

Ex ante Investment, Ex post Remedy, and Product Liability

Yongmin Chen[†] and Xinyu Hua[‡]

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Abstract. Low-quality products may cause consumer harm. A firm can reduce the probability of low quality through ex ante investment before sales, and can take remedy actions such as product recalls if it learns after sales that product quality is low. An increase in product liability increases the firm's incentive for ex post remedy. More ex post remedy, however, may have the "substitution effect" of reducing the firm's ex ante quality investment. On the other hand, more ex post remedy has the "output effect" of increasing consumer demand for the product, and hence greater return to ex ante investment. The trade-off between these two effects can lead to an inverted U-shaped relationship between ex ante investment and product liability. We find that the firm always prefers full liability whereas consumers might be better off with less than full liability. Full product liability tends to be socially optimal when the potential consumer loss from low quality is sufficiently high; otherwise partial liability can be socially optimal.

[†]University of Colorado; yongmin.chen@colorado.edu

[‡]Hong Kong University of Science and Technology; xyhua@ust.hk

1. INTRODUCTION

If the use of a firm's product results in consumer harm due to poor product quality, what should be the firm's liability? Under the rule of full liability, the firm is required to fully compensate the consumer for the harm; whereas under the rule of partial liability, the compensation to the consumer is lower than the consumer's loss. There have been substantial interests in the product liability issue in law and economics, primarily because liability rules can have important impacts on firm incentives and economic efficiency. One literature has focused on the effects of product liability on a firm's incentives for *ex ante* actions. Liability rules can affect a firm's precaution to ensure product safety (Simon, 1981) or its quality choice (Polinsky and Rogerson, 1983). In addition to product quality choice, product liability also affects a firm's incentive to disclose quality information through price and other devices (Daughety and Reinganum, 1995; 2008a; 2008b).¹

With a different focus, another literature has studied the effects of product liability on a firm's incentives for *ex post* actions. Welling (1991) shows that a firm makes recalls in order to build its reputation in the market, whereas Marino (1997) argues that mandatory recalls motivate firms to increase product safety before sales. Spier (2009) analyzes a firm's incentives to buyback unsafe products and finds that the firm offers a lower buyback price than socially desired. Hua (2009) compares strict liability to negligence rules when a firm's recall not only depends on its own costs/liability but also on consumers' return incentives.²

In practice, changes in product liability can affect both firms' *ex ante* investment action and *ex post* actions such as product recalls. For example, in 2008, the US Senate discussed a bill which would give the Consumer Product Safety Commission more power to collect and disclose allegations of injuries. The supporters claimed that the bill would encourage firms to design safer products. However, some other people argued that the bill would increase

¹Furthermore, in relation to product liability, a firm's choices between settlement and litigation and between confidential and open settlement may also affect its *ex-ante* quality choice (Daughety and Reinganum, 2005; 2006).

²For empirical work related to product recalls, see, for example, Jarrell and Peltzman (1985), Hartman (1987), Hoffer, Rruitt, and Reilly (1988), and Rupp and Taylor (2002).

firms' liabilities too much, which might cause more product recalls but also more violations of safety regulations (less safe products).³ These debates are related to the more general question: what is the relationship between firms' ex ante investment in product safety and ex post remedies such as product recalls? Would larger product liability motivate firms to increase or decrease ex ante investment before sales? Also, in the automobile industry, manufacturers could spend more time in safety design, which typically delays the marketing of new models. Alternatively, they could reduce the delay and ex ante investment, but collect further data and modify the design after sales.⁴

In this paper, we bridge and extend the two literatures by analyzing the potential effects of product liability on *both* ex ante quality investment and ex post remedy activities by the firm. We consider a setting where, before selling its product to heterogeneous consumers, a firm can make observable quality investment,⁵ and after sales, it learns about the realization of quality and can take remedy if the product is of low quality or is unsafe (so that there might be consumer harm).⁶ We investigate the interactions between the firm's ex post remedy and ex ante investment, how both activities depend on product liability, and the privately versus socially optimal liability rules. Our analysis identifies two potential effects of product liability on a firm's ex ante investment incentive: (i) *Substitution effect*: An increase in product liability increases the firm's incentive for ex post remedy, which, however, may negatively affect the firm's incentive for ex ante quality investment. (ii) *Output effect*: Higher product liability increases consumer demand for the product and leads to higher equilibrium output *given* ex ante investment (despite a higher expected marginal cost to the firm and a higher price), which increases the return to (incentive for) ex ante

³Business Insurance, March 17, 2008

⁴For example, automobile manufacturers have rushed to promote their new SUVs, though there were many warnings about safety issues. These firms later modified the safety design after Ford and Firestone recalled Explorers with tire problems. (Los Angeles Times, March 14, 2010)

⁵By assuming that ex ante investment is publicly observable, we abstract from considerations such as the signaling role of prices and the firm's incentives for information disclosure, which have been studied in the literature.

⁶We can thus think of a low-quality product in our setting as one with some defects, which could cause consumer harm.

quality investment. These two opposing effects can lead to an inverted U-shaped relationship between ex ante quality investment and product liability, with the highest investment sometimes obtained when there is less than full liability.

We further show that the firm always prefers full liability, whereas consumer surplus and social welfare may be higher under partial liability. The firm's preference for full liability arises in our model primarily because of the endogeneity of consumer demand and the commitment role of product liability. In the absence of reputation considerations and of a legal requirement from the liability law, the firm lacks the ex post incentive to take remedy (such as product recalls) should its product cause consumer harm due to low quality, which lowers consumer demand. Full product liability enables the firm to commit to taking such ex post remedy and internalizing the loss to consumers, which leads to higher consumer demand and to levels of ex post remedy and ex ante investment that are optimal for the firm. If only ex post remedy were feasible, consumers would also prefer full product liability. When ex ante investment is also possible, however, consumers may prefer less than full product liability, and the reason is more subtle. A lower liability may increase the firm's ex ante investment, due to the substitution effect, which improves product quality and possibly to higher equilibrium output. Because the firm is unable to appropriate all the consumer gains from higher product quality and higher output, its ex ante investment tends to be inefficiently low under full liability.⁷ Thus partial liability can result in higher consumer surplus and social welfare than full liability.⁸

In particular, holding all else constant, when the potential consumer loss from low quality is large enough, the output effect dominates the substitution effect on ex ante investment, in which case full liability motivates higher ex ante investment and more sales than partial liability. In this case, it is socially optimal to implement full product liability. In contrast,

⁷Given the equilibrium output, if $L = D$ (full liability), the firm's ex post remedy incentive coincides with that of the society.

⁸Other studies have also shown that full liability may not be socially optimal, but it is usually because of potential inefficient behavior (or negligence) from consumers (e.g., Brown, 1973; Diamond, 1974; Shavell, 1980). Our result is obtained without consumer moral hazard. We discuss the issue of consumer negligence in Section 5.

when the potential loss is at an intermediate level, the substitution effect can dominate the output effect, in which case partial liability leads to higher ex ante investment, and hence can result in higher consumer surplus and social welfare. Similarly, holding all else constant, full liability is socially optimal if ex post remedy cost is large enough or if the effectiveness of remedy is small enough, whereas partial liability can benefit consumers and increase social welfare when neither is too large or too small. These findings provide important policy implications on product liability. Moreover, they also have related policy implications on limited enforcement of warranty, consumer negligence, and punitive damage compensation.

The rest of the paper is organized as follows. Section 2 presents our model and discusses related applications. Section 3 examines how the liability rule affects the firm's ex post remedy choice and ex ante quality investment. Section 4 characterizes the profit-maximizing versus socially optimal liability rules. Section 5 discusses several related policies and possible extensions, including product warranty and return policies, consumer negligence, punitive damage compensation, and the robustness of our results. Section 6 concludes. All proofs are in the appendix.

2. THE MODEL

There are two periods: the ex ante period when a firm sells its product to heterogeneous consumers, and the ex post period when the firm learns additional information about product quality (or safety) and may take remedies such as product recalls or product upgrades.

In the ex ante period, before sales, the true product quality is uncertain, with θ representing the probability that the product is of high quality and $1 - \theta$ the probability that the product is of low quality. The firm can make investment to increase the high-quality probability, $\theta \in [0, \bar{\theta}]$. The investment costs, $k(\theta)$, is increasing and strictly convex in θ , with $k'(0) = 0$. We assume that there is always a non-trivial probability that the product is of low quality. That is, $\bar{\theta} < 1$. This assumption reflects the reality that the firm cannot perfectly control the product quality. Assume that $\bar{\theta}$ is close to one.

