Household Relational Contracts, Fertility and Divorce

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Abstract

The theory of relational contracts is applied to a household model where players decide about the number of children and whether to stay together or get separated. We make precise the idea how cooperation can be enforced within a setup where players are solely driven by their self interest. Since the costs of having children are unequally distributed among spouses, there is a potential conflict between individually optimal and surplus maximizing decisions. Transfers are used to support cooperation but are not court-enforceable and thus have to be part of an equilibrium. This requires stable relationships and a credible punishment threat. Since players can only get punished via a separation, divorce must be rational for a player who did not receive a promised transfer. Therefore, policies that only increase the separation costs of one spouse - like a reduced access to children after divorce or alimony payments - might do a better job of increasing maximum enforceable transfers than generally making divorce more difficult. As stated in the literature, higher divorce costs can be used as a commitment device. But too high separation barriers make divorce threats uncredible, thus reducing punishment possibilities and incentives to cooperate.

JEL Classification: C73, D13, J12, J13, J24

1 Introduction

The casual assumption that family members always act cooperatively and necessarily get Pareto efficient outcomes has been challenged beginning with Ott (1992) and later continued by Konrad and Lommerud (1995) or Lundberg and Pollak (2003). Applying non-cooperative game theory to household decision making in a static environment, they identified sources of inefficient behavior of household members. On the other side, the collective models of household behavior, starting with Manser and Brown (1980), McElroy and Horney (1981) or Chiappori (1988) assume that household allocations are always Pareto efficient. In a cooperative game theory framework, they take individual utilities into account but take cooperative behavior as given.

Instead of assuming either cooperative or non-cooperative behavior, the present paper derives conditions for cooperation in a dynamic setup where each player is solely driven by their self-interest and not able to make binding commitments. We think that the theory of relational contracts[1] gives an appropriate tool to gain new insights of decision making within households. Relational contracts are

[1] Initially developed for labour markets and agency situations, see Bull (1987), MacLeod/Malcomson (1989) or Levin (2003).

dynamic games based on actions or outcomes that are observable but not verifiable, i.e. contracts based on them are not court-enforceable. As agreements in relationship certainly are to a large extent implicit, they present a very good subject to be analyzed with a relational contracts model.

More precisely, we develop a general fertility model where children are seen as public goods. As each partner bears a fixed share of costs of raising children (where the costs adopt several dimensions and also include issues like a loss of human capital), individual utility maximizing often leads to an inefficiently low fertility level, giving room for Pareto improvements: Spouses can make transfers to reward each other for cooperative behavior. But it is not possible to commit to such a transfer ex ante. Therefore, it has to be self-enforcing and be supported by an equilibrium. Although collective household models might implicitly assume a dynamic setting to support efficient decisions, they do not make precise the conditions necessary for cooperation. Our first contribution thus is the explicit statement of these constraints: Cooperative behavior must be individually rational, implying that reneging has to be followed by sufficient and credible punishment. If these constraints do not bind, though, our results do not differ from the collective household approach. Otherwise - i.e. for the case of binding constraints - outcomes are substantially different. We mainly focus on the latter case but also derive differences between both approaches. Whether a constraint binds depends on players´ discount factors as well as the size of punishment, which is determined by players´ outside options to which they are pushed down should they renege on their promises. The outside option in a relationship/marriage is to a large extent captured by separation/divorce. Although we still assume that this decision is made efficiently (applying a Becker-Coase Theorem), a separation threat is crucial to enforce transfers that support cooperation. But low separation utilities alone are not enough to enforce sufficient transfers. First of all, a threat must be credible and thus individually rational. Thus, a player has to favor separation compared to the continuation of a non-cooperative relationship. This implies that a divorce should not be too unattractive. If it becomes too favorable, though, the likelihood that a separation occurs in equilibrium increases, reducing the enforceability of transfers again (which requires punishment off equilibrium).

While agreements during a relationship are to a large extent implicit, this changes after a separation/divorce. When all goodwill is lost, issues like financial support or access to children are mainly governed by law. Therefore - after identifying the feasible equilibrium that maximizes total surplus - we take an institutional perspective and analyze the impact of different policy changes on fertility, marriage stability and the enforceability of transfers. We mainly want to contribute to the discussion of determinants of fertility and reasons for low birth rates in many countries. This discussion often aims at the costs associated with getting children, and most government policies try to reduce these costs. Parents are given direct monetary support or better/subsidized access to child care facilities to reduce the human capital loss caused by child rearing. Although the costs of getting children are an important factor in our analysis as well, we want to show that other issues like divorce or separation laws on fertility and the enforceability of transfers might have an impact on fertility levels.

Thereby, we start with an analysis of divorce costs. By increasing the stability of a relationship, marriage can serve as a commitment device to support voluntary transfers, a thought also captured by Rowthorn (1999) or Matouschek and Rasul (2008). The result that divorce makes cooperation less likely goes back to Lommerud (1989), who assumes that cooperation is driven by "voice enforcement" rather then players´ self interest within a repeated setup and is supported empirically by Lundberg and Rose (1999), where a higher divorce risk is associated with lower levels of specialization. Applied to fertility, Stevenson (2007) finds that a move to a unilateral divorce regime (which can be interpreted as a reduction of divorce costs) decreases fertility. Additionally to stressing the potential benefits of divorce costs, we also show that being too high, they might destroy any chance for cooperation. The reason is that a divorce threat becomes uncredible if it is not optimal for a player to file for divorce after non-cooperation.

Furthermore, we look at alimony payments that are solely based on income differences. Although having no (direct) impact on relationship stability, they can help to increase fertility. As child rearing is associated with a decrease in future income, alimony payments can serve as an insurance against this human capital loss.

Finally, we allow for a reduction of one´s spouse access to his children following a divorce. By increasing marriage stability, reducing ones reservation utility and therefore increasing the punishment following no cooperation, such a law can also help to increase fertility.

If costs are not too high and the reduction of one spouse´s access to their children after a separation is not too high, these policies enforce each other and have an even bigger positive impact on fertility than applied alone.

Generally, if the total costs of child rearing are - for whatever reasons - still unequally distributed in a society, policies making the side bearing a lower share of the costs better off after a separation might lead to lower fertility levels. Since 70-90% of secondary earners in North America and Europe are female[2], the assumption that one side is mainly responsible for raising children seems adequate. Generally, couples with a more traditional role view should be more likely to get married, a claim that is supporated by some empirical evidence[3]. Therefore, although the intention of laws granting divorced husbands a better access to their children following a divorce or reducing their financial obligations to their former wives (not their children!) certainly is honorable, it might lead to lower birthrates.

2 Model Setup

A household, founded at the beginning of the game, consists of a primary and a secondary earner ($i = 1, 2$) and lasts for two periods. In any of these periods $t = 1, 2$, the primary earner spends his total working time (normalized to 1) on the labour market, while player 2 divides her time between working and raising children. In period 1, $n \leq \overline{n}$ children can be born, causing total monetary costs $c(n)$ ($c(0) = 0$ and $c' \geq 0$), where $\gamma c(n)$ is paid by player 1 and $(1 - \gamma)c(n)$ by 2. For convenience, we assume n being continous. Total time needed to raise children amounts to $g(n)$ ($g' \geq 0$), where g only accrues in period 1 and $g(\overline{n}) \leq 1$.

Per period utility functions if the couple lives in a relationship are $u_{it} = x_{it} + \varphi_i(n)$, $\varphi_i(0) = 0$, $\varphi_i' > 0$ and $\varphi_i'' < 0$, where x is a private consumption good and n is the number of children present. The wage both receive depends on previous experience. In $t = 2$, it equals $w_{i2}(e_{i1})$, where $e_{11} = 1$ and $e_{21} = 1 - g(n)$ denote players´ working times in period t. The wage function is increasing and concave, furthermore $w_{i1} > 0$. In the following, we denote 2´s second period wage $w_{22}(n)$ (with $w_{22}' < 0$, since there is a direct mapping of n to experience).

At the beginning of period 1, the partners decide whether to get married or remain together unmarried. The timing in period 1 is:

1. The partners unanimously chose the number of children n.
2. Both work and receive their wages; consumption takes place. Player 2 also raises their children, while player 1 pays $\gamma c(n)$ and 2 $(1 - \gamma)c(n)$.
3. A (voluntary) transfer $b(n)$ is made from player 1 to 2. The transfer can also adopt negative values, meaning that it is effectively paid by the secondary earner.

No explicit contract on $b(n)$ is feasible, it has to be part of an equilibrium within the Household Relational Contract (defined below). In the second period, the couple just makes the decision whether

[2]See Immervoll et. al. (2009) for country specific numbers.

[3]Kaufmann (2004) for example finds out that men with egalitarian counterparts are more likely to cohabit than those with more traditional views

to get separated or remain together. We follow Becker (1991) assuming that the Coase theorem can be applied and the separation decision is thus made efficiently. Therefore, any transfers necessary to support efficient choices are enforceable in period 2. This assumption can be justified when these transfers can be split arbitrarily and if the separation decision can be made at any time (see Wickelgren 2007).

A separation at the beginning of the second period has the following consequences

- Each receives (non-monetary) outside utilities $V = (V_1, V_2)$ which capture issues outside the relationship like potential new partners as well as those within like love or caring. V_i has log-concave density $f_i(V)$ with support $[V^l, V^h]$, $V^l \leq 0 < V^h$, is not restricted to be positive and observed by both at the beginning of the second period. The assumption that spouses know their partners´ outside option fairly well is supported by Peters (1986).

  The common assumption that the couple receives utility just by being together is captured as well in this setup: We just normalized this to zero, distribution and support of outside utilities can be adjusted accordingly.

- Furthermore the per period utility from children of the primary earner after a split amounts to $\theta\varphi(n)$, $\theta \leq 1$. Here, we want to account for potentially different legislations determining the access of the primary earner to his children.

If the couple was married (we assume that divorce without termination and termination without divorce is not possible), a divorce has two further effects:

- If the couple was married (we assume that divorce without termination and termination without divorce is not possible), each of the partners furthers bears divorce costs k.

- Player 2 receives a monetary transfer $\phi(w_{12} - w_{22}(n))$, $0 \leq \phi \leq 1/2$, from player 1 in each subsequent period. Although the transfer does not directly depend on the number of children (monetary education costs have already been paid in the first period), n enters via its impact on 2´s second period wage.

All other parts of the within-relationship utilities remain unchanged and are added to the values just described.

