by a representative agent would be a weighted sum of both as follows:

\[ D_{i,t}^S = \varphi_{i,t} s(p^* - p_t)\kappa_{1,i,t} + (1 - \varphi_{i,t})[p_t^e - (1 + r)p_t] \]

where \( 0 < \varphi_{i,t} < 1 \) is the weight each agent allocates to utility demand.

### 2.1.4 Transaction Cost

As housing market is less liquid than the financial market, agents incur greater transaction costs when they buy or sell houses. The effect of the transaction cost is to diminish agents' transaction volume compared to market with perfect liquidity. As the agent is buying houses when \( D_{i,t}^S > 0 \), the effect of the transaction cost \((C)\) is to decrease the amount the agent will buy with respect to perfectly liquid market; when the agent is selling \((D_{i,t}^S < 0)\), transaction cost decreases the amount of housing units sold. To facilitate quantifying the effect of transaction cost, we set an indicator variable \( \kappa_{2,i,t} \) such that

\[
\kappa_{2,i,t} = \begin{cases} 
1 & D_{i,t}^S > 0 \\
-1 & D_{i,t}^S \leq 0
\end{cases}
\]

Now the individual demand function for a typical agent could be defined as,

\[
n_{i,t} = a_i \frac{D_{i,t}^S - C\kappa_{2,i,t}}{V_{i,t}(p_t)}\kappa_{3,i,t}
\]

where \( a_i > 0 \) is the degree of risk tolerance, \( V_{i,t}(p_t) \) is the variance of forecast error associated with a particular strategy, and \( \kappa_{3,i,t} \) captures the effect of transaction cost: when being too large, an agent would not demand any housing. Therefore, \( \kappa_{3,i,t} \) is an indicator variable such that

\[
\kappa_{3,i,t} = \begin{cases} 
1 & |D_{i,t}^S| > C \\
0 & \text{otherwise}
\end{cases}
\]

### 2.2 Multi-phase Heterogeneous Beliefs

Agents in each period are characterized into two broad groups according to the adoption of different investment strategies, namely fundamentalists (\(\alpha\)-agent) and chartists (\(\beta\)-agent). (see Day and Huang, 1990; Huang et al., 2010) We assume that agents taking the same investment strategy are identical and classify \(N\) agents into two representative groups (\(\alpha\) and \(\beta\)) accordingly.

**Fundamentalists**: Following Huang et al., (2010), there are \(x_\alpha\) fundamentalists in the market, who hold constant price and variance expectations, i.e.,

\[
p_{\alpha,t}^e = \bar{p}_\alpha, \quad V_\alpha(p_t) = \sigma_\alpha^2
\]
where $\bar{p}_\alpha$ is the fundamental value that fundamentalists expect the market to converge in the long run. They will buy in houses when price is below the fundamental and sell houses when price is above.

**Chartists:** There are $x_\beta = N - x_\alpha$ chartists in the market holding constant variance expectations and updating their price expectations as follows,

$$p_{\beta,t}^\beta = p_t + \tau(p_t - \bar{p}_{\beta,t}) \quad V_\beta(p_t) = \sigma_\beta^2$$

where $0 < \tau < 1$ is the speed of adjustment and $\bar{p}_{\beta,t}$ is the short term estimate of fundamental value. Chartists update their price expectations only base on the information of current price. They buy when price is above short term fundamental, and sell when price is below short term fundamental. Chartists chase the trend movement of market price until an interval containing a different short term fundamental is reached.