Consumers can observe the firm's investment level. However, before sales, neither the

firm nor consumers can observe the true quality of the product. These assumptions capture two features of ex ante quality investments for many products. While a firm can invest to have safer product designs, test product quality under different scenarios, or add precaution devices, there can still be uncertainty on product quality, which the firm may learn only after the product is used by (some) consumers. Also, in marketing their products, firms have incentives to disclose their quality investment through product certificates, user manuals, or product tests. Alternatively, government agencies often require firms to disclose their product quality before sales. These disclosures indirectly reflect firms' quality investment. For example, an automobile manufacturer has to disclose the outcomes of car tests and consumers can observe the safety designs.

The total mass of consumers is normalized to 1. Consumers' values for the product, when it is of high quality, are distributed according to c.d.f. $F(v)$, with a corresponding density function $f(v) > 0$ on support $v \in [0, \bar{v}]$. We impose the regularity condition that $f(v)$ is log-concave (i.e., $d^2 \ln f(v) / dv^2 \leq 0$).⁹ This condition implies that the hazard rate, $\lambda(v) \equiv \frac{f(v)}{1-F(v)}$, is non-decreasing (i.e., $\lambda'(v) \geq 0$). Define the inverse hazard rate as $H(v) \equiv \frac{1}{\lambda(v)}$. If the product is of low quality, it may reduce consumers' value or cause harm to consumers (independently) with probability γ , which is also uncertain ex ante and follows a distribution $G(\gamma)$, with a corresponding density function $g(\gamma) > 0$ on $[0, 1]$.¹⁰ When a consumer is harmed, her value is reduced by D .

After making the quality investment, the firm sets its price, and each consumer decides whether to purchase the product based on her realized v , her expectations about θ , γ and potential remedies by the firm if the product is of low quality. The firm's total sales is defined as Q .

In the ex post period, after sales, the firm privately learns the true quality of the product. If the product is of low quality, the firm also privately learns the realization of γ (how serious

⁹This assumption is satisfied by familiar distributions including uniform, exponential, normal, truncated t-distribution, and extreme-value distribution.

¹⁰As we mentioned earlier in the Introduction, we can think of a low-quality product as containing some defect. A more serious defect has a higher probability (γ) to cause consumer harm.

the defect is). The firm may then choose to take ex post remedy to improve the product quality. However, the ex post remedy is not fully effective and can only fix a proportion $\beta < 1$ of the sold product, with remedy costs $C\beta Q$. For example, in most product recalls, not all consumers are informed about the recall or will return the product (possibly due to inconvenience or return costs).¹¹ We assume that β is exogenously given here.¹²

If a consumer is harmed, the firm will give compensation L to the consumer according to product liability. The firm bears "partial liability" if $L < D$, "full liability" if $L = D$, and punitive damage compensation if $L > D$. Note that our framework can be applied in two different scenarios. Under the scenario of product safety, a product of low quality may harm consumers. After learning the potential harm, the firm can make product recalls to fix or replace the defected product. If a consumer is harmed, the firm bears liability L . Under an alternative scenario, a product of low quality may not deliver the expected value to consumers. For example, after selling a medical equipment or drug, a pharmaceutical firm may learn that this equipment or drug could not deliver the surgery effects claimed during sales. Then the firm can upgrade the equipment or replace with another new drug for the consumers. Similarly, after sales, a manufacturer of an industrial machine may find that the machine would depreciate more quickly than the claimed usage life. Then the firm can take remedy to extend the usage life. In both types of applications, L could be the liability determined by courts or given by the firm's warranty. In the basic model, we will focus on policies determined by courts or governments.¹³ In Section 5, we will discuss what

¹¹In practice, most recalls have low return rates. For example, the return rates of many product recalls were between 10% and 30% (New York Times, April 12, 2002). Even for automobiles, the return rates were estimated to be between 20% and 70% (The Philadelphia Inquirer, September 10, 2000).

¹²In Sections 4 and 5, we will also separately examine two policies that affect β : (1) Government may require the firm to keep better records of consumers' information during sales and monitor the firm's information disclosure when taking ex post remedies. Such policies can increase β . (2) Courts may adopt contributory negligence rule or set a lower standard in determining consumer negligence: the firm's liability may be reduced or denied if consumers do not comply with the firm's remedy.

¹³In practice, a firm's ex ante quality investment and ex post remedy may also involve reputation and other dynamic considerations. However, product liability still has important effects given the limitation of those market mechanisms. In this paper, we abstract away from these additional considerations, to focus on

happens if the firm can choose the warranty level itself.

3. EX ANTE INVESTMENT AND EX POST REMEDY

In this section, we first derive the expected costs of low quality to the firm and to the consumers, which determine the expected social cost. We then derive consumer demand and the firm's optimal output. One key observation is that the firm's optimal output can be expressed as a function of investment θ and the expected social cost per unit of output (Δ), which allows us to conveniently characterize two benchmarks where either ex ante investment is not considered (so that θ is given) or ex post remedy is not available. We then analyze our general case where the firm can take both ex ante investment and ex post remedy, and examine how the firm's optimal quality investment depends on the firm's liability L , the unit remedy cost C , the effectiveness of remedy β , and potential damage D .¹⁴

In the ex-post period, suppose that the firm finds product quality to be low. Then the firm will take remedy if and only if $C\beta Q < \gamma\beta LQ$, or equivalently,¹⁵ $\gamma > \frac{C}{L}$. That is, there will be ex post remedy only when it is sufficiently likely for consumers to be harmed. Therefore, from the ex ante point of view, given low product quality, the firm's expected ex post cost per unit of output is

$$x = \int_0^{\frac{C}{L}} \gamma L dG(\gamma) + \int_{\frac{C}{L}}^1 [\beta C + (1 - \beta)\gamma L] dG(\gamma), \quad (1)$$

whereas the expected ex post loss for any consumer using a low-quality product is

the role of product liability.

¹⁴Strictly speaking, we solve for the unique subgame perfect equilibrium of the game. When it will not cause confusion, we simply describe the equilibrium actions of the firm and the consumers as optimal actions.

¹⁵With probability $1 - \beta$, a consumer will not "participate" in the firm's ex post remedy. Under strict liability, the firm will then have the same expected ex post cost from such a consumer, $\gamma(1 - \beta)LQ$, whether or not it takes remedy actions. In Section 5, we discuss how our analysis would be affected if a consumer needs to bear responsibility for negligence.

$$y = \int_0^{\frac{C}{L}} \gamma(D - L)dG(\gamma) + \int_{\frac{C}{L}}^1 (1 - \beta)\gamma(D - L)dG(\gamma). \quad (2)$$

Thus, given low product quality, the expected ex post social cost per unit of output is given by

$$\Delta \equiv x + y = \int_0^{\frac{C}{L}} \gamma D dG(\gamma) + \int_{\frac{C}{L}}^1 [\beta C + (1 - \beta)\gamma D] dG(\gamma). \quad (3)$$

Notice that

$$\begin{aligned} \frac{d\Delta}{dL} &= D \frac{C}{L} g\left(\frac{C}{L}\right) \left(-\frac{C}{L^2}\right) + [\beta C + (1 - \beta)\frac{C}{L}D] g\left(\frac{C}{L}\right) \frac{C}{L^2} \\ &= g\left(\frac{C}{L}\right) \frac{C}{L^2} \beta C \left[1 - \frac{D}{L}\right]. \end{aligned} \quad (4)$$

Simple calculations lead to the following:

Lemma 1 (i) *Ex post*, $d\Delta/dL \leq 0$ if $L \leq D$, and it is socially efficient to have $L = D$.
(ii) $d\Delta/dD > 0$, $d\Delta/dC > 0$, and $d\Delta/d\beta < 0$.

Intuitively, if the firm bears full liability $L = D$, it will make the socially efficient decision on ex post remedy. If $L < D$, then there will be too few remedies relative to the socially desired; if $L > D$, there will be too many remedies.¹⁶ Note that $L > D$ includes punitive damage compensation. Although in theory punitive compensation can be implemented, in practice punitive compensation often cannot be too large perhaps because firms can seek for bankruptcy protection. For convenience, we restrict our analysis to the case with $L \leq D$.¹⁷

Given the anticipated ex post cost, a consumer will buy the product ex ante if and only if her value is large enough:

¹⁶In our model, there is no ex-post heterogeneity among consumers. This simplifies the ex-post efficiency analysis, allowing us to focus on the interaction between ex-ante investment and ex-post remedy. If consumers face heterogeneous harm or return costs, then the firm may not have the right incentives to make socially efficient remedy even when $L = D$ (Hua, 2009; and Spier, 2009).