3 Household Relational Contracts (HRC)

Players have to decide about the form of their relationship, the number of children, voluntary bonus payments and whether they get divorced or stay together. We assume that they formulate a Household Relational Contract (HRC) which specifies all actions players have to take conditional on all possible histories[4]. But it is not possible to write binding contracts that are contingent on actions or outcomes[5]. Since the second period decision whether to get separated or not is made efficiently by assumption, only first period actions have to form a subgame perfect equilibrium. We deviate from the "standard" relational contracts literature and use a finite time horizon. This becomes possible because of the application of the Becker-Coase theorem in the second period (and since the nature of actions in the first and second period are different). Therefore, the last period effect impeding cooperation in such

[4] For a complete characterisation of HRC´s see Fahn and Rees (2009).

[5] Of course, this assumptions can be questioned when it comes to the number of children, which certainly is verifiable. But it is hard to imagine that a court would enforce contracts determining payments contingent on the number of children.

games becomes non-existent and non-cooperation can be punished by the move to a non-favorable equilibrium.

Of main interest in our setup is the number of children the couple decides to get. Since the decision has to be made uninomously and since the distribution of time and monetary costs as well as benefits of getting kids are fixed, it is likely that the individually optimal levels of n (denoted n_i^{**}) differ among spouses. Then, gains from cooperation exist which the partners can try to exploit. The partner bearing relatively high costs might be willing to agree on a higher number of children if compensation for cooperation is promised. This compensation is assumed to happen via the transfer $b(n)$ at the end of period 1. Since no explicit contract can be written, $b(n)$ has to be self enforcing[6]. This implies that a player not willing to cooperate has to be punished in the future. We follow MacLeod, Malcomson (1989) assuming that after someone did not stick to the arrangement, any trust between the partners is lost and the relationship is soured. Thus the harshest possible punishment is used (Abreu (1986)), implying that the equilibrium with the lowest payoff for the player that reneged is played. As standard in this literature, a player is reneging if and only if he did not pay a promised bonus. Not agreeing to the desired level of n is not a contract violation but rather leads to a kind of punishment via an adjusted level of the transfer (which is defined for each level of n).

The harshest feasible punishment for a player reneging on a promised transfer is pushing him down to his reservation utility in period 2. Whether this occurs via a termination of the relationship or pushing one´s utility down to this level within does not play a role. We denote this reservation utility $u_{i2}^R(n)$. It is an expected value, since the realizations of V are not known when the transfer $b(n)$ is made. Before specifying it further, we state conditions for the enforceability of tranfsers $b(n)$. To do this, we further need the expected second period utilities on the equilibrium path, which we denote u_{i2}^C. A positive transfer is enforceable, if it satisfies the dynamic enforcement (DE) constraint

(DE1)

$$b(n) \leq \delta(u_{12}^C(n) - u_{12}^R(n)) \tag{1}$$

If $b(n)$ is supposed to adopt a negative value, we get

(DE2)

$$-b(n) \leq \delta(u_{22}^C(n) - u_{22}^R(n)) \tag{2}$$

The constraints have to hold for each possible value of the transfer. Contrary to papers like MacLeod & Malcomson (1989) or Levin (2003) we cannot replace the left hand sides of conditions 1 and 2 with the potentially highest (max$b(n)$) or lowest (-min$b(n)$) values of the transfer. The reason is that the number of children - and therefore $b(n)$ - has an impact on second period utility realizations.

u_{i2}^R corresponds to the utility this player would get if no cooperation took place in period 2. In the standard relational contract literature, a player´s reservation utility equals the utility he would receive after a termination of the relationship[7]. Therefore, the difference between a player´s cooperation and reservation utility determines the maximal transfer this player might be willing to pay.

Here, a punishment threat is not always credible. To see why, we distinguish three possible outcomes in period 2. The couple can get separated (if realizations of outside utilities are sufficiently high), it can be in the interest of both to remain together (if both realizations of V are low enough) or it can be efficient to stay together, but one partner has to be compensated from the other (if one spouse´s V is high while the other´s is low). Just the last case can be used to enforce a transfer in the first period. Only there exists freedom in surplus distribution, while the others do not impose credible punishment

[6] Note that self-enforcing here does not mean exactly the same as in Abreu´s et al. (1986) work since there the horizon is infinite; instead, we define $b(n)$ to be self-enforcing if it is part of a subgame perfect equilibrium.

[7] Whether the relationship is actually terminated or the surplus is just distributed in a respective manner is of no importance.

threats. In the case of divorce/separation, both players´ utilities are given, and no player is willing to reduce his utility be making a voluntary transfer. If it is in both´s interest to remain together in the second period - implying that no transfer is necessary to prevent a separation - no transfer is even feasible. The reason is that the threat of leaving the relationship after a transfer has not been made is not credible - a separation is not rational for a player in this case. We come back to this issue in the next part.

Finally, a desired level $n*$ will only be chosen by both if $b(n^*)$ satisfies (DE1) and (DE2) and is optimal for each given that they believe the transfer is paid. This is covered by incentive compatibility (IC) constraints, which equal

(ICi)

$$n^* \leq \text{argmax}(u_{i1}(n,b) + \delta u_{i2}^C(n)) \tag{3}$$

The problem of the game then is identical to chose the level n^* that satisfies

$$\text{Max}\, u_{11} + u_{21} + \delta(u_{12}^C + u_{22}^C) \tag{4}$$

s.t. (IC1), (IC2), (DE1), (DE2) are satisfied. Before we are able to derive explicit solutions, we take a closer look at the period 2.

4 Second period

In period 2 - after observing the realizations of their outside utilities - the couple either gets separated or remains together. In the latter case, we further distinguish whether one partner needs to be compensated to remain, since only then there exists some freedom in surplus distribution. There, we do not explicitly model the bargaining process between partners, but refer to MacLeod and Malcomson (1995) for an overview of bargaining in a contracting environment.

Denoting separation utilities $\hat{u}_{12}(V_1) = w_{12} + \theta\varphi_1(n) - \phi(w_{12} - w_{22}(n)) - k + V_1$ and $\hat{u}_{22}(V_2) = w_{22}(n) + \varphi_2(n) + \phi(w_{12} - w_{22}(n)) - k + V_2$ (where divorce costs k and redistribution $\phi(w_{12} - w_{22}(n))$ are only included if the couple was married), we make the already stated assumption

Assumption 1: The couple gets separated at the beginning of the second period if and only if

$$\hat{u}_{12}(V_1) + \hat{u}_{22}(V_2) > u_{12} + u_{22} \tag{5}$$

After a marriage, condition 5 reduces to

$$-2k - (1-\theta)\varphi_1(n) + V_1 + V_2 > 0$$

versus $V_1 + V_2 - (1-\theta)\varphi_1(n) > 0$ after cohabitation.

Define $\tilde{V} = V_1 + V_2$ and $\hat{V} \in \tilde{V} : (V_1, V_2 \mid \hat{u}_{12}(V_1) + \hat{u}_{22}(V_2) = u_{12} + u_{22})$. $\hat{V}$ is the threshold of the sum of outside utilities above which separation occurs. Furthermore, take $\hat{V}_i : (V_i \mid \hat{u}_{i2}(\hat{V}_i) = u_{i2})$ as the individual outside utility thresholds above which a separation would be individually optimal for player i (if no transfer was made).

After a marriage, these thresholds equal

$\hat{V} = 2k + (1-\theta)\varphi_1(n)$

$\hat{V}_1 = k + \phi(w_{12} - w_{22}(n)) + (1-\theta)\varphi_1(n)$

$\hat{V}_2 = k - \phi(w_{12} - w_{22}(n))$

Assumption 1 is not necessary, if it is in the interest of both to remain together, i.e. if

$$u_{i2} \geq \hat{u}_{i2}(V_i),\ i = 1, 2 \tag{6}$$

Then, no transfer is needed to maintain the relationship. Furthermore, no redistribution is possible for those realizations of outside utilities that satisfy 6, which we state in the following corollary.

Corollary: Assume condition 6 is satisfied. Then, player i receives $w_{i2}(n) + \varphi_i(n)$ in period 2.

Proof: Assume the HRC specifies a transfer from 1 to 2 together with a separation threat of 2 following a renegingt. This threat is not credible since it is never rational for her to file for divorce, also if the transfer is not made.

For a general statement of the corollary in a bargaining environment, see MacLeod and Malcomson (1995).

In the following, we will use the following version of 6: $V_i < \hat{V}_i,\ i = 1, 2$.

Finally, we consider the case where one player would file for divorce but equation 5 is not satisfied. This is characterized by

$$V_i > \hat{V}_i,\ \tilde{V} < \hat{V}$$

Then, a transfer is needed to prevent a separation. Due to assumption 1, we do not have to worry about the enforceability of this transfer and apply Nash bargaining. This implies a fixed distribution of the resulting surplus. Assume that in equilibrium, player 1 receives the share α of this surplus, while 2 keeps $(1 - \alpha)$. Equilibrium utilities in the case were $\tilde{V} \leq \hat{V}$ and either $V_1 \geq \hat{V}_1$ or $V_2 \geq \hat{V}_2$ in period 2 then equal

$\hat{u}_{12}(V_1) + \alpha[u_{12} + u_{22} - (\hat{u}_{12}(V_1) + \hat{u}_{22}(V_2))]$

for the primary and

$\hat{u}_{22}(V_2) + (1 - \alpha)[u_{12} + u_{22} - (\hat{u}_{12}(V_1) + \hat{u}_{22}(V_2))]$

for the secondary earner (if the couple is married). As α can adopt any value between 0 and 1 in equilibrium, it is straightforward to set it equal to 0 in u_{12}^R and 1 in u_{22}^R. This maximizes punishment after a deviation and thereby the "power" of incentives $b(n)$ that can be given in period 1.

We are now ready to take a closer look at second period (expected) utilities u_{i2}^C and u_{i2}^R.

Define $P_i := \text{Prob}(V_i \geq \hat{V}_i, \tilde{V} \leq \hat{V})$ and $P := \text{Prob}(V_1 < \hat{V}_1, V_2 < \hat{V}_2)$.

To characterize all possible cases in period 2, we finally need $(1 - F(\hat{V}))$.