Similar to Huang et al., (2010), the short term fundamental value ($\bar{p}_{\beta,t}$) is obtained by first dividing price range into different intervals, $\bar{p}_{\beta,t}$ is taken to be the mid-point of each interval. The rationale behind such setting is that chartists trade according to experience and believe that price would follow the current trend to diverge further from short term fundamental. Mathematically, the whole price interval $\mathbb{P}$ could be divided into $n$ subintervals with equal length such that

$$\mathbb{P} = \bigcup_{j=1}^{n} \mathbb{P}_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \cdots \cup [\bar{p}_{n-1}, \bar{p}_n],$$

and

$$\bar{p}_{\beta,t} = (\bar{p}_{j-1} + \bar{p}_j)/2 \quad \text{if} \quad p_t \in [\bar{p}_{j-1}, \bar{p}_j) \quad \text{where} \quad j = 1, 2, \ldots, n$$

By setting $\lambda = \bar{p}_j - \bar{p}_{j-1}$ and $\bar{p}_0 = 0$, we have $\mathbb{P} = \bigcup_{j=1}^{n} \mathbb{P}_j = [0, \lambda) \cup [\lambda, 2\lambda) \cup \cdots \cup [(n-1)\lambda, n\lambda]$ and

$$\bar{p}_{\beta,t} = ([\bar{p}_t/\lambda] + [\bar{p}_t/\lambda])\lambda/2 \quad \text{if} \quad p_t \in [\bar{p}_{j-1}, \bar{p}_j) \quad \text{where} \quad j = 1, 2, \ldots, n \quad (2)$$

### 2.3 Strategy Switch and Endogenous Weighting

Following Huang et al. (2010), we assume that agents in housing market choose their investment strategies according to expected future gains from investment. Specifically, agents will cluster to the strategy or demand part with higher expected future gains. In a similar manner, agents decide on the weight assigned to utility demand after comparing future expected returns from investment and utility.

#### 2.3.1 Investment Strategy Switch

Expected future profits is the criterion used when comparing the superiority of different trading strategies. As housing market is not forced to clear every period, agents might hold

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2For detailed explanation and derivation see Huang et al., 2010.
housing units for more than one period. Therefore, agents care the discounted expected future profit of the investment strategy. As chartists are assumed to realize their profit every period (see Lux and Marchesi, 1999), so the discount factor $S_\beta(p_t) = 1$. However, fundamentalists would hold housing units for multiple periods unless price is expected to return to fundamental value imminently. The further the price deviates from fundamental value, the more likely the price reversion will take place in the next period, therefore the larger the discount factor. Following Huang et al., (2010), the discount factor for fundamentalists is $S_\alpha(p_t) = (\tilde{p}_\alpha - p_t)^2 / 3\tilde{p}_\alpha^2$. We assume that fundamentalists incur costs when searching for information on market fundamentals. Thus, for fundamentalists, the discounted expected future profit per unit housing is

$$\pi^{e_\alpha,t} = S_\alpha(p_t) |\tilde{p}_\alpha - (1 + r)p_t| - C_\alpha,$$

where $C_\alpha$ is the information cost to be a fundamentalist. As chartists are myopic, no information costs are involved and their expected future profit per unit housing is

$$\pi^{e_\beta,t} = |\tau(p_t - \tilde{p}_\beta,t) - rp_t|.$$

Now define $\eta_{i,t}$ to be the market fraction of a particular type of agents. Following Brock and Hommes (1998), the market fraction is updated according to discrete choice probability, that is,

$$\eta_{i,t} = \frac{\exp(\rho\pi^{e_\alpha,t})}{\exp(\rho\pi^{e_\alpha,t}) + \exp(\rho\pi^{e_\beta,t})}, \ i = \alpha, \beta$$

where $0 < \rho < 1$ is the intensity of switching between the two investment strategies. With some algebraic manipulations, the market fraction index is defined as

$$m_t = \eta_{\alpha,t} - \eta_{\beta,t} = \tanh \left[ \frac{\rho}{2} \cdot (\pi^{e_\alpha,t} - \pi^{e_\beta,t}) \right]$$

When the profit of being a fundamentalist is higher ($\pi^{e_\alpha,t} > \pi^{e_\beta,t}$), $m_t$ is positive, meaning fundamentalists dominate the market in terms of the number of agents. However, there are still chartists in the market, indicating that not all agents will cluster to the more profitable alternative strategy.