¹⁷In Section 5, we will discuss the possibility of allowing $L > D$. For any given $L' > D$, as long as L' is not too large, there exists a certain level $L < D$ which leads to the same ex post social cost, that is, $\Delta(L) = \Delta(L')$. As it will become clear shortly, the firm's ex ante quality investment only depends on the expected social cost Δ , and consequently it is without loss of generality to assume $L \leq D$ as long as L cannot be much higher than D .

$$v - p - (1 - \theta)y \geq 0.$$

Correspondingly, the total demand for the firm's product is

$$Q = 1 - F(p + (1 - \theta)y),$$

or, the inverse demand is

$$p = F^{-1}(1 - Q) - (1 - \theta)y. \quad (5)$$

Given the ex ante quality investment θ , the firm chooses Q to maximize its profit

$$\begin{aligned} \Pi(\theta) &\equiv \max_{Q \leq 1} Q [p - (1 - \theta)x] = \max_{Q \leq 1} Q [F^{-1}(1 - Q) - (1 - \theta)(x + y)] \\ &= \max_{Q \leq 1} Q [F^{-1}(1 - Q) - (1 - \theta)\Delta]. \end{aligned} \quad (6)$$

Under the monotone hazard rate, it is easy to verify that the above objective function is concave. The first order condition is

$$F^{-1}(1 - Q) - \frac{Q}{f[F^{-1}(1 - Q)]} = (1 - \theta)\Delta. \quad (7)$$

Define

$$t = F^{-1}(1 - Q), \quad (8)$$

then t monotonically decreases in Q , and equation (7) becomes

$$t - \frac{1 - F(t)}{f(t)} = (1 - \theta)\Delta. \quad (9)$$

Define the firm's profit-maximizing output as $Q(\theta, \Delta)$. Given θ , t in equation (9) increases in Δ . That is, $Q(\theta, \Delta)$ decreases in Δ , the unit social cost from low product quality. Note that, for a consumer with $v = t$, she is indifferent between purchasing and not purchasing the product. In fact, since the expected social cost from low quality under output $Q(\theta, \Delta)$ is $(1 - \theta)Q\Delta$, the same as the reduction to the firm's maximum profit, given ex ante

investment θ , the firm fully internalizes the social cost from low quality when setting its price. Intuitively, a lower liability L reduces each consumer's expected utility from the product, which reduces consumer demand; but it also decreases the firm's expected marginal (ex post) cost from selling the product. These two effects happen to exactly offset each other. Hence, given θ , the firm bears the full social cost from low quality. Furthermore, by the envelope theorem

$$\frac{d\Pi(\theta)}{dL} = -Q(1-\theta) \frac{d\Delta}{dL} = 0$$

when $L = D$. That is, if there were no ex ante investment on quality (so that θ is given), the firm's profit would be maximized if there is full liability ($L = D$). We thus have:

Lemma 2 *Given ex ante quality investment θ , the expected cost from low quality is the same for the firm as for the society, and the profit-maximizing liability is $L = D$, same as the ex post socially efficient liability.*

The firm's ex ante quality investment is determined by the following problem:

$$\max_{\theta} Q(\theta, \Delta)[F^{-1}(1 - Q(\theta, \Delta)) - (1 - \theta)\Delta] - k(\theta).$$

From the envelop theorem, the optimal ex ante quality investment satisfies

$$Q(\theta, \Delta)\Delta - k'(\theta) = 0. \tag{10}$$

Define the firm's optimal ex ante investment as $\theta(\Delta)$. Note that, if the firm has no opportunity to take ex post remedy, the unit ex post social cost Δ from low quality would be fixed. The firm's liability would then be merely a transfer between the firm and consumers. If the firm bears larger liability, the firm's ex post cost would be increased by a certain amount; at the same time, consumers' willingness to pay would be increased by the same amount. The effects of these two changes on the firm's profit exactly cancel each other, as $\pi(\theta)$ in (6) is unchanged when Δ is unchanged. We thus have:

Lemma 3 *If the firm could not take ex post remedy, its optimal ex ante quality investment, the equilibrium output, and social welfare would be independent of ex post liability L .*

Now consider our general case where θ is determined endogenously when both ex ante investment *and* ex post remedy are possible. From condition (10), we see that a reduction in Δ can either increase or decrease θ , depending on how $Q(\theta, \Delta)\Delta$ varies with θ . Therefore, the firm's optimal ex ante quality investment may not be monotone in the expected social cost. In fact, the equilibrium θ is an inverted U-shaped function of Δ , as shown in the following proposition.

Proposition 1 *There exists a unique cut-off $\hat{\Delta} > 0$, such that the firm's optimal ex ante quality investment $\theta(\Delta)$ increases in Δ when $\Delta < \hat{\Delta}$ and decreases in Δ when $\Delta > \hat{\Delta}$.*

Note that, from condition (10), the unit social cost Δ from low quality affects θ both directly as a cost for each unit of output and indirectly through its effect on the output. As Δ increases, the direct cost effect tends to raise θ , whereas the indirect effect through a reduction in output tends to lower θ . When Δ is small, the direct effect dominates; but as Δ increases, the indirect effect eventually must dominate since θ will become zero when Δ is high enough. Therefore there will be some $\hat{\Delta}$ at which $d\theta/d\Delta = 0$, and the monotonicity of $H(\cdot)$ ensures that $\hat{\Delta}$ is unique. Thus the equilibrium θ is an inverted U-shaped function of Δ .¹⁸

Since from Lemma 1 $d\Delta/dL < 0$ for $L < D$, the firm's ex ante quality investment may also be non-monotonic in the firm's liability:

Corollary 1 *Consider the range $L \leq D$. There exists a unique cut-off $\hat{L} \in [0, D]$, such that the firm's optimal ex-ante quality investment $\theta(\Delta)$ increases in L when $L < \hat{L}$; and, if $\hat{L} < D$, $\theta(\Delta)$ decreases in L when $L > \hat{L}$. Furthermore, $(1 - \theta)\Delta$ decreases in L and $Q(\theta, \Delta)$ increases in L when $L < \hat{L}$.*

A change in product liability can potentially have two opposing effects on ex ante quality investment. On one hand, given output Q , there is a "substitution effect": more ex post remedy due to a larger L reduces Δ , which in turn leads to lower ex ante investment (so θ

¹⁸However, it is possible that the relevant Δ in a given problem is below $\hat{\Delta}$, in which case θ monotonically increases in Δ .

is lower). On the other hand, there is an "output effect": lower Δ leads to a larger quantity of sales Q , which in turn increases the firm's ex ante investment (so θ is higher). Suppose that there is some interior $\hat{L} \in (0, D)$ such that $\hat{\Delta} = \Delta(\hat{L})$. Then, when product liability (L) is small enough, the firm has low incentive to take ex post remedy and thus the unit ex post social cost from low quality is high. In this case, an increase in L lowers Δ , which in turn raises θ (since $d\theta/d\Delta < 0$ when $\Delta > \hat{\Delta}$, or $L < \hat{L}$). That is, the output effect from the increase in product liability dominates the substitution effect, so that the firm's ex ante quality investment is higher. In contrast, when the firm's liability is large ($L > \hat{L}$), the firm has high incentives to take ex post remedy, which reduces the unit ex post social costs ($\Delta < \hat{\Delta}$). In this case, the substitution effect dominates, so that an increase in L lowers Δ and also θ .¹⁹ Notice that \hat{L} can be at the corner, in which case θ monotonically increases in L if $\hat{L} = D$ and θ monotonically decreases in L when $\hat{L} = 0$. We shall later give examples where \hat{L} can be interior or at one of the corners.

Since the unit ex post remedy cost (C), the remedy effectiveness parameter (β), and the potential damage level (D) all affect the unit ex post social cost of low quality (Δ), the firm's ex ante quality investment may also not be monotone in these variables, similarly as with product liability. The result below follows straightforwardly from Lemma 1 and Proposition 1.