Explicit formulations: As V_1 and V_2 are independent, $f(\tilde{V}) = (f_1 * f_2)(\tilde{V}) = \int_{V^l}^{V^h} f_1(\tilde{V} - V_2) f_2(V_2) dV_2 = \int_{V^l}^{V^h} f_2(\tilde{V} - V_1) f_1(V_1) dV_1$

Then,

$$F(\hat{V}) = \int_{2V^l}^{\hat{V}} f(\tilde{V}) d\tilde{V} = \int_{2V^l}^{\hat{V}} \left(\int_{V^l}^{V^h} f_1(\tilde{V} - V_2) f_2(V_2) dV_2 \right) d\tilde{V}$$

$$P = \int_{V^l}^{\hat{V}_1} f_1(V_1) dV_1 \int_{V^l}^{\hat{V}_2} f_2(V_2) dV_2 = \int_{V^l}^{\hat{V}_1} \int_{V^l}^{\hat{V}_2} f_1(V_1) f_2(V_2) dV_1 dV_2$$

Since we only need the sum $P_1 + P_2$, we will work with $P_1 + P_2 = F(\hat{V}) - P$ in the following.

This enables us to explicitly formulate second period utilities.

They equal

$u_{12}^C = (1 - F(\hat{V}))\hat{u}_{12}(V_1 \mid \tilde{V} > \hat{V}) + Pu_{12}$

$= w_{12} + \varphi_1(n) - (1 - P)[(1 - \theta)\varphi_1(n) + k + \phi(w_{12} - w_{22}(n))] + \mathrm{E}(V_1) - P\mathrm{E}(V_1 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)$

$+\alpha(F(\hat{V}) - P)[2k + (1 - \theta)\varphi_1(n)] - \alpha[F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) - P\mathrm{E}(V_1 + V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)]$

and

$u_{22}^C = w_{22}(n) + \varphi_2(n) - (1 - P)[k - \phi(w_{12} - w_{22}(n))] + \mathrm{E}(V_2) - P\mathrm{E}(V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)$

$+(1-\alpha)(F(\hat{V}) - P)[2k + (1 - \theta)\varphi_1(n)] - (1-\alpha)[F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) - P\mathrm{E}(V_1 + V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)]$

Furthermore,

$u_{12}^R = w_{12} + \varphi_1(n) - (1 - P)[k + (1 - \theta)\varphi_1(n) + \phi(w_{12} - w_{22}(n))] + \mathrm{E}(V_1) - P\mathrm{E}(V_1 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)$

and

$u_{22}^R = w_{22}(n) + \varphi_2(n) - (1 - P)[k - \phi(w_{12} - w_{22}(n))] + \mathrm{E}(V_2) - P\mathrm{E}(V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)$

5 Results

Now, we are able to explicitly define dynamic enforcement constraints, which have to be satisfied for each $b(n)$:

(DE1): $b(n) \leq \delta\alpha(F(\hat{V}) - P)[2k + (1 - \theta)\varphi_1(n)] - \alpha[F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) - P\mathrm{E}(V_1 + V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)]$

(DE2): $-b(n) \leq \delta(1 - \alpha)(F(\hat{V}) - P)[2k + (1 - \theta)\varphi_1(n)] - \alpha[F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) - P\mathrm{E}(V_1 + V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)]$

The next proposition allows us to simplify the analysis:

Propostion 1: The value of α has no impact on the solution to 4, the equilibrium number of children n^*.

Proof: See Appendix.

Therefore, we make

Assumption 2: $\alpha = 1$

The distribution of the total surplus will be affected by the assumption (if DE constraints do not bind), but upfront payments would allow us to receive any desired surplus distribution.

Note that here the surplus of remaining together - which is only partially connected to the number of children n - is used to provide incentives for cooperative fertility behavior, an issue different from the standard relational contract models.

Due to assumption 2, $b(n)$ cannot be negative. Thus we can concentrate on transfers from 1 to 2 and omit (DE2).

Finally, we make

Assumption 3:

A) $b(n) = \begin{cases} b \text{ if } n = n* \\ 0 \text{ otherwise} \end{cases}$ for $n_1^{**} > n_2^{**}$

B) $b(n) = \begin{cases} 0 \text{ if } n = n* \\ b \text{ otherwise} \end{cases}$ for $n_1^{**} < n_2^{**}$

Remember that n_i^{**} characterizes the number of children player 1 would like to get conditional on no transfer paid.

Assumption 3 can also be made without loss of generality since the "production process" is non-stochastic.

In the first case - where player 1 wants more children than 2 - the transfer b serves as a reward for 2´s cooperation. In case B) - where 2 wants more children than 1 - the transfer is used as a punishment for non-cooperation. Note that although b is not paid in equilibrium in B), it is needed to support cooperation.

Therefore, second period utilities equal

$$u_{12}^C = w_{12} + \varphi_1(n) - (1-P)[(1-\theta)\varphi_1(n) + k + \phi(w_{12} - w_{22}(n))] + \mathrm{E}(V_1) - P\mathrm{E}(V_1 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)$$
$$+(F(\hat{V}) - P)[2k + (1-\theta)\varphi_1(n)] - [F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) - P\mathrm{E}(V_1 + V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)]$$
$$u_{12}^R = w_{12} + \varphi_1(n) - (1-P)[k + (1-\theta)\varphi_1(n) + \phi(w_{12} - w_{22}(n))] + \mathrm{E}(V_1) - P\mathrm{E}(V_1 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)$$

and

$$u_{22}^C = u_{22}^R = w_{22}(n) + \varphi_2(n) - (1-P)[k - \phi(w_{12} - w_{22}(n))] + \mathrm{E}(V_2) - P\mathrm{E}(V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)$$

Furthermore,

$$b(n) \leq \delta((F(\hat{V}) - P)[2k + (1-\theta)\varphi_1(n)] - [F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) - P\mathrm{E}(V_1 + V_2 \mid V_1 < \hat{V}_1, V_2 < \hat{V}_2)])$$

Defining $U_i(n, b(n))$ as the discounted payoff streams conditional on n and the transfer and $n^{**} = \min[n_1^{**}, n_2^{**}]$ as the resulting number of children if no transfer is made, we can state the final version of the problem.

It solves

$$\underset{b,\, n^*}{\mathrm{Max}}\, U_1 + U_2 = u_{11}(n) + u_{21}(n) + \delta(u_{12}^C(n) + u_{22}^C(n))$$

s.t.

(DE1) $b \leq \delta(u_{12}^C - u_{12}^R)$

A) if $n_1^{**} > n_2^{**}$

(ICi) $U_i(n^*, b) \geq U_i(n^{**}, 0)$

B) if $n_1^{**} < n_2^{**}$

(ICi) $U_i(n^*, 0) \geq U_i(n^{**}, b)$

We assume the outside utilities are distributed such that an interior solution always exists.

Proposition 2: $n^* \leq n^e$, where n^e characterizes the efficient or unconstrained number of children. If the inequality is strict, (DE1) is satisfied as an equality and n^* is determined by the binding (IC2) (case A) or (IC1) (case B).

Proof:

Setting the Lagrangian and solving for the optimal b yields first order conditions (respective Lagrange parameters are denoted λ)

A): $-\lambda_{IC1} + \lambda_{IC2} - \lambda_{DE} = 0$

B): $\lambda_{IC1} - \lambda_{IC2} - \lambda_{DE} = 0$

A): If (DE) binds, (IC2) has to bind as well. If $\lambda_{DE} = 0$, $\lambda_{IC1} = \lambda_{IC2} = 0$ (can only adopt a positive value with zero measure)

B): If (DE) binds, (IC1) is binding as well. If $\lambda_{DE} = 0$, $\lambda_{IC1} = \lambda_{IC2}$.

$n^* \leq n^e$ follows from concavity of the problem and the constraints. Q.E.D.

The proposition states that if the dynamic enforcement constraint binds (implying that the efficient result is not enforceable), n^* is determined by an incentive compatibility constraint that is satisfied as an equality (again: assumption on distribution function of outside utilites). In the first case - where 2 needs to be rewarded for a higher fertility level - it is player 2´s IC constraint. This case is of special interest when analyzing societies where women are mainly burdened with child rearing and where no sufficient child care facilities exist to allow a reduction of the human capital loss. Especially for highly educated women, this present s a barrier to higher fertility levels. When analyzing the impact of policies on the equilibrium number of children, we therefore want to focus on the case $n_1^{**} > n_2^{**}$ from now on.

6 Divorce Laws and their impact on n^*

When discussing the determinants of fertility (and possible approaches to increase it), public opinion and economic research often focus on a simple benefit-cost analyses. Children are costly and force parents to stay at home for some time. Governments try to support families by granting child allowances or offering and subsidizing child care facilities. Many studies analyze the success of different policies. All these cost determinants are part of our model as well and have an intuitive impact on equilibrium fertility n^*. But we want to go one step further and analyze the impact of some divorce/separation laws on n^*. The reason is that costs of child rearing are quite unequally distributed among spoused in many countries. There, it seems that this unequal distribution is not driven by optimality but rather by cultural reasons (countries like Germany or Sweden try to induce fathers to spend some time at home for child rearing). We assume that this cost distribution is somehow fixed and focus on the extreme case where only one partner stays at home. This will generally lead to a situation where the individually optimally numbers of children, n_i^{**}, do not coincide, requiring some redistribution among partners. Since the necessary transfers have to be self enforcing, outside options play a crucial role on the maximum amount this transfer can take. Our selection of divorce laws and their impact on equilibrium of course is not exclusive but is meant to provide a first insight of the impact these laws can have on the enforceability of trnasfers.

We start with the benchmark case of non-binding constraints and there receive the same results one would get applying the collective approach to our setup. The case of binding constraints then gives the main contribution of our paper, since results can be substantially different. Generally, we distinguish between the isolated impact of the policies and interactions.

6.1 Benchmark: The Collective Approach

The collective approach (as a starting point, see Chiappori 1988) assumes that household decisions are made cooperatively and that the surplus distribution is made according to a bargaining rule. Our approach does not assume efficient actions but rather derives conditions for cooperative behavior when players maximize their individual utilities. If the (DE) constraint does not bind and the resulting n^* thus maximizes the sum of individual utilities, both approaches provide identical results. Therefore, we derive the unconstrained outcome in this section as a benchmark and analyze the impact of divorce laws on fertility.

For this section, we thus make

Assumption 4: The (DE) constraint does not bind and the couple maximizes the unconstrained sum of individual utilties.