### 2.3.2 Endogenous Weighting

After deciding which investment strategy to adopt, each agent has to determine the weight that is assigned to the utility demand. It is reasonable to assume the expected gains from utility demand are homogeneous across all agents, be it a fundamentalist or chartist. Specifically,
each agent expects gains from utility demand at each period to be \(^3\)

\[
U_{i,t}^e = e(D_{i,t}^U)^f \kappa_{i,t}, \ i = \alpha, \beta
\]

where \(e > 0\) is a constant and \(0 < f \leq 1\) is a scaling factor characterizing the fact of non-increasing marginal returns. As discounted expected future profit from housing investment is different between fundamentalists and chartists, the endogenous weighting factor \(\varphi_{i,t}\) is different in general as well.

Again employing discrete choice probability, the weighting factor for each type of agent is

\[
\varphi_{i,t} = \frac{\exp(l \cdot U_{i,t}^e)}{\exp(l \cdot U_{i,t}^e) + \exp(l \cdot \pi_{i,t}^e)} \ i = \alpha, \beta
\]  \((6)\)

where \(0 < l < 1\) is the intensity of allocation between utility and investment demand.

### 2.4 Price Dynamics

Before moving on to price impact function, we define \(\delta > 0\) to be the relative risk tolerance such that \(a_{\beta} = \delta a_{\alpha}\). Following Gennotte and Leland (1990), aggregate excess demand is normalized by the weighted sum of all agents’ risk tolerance, and substituting equation (1) to (6), we have

\[
D_t = \frac{n_{\alpha,t}x_{\alpha,t} + n_{\beta,t}x_{\beta,t}}{a_{\alpha}x_{\alpha,t} + a_{\beta}x_{\beta,t}} = \frac{s(p_t^* - p_t)^h \kappa_{1,i,t} \varphi_{\alpha,t} \sigma_2^2(1 + m_t) \kappa_{3,\alpha,t} + \delta \varphi_{\beta,t} \sigma_2^2(1 - m_t) \kappa_{3,\beta,t}}{\sigma_2^2 \sigma_2^2[(1 + \delta) + (1 - \delta)m_t]}
\]

\[
+ \frac{(1 - \varphi_{\alpha,t}) \sigma_2^2(1 + m_t)[\bar{p}_\alpha - (1 + r)p_t] \kappa_{3,\alpha,t}}{\sigma_2^2 \sigma_2^2[(1 + \delta) + (1 - \delta)m_t]}
\]

\[
+ \frac{(1 - \varphi_{\beta,t}) \delta \sigma_2^2(1 - m_t)[\tau(p_t - \bar{p}_{\beta,t}) - r_p] \kappa_{3,\beta,t}}{\sigma_2^2 \sigma_2^2[(1 + \delta) + (1 - \delta)m_t]}
\]

\[
- \frac{C[\sigma_2^2(1 + m_t) \kappa_{2,\alpha,t} \kappa_{3,\alpha,t} + \delta \sigma_2^2(1 - m_t) \kappa_{2,\beta,t} \kappa_{3,\beta,t}]}{\sigma_2^2 \sigma_2^2[(1 + \delta) + (1 - \delta)m_t]}
\]  \((7)\)

If \(D_t = 0\), the market is cleared; if \(D_t > 0\), demand exceeds supply so price tends to rise; if \(D_t < 0\), supply exceeds demand so price tends to fall. Therefore the price impact function could be defined as

\[
p_{t+1} = p_t + \gamma D_t, \quad \text{where } \gamma > 0\text{ is the speed of adjustment.}
\]  \((8)\)

\(^3\)We may also refer to the expected gains from utility demand as the (cardinal) utility from housing consumption.
3 Comparative Dynamics

In this part, we perform theoretical analysis on how changes in various factors affect the magnitude of price fluctuations, which we shall denote as $\Delta_t p = p_{t+1} - p_t$. With the aid of such analysis, we aim to 1) understand the price responses to various external shocks; 2) to justify the rationale behind price mechanisms during crisis period. To facilitate the analysis, we define two crucial price intervals as bull and bear markets respectively, where prices are too high or too low to be sustained. We believe that price movements outside these intervals are tolerable. Therefore, we focus on bull and bear market only in this part of analysis.