Corollary 2 *Given L , there exist unique cut-offs $C(L), \beta(L), D(L)$, such that the firm's optimal ex ante quality investment $\theta(\Delta)$ increases in C , β , and D respectively when $C < C(L)$, $\beta < \beta(L)$, or $D < D(L)$, whereas $\theta(\Delta)$ decreases in C , β , and D respectively when $C \geq C(L)$, $\beta \geq \beta(L)$, or $D \geq D(L)$.*

As potential damage D increase, the firm has more incentive to take ex ante quality investment as long as D is below the cut-off. However, when D becomes too large (above the cut-off), the firm would rather reduce its output to lower its expected ex post cost from low product quality, and the lower output reduces the incentive for ex ante investment. Similarly, increasing β , the effectiveness of ex post remedy, may not always increase the ex

¹⁹Note that if $\hat{L} < D$, an increase in L has ambiguous effects on $(1 - \theta)\Delta$ and $Q(\theta, \Delta)$ when $L > \hat{L}$.

ante quality investment or decrease the expected ex post social cost. Therefore, policies that increase β , such as requiring firms to keep better record of consumers' information or monitoring firms' disclosure of ex post remedy information, may not always increase (expected) product quality.

We now illustrate our findings in this section with the following examples. For both examples, suppose that γ is distributed uniformly on $[0, 1]$,

Example 1 *Suppose that consumers' value follows the uniform distribution on $[0, 1]$. In addition, $k(\theta) = \theta^2/2$ if $\theta \leq 0.9$ and $k(\theta) = M$ if $\theta > 0.9$, where M is sufficiently large (for example, $M \geq D$) so that $\bar{\theta} = 0.9$. Let $C = 0.5$ and $\beta = 1$. We have:*

(1) *If $D \leq 1.172$, then θ decreases in L for any $L \leq D$.*

(2) *If $D > 1.172$: then there exists $\hat{L} < D$ such that θ strictly increases in L for $L < \hat{L}$ and strictly decreases in L for $L > \hat{L}$. For instance, when $D = 2$, $\hat{L} = 0.787$.*

In Example 1, the equilibrium θ is an inverted U-shaped function of $L \leq D$ when D is high enough. Example 2 below illustrates that the equilibrium θ may only weakly increase or decrease in L .

Example 2 *Suppose that consumers' value follows the uniform distribution on $[0, 1]$. In addition, $k(\theta) = \theta^2/8$ if $\theta \leq 0.9$ and $k(\theta) = M$ if $\theta > 0.9$, where M is sufficiently large (for example, $M \geq D$) so that $\bar{\theta} = 0.9$. Let $C = 0.5$ and $\beta = 1$.*

(1) *If $D \leq 0.944$, then θ decreases in L for any $L \leq D$ (i.e. the substitution effect dominates).*

(2) *If $0.944 < D \leq 4.464$: then there exists $\hat{L} < D$ such that θ increases in L for $L < \hat{L}$ and decreases in L for $L > \hat{L}$.*

(3) *If $D \geq 4.464$: then θ weakly increases in L for any $L \leq D$ (i.e., the output effect dominates).²⁰*

In sum, this section has shown that there may be a non-monotone relationship between the firm's ex ante quality investment and ex post remedy incentives. Increasing the firm's

²⁰If D is too large, for any $L \leq D$, the firm would not product at all and, correspondingly, $\theta = 0$.

liability may not always increase the ex ante quality investment or reduce the expected ex post social cost. This observation will have important implications for determining the socially optimal liability policy, which we next turn to.

4. PRIVATE V.S. SOCIAL INCENTIVES FOR PRODUCT LIABILITY

In this section, we examine how product liability L affects firm profit, consumer surplus, and social welfare. These discussions will shed light on whether it is socially optimal to impose full liability or partial liability for the firm. As discussed in Section 3, for ease of exposition we assume $L \leq D$.

Define social welfare as $W = \Pi + U$, where Π is the firm's expected profit and U is the aggregate consumer surplus. From the analysis in Section 3, we have

$$\Pi = \max_{\theta} Q(\theta, \Delta) [F^{-1}(1 - Q(\theta, \Delta)) - (1 - \theta)\Delta] - k(\theta),$$

$$U = \int_t^{\bar{v}} (v - t) dF(v) = \int_{F^{-1}(1-Q(\theta, \Delta))}^{\bar{v}} [v - F^{-1}(1 - Q(\theta, \Delta))] dF(v).$$

Note that consumer surplus only depends on total output $Q(\theta, \Delta)$. Intuitively, the marginal consumer with $v = t$ is indifferent between purchasing and not purchasing the product. From the ex ante point of view, all consumers face the same expected harm if the product quality is low. When there are more sales, there is more information rent for consumers.

Proposition 2 *Firm profit is maximized under full liability $L = D$.*

Thus, ex ante, the firm always prefers full liability, although ex post the firm would prefer no liability. Intuitively, if the firm bears only partial liability, it may not take ex post remedy as socially desired. The unit ex post social cost from low quality would be larger. As discussed in Section 3, the firm fully internalizes the social cost from low quality. Full liability allows the firm to create the intertemporal commitment to take efficient ex post remedy that minimizes expected unit social cost from low quality.

However, given that the firm has market power, to minimize unit ex post social cost of low quality, the firm may reduce its output and therefore consumer surplus would be decreased. Note that the firm's output depends on its ex ante quality investment. Some of our results below will use the following condition.

Condition T1: $k'''(\theta) \leq 0$ and $\sup_{\Delta} \theta(\Delta) > 1/2$.

Condition **T1** is satisfied as long as the firm's ex-ante investment costs are not too large, and the product is more likely to have high quality than to have low quality in equilibrium. For example, it is satisfied if consumers' value follows the uniform distribution and $k(\theta) = a\theta^2$, for any $a \leq \frac{1}{8}$.

The following lemma shows that the firm's optimal quantity of sales may not be monotone in the unit social cost of low quality.

Lemma 4 $Q(\theta, \Delta)$ and U decrease in Δ when $\Delta \geq \widehat{\Delta}$; but they can increase in Δ when $\Delta < \widehat{\Delta}$. In particular, if condition **T1** holds, then there exists a cut-off $\widetilde{\Delta} < \widehat{\Delta}$ such that $Q(\theta, \Delta)$ and U increase in Δ if $\widetilde{\Delta} \leq \Delta < \widehat{\Delta}$.

Intuitively, when Δ is large enough, as discussed in Section 3, the output effect dominates the substitution effect: when Δ increases further, the firm takes less ex ante quality investment. Correspondingly, the expected unit social cost $(1 - \theta)\Delta$ becomes larger and reduces output even further. On the other hand, when Δ is small enough, the substitution effect dominates the output effect: as Δ increases, the firm takes more ex ante quality investment. If such ex ante investment increases more quickly than the increase in Δ , then the expected unit social cost $(1 - \theta)\Delta$ becomes smaller and output increases.

Proposition 3 Suppose that condition **T1** holds. (1) Given C and β , there exist two cut-offs $\widetilde{D} < \widehat{D}$: If $D \geq \widehat{D}$, consumer surplus is maximized under full liability; if $\widetilde{D} \leq D < \widehat{D}$, consumer surplus is maximized under partial liability. (2) Given D and β , there exist two cut-offs $\widetilde{C} < \widehat{C}$: If $C \geq \widehat{C}$, consumer surplus is maximized under full liability; if $\widetilde{C} \leq C < \widehat{C}$, consumer surplus is maximized under partial liability. (3) Given D and C ,

there exist two cut-offs $\tilde{\beta} > \hat{\beta}$: If $\beta \leq \hat{\beta}$, consumer surplus is maximized under full liability; if $\hat{\beta} < \beta \leq \tilde{\beta}$, consumer surplus is maximized under partial liability.

The above results provide clear policy implications. In order to increase consumer surplus, it may not be optimal to impose full liability for the firm. Under full liability, the firm would take ex post remedy to minimize the unit ex-post social cost from low quality. Anticipating this, as long as the potential damage level and remedy cost are not too large, and ex post remedy is effective enough, the firm might reduce output ex ante, which in turn would reduce consumer surplus.

The previous analysis suggests that the firm and consumers may have conflicting incentives for full product liability. Therefore, full liability may not maximize social welfare. Suppose that $H'(v)$ is bounded for any $v \in [0, \bar{v}]$. Define $h = \max_v [-H'(v)]$.

Condition T2: $k'''(\theta) \leq 0$ and $\sup_{\Delta} \theta(\Delta) > \frac{2+h}{3+h}$.

For many familiar log-concave distributions, $H'(v)$ is bounded for any $v \in [0, \bar{v}]$. The above condition is satisfied as long as the firm's ex ante investment costs are not too large or the probability of high quality is not always lower than $\frac{2+h}{3+h}$ in equilibrium. For example, **T2** is satisfied if consumers' value follows the uniform distribution and $k(\theta) = a\theta^2$, for any $a \leq \frac{1}{8}$.