$U_1 + U_2 = w_{11} + \varphi_1(n) + w_{21}(1 - g(n)) + \varphi_2(n) - c(n)$

$+\delta[w_{12}+\varphi_1(n)+w_{22}(n)+\varphi_2(n)-(1-F(\hat{V}))(2k+(1-\theta)\varphi_1(n))+\mathrm{E}(\tilde{V})-F(\hat{V})\mathrm{E}(\tilde{V}\mid\tilde{V}<\hat{V})]$ and
$\hat{V}=2k+(1-\theta)\varphi_1(n)$

In isolation, divorce costs have no impact on efficient fertility, formalized by

Proposition 3: Assume $\phi=(1-\theta)=0$. Then, divorce costs k have no impact on the efficient number of children, n^e but decrease total surplus.

Proof:

n^e is determined by $\frac{\partial(U_1+U_2)}{\partial n}=\varphi_1'+\varphi_2'-c'-g'w_{21}+\delta[\varphi_1'+w_{22}'+\varphi_2'-(1-\theta)\varphi_1'(1-F(\hat{V}))]=0$. If $\theta=1$, the FOC is independent of divorce costs.

Furthermore, the envelope theorem gives
$\frac{d(U_1(n^e)+U_2(n^e))}{dk}=\frac{\partial(U_1(n^e)+U_2(n^e))}{\partial k}=-\delta 2(1-F(\hat{V}))<0$. Q.E.D.

This is intuitive since the total utility gained from children is the same, no matter whether the couple is married or not. The case is different when we also reduce 1´s access to his children after a divorce (alimony payments to do not play a role here). Then, 1´s utility from children is lower after a separation. As higher costs decrease divorce probabilities, they lead to an increased fertility:

Proposition 4: Assume $\theta<1$. Then, $\frac{dn^e}{dk}>0$ for $\hat{V}\leq 2V^h$.

Proof: $\frac{dn^e}{dk}=-\frac{\frac{\partial FOC}{\partial k}}{\frac{\partial FOC}{\partial n}}$. The denominor has to be negative, as otherwise the FOC would not characterize a maximum. Therefore, we are interested in $\frac{\partial FOC}{\partial k}=2\delta f(\hat{V})(1-\theta)\varphi_1'>0$

Q.E.D.

As already indicated, alimony payments have no impact on efficient fertility, since they represent a pure redistributed among spouses and therefore cancel out in U_1+U_2.

Proposition 5: Alimony payments $\phi(w_{12}-w_{22}(n))$ have no impact on n^e and the total surplus.

Reducing the primary earner´s access to his children after a separation will an impact on efficient fertilty n^e. There are two opposing effects and it is not clear which dominates:

Proposition 6: A lower θ leads to less or more children but reduces total surplus.

Proof:

$\frac{dn^e}{d\theta}=-\frac{\frac{\partial FOC}{\partial\theta}}{\frac{\partial FOC}{\partial n}}$. Still, the denominator has to be negative and we are only interested in

$\frac{\partial FOC}{\partial\theta}=\delta\varphi_1'[(1-F(\hat{V}))-(1-\theta)\varphi_1(n)f(\hat{V})]$, where both terms are positive

$\frac{d(U_1+U_2)^e}{d\theta}=\frac{\partial(U_1+U_2)^e}{\partial n}\frac{\partial n^e}{\partial\theta}+\frac{\partial(U_1+U_2)^e}{\partial\theta}=-\delta\hat{V}_\theta(1-F(\hat{V}))>0$. Q.E.D.

On the one hand, a lower θ reduces fertility simply due to the fact that in case of a divorce, the utility from children decreases. On the other hand, a lower θ can also increase fertility. If $\theta<1$, children might be used to prevent a separation. The higher n, the bigger is the difference between utilities in and outside the relationship. The lower θ, the stronger is this effect. Which of these effect dominates depends on the current size of costs and access reduction parameter θ.

6.2 Binding (DE) constraint

We now assume that (DE) constraints are binding and n^*therefore is smaller than the efficient number of children, n^e.

Furthermore, we make

Assumption 5: $n_1^{**} > n_2^{**}$

Assumption 5 indicates that the costs of child rearing are unequally distributed and disadvantage the secondary earner. This assumption seems reasonable if the associated human capital loss is sufficiently high. Whether the (DE) constraint actually binds depends on the difference between levels n_i^{**} (which will generally be larger with a higher unequalitiy in the distribution of costs) and on the maximal feasible transfer level. This is contingent on the discount factor $\delta$and the difference $u_{12}^C - u_{12}^R$.

In sections 6.2.1 - 6.2.3, we focus on the isolated effect of each divorce law which means that the others are assumed to be non-existent. Their interaction is analyzed in 6.2.4.

6.2.1 Divorce costs k

We start with divorce costs k and for now assume $\phi = 0$ and $\theta = 1$. Higher costs obviously make a divorce less likely. Holding the number of children fixed, they further decrease total surplus. But due to the increased marriage stability, divorce costs can serve as a commitment device for promises to reward cooperation within the relationship. This thought is captured by Rowthorn (1999) and especially Matouschek and Rasul (2008) who show empirically that a marriage is indeed sometimes used as a commitment device. Analyzing the changes of US states from consent to unilateral divorce regimes and assuming that the difference between both is that consent divorce inhabits higher separation costs, they show that a commitment effect exists.

We are interested in the impact of divorce costs k on equilibrium fertility n^*. Since divorce costs do not directly influence costs or benefits associated with getting children, they only have an impact on the enforceability of transfers. Since they reduce the probability of divorce, costs can also serve as a commitment device to enforce promised transfers. But they also increase individual thresholds $\hat{V}_i$. Recall that any redistribution in period 2 is only feasible if one partner needs to be compensated to remain with the relationship. Therefore, too high costs make punishment threats uncredible and reduce the possibility of redistribution in the second period.

Similar to Matouschek and Rasul (2008) we do not make a difference between unilateral and consent divorce regimes but just apply unilateral divorce. Consent divorce can be interpreted as imposing higher divorce costs.

Proposition 7: Assume $\phi = 0$, $\theta = 1$. Then, the effect of k on n^* is solely determined by its impact on the enforceability of transfers, $\frac{\partial b}{\partial k}$.

Proof:

As (IC2) binds, n^* is determined by

$U_2(n^*, 0) + b - U_2(n^{**}, 0) = 0$ and

$$\frac{dn*}{dk} = -\frac{\frac{\partial(U_2(n^*,0) - U_2(n^{**},0))}{\partial k} + \frac{\partial b}{\partial k}}{\frac{\partial(U_2(n^*,b) - U_2(n^{**},0))}{\partial n}}$$

As $n_1^{**} > n_2^{**}$, $n^{**} = n_2^{**}$ and $\frac{U_2(n^{**},0)}{\partial n} = 0$. Furthermore $\frac{\partial U_2(n^*,0)}{\partial n} + \frac{\partial b}{\partial n}$ has to be negative. First of all, concavity of the problem implies that $\frac{\partial U_2(n^*,0)}{\partial n} < 0$. Even if $\frac{\partial b}{\partial n}$ is positive, the sum has to be negative. If it were positive, n^* would not be part of an equilibrium, since a higher n would increase the potential transfer by more than the associated utility loss of player 2. Therefore, the sign of $\frac{dn^*}{dk}$ is determined by the nominator.

Since $\hat{V}_1 = \hat{V}_2 = k$ and $\hat{V} = 2k$, divorce threshold are not contingent on n. Therefore $\frac{\partial U_2(n,0)}{\partial k} = \frac{\partial b}{\partial n} = 0$ and $\frac{\partial(U_2(n^*,0) - U_2(n^{**},0))}{\partial n}$ is independent of k.

Proposition 8: Assume $\phi = 0$, $\theta = 1$. Then, the maximum feasible transfer b is increasing or decreasing in k. Furthermore, there exists a $\overline{k}$ such that if $k > \overline{k} \Rightarrow b = 0$.

Proof: Note that in this case,

$b = \delta[2k(F(\hat{V}) - P) - [F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) - P\mathrm{E}(\tilde{V} \mid V_1 < \hat{V}_1,\, V_2 < \hat{V}_2)]]$

$= \delta[2k(F(\hat{V}) - F_1(\hat{V}_1)F_2(\hat{V}_2)) - \int_{2V^l}^{\hat{V}}\int_{V^l}^{V^h} \tilde{V} f_2(\tilde{V} - V_1) f_1(V_1) dV_1 d\tilde{V} + \int_{V^l}^{\hat{V}_1}\int_{V^l}^{\hat{V}_2} (V_1 + V_2) f_1(V_1) f_2(V_2) dV_1 dV_2]$

with $\hat{V} = 2k$ and $\hat{V}_1 = \hat{V}_2 = k$. Take $\overline{k} = V^h$. Then, $F(\hat{V}) = P = 1$ and $F(\hat{V})\mathrm{E}(\tilde{V} \mid \tilde{V} < \hat{V}) = P\mathrm{E}(\tilde{V} \mid V_1 < \hat{V}_1,\, V_2 < \hat{V}_2)$.

Otherwise, we get

$\frac{\partial b}{\partial k} = \delta[2(F(\hat{V}) - P) + 2k(f(\hat{V})\hat{V}_k - \hat{V}_{1k} f_1(\hat{V}_1)F_2(\hat{V}_2) - \hat{V}_{2k} F_1(\hat{V}_1) f_2(\hat{V}_2))$

$-\hat{V}_k \int_{V^l}^{V^h} \hat{V} f_2(\hat{V} - V_1) f_1(V_1) dV_1 + \hat{V}_{1k} \int_{V^l}^{\hat{V}_2} (\hat{V}_1 + V_2) f_1(\hat{V}_1) f_2(V_2) dV_2 + \hat{V}_{2k} \int_{V^l}^{\hat{V}_1} (V_1 + \hat{V}_2) f_1(V_1) f_2(\hat{V}_2) dV_1]$

$= \delta[2(F(\hat{V}) - P) + f_1(\hat{V}_1) \int_{V^l}^{\hat{V}_2} (V_2 - \hat{V}_2) f_2(V_2) dV_2 + f_2(\hat{V}_2) \int_{V^l}^{\hat{V}_1} (V_1 - \hat{V}_1) f_1(V_1) dV_1]$

as $\hat{V}_{1k} = \hat{V}_{2k} = 1$, $\int_{V^l}^{V^h} f_2(\hat{V} - V_1) f_1(V_1) dV_1 = f(\hat{V})$ $\hat{V}_k = 2$ and $\hat{V} = 2k$.