3.1 Definition of Bull and Bear Market

We first define the scaled difference between the number of shares demanded by a chartist and a fundamentalist when they bear the same risk tolerance as proposed by Huang et al. (2010):

$$ Y = n_{\beta,t}/a_{\beta} - n_{\alpha,t}/a_{\alpha} $$

We identify two price levels, $p_{bull}$ and $p_{bear}$, such that when price goes above $p_{bull}$ (below $p_{bear}$), $Y > 0$ ($Y < 0$) always holds true. Mathematically we can show that if price is below $p^*$, min $Y > 0$ cannot be true. Thus, we set $p_{bull} = max\{\hat{p}, p^*\}$, where $\hat{p}$ satisfies

$$ \hat{p} = \frac{\sigma_2^2(1 - \varphi_{\alpha,t})\bar{p}_{\alpha} + \sigma_1^2(1 - \varphi_{\beta,t})\tau \cdot \lambda + (\sigma_1^2 - \sigma_2^2)C}{\sigma_2^2(1 + r)(1 - \varphi_{\alpha,t}) + \sigma_1^2(1 - \varphi_{\beta,t})r} $$

With some parameter approximation, $p_{bear}$ could be simplified as:

$$ p_{bear} = p^* - \frac{\tau \lambda}{2s} $$

It is observed that $p_{bull} \geq p^* > p_{bear}$. We define the bull market by $p_t \in [p_{bull}, p_{max}]$ where price is overestimated and the bear market by $p_t \in [p_{min}, p_{bear}]$ where price is underestimated.

3.2 The Impact of Various Factors on Price Fluctuations

To analyze how changes in each factor may affect the price fluctuations, we differentiate $\Delta_t p$ directly with respect to each factor. The results are summarized in Table 1 and Table 2. As when price falls into the bull market, we would expect it to drop in near future back closer to market fundamental value. Thus, for $p_t \in [p_{bull}, p_{max}]$, we are interested in the impact on price drops and focus on the interpretation of $-d\Delta_t p/dk$. In contrast, for $p_t \in [p_{min}, p_{bear}]$, we expect future price rebounds, thus are interested in the impact on price rises and shall interpret $d\Delta_t p/dk$. Here, $k \in \{m_t, \delta, C, \rho, C, \varphi_{\beta,t}\}$.

Market fraction index, $m_t$: $-d\Delta_t p/dm_t$ ($d\Delta_t p/dm_t$) is positive in the bull (bear) market. A larger $m_t$ implies a larger proportion of fundamentalists in the market. The larger the fraction of fundamentalists, the quicker the price gets back to the fundamental value. We may
Table 1: The impact of various factors on price fluctuations

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>Y</th>
<th>$d\Delta_t p/dm_t$</th>
<th>$d\Delta_t p/d\delta$</th>
<th>$d\Delta_t p/dC_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t \in [p_{bull}, p_{max}]$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$p_t \in [p_{min}, p_{bear}]$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Remarks

- $d\Delta_t p/dm_t = -2\gamma d\delta Y/[(1 + \delta) + (1 - \delta)m_t]^2$
- $d\Delta_t p/d\delta = \gamma (1 - m_t^2) Y/[(1 + \delta) + (1 - \delta)m_t]^2$
- $dm_t/dC_\alpha = -\rho(1 - m_t^2)/2$

conclude that fundamentalists play a crucial role in bringing the price back to fundamental level.