The following proposition shows that partial liability ($L < D$) can be socially optimal.

Proposition 4 *Suppose that condition T2 holds. (1) Given C and β , there exist a cut-off $D' \in [\tilde{D}, \hat{D}]$: If $D \geq \hat{D}$, social welfare is maximized under full liability; if $D' \leq D < \hat{D}$, social welfare is maximized under partial liability. (2) Given D and β , there exists a cut-off $C' \in [\tilde{C}, \hat{C}]$: If $C \geq \hat{C}$, social welfare is maximized under full liability; if $C' \leq C < \hat{C}$, social welfare is maximized under partial liability. (3) Given D and C , there exists a cut-off $\beta' \in (\hat{\beta}, \tilde{\beta})$: If $\beta \leq \hat{\beta}$, social welfare is maximized under full liability; if $\hat{\beta} < \beta \leq \beta'$, social welfare is maximized under partial liability.*

We illustrate the findings with examples which are continuations of Example 2 in Section 3:

Example 3 Suppose that consumers' value follows the uniform distribution on $[0, 1]$. In addition, $k(\theta) = \theta^2/8$ if $\theta \leq 0.9$ and $k(\theta) = M$ if $\theta > 0.9$, where M is sufficiently large (for example, $M \geq D$). That is, $\bar{\theta} = 0.9$. Assume that γ also follows the uniform distribution on $[0, 1]$. Then we have the following relationship between social welfare and the unit expected social costs from low quality:

$$W \begin{cases} \text{decreases in } \Delta & \text{if } \Delta \in [0, 0.399] \\ \text{increases in } \Delta & \text{if } \Delta \in (0.399, 0.472] \\ \text{decreases in } \Delta & \text{if } \Delta \in (0.472, \infty) \end{cases} .$$

The above relationship provides the base for the following results on how damage level D , remedy costs C , or effectiveness of remedy β affects the socially optimal liability. Recall that

$$\Delta \equiv \int_0^{\frac{C}{L}} \gamma D dG(\gamma) + \int_{\frac{C}{L}}^1 [\beta C + (1 - \beta)\gamma D] dG(\gamma).$$

First, given β , Figure 1 illustrates how the socially optimal liability depends on the damage level D and the remedy costs C , where the red curve is defined by $C = D - D\sqrt{1 - \frac{0.944}{D}}$, the green curve is defined by $C = D - D\sqrt{1 - \frac{0.614}{D}}$, and the purple curve is from simulation such that no liability and full liability lead to the same social welfare for $D < 0.944$. Notice that full liability $L = D$ is more efficient in Range F where either D or C is sufficiently high; partial liability $L \in (0, D)$ is more efficient in Range P where both D and C are intermediate; and it is more efficient to impose no liability for the firm in Range N. Also, in this particular example, full liability is more efficient in Range F* where C is sufficiently small. These results are consistent with the general predictions in Proposition 4.

In particular, let $C = 0.5$ and $\beta = 1$. Then full liability is more efficient when $D \leq 0.870$ or $D \geq 4.464$; partial liability $L \in (0, D)$ is more efficient when the damage level is intermediate ($0.944 \leq D \leq 4.464$); zero liability is more efficient when $0.870 \leq D \leq 0.944$.

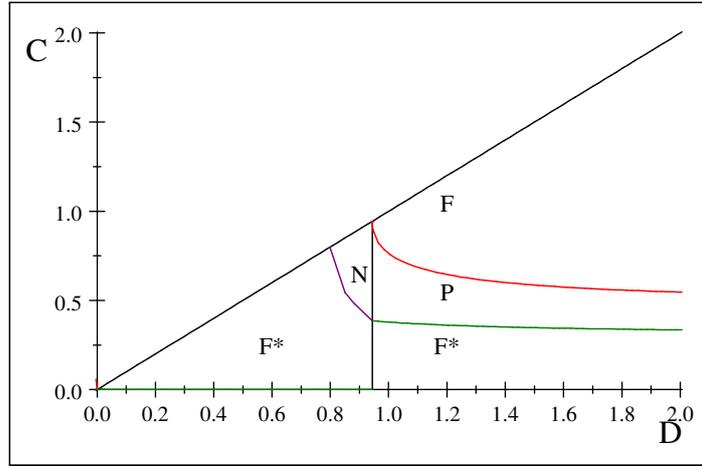


Figure 1: Optimal Liability (Given $\beta = 1$)

Second, given D , Figure 2 illustrates how the socially optimal liability depends on the effectiveness (β) and the unit cost (C) of ex-post remedy, where the red curve is defined by $\beta = \frac{0.056}{(C-1)^2}$; the green curve is defined by $\beta = \frac{0.385}{(C-1)^2}$. As shown in Figure 2, partial liability is more efficient when β or C is intermediate; otherwise full liability is more efficient.

In particular, let $D = 1$ and $C = 0.2$. Then full liability is more efficient when $\beta > 0.602$ or $\beta < 0.088$; otherwise partial liability is more efficient. Furthermore, let $D = 1$ and $\beta = 0.5$. Then full liability is more efficient when $C > 0.665$ or $C < 0.122$; otherwise partial liability is more efficient.

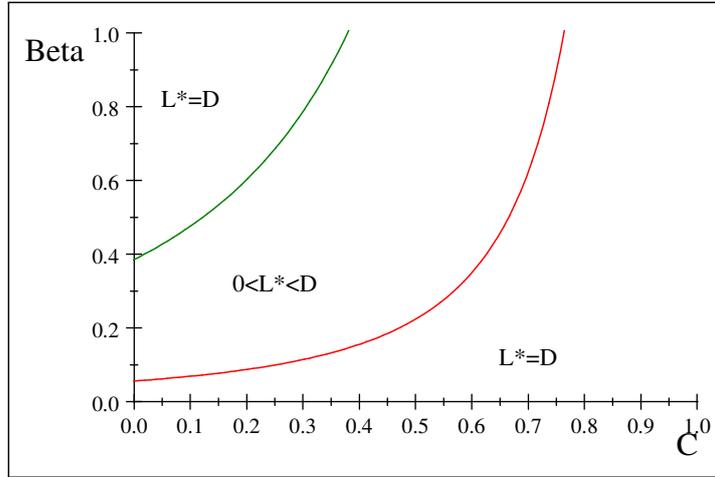


Figure 2: Optimal Liability (Given $D = 1$)

Intuitively, when the potential damage is large enough, the ex-post remedy is too costly or not very effective, the unit ex post social cost from low quality is large enough. In this scenario, the output effect dominates the substitution effect of larger liability. Therefore, when the firm bears larger liability, both output Q and the ex ante investment θ would increase, which increases overall social welfare. That is, full liability is socially optimal. In contrast, when the potential damage, the remedy cost or the effectiveness of remedy is at intermediate level, the substitution effect dominates the output effect. Moreover, when the firm bears larger liability, the output and the ex-ante investment may decrease, which may reduce social welfare. Therefore, partial liability can be more efficient than full liability.

Proposition 4 and the above illustrations provide several interesting policy implications. A policy or liability rule should consider its effects on output and address the potential non-monotone relationship between ex ante investments and ex post remedies. Although a larger product liability can motivate the firm to take more ex post remedies, it may reduce ex ante investments and output. As shown in the above numeric examples, for a range of parameter values, partial liability is more efficient than full liability. In practice, the liability rule should depend on the potential damage level and ex post remedy costs. For products with large consumer damage (perhaps such as cars) relative to ex post remedy

costs, full liability is more efficient than partial liability; for products with intermediate damage compared to ex post remedy costs, partial liability could be more efficient.

Government agencies such as the US Food and Drug Administration have required firms to keep better record of consumer information or monitor firms' disclosure of ex post remedies, in order to increase the effectiveness of remedies. While we find such policies can often increase consumer surplus and social welfare by increasing β or potentially reducing C , it is intriguing that a higher β or a lower C sometimes can reduce firms' ex ante investment and output, which may reduce social welfare.

5. DISCUSSIONS

5.1 Product Warranty

The previous analysis assumes that the firm cannot commit to making ex post remedy if it bears no liability. One way for the firm to create such inter-temporal commitment is to offer warranties. The existing literature on warranty mainly views warranty as a signaling device for a firm's unobservable quality choice or quality investment (e.g. Grossman, 1981; Cooper and Ross, 1985; Lutz, 1989).²¹ It typically considers a firm's expected compensation to consumers when a quality problem arises, but does not address the possibility for the firm to take ex post remedies before consumers are harmed.