Since $F(\hat{V}) - P \geq 0$ and the last two terms are negative, this proves the proposition. Q.E.D.

Proposition 9: Assume $\phi = 0$, $\theta = 1$ and that outside utilities V_i are distributed uniformly. Then, there exists a cost level k^*maximizing b and therefore n^*.

Proof:

$\frac{\partial b}{\partial k} = \frac{\delta}{(V^h - V^l)^2}[2(V^h - V^l)^2 F(\hat{V}) - 2(\hat{V}_1 - V^l)(\hat{V}_2 - V^l) + \frac{(\hat{V}_2)^2 - (V^l)^2}{2} \int_{V^l}^{\hat{V}_2} V_2 dV_2 - \int_{V^l}^{\hat{V}_2} \hat{V}_2 dV_2 + \int_{V^l}^{\hat{V}_1} V_1 dV_1 -$
$\int_{V^l}^{\hat{V}_1} \hat{V}_1 dV_1]$

(I) $\hat{V} \leq V^h + V^l$:

$b = \delta[2k(F(\hat{V}) - F_1(\hat{V}_1)F_2(\hat{V}_2)) - \int_{2V^l}^{\hat{V}}\int_{V^l}^{V^h} \tilde{V} f(\tilde{V} - V_2) f_2(V_2) dV_2 d\tilde{V} + \int_{V^l}^{\hat{V}_1}\int_{V^l}^{\hat{V}_2} (V_1 + V_2) f_1(V_1) f_2(V_2) dV_1 dV_2]$

$= \frac{\delta}{(V^h - V^l)^2}[\frac{1}{3}(k)^3 - k^2 V^l + k(V^l)^2 + \frac{1}{3}(V^l)^3]$

$\frac{\partial b}{\partial k} = \frac{\delta}{(V^h - V^l)^2}[k - V^l]^2 \geq 0$

(II) $\hat{V} > V^h + V^l$:

$b = \delta[2k[\frac{1}{(V^h - V^l)^2}((V^h + V^l)^2 - 4V^h V^l + 2V^h \hat{V} - 2(V^h)^2 - \frac{(\hat{V})^2}{2}) - \frac{1}{(V^h - V^l)^2}(\hat{V}_1 - V^l)(\hat{V}_2 - V^l)]$

$- \int_{2V^l}^{V^h + V^l} \tilde{V} \frac{\tilde{V} - 2V^l}{(V^h - V^l)^2} d\tilde{V} - \int_{V^l + V^h}^{\hat{V}} \tilde{V} \frac{2V^h - \tilde{V}}{(V^h - V^l)^2} d\tilde{V} + \frac{1}{(V^h - V^l)^2} \int_{V^l}^{\hat{V}_1}\int_{V^l}^{\hat{V}_2} (V_1 + V_2) dV_1 dV_2]$

$= \frac{\delta}{(V^h - V^l)^2}[2k[-(V^h)^2 + (V^l)^2 - 2V^h V^l + 4V^h k - 2k^2 - (k - V^l)^2]$

$-\frac{(V^h + V^l)^3 - (2V^l)^3}{3} + 2V^l \frac{(V^h + V^l)^2 - (2V^l)^2}{2} - 2V^h \frac{(2k)^2 - (V^l + V^h)^2}{2} + \frac{(2k)^3 - (V^l + V^h)^3}{3} + k^3 - (k)^2 V^l - (V^l)^2 k +$
$(V^l)^3]$

$\frac{\partial b}{\partial k} = \frac{\delta}{(V^h - V^l)^2}[-2(V^h)^2 - 4V^h V^l - (V^l)^2 + k(8V^h + 6V^l - 7k)]$

Optimal k:

$\frac{\partial b}{\partial k} = \frac{\delta}{(V^h - V^l)^2}[-2(V^h)^2 - 4V^hV^l - (V^l)^2 + k(8V^h + 6V^l - 7k)] = 0$ and

$7k^2 - k(8V^h + 6V^l) + 2(V^h)^2 + 4V^hV^l + (V^l)^2 = 0$

$k_{1,2} = \frac{(8V^h+6V^l)\pm\sqrt{(8V^h+6V^l)^2-28(2(V^h)^2+4V^hV^l+(V^l)^2)}}{14} = \frac{(8V^h+6V^l)\pm\sqrt{64(V^h)^2+96V^hV^l+36(V^l)^2-56(V^h)^2-112V^hV^l-28(V^l)^2)}}{14} = \frac{(8V^h+6V^l)\pm(V^h-V^l)\sqrt{8}}{14}$

We know that $k \geq \frac{V^h+V^l}{2}$. Therefore, we need $\frac{8V^h+6V^l-(V^h-V^l)\sqrt{8}}{14} \geq \frac{V^h+V^l}{2}$ and $(V^h - V^l)(1-\sqrt{8}) \geq 0$ which is not the case.

For $k = \frac{(8V^h+6V^l)+(V^h-V^l)\sqrt{8}}{14}$, we get $\frac{\partial^2 b}{\partial k^2} = \frac{\delta}{(V^h-V^l)^2}[-(V^h - V^l)\sqrt{8})] < 0$. Q.E.D.

Therefore, divorce costs can serve as a commitment device to enforce transfers. If they are relatively low, an increase helps to relax the (DE) constraint and increase equilibrium fertility. If they are too high, divorce threats become uncredible and non-cooperative behavior cannot be punished anymore.

Recall that using the collective approach (when the (DE) constraint does not bind), divorce costs in isolation have no impact on equilibrium fertility. Their impact on the maximum transfer b makes costs play a crucial role when the constraint binds.

6.2.2 Alimony payments

Now, we assume that a divorce causes alimony payments. Furthermore, $k = 0$ and $\theta = 1$. Player 2 receives a linear monetary transfer $\phi(w_{12} - w_{22})$ from player 1, $0 \leq \phi \leq 1/2$.

Since the divorce decision is made efficiently, alimony payments as a pure redistribution have no impact on separation probabilities. They rather influence bargaining powers and naturally increase 2´s share of the surplus; generally, they also give a better enforceability of transfers, since they decrease 1´s reservation utility in states where separation does not occur but when a partner has to be compensated for remaining together. Now, the thresholds equal $\hat{V}_1 = \phi(w_{12} - w_{22}(n))$, $\hat{V}_2 = -\phi(w_{12} - w_{22}(n))$ and $\hat{V} = 0$.

As in the previous section, n^* is again determined by binding (IC2) & (DE):

$U_2(n^*, 0) + b = U_2(n^{**}, 0)$.

Therefore,

$$\frac{dn^*}{d\phi} = -\frac{\frac{\partial U_2(n^*,0)}{\partial \phi} - \frac{\partial U_2(n^{**},0)}{\partial \phi} + \frac{\partial b}{\partial \phi}}{\frac{\partial U_2(n^*,0)}{\partial n} + \frac{\partial b}{\partial n}} \tag{7}$$

Since the denominator has to be negative ($\frac{\partial U_2(n^{**},0)}{\partial n} = 0$), the sign of $\frac{dn^*}{d\phi}$ is solely determined by the nominator of 7.

Although $\frac{\partial U_2(n,0)}{\partial \phi}$ is not necessarily zero, we first analyze the impact alimony payments have on the enforceability of transfers.

$\frac{\partial b}{\partial \phi} = -\delta[-\hat{V}_{1\phi} \int_{V^l}^{\hat{V}_2} (\hat{V}_1 + V_2) f_1(\hat{V}_1) f_2(V_2) dV_2 - \hat{V}_{2\phi} \int_{V^l}^{\hat{V}_1} (V_1 + \hat{V}_2) f_1(V_1) f_2(\hat{V}_2) dV_1]$

$= \delta(w_{12} - w_{22}(n))[f_1(\hat{V}_1) \int_{V^l}^{\hat{V}_2} \hat{V}_1 f_2(V_2) dV_2 - f_2(\hat{V}_2) \int_{V^l}^{\hat{V}_1} \hat{V}_2 f_1(V_1) dV_1 + f_1(\hat{V}_1) \int_{V^l}^{\hat{V}_2} V_2 f_2(V_2) dV_2 - f_2(\hat{V}_2) \int_{V^l}^{\hat{V}_1} V_1 f_1(V_1) dV_1]$

Assuming a uniform distribution, we can state

Proposition 10: Assume $k=0$, $\theta=1$ and outside utilities are uniformly distributed between V^l and V^h. Then, $\frac{\partial b}{\partial \phi} \geq 0$.

Proof:

Now,

$\frac{\partial b}{\partial \phi} = \delta(w_{12}-w_{22}(n))[f_1(\hat{V}_1)\hat{V}_1F_2(\hat{V}_2)-f_2(\hat{V}_2)\hat{V}_2F_1(\hat{V}_1)+f_1(\hat{V}_1)\int\limits_{V^l}^{-\phi(n)} V_2f_2(V_2)dV_2-f_2(\hat{V}_2)\int\limits_{V^l}^{-\phi(n)} V_1f_1(V_1)dV_1-$

$f_2(\hat{V}_2)\int\limits_{-\phi(n)}^{\phi(n)} V_1f_1(V_1)dV_1]$

$= \frac{\delta(w_{12}-w_{22}(n))}{(V^h-V^l)^2}[V^l(\hat{V}_2-\hat{V}_1)]$

$= -2\phi\frac{\delta(w_{12}-w_{22}(n))^2}{(V^h-V^l)^2}V^l \geq 0$. Q.E.D.

This positive impact is not caused by the increased payments 1 has to make in u_{12}^R, since this is offset by an identical increase of transfers in u_{12}^C, therefore not entering the difference $u_{12}^C - u_{12}^R$. It is solely caused by the increased expected outside utility in the state where it is individually optimal for both to remain together and where surplus redistribution thus is not feasible.

The fact that alimony payments increase 2´s outside utility for a given n and thus make her more willing to increase n^* (supermodularity of n and ϕ) has no impact on the maximum b. It nevertheless affects her general propensity to get more children, which is stated in

Proposition 11: Assume $k=0$, $\theta=1$ and outside utilities are uniformly distributed between V^l and V^h. Then, $\frac{dn^*}{d\phi} \geq 0$.