*Relative risk tolerance, $\delta$: $-d\Delta_t p/d\delta$ ( $d\Delta_t p/d\delta$ ) is negative in the bull (bear) market.* Typically, all types of investors become less risk tolerant when the market grooms. If certain types of investors are more sensitive to market changes, their risk tolerance may reduce more than the other type when housing bubble bursts. An increase in $\delta$ implies greater risk tolerance of chartists compared with fundamentalists. Actions taken by chartists crowds out the pulling back effect imposed by fundamentalists.

*Information cost, $C_\alpha$: $-d\Delta_t p/dC_\alpha$ ( $d\Delta_t p/dC_\alpha$ ) is negative in the bull (bear) market.* As it is costly to be fundamentalists, an increase in information cost further reduces the temptation of being fundamentalists, resulting in reduced market power. Thus the implications are consistent with what we arrived at with $d\Delta_t p/d\delta$.

Table 2: The impact of various factors on price fluctuations (Cont’d)

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$d\Delta_t p/d\rho$</th>
<th>$d\Delta_t p/dC$</th>
<th>$d\Delta_t p/d\varphi_{\beta,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t \in [p_{bull}, p_{max}]$</td>
<td>- if $m_t &gt; 0$ Independent of $p_t$ Ambiguous</td>
<td>+ if $m_t &lt; 0$</td>
<td></td>
</tr>
<tr>
<td>$p_t \in [p_{min}, p_{bear}]$</td>
<td>+ if $m_t &gt; 0$ Independent of $p_t$ +</td>
<td>- if $m_t &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

Remarks

- $dm_t/d\rho = -\pi_{\alpha,t}^e - \pi_{\beta,t}^e (1 - m_t^2)/2$
- $d\Delta_t p/dC = -\gamma \sigma_\alpha^2 (1 + m_t) k_{2,\alpha}^2 + \sigma_\beta^2 (1 + m_t) k_{2,\beta}^2 (1 + \delta)^2 (1 + \delta) m_t |
- (when $k_{2,\alpha} = k_{2,\beta} = 1$)
- - if $m_t > 0$, + if $m_t < 0$
- (when $k_{2,\alpha} = k_{2,\beta} = 1$)

*Intensity of choice, $\rho$: When $m_t > 0$ ($m_t < 0$), $-d\Delta_t p/d\rho$ is positive in the bear market (negative in the bull market) and so is $d\Delta_t p/d\rho$. Higher intensity of choice enhances herding behavior. Since fundamentalists dominate when $m_t > 0$, the enhanced herding behavior implies stronger pulling back effect. Contrarily, when chartists dominate, enhanced herding behavior weakens the pulling back effect.*
Transaction cost, $C^4$: Both groups have net demand in housing when $\kappa_{2,\alpha,t} = \kappa_{2,\beta,t} = 1$. Price is likely to shoot up with excess demand from both groups. The rise in transaction cost works against the effect of overheating in the market (negative $d\Delta p/dC$). Similarly, we can show that increase in transaction cost prevents the housing price from dropping drastically when both parties have excess supply (negative $-d\Delta p/dC$). If one group is buying while the other is selling, the effect of transaction cost depends on which group dominates the market. For example, when $m > 0$, i.e. fundamentalists dominate, $d\Delta p/dC$ is negative.

Weighting factor for chartists, $\varphi_{\beta,t}$: the sign of $d\Delta p/d\varphi_{\beta,t}$ is ambiguous except when price falls down into the bull market. When price is low enough, the pulling back effect is enhanced with an increase in the allocation of resources that chartists place on utility demand relative to investment demand. This is consistent with our expectation that the presence of utility demand tends to stabilize the market.

4 Housing Market Crises Demonstrations

In this section we use the model developed in Section 2 to simulate the price movement during smooth crisis and sudden crisis. While the previous section illustrates how various factors can affect the magnitude of price fluctuations, the internal price dynamics remains unresolved. We thus examine sudden crisis and smooth crisis with the same standard parameters set$^5$ but with different initial prices in order to illustrate that the differences among crises are endogenously caused. To show that the model captures the fundamental factors of housing markets, we further match the simulated price series with the real historical scenarios of housing market crises.