Suppose that the firm could issue a warranty which specifies compensation L to a consumer when she has utility loss due to low quality. The firm can take ex post remedy after sales. As shown in Section 4, the firm prefers full warranty $L = D$, which provides commitment that the firm would take efficient ex post remedy. Such a full warranty, if perfectly enforced, maximizes the firm's profits.

In practice, firms often offer limited warranty, because a full warranty may unduly reduce consumer care in using the product. Our findings in Section 4 suggest that a limited warranty can sometimes be what consumers prefer, because it can lead to higher ex ante

²¹Similarly, during sales, firms may offer return policies as signaling devices (Moorthy and Srinivasan, 1995). Return policies may also enhance risk sharing between a firm and its consumers (Che, 1996).

quality investment and output, which benefits consumers.

Another potential welfare loss from full warranty is that consumers may not comply with the firm's ex post remedies. On one hand, with full warranty, consumers' negligence would reduce the effectiveness of the firm's ex post remedies; on the other hand, it may induce the firm to make more ex ante investment. The next subsection provides more general discussions on whether consumers' negligence should be considered in determining the firm's liability.

5.2 Effectiveness of ex post Remedy and Consumer Negligence

Our main model assumes that β is exogenously given. In practice, the effects of ex post remedies may hinge on consumers' awareness or incentives to comply with the firm's ex post remedies. Consumers' incentives can be affected by liability rules. For example, courts may use either strict liability or negligence rules.²² Under strict liability, the firm bears the same liability L no matter whether it has taken ex post remedies or not. Under negligence rules, the firm's liability may be reduced if the firm has taken ex post remedies and consumers get informed but do not comply.²³ Under negligence rules, consumers have more incentives to comply with the firm's remedies such as recalls.

Assume that, under strict liability as analyzed in Section 3, the ex post remedy can fix a proportion β of the sold product; under the negligence rule, the ex post remedy can fix a proportion $\beta_N > \beta$ of the sold product.

Under the negligence rule, the firm no longer bears liability if consumers do not comply with ex post remedies. Hence, the firm will take ex post remedies if and only if $C\beta_N Q < \gamma LQ$. Given low product quality, the expected ex post social costs per unit of the product are defined as

²²There is a large literature comparing strict liability and negligence rules. For examples, see Brown (1973), Green (1976), Shavell (1980), Rubinfeld (1987), Emons (1990), Emons and Sobel (1991), Bar-Gill and Ben-Shahar (2003),

²³The choice between strict liability and negligence rules may affect the firm's incentives to take ex-post remedies, for example, as discussed in Hua (2009).

$$\Delta_N = \int^{\frac{C\beta_N}{L}} \gamma D dG(\gamma) + \int_{\frac{C\beta_N}{L}} [\beta_N C + (1 - \beta_N)\gamma D] dG(\gamma).$$

In contrast, as shown in Section 3, under strict liability, the firm will take ex post remedies if and only if $C\beta Q < \gamma\beta LQ$. Given low product quality, the expected ex post social costs per unit of the product are defined as

$$\Delta_S = \int^{\frac{C}{L}} \gamma D dG(\gamma) + \int_{\frac{C}{L}} [\beta C + (1 - \beta)\gamma D] dG(\gamma).$$

When the legal system is changed from strict liability to negligence rules, the firm is more likely to take ex post remedies, since $\frac{C\beta_N}{L} < \frac{C}{L}$. However, this change may not necessarily increase ex post efficiency. To see this, suppose that $\beta_N D < L$. Then $\frac{C\beta_N}{L} < \frac{C}{D}$, that is, the negligence rule leads the firm to take too many ex post remedies relative to socially desired; but the ex post remedies become more effective. In this case, negligence rules may lead to either higher or lower ex post social costs than strict liability.

If $\beta_N D \geq L$, then $\frac{C}{D} \leq \frac{C\beta_N}{L} < \frac{C}{L}$. That is, the negligence rule leads the firm to take more efficient ex post remedies and furthermore, the ex post remedies become more effective. In this case, ex post social costs are reduced, i.e., $\Delta_N < \Delta_S$. However, according to Proposition 4, if condition **T2** holds, for any $\Delta \in (\Delta', \widehat{\Delta})$, reducing ex post social costs would decrease social welfare. Therefore, if $\Delta' < \Delta_N < \Delta_S < \widehat{\Delta}$, strict liability is socially more efficient. The full-fledged comparison between strict liability and negligence rules is ambiguous in this general framework. However, this section illustrates that the choice between strict liability and negligence rules should also take firms' ex ante quality investment and ex ante output into consideration. If negligence rules increase ex post efficiency but reduce firms' ex ante quality investment and ex ante output significantly, then it may be more efficient to impose strict liability or set a higher standard for evidence in determining consumers' negligence.

5.3 Punitive Damage Compensation

Our main analysis in Sections 3 and 4 focuses on the scenario with $L \leq D$. In practice, sometimes courts may impose punitive damage compensation so that $L > D$. If there is

punitive compensation, as shown in Lemma 1, the firm would take ex post remedy more frequently than socially desired. In addition, the following proposition shows that, as long as punitive damage compensation cannot be too large (perhaps because firms can resort to bankruptcy protection), partial liability $L < D$ can lead to the same social welfare as punitive damage compensation.

Proposition 5 *Suppose that condition T2 holds. (1) If $\int_0^1 (C - \gamma D) dG(\gamma) \leq 0$, then for any punitive damage compensation $L > D$, there exists $L' < D$ which leads to the same social efficiency. (2) If $\int_0^1 (C - \gamma D) dG(\gamma) > 0$, then there exists $\bar{L} > D$ such that punitive damage compensation $L > \bar{L}$ can be more efficient than $L \leq D$.*

Intuitively, the firm's liability only affects the level of unit ex post social cost Δ , but does not directly affect the firm's optimal output and the ex ante quality investment. Therefore, the firm's liability only influences social welfare through the change of Δ . For punitive damage compensation $L \in (D, \bar{L}]$, we can always find a partial liability level which leads to the same ex post social cost and correspondingly, the same social welfare. Furthermore, if punitive damage compensation is very large ($L > \bar{L}$), it causes the firm to take more ex post remedy than socially desired. However, given intermediate damage levels, it may increase the firm's ex ante quality investment and the output level. Therefore, very larger punitive damage compensation may increase social welfare in this case. In practice, it is often difficult to impose very large punitive damage compensation, especially when firms can seek bankruptcy protection or the legal enforcement is not perfect.

5.4 Alternative Quality Investment Technology

In the main model, the firm's ex ante quality investment θ affects the probability for the product to have low quality. Given the product of low quality, the likelihood for consumers to be harmed, γ , is uncertain but not contingent on θ . For example, before sales, the firm may take R&D investment to have safer product design. However, there is still non-trivial probability for the product to have design defect. Given the design defect, the likelihood for consumers to be harmed depends on the nature of the defect as well as consumers' usage.

The results on the relationship between ex ante quality investment and ex post remedies can still hold in an alternative scenario where the firm's ex ante investment θ affects the distribution of γ .

In particular, suppose that the firm can take ex ante investment θ and the likelihood for consumers to be harmed, γ , follows a distribution $G(\gamma | \theta)$. After sales, the firm privately learns the realization of γ and then decides whether to take ex post remedy. Assume $G'_\theta(\gamma | \theta) \geq 0$. That is, $G(\gamma | \theta)$ first-order stochastically dominates $G(\gamma | \theta')$ for any $\theta < \theta'$. Without loss of generalization, we focus on the scenario with $L \leq D$. It is easy to verify that the firm's optimal ex ante quality investment satisfies

$$Q(\Delta(\theta, L))\Delta(\theta, L) - k'(\theta) = 0. \tag{11}$$

Similar to the analysis in Section 3, when the firm's liability L increases, there are again two conflicting effects on the ex ante quality investment. On one hand, given the output Q , there is a substitution effect between ex ante quality investment and ex post remedy. On the other hand, there is an output effect: lower $\Delta(\theta, L)$ may lead to a larger output Q , which in turn increases the firm's quality investment θ . Therefore, the firm's optimal ex ante investment may not be monotone in its liability.