Proof:

$\frac{\partial U_2(n,0)}{\partial \phi} = \delta[-(\hat{V}_{1\phi}f_1(\hat{V}_1)F_2(\hat{V}_2)+\hat{V}_{2\phi}F_1(\hat{V}_1)f_2(\hat{V}_2))\phi(w_{12}-w_{22}(n))+(w_{12}-w_{22}(n))(1-P)$

$-\hat{V}_{1\phi}\int\limits_{V^l}^{\hat{V}_2} V_2f_2(V_2)f(\hat{V}_1)dV_2 - \hat{V}_{2\phi}\int\limits_{V^l}^{\hat{V}_1}\hat{V}_2f_2(\hat{V}_2)f(V_1)dV_1]$

$= \delta(w_{12}-w_{22}(n))[f(\hat{V}_1)\int\limits_{V^l}^{\hat{V}_2}(\hat{V}_2-V_2)f_2(V_2)dV_2+(1-P)]$

as $\hat{V}_{1\phi} = -\hat{V}_{2\phi} = (w_{12}-w_{22}(n))$, $-\phi(w_{12}-w_{22}(n)) = \hat{V}_2$ and $\int\limits_{V^l}^{\hat{V}_1} f(V_1)dV_1 = F_1(\hat{V}_1)$.

With a uniform distribution, we get

$\frac{\partial U_2(n,0)}{\partial \phi} = \frac{\delta(w_{12}-w_{22}(n))}{(V^h-V^l)^2}[(V^h-V^l)^2-\frac{(V^l)^2}{2}+\frac{3(\phi(w_{12}-w_{22}(n)))^2}{2}+\phi(w_{12}-w_{22}(n))V^l)]$

$\frac{\partial U_2(n,0)}{\partial \phi}+\frac{\partial b(n^*)}{\partial \phi}-\frac{\partial U_2(n,0)}{\partial \phi}$

$= \frac{\delta[(V^h-V^l)^2-\frac{1}{2}(V^l)^2]}{(V^h-V^l)^2}[w_{22}(n^{**})-w_{22}(n^*)]+\frac{3\delta(\phi(w_{12}-w_{22}(n^*)))^3}{2(V^h-V^l)^2}-\frac{3\delta(\phi(w_{12}-w_{22}(n^{**})))^3}{2(V^h-V^l)^2}$

$-\phi\frac{\delta(w_{12}-w_{22}(n*))^2}{(V^h-V^l)^2}V^l-\phi\frac{\delta(w_{12}-w_{22}(n^{**}))^2}{(V^h-V^l)^2}V^l$.

As $n^* > n^{**}$, therefore $w_{22}(n^*) < w_{22}(n^{**})$ and $V^l \leq 0$, this is positive. Q.E.D.

This result is completely different from an application of the collective approach, where alimony payments cancel out and therefore have no impact on the equilibrium outcome.

Explicit versus implicit contracts

If player 2 needs incentives to increase n, higher alimony payments may help to induce her to do so, since this at least partly compensates her for her human capital loss and further increases the maximum feasible transfer. This is also true if no divorce occurs and if we are in states were a divorce threat is be credible. It is interesting to compare this result to the literature analysing the interaction of explicit and implicit contracts. Let us focus on the important paper of Baker, Gibbons, Murphy (1994). There, the ability to also write explicit (yet depending on a less precise signal) contracts can destroy the ability to enforce (even otherwise perfect) implicit contracts. Note that the enforceability of transfers rewarding cooperation in relational contracts relies on the difference between equilibrium and outside surplus. If a potential explicit contract (which the players are assumed to conclude off equilibrium) increases this outside surplus sufficiently, the power of implicit incentives becomes too low.

In our setup, the existence of the explicit contract enforcing compensation for the human capital loss associated with children can increase cooperation. Two issues drive the different outcomes. On the one hand, our model is different as "production" only occurs once while it happens repeatedly in BGM. Furthermore, before deciding on n, by getting married the couple already chooses the explicit contract in place after a termination of the relationship (we do not explicitly analyze marriage contracts here, as they actually state alimony payments where the level of ϕ can be chosen - a tool that can further increase efficiency), while this contract is written after the termination in BGM and is therefore only affected by future outcomes.

To our knowledge, there exists no model in the relational contracts literature assuming two effort dimension in a way that one dimension is applied frequently and one only from time to time and therefore being more similar to ours - it should be interesting to analyze such a setting generally and then also further derive the interaction between explicit and implicit contracts.

6.2.3 Reducing 1´s access to his children after a separation

Finally, we assume that laws restrict the primary earner´s access to his children after a separation. Then, the per period utility from children of the primary earner after a split amounts to $\theta\varphi(n)$, $\theta \leq 1$. Similarly to divorce costs, a lower value of θ reduces efficiency on first sight, since it decreases the sum of expected utilities. If this effect is offset by a better enforceability of transfers and thus leads to more children, a low value θ might nevertheless be justified in our model. To get the isolated effect of a access reduction, we now assume $k = \phi = 0$. This reduced access can lead to a better enforceability of transfers, especially if θ is not too small. Assuming a uniform distribution, we can again find a value of θ that maximizes b. Generally, it can be said that 1´s access to his children should at least be marginally reduced after a separation. Nevertheless, neither full nor no access at all is optimal.

The thresholds $\hat{V}_i$ and $\hat{V}$ are now defined

$\hat{V} = \hat{V}_1 = (1-\theta)\varphi(n)$

$\hat{V}_2 = 0$ and

$$b = \delta[(1-\theta)\varphi_1(n)(F(\hat{V}) - F_1(\hat{V}_1)F_2(\hat{V}_2)) - \int_{2V^l}^{\hat{V}}\int_{V^l}^{V^h} \tilde{V} f_1(\tilde{V} - V_2) f_2(V_2) dV_2 d\tilde{V} + \int_{V^l}^{\hat{V}_1}\int_{V^l}^{\hat{V}_2} (V_1 + V_2) f_1(V_1) f_2(V_2) dV_1 dV_2]$$

$$\frac{\partial b}{\partial \theta} = \delta[-\varphi_1(n)(F(\hat{V}) - F_1(\hat{V}_1)F_2(\hat{V}_2)) + (1-\theta)\varphi_1(n)(\hat{V}_\theta f(\hat{V}) - \hat{V}_{1\theta} f_1(\hat{V}_1)F_2(\hat{V}_2))$$

$$-\hat{V}_\theta \hat{V} \int_{V^l}^{V^h} f_1(\hat{V} - V_2) f_2(V_2) dV_2 + \hat{V}_{1\theta} \int_{V^l}^{\hat{V}_2} (\hat{V}_1 + V_2) f_1(\hat{V}_1) f_2(V_2) dV_2]$$

$$= \delta\varphi_1(n)[-f_1(\hat{V}_1)\int_{V^l}^{\hat{V}_2} V_2 f_2(V_2)dV_2 - (F(\hat{V}) - P)]$$

This is either positive or negative, since the first part is positive due to $V^l \leq 0$ and $\hat{V}_2 = 0$. If the difference between $F(\hat{V})$ and P is sufficiently large, a reduction of θ increases maximum b, since it decreases 1´s reservation utility. The higher stability of marriage more than offsets the lower ability of 1 to make credible divorce threats: The impact of θ on $F(\hat{V})$ and P cancels out against its effect on expected outside utilities (a higher θ decreases marriage stability and thus increases the expected realized value of $\tilde{V}$; as it further increases P_1 (the probability that 1 would file for divorce if not compensated), the expected "usable" value of V_1 is reduced). What remains and may lead to a positive impact of a higher θ on b (if $F(\hat{V})$ and P are too close) is that via the increase of P_1, 2´s expected outside utility (which is negative) can be used more often. The sign also depends on the value the partners address to the stability of the relationship. This argument will become clearer when using uniform distributions.

Proposition 12: Assume outside utilities are uniformly distributed. Then, there exists a (unique) θ^* maximizing b. If $2V^h + V^l + \sqrt{2(V^h)^2 + 2(V^l)^2} > 0$, $\theta^* < 1$. For $2V^h + V^l + \sqrt{2(V^h)^2 + 2(V^l)^2} \leq 0$, $\theta^* = 1$.

Proof:

$\frac{\partial b}{\partial \theta} = \frac{\delta\varphi_1(n)}{(V^h - V^l)^2}[\frac{3}{2}(V^l)^2 - \hat{V}_1 V^l - (V^h - V^l)^2 F(\hat{V})]$

(A) $\hat{V} = (1-\theta)\varphi_1(n) \leq V^h + V^l$: $F(\hat{V}) = \frac{1}{(V^h - V^l)^2}(\frac{(\hat{V})^2}{2} - 2\hat{V}V^l + 2(V^l)^2)$ and

$\frac{\partial b}{\partial \theta} = -\frac{1}{2}\frac{\delta\varphi_1(n)}{(V^h - V^l)^2}(\hat{V}_1 - V^l)^2 < 0$

(B) $\hat{V} > V^h + V^l$: $F(\hat{V}) = \frac{1}{(V^h - V^l)^2}[-(V^h)^2 - 2V^h V^l + (V^l)^2 - \frac{(\hat{V})^2}{2} + 2V^h\hat{V}]$

$\frac{\partial b}{\partial \theta} = \frac{\delta\varphi_1(n)}{(V^h - V^l)^2}[\frac{1}{2}(V^l)^2 + (V^h)^2 + 2V^h V^l - 2V^h\hat{V}_1 + \frac{(\hat{V}_1)^2}{2} - \hat{V}_1 V^l]$

$= \frac{\delta\varphi_1(n)}{2(V^h - V^l)^2}[\left(2V^h + V^l - (1-\theta)\varphi_1(n)\right)^2 - 2(V^h)^2]$

θ maximizing b:

FOC: $0 = \frac{\partial b}{\partial \theta} = \frac{\delta\varphi_1(n)}{(V^h - V^l)^2}[\frac{1}{2}(V^l)^2 + (V^h)^2 + 2V^h V^l - 2V^h(1-\theta)\varphi_1(n) + \frac{((1-\theta)\varphi_1(n))^2}{2} - V^l(1-\theta)\varphi_1(n)]$

and $(1-\theta)^2\frac{(\varphi_1(n))^2}{2} - (1-\theta)\varphi_1(n)[2V^h + V^l] - \frac{1}{2}(V^l)^2 + (V^h)^2 + 2V^h V^l = 0$

$(1-\theta)\varphi_1(n)_{1,2} = [2V^h + V^l] \pm \sqrt{(2V^h + V^l)^2 - 2(-\frac{1}{2}(V^l)^2 + (V^h)^2 + 2V^h V^l)}$

$= 2V^h + V^l \pm \sqrt{2(V^h)^2 + 2(V^l)^2}$

$(1-\theta)\varphi_1(n) = 2V^h + V^l - \sqrt{2(V^h)^2 + 2(V^l)^2}$ cannot be a solution, since $\hat{V} = (1-\theta)\varphi_1(n) > V^h + V^l$ and $2V^h + V^l - \sqrt{2(V^h)^2 + 2(V^l)^2} < V^h + V^l$.