As demonstrated by Negro and Otrok (2007) using a VAR, historical U.S. housing market price movements were mainly driven by local (state- or region-specific) shocks rather national shocks. Therefore, rather than examine the U.S. housing market crises in integrity, we investigate into state-wise cases. Fortunately, during the recent U.S. housing market crises, states across the U.S. exhibited different patterns of price drops, providing us a sample pool of the two typical crises: the sudden crisis and smooth crisis. Under our belief that housing market crises are results of internal interactions, external shocks serve only as triggering effects, which are captured by our specification of initial values.

4.1 Sudden Crisis

In a sudden crisis, price drops drastically from the crest down to the trough and the duration of such crisis is typically short. A typical example of the sudden crisis is the recent crash in

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$^4$We analyze the case when $\kappa_{3,\alpha,t} = \kappa_{3,\beta,t} = 1$ only. $\kappa_{3,t} = 0$ when transaction cost is too large. Decreases in transaction cost down to a threshold value will reactivate the transactions in the market. However, such effect is discontinuous and therefore cannot be analyzed with differentiation.

$^5$Standard parameters set: $p_\alpha = 50$, $p^* = 50$, $\gamma = 3.5187$, $\delta = 21.3591$, $\tau = 0.5$, $r = 0.00001$, $\rho = 0.9$, $C_a = 3$, $C = 0.02$, $\lambda = 13.17871$, $\sigma_\alpha = 1$, $\sigma_\beta = 1$, $l = 0.8$, $s = 0.2$, $e = 0.5$, $f = 1$, $h = 0.2$. Data on rental price suggest it to be stable over this period, we assume rental price is fixed. So is the reservation price $p^*$. 

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Figure 1: Sudden crisis: the upper panel presents the comparison of the simulated price series (dotted line) with California housing prices indicators from FHFA (solid line) from 2005Q1 to 2011Q1; the lower panel shows the track of market faction index $m$

Californias housing market. Crash in Florida housing market serves as another example.

The upper panel of Fig. 1 shows the housing prices series (solid line) in California from 2005Q1 to 2011Q1. We employ the quarterly Housing Price Index (HPI) developed by the Freddie Mac Housing Agency (FHFA) for different U.S. states. The price index dropped by 39.3% within a short period of 20 months starting on June, 2007. The simulated prices series with the initial price $p_0 = 70.93851968$ (dotted line) is simultaneously shown in the figure. The lower panel of Fig. 1 records the corresponding track of the market fraction index. It is worth noting that prior to such a fast drop in price, a sharp increase in $m$ is observed. The shoot in $m$ occurs when the simulated price rises up to a point higher enough above the fundamental price — the expected profit of the fundamentalists strategy is so large that investors cluster to fundamental trading strategy. Subsequently, when enormous selling orders are placed by fundamentalists, crash of the housing market occurs.

As illustrated in the previous section, an increase in the value of $m$, i.e. an increase in the proportion of fundamentalists in the market, enlarges the magnitude of the price drop in the bull market. This due to the fact that at a high price level, increased number of fundamentalists enhances the accumulation of each individual fundamentalist’s selling motivation.
Hence, when fundamentalists suddenly dominate the market, the pull-back effect is so huge that trigger the price plunge.

Fig. 2 shows the price dynamics which helps to illustrate the fundamental mechanism. Similar to the sudden crises in financial markets, when the price is high above the fundamental value, fundamentalists become more certain about the price reversal and hence improve their discounting factor \( S(p_t) \). Consequently, the discounted expected profit of fundamentalists, \( \pi_\alpha \), spurs significantly to exceed that of chartists, \( \pi_\beta \). Investors are largely attracted by such an outstanding expected profit that they update their investment beliefs and switch to fundamentalists dramatically. However, different from the financial markets’ sudden crises, price drops do not take place within one single period but are outcomes of a sequence of quick decline in multiple periods. This accounts for the differences in the patterns of price dynamics exhibited by financial and housing market crises.