6. CONCLUSION

This paper has studied how product liability affects product quality/safety as well as consumer and social welfare. We find that the interactions between a firm's ex post remedy for low product quality and its incentive for ex ante quality investment have important implications for the effects of product liability. Higher liability increases ex post remedy activities, and this can in turn reduce the incentive for ex ante investment. On the other hand, higher liability increases consumer demand for the product, which in turn increases the firm's incentive for ex ante quality investment. The presence of these two opposing effects, the substitution effect and the output effect, implies that the ex ante quality investment may not monotonically increase in product liability—the relationship may be an

inverted U-shaped curve. While full liability maximizes profit since it allows the firm to make the intertemporal commitment for ex post remedy, it may not be optimal for consumers when it reduces ex ante quality investment and output by the firm. Full product liability tends to be socially optimal when the potential consumer loss from low quality is sufficiently high; otherwise partial liability can be socially optimal.

There are several directions for further research. First, given the trade-offs between the substitution effect and the output effect, firms' ex ante investment and ex post remedy effort could be either substitutes or complements. It would be useful to empirically verify which effect would dominate under different liability rules. Second, competition among different firms may affect their ex ante investment and ex post remedies such as recalls. It would be desirable to study the potential trade-offs involved and the implications for product liability rules. Third, firms may continue their sales after product recalls. It would be interesting to explore how reputation concern interacts with liability rules and how they jointly affect firms' ex ante investments and ex post remedies. Finally, our current paper assumes that consumers can observe the firm's ex ante investment. If that is not the case, the firm's pricing may signal its quality investment as shown in the literature. Product liability may then have additional implications for the interaction between ex ante investment and ex post remedies.

APPENDIX

Proof of Proposition 1: Given θ , the firm's optimal output $Q(\theta, \Delta)$ satisfies (9), from which we obtain

$$Q'_\Delta(\theta, \Delta) = \frac{\partial Q}{\partial t} \frac{\partial t}{\partial \Delta} = -f(t) \frac{1 - \theta}{1 - H'(t)}.$$

The optimal θ satisfies either $\theta = \bar{\theta}$ or condition (10).

First, when $\Delta = 0$, condition (10) implies that $\theta(\Delta) = 0$. (If there is no ex post social cost, the firm would not make any ex ante investment.) When Δ goes to infinity, condition (9) and (8) imply $Q(\theta, \Delta) = 0$, which also implies $\theta(\Delta) = 0$. Therefore $d\theta/d\Delta > 0$ when Δ is sufficiently small but θ eventually decreases in Δ when Δ is sufficiently large. It suffices

to consider two cases as follows.

(1) Consider the case with interior solution: There exists some $\widehat{\Delta}$ at which $d\theta/d\Delta = 0$: From (10), we have

$$\frac{d\theta}{d\Delta} = -\frac{Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta)}{\Delta Q'_\theta(\theta, \Delta) - k''(\theta)}.$$

Note that the second order condition

$$\Delta Q'_\theta(\theta, \Delta) - k''(\theta) \leq 0$$

for interior solutions $\theta(\Delta) \in (0, \bar{\theta})$. Thus, the sign of $\frac{d\theta}{d\Delta}$ is determined by that of

$$Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta).$$

In particular, $Q(\theta, \Delta) = 1 - F(t)$ by definition. Thus,

$$Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta) = 1 - F(t) - \Delta f(t) \frac{1 - \theta}{1 - H'(t)}.$$

Equivalently,

$$\frac{Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta)}{f(t)} = \frac{1 - F(t)}{f(t)} - \Delta \frac{1 - \theta}{1 - H'(t)} = H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)}.$$

Therefore, the sign of $\frac{d\theta}{d\Delta}$ is the same as that of $H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)}$. Since by assumption $f(v)$ is log-concave, $H'(t) \leq 0$ and $H''(t) \geq 0$.

Suppose that, given a particular Δ , $\frac{d\theta}{d\Delta} \leq 0$, or equivalently, $H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)} \leq 0$. When Δ increases marginally, $\theta(\Delta)$ cannot increase. This implies that $(1 - \theta)\Delta$ would increase. According to condition (9), if $(1 - \theta)\Delta$ increases, t must increase as well. Correspondingly, $H(t)$ and $1 - H'(t)$ would decrease. Hence $H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)}$ would decrease and become even more negative. Therefore, if we start at some Δ such that $\frac{d\theta}{d\Delta} \leq 0$, for any larger Δ , $\frac{d\theta}{d\Delta} < 0$. This implies that there must be a unique cut-off $\widehat{\Delta}$ such that $\frac{d\theta}{d\Delta} \geq 0$ when $\Delta \leq \widehat{\Delta}$.

(2) Now consider the case with corner solution or discontinuity in $\theta(\Delta)$: that is, there exists some $\widehat{\Delta}$ at which $\theta = \bar{\theta}$ but $d\theta/d\Delta > 0$. Since $\theta = \bar{\theta}$, increasing Δ marginally from $\widehat{\Delta}$ cannot increase the firm's ex-ante investment θ further. Therefore, according to condition (9), t must increase, or equivalently, $Q(\bar{\theta}, \Delta)$ must decrease. When Δ increases further,

$Q(\bar{\theta}, \Delta)$ decreases and gets closer to zero, so that $Q(\bar{\theta}, \Delta)\Delta - k'(\bar{\theta})$ eventually becomes negative, that is, $\frac{d\theta}{d\Delta} \leq 0$. The rest of the proof is similar to that in part (1). Q.E.D.

Proof of Corollary 1: According to Lemma 1, Δ decreases in L given $L \leq D$. Let \hat{L} be such that

$$\hat{L} = \begin{cases} \hat{L} & \text{if } \hat{\Delta} = \Delta(\hat{L}) \text{ for } \hat{L} \in (0, D) \\ D & \text{if } \Delta(D) \geq \hat{\Delta} \\ 0 & \text{if } \Delta(0) \leq \hat{\Delta} \end{cases}$$

Then, when $L < \hat{L}$, $\Delta(L) > \hat{\Delta}$. Within this range, a higher liability L , through decreasing Δ , increases θ . Therefore, $(1 - \theta)\Delta$ decreases in L . According to condition (9), $t - H(t) = (1 - \theta)\Delta$, where $t = F^{-1}(1 - Q)$, it then follows that $Q(\theta, \Delta)$ increases in L . In contrast, if $\hat{L} < D$, $\Delta(L) < \hat{\Delta}$ when $L > \hat{L}$, in which case $\theta(\Delta)$ decreases in L . We have $\hat{L} = 0$ if for any $L \leq D$, $\Delta(L) \leq \Delta(0) \leq \hat{\Delta}$. Similarly, $\hat{L} = D$ if for any $L \leq D$, $\Delta(L) \geq \Delta(D) \geq \hat{\Delta}$. Q.E.D.

Proof of Proposition 2: The firm's optimal investment satisfies

$$\max_{\theta} Q(\theta, \Delta)[F^{-1}(1 - Q(\theta, \Delta)) - (1 - \theta)\Delta] - k(\theta).$$

For any θ and Q , the objective function is higher with lower Δ . Therefore, the maximal profit also decreases in Δ . When $L = D$, Δ is the lowest and Π is the highest. Q.E.D.

Proof of Lemma 4: The firm's optimal output is determined by

$$t - \frac{1 - F(t)}{f(t)} = t - H(t) = (1 - \theta)\Delta,$$

where $t = F^{-1}(1 - Q)$. The optimal quality investment θ satisfies either $\theta = \bar{\theta}$ or the following condition

$$Q(\theta, \Delta)\Delta - k'(\theta) = 0$$

According to Proposition 1, when $\Delta \geq \hat{\Delta}$, $\theta(\Delta)$ weakly decreases in Δ . Therefore, when Δ increases, $(1 - \theta)\Delta$ would increase. Correspondingly, $Q(\theta, \Delta)$ would decrease. When $\Delta < \hat{\Delta}$, $\theta(\Delta)$ increases in Δ and therefore $(1 - \theta)\Delta$ may increase or decrease in Δ . Similar

to the proof of Proposition 1, we have

$$Q'_\Delta(\theta, \Delta) = -f(t) \frac{1-\theta}{1-H'(t)}$$

and

$$Q'_\theta(\theta, \Delta) = \frac{\Delta f(t)}{1-H'(t)}.$$

Therefore,

$$\begin{aligned} \frac{d[(1-\theta)\Delta]}{d\Delta} &= (1-\theta) - \Delta \frac{d\theta}{d\Delta} \\ &= (1-\theta) + \Delta \frac{Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta)}{\Delta Q'_\theta(\theta, \Delta) - k''(\theta)} = (1-\theta) + \Delta \frac{(1-F(t)) - \Delta f(t) \frac{1-\theta}{1-H'(t)}}{\frac{\Delta^2 f(t)}{1-H'(t)} - k''(\theta)}. \end{aligned}$$