Therefore, only $(1-\theta)\varphi_1(n) = 2V^h + V^l + \sqrt{2(V^h)^2 + 2(V^l)^2}$ might be valid.

It is indeed a maximum, since

$\frac{\partial^2 b}{\partial \theta^2} = \frac{\delta\varphi_1(n)}{(V^h - V^l)^2}[\varphi_1(n)(2V^h + V^l) - \varphi_1(n)(1-\theta)\varphi_1(n)]$

$= \frac{\delta(\varphi_1(n))^2}{(V^h - V^l)^2}[-\sqrt{2(V^h)^2 + 2(V^l)^2}] < 0$. Q.E.D.

It will generally be optimal with respect to a transfer to set a $\theta < 1$. With a uniform distribution, this is also superior with respect to equilibrium fertility n^* (remember that $\frac{dn^*}{d\theta} = -\frac{\frac{\partial U_2(n^*,0)}{\partial \theta} - \frac{\partial U_2(n^{**},0)}{\partial \theta} + \frac{\partial b}{\partial \theta}}{\frac{\partial FOC}{\partial n}}$ and the denominator is negative):

$\frac{\partial U_2(n,0)}{\partial\theta} = \delta\varphi_1(n) f_1(\hat{V}_1) \int\limits_{V^l}^{\hat{V}_2} V_2 f_2(V_2) dV_2 \leq 0$

$= -\delta\varphi_1(n) \frac{1}{(V^h - V^l)^2} \frac{(V^l)^2}{2}$

$\frac{\partial U_2(n^*,0)}{\partial\theta} - \frac{\partial U_2(n^{**},0)}{\partial\theta} = \frac{\delta (V^l)^2}{2(V^h - V^l)^2} [\varphi_1(n^{**}) - \varphi_1(n^*)] < 0$

Marriage versus Cohabitation

Until now, we did not explicitly analyze the couple´s decision whether it wants to get married or just live together. If $k = \phi = 0$ (1´s access to his children is also reduced after the termination of a relationship without marriage), this decision is irrelevant in our model (since we do not assume any utility of being married per se - which would translate to our setup in different distributions of the outside utilities V_i). Therefore, couples should only get married if positive values actually help them to increase their utilities via a better enforceability of transfer and therefore a higher n^*.

Abstracting from other advantages and roles a marriage has in our society, government should leave costs and alimony payments at substantial levels such that couples really have the choice between different regimes and can apply the one maximizing their utilities.

Now, assume that k and ϕ are positive in a way they help to induce cooperation in a way described above. Remember that the assumption of an exogenous and fixed cost distribution (driven by factors outside our model) is crucial for a potentially positive effect of these policies. If the couple was able to independently decide about it and also partition the time both stay at home somehow fairly, no redistribution would be necessary and the partners would never marry for positive values of k (since separation costs just reduce efficiency). Therefore, applying our model, mainly couples where partners still believe in traditional role models should marry - while more “modern” partners would be expected to live together in cohabitation. This difference should be smaller the more institutionally harmonized marriage and cohabitation get.

6.3 Interactions between policies

Until now, we have focused on isolated policy effects. Finally, we want to consider possible interactions and analyze whether and when they amplify or mitigate each other. One outcome is straightforward: If divorce costs k are too high, no redistribution is possible at all. Then, alimony payments or a low θ do not play any role since it is never optimal for anyone to file for divorce.

Deriving our further results, we focus on a uniform distribution of outside utilities and just concentrate on the impact on transfers b. This is made to simplify the analysis but will not restrict our results too much, since the transfers played the major role in determining whether (DE) constraints bind and whether they can be relaxed until.

If costs are not too high and 1´s access reduction os not too large, all policies enforce each other. This means that the positive impact of a sufficiently small cost level is stronger when 1 also has to suffer alimony payments and if his access to the children after a separation is slightly reduced. This has an important policy implication: Generally, it will be almost impossible to find the optimal levels of k, ϕ or θ for a legislation (where there is the danger that k is too high and θ too low). Then, it can already have substantial positive consequences on equilibrium fertility of all tools are applied only slightly but in combination. The reason is that the reaction functions are convex for sufficiently small values of $\hat{V}$.

Similar arguments can be found in the tax literature (see ....)

Proposition 13: Assume outside utilities are uniformly distributed. Then, $\frac{\partial^2 b}{\partial\phi\partial k} \geq 0$ and $\frac{\partial^2 b}{\partial\phi\partial\theta} \leq 0$. For small enough $\hat{V}$ ($\hat{V} \leq V^h + V^l$ is a sufficient condition), $\frac{\partial^2 b}{\partial k\partial\theta} \leq 0$.

Proof:

Recall that if $k > 0$, $\phi > 0$ and $\theta < 1$,

$\hat{V} = 2k + (1-\theta)\varphi_1(n)$

$\hat{V}_1 = k + \phi(w_{12} - w_{22}(n)) + (1-\theta)\varphi_1(n)$

$\hat{V}_2 = k - \phi(w_{12} - w_{22}(n))$

Furthermore,

$$b = \delta((F(\hat{V})-P)[2k+(1-\theta)\varphi_1(n)] - \int_{2V^l}^{\hat{V}}\int_{V^l}^{V^h} \tilde{V} f_2(\tilde{V}-V_1) f_1(V_1) dV_1 d\tilde{V} + \int_{V^l}^{\hat{V}_1}\int_{V^l}^{\hat{V}_2} (V_1+V_2) f_1(V_1) f_2(V_2) dV_1 dV_2)$$

$$\frac{\partial b}{\partial k} = \delta(2(F(\hat{V})-P) - \hat{V}_2 f_1(\hat{V}_1)F_2(\hat{V}_2) - \hat{V}_1 F_1(\hat{V}_1) f_2(\hat{V}_2) + \int_{V^l}^{\hat{V}_2} V_2 f_1(\hat{V}_1) f_2(V_2) dV_2 + \int_{V^l}^{\hat{V}_1} V_1 f_1(V_1) f_2(\hat{V}_2) dV_1)$$

$= \frac{\delta}{(V^h-V^l)^2}(2(V^h-V^l)^2 F(\hat{V}) - 2\hat{V}_1\hat{V}_2 + 3\hat{V}_1 V^l + 3\hat{V}_2 V^l - 3(V^l)^2 - \frac{(\hat{V}_2)^2}{2} - \frac{(\hat{V}_1)^2}{2})$

$\hat{V} \leq V^h + V^l$:

$\frac{\partial b}{\partial k} = \frac{\delta}{(V^h-V^l)^2}(\frac{(\hat{V}_2)^2}{2} + \frac{(\hat{V}_1)^2}{2} - \hat{V}V^l + (V^l)^2)$

$= \frac{\delta}{(V^h-V^l)^2}(k^2 + (\phi(w_{12} - w_{22}(n)))^2 + (k + \phi(w_{12} - w_{22}(n)))(1-\theta)\varphi_1(n) + \frac{((1-\theta)\varphi_1(n))^2}{2} - (2k + (1 - \theta)\varphi_1(n))V^l + (V^l)^2)$

$\frac{\partial^2 b}{\partial k \partial \phi} = \frac{\delta(w_{12}-w_{22}(n))}{(V^h-V^l)^2}(2\phi(w_{12} - w_{22}(n)) + (1-\theta)\varphi_1(n)) \geq 0$

$\frac{\partial^2 b}{\partial k \partial \theta} = -\frac{\delta\varphi_1(n)}{(V^h-V^l)^2}(k + \phi(w_{12} - w_{22}(n)) + (1-\theta)\varphi_1(n) - V^l) \leq 0$

$\hat{V} > V^h + V^l$:

$\frac{\partial b}{\partial k} = \frac{\delta}{(V^h-V^l)^2}(-2(V^h)^2 - 4V^h V^l - (V^l)^2 - \frac{3(\hat{V}_2)^2}{2} - \frac{3(\hat{V}_1)^2}{2} - 4\hat{V}_1\hat{V}_2 + \hat{V}(4V^h + 3V^l))$

$= \frac{\delta}{(V^h-V^l)^2}(-2(V^h)^2 - 4V^h V^l - (V^l)^2 - 7k^2 + (\phi(n))^2 - (1-\theta)\varphi_1(n)[7k - \phi(n))]$

$-\frac{3((1-\theta)\varphi_1(n))^2}{2} + (2k + (1-\theta)\varphi_1(n))(4V^h + 3V^l))$

$\frac{\partial^2 b}{\partial k \partial \phi} = \frac{\delta}{(V^h-V^l)^2}(2\phi(n) + (1-\theta)\varphi_1(n)) \geq 0$

$\frac{\partial^2 b}{\partial k \partial \theta} = \frac{\delta\varphi_1(n)}{(V^h-V^l)^2}(7k - \phi(n) + 3(1-\theta)\varphi_1(n) - (4V^h + 3V^l)) \lesseqgtr 0$

$$\frac{\partial b}{\partial \phi} = \delta(w_{12}-w_{22}(n))(-f_1(\hat{V}_1)F_2(\hat{V}_2)\hat{V}_2 + F_1(\hat{V}_1) f_2(\hat{V}_2)\hat{V}_1 + f_1(\hat{V}_1)\int_{V^l}^{\hat{V}_2} V_2 f_2(V_2) dV_2 - \int_{V^l}^{\hat{V}_1} V_1 f_1(V_1) f_2(\hat{V}_2) dV_1)$$

$= \frac{\delta(w_{12}-w_{22}(n))}{(V^h-V^l)^2}(2k\phi(w_{12} - w_{22}(n)) + \frac{((1-\theta)\varphi_1(n))^2}{2} + (k + \phi(w_{12} - w_{22}(n)))(1-\theta)\varphi_1(n) - V^l 2\phi(w_{12} - w_{22}(n)) - V^l(1-\theta)\varphi_1(n)))$

Then,

$\frac{\partial^2 b}{\partial \phi \partial \theta} = -\frac{\delta(w_{12}-w_{22}(n))\varphi_1(n)}{(V^h-V^l)^2}(k + \phi(w_{12} - w_{22}(n))) - (1-\theta)\varphi_1(n) + V^l) \leq 0$

Q.E.D.