## 4.2 Smooth Crisis

During the smooth crisis, price falls smoothly and persistently from the crest to the trough. The length of such crisis is typically long. The recent price fall in Massachusetts housing market is a typical example of smooth crisis.

The upper panle of Fig. 3 shows the housing prices series (solid line) in Massachusetts from 2005Q1 to 2011Q1 using the quarterly HPI from FHFA. The price dropped smoothly and
Figure 3: Smooth crisis: the upper panel presents the comparison of the simulated price series (dotted line) with Massachusetts housing prices indicators from FHFA (solid line) from 2005Q1 to 2011Q1; the lower panel shows the track of market faction index $m$. 
Figure 4: Smooth crisis: the price dynamics

persistently throughout 72 months starting from January 2005. The simulated prices series with the standard parameters set and an initial price of $p_0 = 64.3454014591$ (dotted line) is simultaneously shown in the figure. The corresponding track of the market fraction index is shown in the lower panel of Fig. 3. It must be noted that the value of $m$ stays slightly higher above -1, indicating that chartists dominate during the crisis. The reason why the market fraction of fundamentalists remains low is that the prices never reach a point where $\pi^e_\alpha$ exceed $\pi^e_\beta$.

Fig. 4 shows the price dynamics of this crisis scenario. During the crisis, the arbitrage behavior of chartists dominates the housing markets. The magnitude and direction of the price movement are dominated by the chartists’ short-term fundamental value, $\bar{p}_{\beta,t}$. It is crucial to note that in this particular case, the sell (when $p_t > \bar{p}_{\beta,t}$) and buy (when $p_t < \bar{p}_{\beta,t}$) decisions made by chartists lead to a persistent drop of price with episodic fluctuations, which is in correspondence with the lower panel of Fig. 5. It is also important to note that once the price falls below the reservation price, $p^*$, agents’ utility demands exert a pulling back effect to moderate the price drop driven by the chartists’ investment demand.
5 Conclusion

Our endogenous housing market model captures some, if not all, of internal factors that are essential to the bursts of housing market bubbles. It is capable of generating two types of housing market crises that resemble crashes of real housing market. Most of our findings are consistent with existing literature on heterogeneous agent models. In such models, interactions between different market investors driven by discounted profit expectations are crucial to the occurrence of housing market crash. Moreover, we are successful in demonstrating that investment needs increase during the market crash, echoing with previous study which claims that investment demand is the driving force of market bubbles.

Our model contributes to existing literature by incorporating a few distinguished features of housing market. First, we adopted a new strategy in analyzing real housing market crises by introducing the interaction mechanism in financial market to housing market. Under such mechanism, we discovered that the occurrence of sudden crisis is initiated by fundamentalists' dominance. However, such phenomenon is invisible in smooth crisis, which is solely due to the buy and sell decisions of chartists. Subsequently, we identify a few prominent features of housing market and incorporate them into the existing model. The enriched model takes into account individual utility demand and transaction costs. Analysis reveals that investment demand are vital to crash of housing bubbles but presence of both utility demand and transaction costs tends to smooth out the price volatility in the market. Furthermore, we endogenize the weighting factor so that it varies with expected profit and utility.

Our model builds upon several assumptions that might be released in future study. Firstly, we restricted the rental price to be constant. In real housing market, rental prices tend to be stable but not necessarily fixed. By letting the rent vary, we allow for more flexibility in utility demand. Secondly, we treat each unit of housing the same. Our model would exhibit more realistic price dynamics by allowing for some degree of heterogeneity in housing. Thirdly, we failed to account for both the down-payment and income constraints each individual faces when making investment decisions. These toss bricks for further study on endogenous housing market models.

References