Note that the second order condition implies $\Delta Q'_\theta(\theta, \Delta) - k''(\theta) \leq 0$ for interior solutions. Thus, $(1-\theta) + \Delta \frac{(1-F(t)) - \Delta f(t) \frac{1-\theta}{1-H'(t)}}{\frac{\Delta^2 f(t)}{1-H'(t)} - k''(\theta)} < 0$ is equivalent to

$$\Delta(1-F(t)) - \Delta^2 f(t) \frac{1-\theta}{1-H'(t)} > (1-\theta)(k''(\theta) - \frac{\Delta^2 f(t)}{1-H'(t)}), \text{ or}$$

$$\Delta(1-F(t)) > (1-\theta)k''(\theta).$$

Given

$$Q(\theta, \Delta)\Delta - k'(\theta) = 0,$$

the above condition becomes $k'(\theta) > (1-\theta)k''(\theta)$. Therefore, as long as $k'(\theta) > (1-\theta)k''(\theta)$, $\frac{d[(1-\theta)\Delta]}{d\Delta} < 0$ so that $Q(\theta, \Delta)$ increases in Δ . If $k'''(\theta) \leq 0$, for any $\theta > 1/2$, we have

$$k'(\theta) \geq \theta k''(\theta) > (1-\theta)k''(\theta).$$

According to Proposition 1, when $\Delta < \widehat{\Delta}$, θ increases in Δ . Therefore, since condition **T1** holds, there exists a cut-off $\widetilde{\Delta} < \widehat{\Delta}$ such that $\theta > 1/2$ when $\Delta \in (\widetilde{\Delta}, \widehat{\Delta}]$. Correspondingly, $Q(\theta, \Delta)$ increases in Δ when $\Delta \in (\widetilde{\Delta}, \widehat{\Delta}]$. Q.E.D.

Proof of Proposition 3: According to Lemma 1, Δ decreases in β and increases C and D . Given C and β , there always exists \widehat{D} such that $\Delta(L = \widehat{D}) = \widehat{\Delta}$. According to Proposition 1, $\widehat{\Delta} > 0$ and therefore $\widehat{D} > 0$. Given condition T1, there exists \widetilde{D} such that $\Delta(L = \widetilde{D}) = \widetilde{\Delta}$.

Then If $D \geq \widehat{D}$, $\Delta(L < D) > \Delta(L = D) \geq \widehat{\Delta}$. In this range, according to Lemma 4, U decrease in Δ and therefore consumer surplus is maximized under $L = D$. If $\widetilde{D} \leq D < \widehat{D}$, then $\widetilde{\Delta} \leq \Delta < \widehat{\Delta}$. In this range, according to Lemma 4, U increase in Δ and therefore consumer surplus is maximized under $L < D$. The proof for part (2) and part (3) is similar to the above. Q.E.D.

Proof of Proposition 4: First, when $D \geq \widehat{D}$, Propositions 2 and 3 imply that both the firm and consumers prefer $L = D$. Therefore, full liability is socially optimal. In the following analysis, suppose $D < \widehat{D}$.

Note that

$$\frac{d\Pi}{d\Delta} = -(1 - \theta)Q(\theta, \Delta)$$

and

$$\frac{dU}{d\Delta} = \frac{dU}{dt} \frac{dt}{d\Delta} = -(1 - F(t)) \frac{dt}{d\Delta} = -Q(\theta, \Delta) \frac{dt}{d\Delta}.$$

Given $t - \frac{1-F(t)}{f(t)} = t - H(t) = (1 - \theta)\Delta$,

$$\frac{dt}{d\Delta} = \frac{1}{1 - H'(t)} \frac{d[(1 - \theta)\Delta]}{d\Delta} = \frac{1}{1 - H'(t)} \left\{ (1 - \theta) - \Delta \frac{(1 - F(t)) - \Delta f(t) \frac{1-\theta}{1-H'(t)}}{k''(\theta) - \frac{\Delta^2 f(t)}{1-H'(t)}} \right\}.$$

Note that

$$\Delta(1 - F(t)) = \Delta Q(\theta, \Delta) = k'(\theta).$$

Then we have

$$\frac{dU}{d\Delta} = Q(\theta, \Delta) \left\{ \frac{k'(\theta) - \Delta^2 f(t) \frac{1-\theta}{1-H'(t)}}{k''(\theta)(1 - H'(t)) - \Delta^2 f(t)} - \frac{1 - \theta}{1 - H'(t)} \right\}.$$

If $\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0$, then increasing Δ would increase social welfare.

$$\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0$$

is equivalent to

$$\frac{k'(\theta) - \Delta^2 f(t) \frac{1-\theta}{1-H'(t)}}{k''(\theta)(1 - H'(t)) - \Delta^2 f(t)} - \frac{1 - \theta}{1 - H'(t)} > 1 - \theta.$$

The second order condition of (10) implies

$$k''(\theta)(1 - H'(t)) - \Delta^2 f(t) > 0.$$

Therefore, the above inequality is equivalent to

$$k'(\theta) > (1 - \theta)k''(\theta)[2 - H'(t)] - (1 - \theta)\Delta^2 f(t).$$

If $k'''(\theta) \leq 0$, for any $\theta > \frac{2+h}{3+h}$,

$$\begin{aligned} k'(\theta) &\geq \theta k''(\theta) > (1 - \theta)k''(\theta)[2 + h] \\ &\geq (1 - \theta)k''(\theta)[2 - H'(t)] \geq (1 - \theta)k''(\theta)[2 - H'(t)] - (1 - \theta)\Delta^2 f(t). \end{aligned}$$

Therefore, given condition T2, there exists a non-empty set $(\Delta', \widehat{\Delta})$ such that, if $\Delta \in (\Delta', \widehat{\Delta})$, $\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0$. Since condition T2 implies condition T1 but the reverse might not be true, it can be verified that $\Delta' \geq \widetilde{\Delta}$, where $\widetilde{\Delta}$ is defined in Lemma 4. Given C and β , define D' such that $\Delta(L = D') = \Delta'$. Then if $D \geq \widehat{D}$, $\frac{dU}{dL} + \frac{d\Pi}{dL} < 0$; if $D' \leq D < \widehat{D}$, $\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0$. The proof for parts (2) and (3) is similar to the above. Q.E.D.

Proof of Proposition 5: According to Lemma 1, Δ decreases in L when $L \leq D$ and increases in L when $L > D$. Note that $\Delta(L = 0) = \int_0^1 \gamma D dG(\gamma) > \Delta(L = D)$. When L goes to infinity, $\Delta(L \rightarrow \infty) = \int_0^1 [\beta C + (1 - \beta)\gamma D] dG(\gamma)$. (1) If $\int_0^1 (C - \gamma D) dG(\gamma) \leq 0$, then $\Delta(L \rightarrow \infty) \leq \Delta(L = 0)$. In this case, for any punitive damage compensation $L > D$, there exists $L' < D$ such that $\Delta(L) = \Delta(L')$. (2) If $\int_0^1 (C - \gamma D) dG(\gamma) > 0$, then $\Delta(L \rightarrow \infty) > \Delta(L = 0)$. Given continuity, there exists $\bar{L} > D$ such that $\Delta(L = \bar{L}) = \Delta(L = 0)$. Therefore, for any $L \in (D, \bar{L}]$, there exists $L' < D$ such that $\Delta(L) = \Delta(L')$. Similar to the proof for Proposition 4, if condition T2 holds, there exists a non-empty set $(\Delta', \widehat{\Delta})$ such that, if $\Delta \in (\Delta', \widehat{\Delta})$, $\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0$. Note that, if $L = 0$, $\Delta = E[\gamma]D$. Define D'' as $E[\gamma]D'' = \Delta'$. Then according to Proposition 4, when $D \in (D', D'')$, $C \in (C', \widehat{C})$, for $\beta \in (\widehat{\beta}, \beta')$, $\Delta(L) \in (\Delta', \widehat{\Delta})$ for any $L \leq D$. Thus, $\Delta(L) \in (\Delta', \widehat{\Delta})$ for any $L \in (D, \bar{L}]$: since $\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0$, $L > \bar{L}$ is socially more efficient than any $L \leq \bar{L}$. Q.E.D.

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