7 Conclusions

We use the concept of relational contracts to analyse the importance of the enforceability of voluntary within-household transfers. Of special interest are the impacts of policies following a divorce on this enforceability and fertility.

Divorce costs might serve as a commitment device but must not be too high to allow for credible punishment threats which are needed to support a cooperative equilibrium. If costs associated with child rearing - monetary costs, time requirements and associated human capital reductions - are unequally distributed and transfers to the side bearing the majority of these costs are necessary to increase fertility, policies reducing the other´s utility after a separation might do a better job for transfer enforcement. We analyzed alimony payments and a reduction of the primary earner´s access to his children following a separation. While the first served as a partial insurance against the human capital loss, the second increased the punishment following no-cooperation. When making or adjusting divorce laws, a government therefore has to look at all consequences this might have. It further has to consider the willingness to distribute burdens associated with child rearing equally in a society. If traditional role models are still prevalent and if families only have insufficient access to child care facilities, potential positive effects of these policies are stronger (especially for highly educated women). But if males are willing to bear a sufficient share of the burdens associated with child rearing, these policies are more likely to decrease efficiency. Then, a separation should be constrained as little as possible.

Appendix

Proof of Proposition 1:

For general levels of α, the problem equals

$\max U_1 + U_2$

s.t.

A): $n_2^{**} < n_1^{**}$

(IC1) $U_1(n^*, 0) - b_1 \geq U_1(n^{**}, 0) - b_2$

(IC2) $U_2(n^*, 0) + b_1 \geq U_2(n^{**}, 0) + b_2$

B): $n_1^{**} < n_2^{**}$

(IC1) $U_1(n^*, 0) - b_2 \geq U_i(n^{**}, 0) - b_1$

(IC2) $U_2(n^*, 0) + b_2 \geq U_i(n^{**}, 0) + b_1$

(DE1): $b_1 \leq \delta\alpha[(P_1 + P_2)((1-\theta)\varphi_1(n) + 2k) - P_1 E[\tilde{V} \mid P_1] - P_2 E[\tilde{V} \mid P_2]]$

(DE2): $-b_2 \leq \delta(1-\alpha)[(P_1 + P_2)((1-\theta)\varphi_1(n) + 2k) - P_1 E[\tilde{V} \mid P_1] - P_2 E[\tilde{V} \mid P_2]]$

Then,

A)

$\frac{\partial U}{\partial b_1} = -\lambda_{IC1} + \lambda_{IC2} - \lambda_{DE1} = 0$

$\frac{\partial U}{\partial b_2} = \lambda_{IC1} - \lambda_{IC2} + \lambda_{DE2} = 0$

Assume λ_{DE1} is positive. Then, the same is true for λ_{IC2}, as well as λ_{DE2}.

n^* is determined by binding (IC2):

$w_{21} + \varphi_2(n^*) + \delta[w_{22}(n^*) + \varphi_2(n^*) - (1-P(n^*))(k - \phi(w_{12} - w_{22}(n^*))) + (1-F(\hat{V}(n^*)))E[V_2 \mid \tilde{V} \geq \hat{V}(n^*)]$

$+P_1(n^*)[E[V_2 \mid P_1(n^*)] + (1-\alpha)((1-\theta)\varphi_1(n^*) + 2k - E[V_1 + V_2 \mid P_1(n^*)])]]$

$+P_2(n^*)[E[V_2 \mid P_2(n^*)] + (1-\alpha)((1-\theta)\varphi_1(n^*) + 2k - E[V_1 + V_2 \mid P_2(n^*)])]]$

$+\delta\alpha[(P_1(n^*) + P_2(n^*))((1-\theta)\varphi_1(n^*) + 2k) - P_1(n^*)E[\tilde{V} \mid P_1(n^*)] - P_2(n^*)E[\tilde{V} \mid P_2(n^*)]]$

$= w_{21} + \varphi_2(n^{**}) + \delta[w_{22}(n^{**}) + \varphi_2(n^{**}) - (1-P(n^{**}))(k - \phi(w_{12} - w_{22}(n^{**}))) + (1-F(\hat{V}(n^{**})))E[V_2 \mid \tilde{V} \geq \hat{V}(n^{**})]$

$+P_1(n^{**})[E[V_2 \mid P_1(n^{**})] + (1-\alpha)((1-\theta)\varphi_1(n^{**}) + 2k - E[V_1 + V_2 \mid P_1(n^{**})])]]$

$+P_2(n^{**})[E[V_2 \mid P_2(n^{**})] + (1-\alpha)((1-\theta)\varphi_1(n^{**}) + 2k - E[V_1 + V_2 \mid P_2(n^{**})])]]$

$-\delta(1-\alpha)[(P_1(n^{**}) + P_2(n^{**}))((1-\theta)\varphi_1(n^{**}) + 2k) - P_1E[\tilde{V} \mid P_1(n^{**})] - P_2E[\tilde{V} \mid P_2(n^{**})]]$

where all parts containing α cancel out

All further steps follow. Q.E.D.

$U_2(n,0) = w_{21}(1-g(n)) + \varphi_2(n) - (1-\gamma)c(n) + \delta[w_{22}(n) + \varphi_2(n) - (1 - F_1(\hat{V}_1)F_2(\hat{V}_2))\hat{V}_2 + \mathrm{E}(V_2) - \int\limits_{V^l}^{\hat{V}_1}\int\limits_{V^l}^{\hat{V}_2} V_2 f_1(V_1) f_2(V_2) dV_1 dV_2]$

$\frac{\partial U_2(n,0)}{\partial k} = \delta[-(1 - F_1(\hat{V}_1)F_2(\hat{V}_2)) + f_1(\hat{V}_1)F_2(\hat{V}_2)\hat{V}_2 - \int\limits_{V^l}^{\hat{V}_2} V_2 f_1(\hat{V}_1) f_2(V_2) dV_2]$

$= \delta[-1 + \frac{1}{2(V^h-V^l)^2}[2\hat{V}_1\hat{V}_2 - 2\hat{V}_1V^l - 4V^l\hat{V}_2 + 3(V^l)^2 + (\hat{V}_2)^2]]$

$= \delta[-1 + \frac{1}{2(V^h-V^l)^2}[3k^2 - 6V^lk - 2k\phi(n) - (\phi(n))^2 + 2(1-\theta)\varphi_1(n)k - 2(1-\theta)\varphi_1(n)\phi(n) + 2V^l\phi(n) - 2V^l(1-\theta)\varphi_1(n) + 3(V^l)^2]]$ and

$\frac{\partial U_2(n^*,0)}{\partial k} - \frac{\partial U_2(n^{**},0)}{\partial k} = \frac{\delta}{2(V^h-V^l)^2}[2(k - V^l)\,((1-\theta)[\varphi_1(n^*) - \varphi_1(n^{**})] - [\phi(n^*) - \phi(n^{**})])$

$+(\phi(n^{**}))^2 - (\phi(n^*))^2 + 2(1-\theta)[\varphi_1(n^{**})\phi(n^{**}) - \varphi_1(n^*)\phi(n^*)]]$ can be positive or negative

$\frac{\partial U_2(n,0)}{\partial \phi} = \delta\hat{V}_{1\phi}[1 - F_1(\hat{V}_1)F_2(\hat{V}_2) + f_1(\hat{V}_1)F_2(\hat{V}_2)\hat{V}_2 - \int\limits_{V^l}^{\hat{V}_2} V_2 f_1(\hat{V}_1) f_2(V_2) dV_2]$

$= \delta(w_{12} - w_{22}(n))[1 - \frac{1}{(V^h-V^l)^2}(\hat{V}_1\hat{V}_2 - \hat{V}_1V^l + \frac{(V^l)^2}{2} - \frac{(\hat{V}_2)^2}{2})]$

$= \delta(w_{12} - w_{22}(n))[1 - \frac{1}{(V^h-V^l)^2}(\frac{k^2}{2} - \frac{3(\phi(n))^2}{2} + (1-\theta)\varphi_1(n)k - (1-\theta)\varphi_1(n)\phi(n) + k\phi(n) - kV^l - V^l\phi(n) - V^l(1-\theta)\varphi_1(n) + \frac{(V^l)^2}{2})]$

and $\frac{\partial U_2(n^*,0)}{\partial \phi} - \frac{\partial U_2(n^{**},0)}{\partial \phi}$

$= \delta[1 - \frac{1}{2(V^h-V^l)^2}(k - V^l)^2](w_{22}(n^{**}) - w_{22}(n^*))$

$-\frac{\delta(w_{12}-w_{22}(n^*))}{(V^h-V^l)^2}(-\frac{3(\phi(n^*))^2}{2} + (1-\theta)\varphi_1(n^*)k - (1-\theta)\varphi_1(n^*)\phi(n^*) + k\phi(n^*) - V^l\phi(n^*) - V^l(1-\theta)\varphi_1(n^*))]$

$+\frac{\delta(w_{12}-w_{22}(n^{**}))}{(V^h-V^l)^2}(-\frac{3(\phi(n^{**}))^2}{2} + (1-\theta)\varphi_1(n^{**})k - (1-\theta)\varphi_1(n^{**})\phi(n^{**}) + k\phi(n^{**}) - V^l\phi(n^{**}) - V^l(1-\theta)\varphi_1(n^{**}))]$

$\frac{\partial U_2(n,0)}{\partial \theta} = \delta\varphi_1(n)f_1(\hat{V}_1)[-F_2(\hat{V}_2)\hat{V}_2 + \int\limits_{V^l}^{\hat{V}_2} V_2 f_2(V_2) dV_2] < 0$

$\frac{\partial U_2(n,0)}{\partial \theta} = -\frac{1}{2}\frac{\delta\varphi_1(n)}{(V^h-V^l)^2}(\hat{V}_2 + V^l)^2$

$= -\frac{1}{2}\frac{\delta\varphi_1(n)}{(V^h-V^l)^2}(k - \phi(w_{12} - w_{22}(n)) + V^l)^2$ and

$\frac{\partial U_2(n^*,0)}{\partial \theta} - \frac{\partial U_2(n^{**},0)}{\partial \theta} = \frac{1}{2}\frac{\delta}{(V^h-V^l)^2}[\varphi_1(n^{**})(k - \phi(n^{**}) + V^l)^2 - \varphi_1(n^*)(k - \phi(n^*) + V^l)^2]$

θ and ϕ work into different directions.

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